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Gamma distribution parameter estimation for field reliability data with missing failure times

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Maximum likelihood estimators have been developed for the gamma distribution when there is missing time-to-failure information. Data sets with missing time-to-failure data can arise from field data collection systems that rely on recorded observations of the system by the operators and maintenance personnel. In many regards, this type of data is highly desirable because it implicitly accounts for all actual usage and environmental stresses. Unfortunately the component times-to-failure are not always recorded for fielded systems because of a lack of elapsed time meters, unsatisfactory data reporting requirements, or incomplete or lost information. When only data of this type is available, it creates a non-standard form of data censoring and it has generally not been possible to fit most common time-to-failure distributions. Reliability practitioners have sometimes made unsubstantiated simplifying assumptions so the data can be used. In this paper, a more rigorous approach is presented. Maximum likelihood estimators are derived and demonstrated for the gamma distribution based on merged data records where the individual failure times have not been recorded. These results are important because the gamma distribution can model diverse time-to-failure behavior. This provides a particularly useful tool for data sets that may otherwise not be satisfactorily analyzed.

1. Introduction

Timely and accurate reliability data is needed to analyze system designs and to aid in decision making. Reliability data pertaining to components installed in fielded systems is desirable because this type of data inherently captures the usage and environmental stresses associated with the actual usage conditions. It is not always possible to accurately anticipate or simulate these stresses in a laboratory or even in a field test.

Data from fielded systems is often recorded by the actual system owners as they use the system for its intended purposes and then maintain the system upon failure. Data collection is generally performed passively by the system owner or user. This type of data collection is often uncontrolled and important details are not always recorded or they can be lost. In particular, the actual times-to-failure are often not recorded even though the failure itself has been faithfully noted. This is somewhat understandable because the elapsed time meter records time for the entire assembled system and not the individual components. Additionally, many maintenance personnel understand that their primary responsibility is to maintain systems, and not to record data. While this is understandable, it is also unfortunate because reliability data is required to support many design and logistics decisions.

Many large organizations such as the US Air Force and Navy, and utility companies develop ambitious reliability databases to track the field reliability on the systems they operate and maintain. Inputs to the database come from a variety of sources but generally include maintenance and operator records. The magnitude of such efforts often leads to compromises in the level of details tracked on system and component failures. The Reliability Analysis Center (RAC) is a US DoD Information Analysis Center, operated by IIT Research Institute, dedicated to the collection and dissemination of reliability data for components operated in fielded systems [1,2]. It is the largest publicly available source of reliability data. For a variety of reasons, over 90% of their data does not have the individual failure times recorded. Table 1 presents data on aircraft indicator lights [1] from this database. Individual time-to-failure information is not available. However, the total number of failures and the cumulative operating hours are available.

Parametric assumptions are often made to model component time-to-failure and to estimate component reliability based on empirical data. Estimation of the distribution parameters relies on the explicit knowledge of observed times-to-failure, censor times and the applicable form of data censoring. If failures are observed in fielded systems, but the failure times are not faithfully recorded, it results in a non-standard form of random

Table 1. Airplane indicator light reliability data [1]

Failures	*Cumulative operating time (hours)*
2	T_{21} = 51 000
9	T_{91} = 194 900
8	T_{81} = 45 300
8	T_{82} = 112 400
6	T_{61} = 104 000
5	T_{51} = 44 800

data censoring which can not be accommodated by available techniques. To address this problem, Maximum Likelihood Estimates (MLE) for the gamma distribution with missing times-to-failure are developed and demonstrated in this paper.

There has been other research concerned with the use of data with missing attributes. Dey [3] has developed a simulation model to observe the behavior of grouped data and test an exponential distribution assumption. Coit and Dey [4] have developed and demonstrated a hypothesis test to evaluate an exponential distribution assumption when there is missing time-to-failure data. In this work, the grouped exponential data was modeled using a k-Erlang distribution. The hypothesis test was demonstrated to successfully reject the exponential distribution when it was not appropriate even without detailed knowledge of component time-to-failure.

Usher [5] and Lin *et al.* [6] have developed techniques to analyze time-to-failure data and determine *component* Weibull distribution parameters when every *system* time-to-failure is observed and recorded, but not all system failures have been sufficiently diagnosed to identify the failed component. Wang [7] has developed a graphical approach for evaluating repairable system reliability when the data is grouped together and the populations are changing.

2. Assumptions and notation

The following assumptions were made concerning the component failures and data collection system.

- Component times-to-failure are *iid*.
- Component time-to-failure is distributed in accordance with a gamma distribution.
- Repair times are insignificant compared to operating time.
- System repair does not degrade or otherwise affect the reliability of the unfailed components.

The following notation is used in the development of the gamma distribution MLEs.

λ, k = gamma distribution parameters, $f(t) = \lambda^k t^{k-1} \mathrm{e}^{-\lambda t}/\Gamma(k)$;

$\hat{\lambda}, \hat{k}$ = Maximum Likelihood Estimates (MLE) of λ and k;

r = number of observed failures;

T_{rj} = jth cumulative operating time with r failures = $X_1 + X_2 + \cdots + X_r$

X_i = ith time-to-failure;

n_r = number of data records with exactly r failures;

$\mathbf{n}$ = $(n_1, n_2, \ldots, n_m)$;

m = maximum number of failures for any data record within a data set = $\max\{r \mid n_r > 0\}$;

N = number of data records, $N = \sum_{r=1}^{m} n_r$;

M = total number of failures associated with all N data records, $M = \sum_{r=1}^{m} r\, n_r$;

$\bar{t}$ = average time-to-failure = $\sum_{r=1}^{m} \sum_{j=1}^{n_r} T_{rj}/M$;

$\Gamma(z)$ = gamma function = $\int_0^\infty u^{z-1} \mathrm{e}^{-u} \mathrm{d}u$.

3. Attributes of field data

For the assessment of component reliability, field data has many distinct advantages. The principal advantage is that the operational and environmental stresses are those which are of most importance, i.e., the actual usage environment. In the field, the stresses are applied simultaneously and variable interactions are implicitly considered. Alternatively, even the most faithful and rigorous laboratory testing will fail to precisely simulate all field conditions. For this reason, there are many dedicated company-specific and industry-specific field data collection programs.

For all of the advantages of field data, there are also disadvantages, including incomplete or inaccurate data reporting, unconfirmed component failures and others. Several of these disadvantages are described in more detail by Coit *et al.* [8]. The disadvantage to be addressed in this paper is the fact that the individual times-to-failure are often missing. The data is often only available in the form of r collective failures observed in T_{rj} cumulative hours with no further delineation or detail available [1,2]. r and T_{rj} are known but the individual failure times are not. Analysts may have many of these merged data records available for the same component. Table 1 presents a data set of this type.

3.1. *Missing failure times*

Generally, elapsed time meters record the operating hours (or other measure) of an assembled *system* as opposed to individual *components* within the system. Component-level time-to-failure is then determined as the difference between the system operating time when the component was installed (perhaps as a replacement for a previous failure) and the system operating time when the same component fails. However, the system operating times are not always faithfully recorded by field operators and maintenance technicians. This is unfortunate because accurate estimation of component time-to-failure distribution parameters is needed to determine preventive maintenance intervals and warranty periods, to study the failure patterns of various failure mechanisms, and to evaluate whether the component should be used in future designs.

When the *system* operating time associated with a *component* field failure is observed and accurately recorded, the only information that may be known is the total number of component failures since the previously recorded failure time for that component. The intermediate times-to-failure are not known although the total number of failure events is known. The system operating times for the other failures, since the last one recorded, may not have been recorded or were lost due to poor record keeping.

If only the cumulative number of failures and operating hours are known, the MLE for the exponential distribution can still be used because this information is sufficient. However, the exponential distribution may be inappropriate for the failure mechanisms being studied.

Consider the graphical depiction of a data set given in Fig. 1 for a system that is designed with a single critical component of interest. In the figure, S represents the elapsed time recorded for the system. When the component fails, the repair action is to remove the failed component and install a replacement. In the figure, eight successive failures are observed for the particular component corresponding to eight system failures. The total number of failures is known from maintenance logs but the system operating time was only accurately recorded for the first, fourth, sixth and eighth failure. Therefore all component times-to-failure are missing (i.e., unknown) except the first, X_1. For this data set $m = 3$, $N = 4$, $M = 8$ and $\mathbf{n} = (1, 2, 1)$. The resulting data set consists of T_{11}, T_{21}, T_{22} and T_{31}.

More advanced analytical techniques are presented here to address this non-standard data type. However, it needs to be emphasized that the best solution to this problem is not achieved via analytical means. This problem can only truly be resolved through the implementation of better data collection policies and real-time data-capture technology to minimize the reliance on maintenance and operator personnel records.

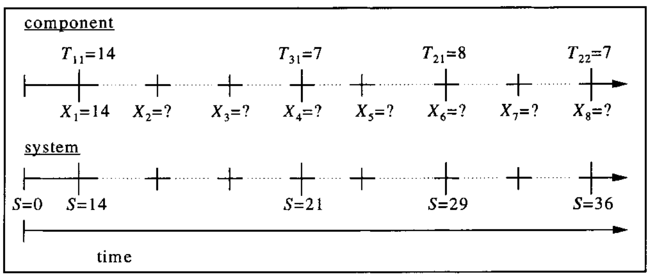

Fig. 1. Example with missing component time-to-failure.

3.2. *Time-to-failure distributions*

For standard data sets, there are well-known techniques to estimate parameters for many distributions and types of censoring, and to objectively evaluate the applicability of these distributions. Thorough descriptions of these techniques are presented in Mann *et al.* [9], and also Barlow and Proschan [10]. However, the analysis of reliability data with missing times-to-failure is a non-standard problem and it is not addressed in these references.

Without individual time-to-failure data, it is impossible to fit the data to most popular distributions (e.g., gamma, Weibull, lognormal) using standard techniques such as MLE or regression analysis. However, the MLE for the exponential distribution with Type I or II censoring only depends on the number of failures and the cumulative hours. This is a product of the well known memoryless property associated with the exponential distribution.

The limitations of the available field data and the simplicity of the exponential MLE have been used to rationalize the exponential distribution in applications where it would seemingly be a poor choice. The constant hazard function associated with the exponential distribution is not intuitively appropriate for most failure mechanisms which can be attributed to the accumulation of stress, such as fracture, fatigue, corrosion and wear mechanisms. Unfortunately, it is difficult to challenge the exponential assumption empirically because of the nature of the data that is often available from the field data systems.

Incorrect assumptions of the underlying distribution can have dire consequences. For many fledgling companies, major decisions are made with limited data being used as rationale. When an incorrect distribution is assumed, particularly for reasons of convenience, it is particularly dangerous.

The gamma distribution is a well-known, flexible distribution that can model many particular component failure mechanisms. Methods to determine MLE estimates are available [11] for Type I, Type II and chronologically grouped failure data. However, none of the available MLE estimators pertain to the data type described in this paper.

4. Gamma distribution maximum likelihood estimates

4.1. *Distribution of cumulative time-to-failure*

The approach to determine gamma distribution MLEs is to develop a likelihood function based on the distribution parameters (λ, k) and observed r and T_{rj} values rather than times-to-failure and censor times (which are not known). When time-to-failure, X_i, is distributed according to a gamma distribution, then T_{rj} is distributed in accordance with a related gamma distributions for fixed r.

X_i = component time to failure;
$X_i \sim$ gamma(λ, k), $E(X_i) = k/\lambda$, $\text{Var}(X_i) = k/\lambda^2$;
$T_{rj} = \sum_{i=1}^{r} X_i$;
$T_{rj} \sim$ gamma(λ, rk), $E(T_{rj}) = rk/\lambda$, $\text{Var}(T_{rj}) = rk/\lambda^2$.

T_{rj} is a random variable depending on specified r. If r is also a random variable, then consider T as the associated cumulative time-to-failure for random r. The density function for T_{rj} is then a conditional probability density function for T. Furthermore, the density function for T can be expressed as a sum of conditional terms.

$$f_{T_r}(t) = f_T(t\,|r) = \frac{\lambda^{rk} t^{rk-1}\exp(-\lambda t)}{\Gamma(rk)}, \tag{1}$$

$$f_T(t) = \sum_{r=1}^{\infty} f_T(t\,|r)\Pr(R=r),$$

where $\Pr(R = r)$ is the probability that a randomly selected data record includes exactly r failures.

Consider a specific observed data set with N data records. For each of the N data records, r and T_{rj} are known. The set of cumulative operating times (T_{rj}) form a non-homogeneous population that can be modeled using a mixture model [11]. The T_{rj} variables are associated with different distributions (depending on r), but the distributions are related gamma distributions.

Mixture models are generally used when there is a non-homogeneous population composed of distinct subpopulations. The most common situation is when the proportion of each subpopulation is known, or can be estimated, but it is unknown which members belong to which subpopulations. A common example is in semiconductor manufacturing where there is a subpopulation of weak or flawed devices but it is difficult or impossible to sort the population into the two subpopulations (flawed and good) prior to introducing the components into the field.

The probability density function for a non-homogeneous population can be expressed as a weighted sum of the respective probability density functions. The weights represent the probability that a randomly selected member of the population is from a particular subpopulation. For the problem being addressed here, the subpopulations are characterized by the number of failures within the merged data set. The weights are the probabilities that a randomly selected T_{rj}, from the N data records, has r failures. This probability is n_r/N.

$$f_T(t) = \sum_{r=1}^{\infty} \frac{\lambda^{rk} t^{rk-1} \mathrm{e}^{-\lambda t}}{\Gamma(rk)} \Pr(R=r),$$

$$f_T(t|\mathbf{n}) = \sum_{r=1}^{m} \frac{\lambda^{rk} t^{rk-1} \mathrm{e}^{-\lambda t}}{\Gamma(rk)} \frac{n_r}{N},$$

$$f_T(t|\mathbf{n}) = \frac{1}{N}\sum_{r=1}^{m} \frac{n_r \lambda^{rk} t^{rk-1} \mathrm{e}^{-\lambda t}}{\Gamma(rk)}. \tag{2}$$

4.2. *Maximum likelihood estimate*

A likelihood function for λ and k can be presented directly based on the observed data and Equation 2. The likelihood function, $L(k, \lambda)$, is expressed as a product of $f_T(t|\mathbf{n})$ for the N data records within a data set.

$$L(k,\lambda) = \prod_{r=1}^{m}\left\{\prod_{j=1}^{n_r}\left[\frac{1}{N}\sum_{i=1}^{m}\frac{n_i \lambda^{ik} T_{rj}^{ik-1}\mathrm{e}^{-\lambda T_{rj}}}{\Gamma(ik)}\right]\right\}. \tag{3}$$

Estimates of k and λ can be obtained using a Newton search to minimize $L(k, \lambda)$. An alternative and preferred likelihood function can be developed by exploiting the observation that the subpopulations are clearly identified within the overall population. The alternative likelihood function can be expressed as the product of the conditional density functions for T_{rj}, $f_T(t|r)$, from Equation 1.

$$L(k,\lambda) = \prod_{r=1}^{m}\prod_{j=1}^{n_r} f_T(k,\lambda|T_{rj}) = \prod_{r=1}^{m}\prod_{j=1}^{n_r}\frac{\lambda^{rk}T_{rj}^{rk-1}\mathrm{e}^{-\lambda T_{rj}}}{\Gamma(rk)},$$
$$= \left(\exp\left(-\lambda\sum_{r=1}^{m}\sum_{j=1}^{n_r}T_{rj}\right)\right)\prod_{r=1}^{m}\prod_{j=1}^{n_r}\frac{\lambda^{rk}T_{rj}^{rk-1}}{\Gamma(rk)}. \tag{4}$$

$$\ln L(k,\lambda) = -\lambda M\bar{t} + Mk\ln\lambda + k\sum_{r=1}^{m}\sum_{j=1}^{n_r} r\ln T_{rj} - \sum_{r=1}^{m}\sum_{j=1}^{n_r}\ln T_{rj} - \sum_{r=1}^{m} n_r \ln\Gamma(rk). \tag{5}$$

Taking partial derivatives of the log-likelihood function with respect to k and λ yields,

$$\lambda: \quad \frac{\partial}{\partial\lambda}\ln L(k,\lambda) = -M\bar{t} + \frac{Mk}{\lambda} = 0, \quad \lambda = \hat{k}/\bar{t}. \tag{6}$$

$$k: \quad \frac{\partial}{\partial k}\ln L(k,\lambda) = M\ln(k/\bar{t}) + \sum_{r=1}^{m}\sum_{j=1}^{n_r} r\ln T_{rj} - \sum_{r=1}^{m} r n_r \psi(rk) = 0, \tag{7}$$

where, $\psi(rk) = \Gamma'(rk)/\Gamma(rk)$.

The digamma function, $\psi(rk)$, can be approximated very accurately using the following Equation [11].

Table 2. Data for simulated examples

Example 2 (k = 1, λ = 0.000 068 79)		*Example 3 (k = 3, λ = 0.000 2064)*		*Example 4 (k = 1, λ = 0.000 068 79)*		*Example 5 (k = 3, λ = 0.000 2064)*	
Failures	T_{rj}	*Failures*	T_{rj}	*Failures*	T_{rj}	*Failures*	T_{rj}
2	28 131	2	46 170	2	32 054	2	34 400
5	61 363	5	83 170	2	30 832	2	13 950
6	64 995	6	88 950	5	85 771	5	47 060
8	98 859	8	110 530	5	31 147	5	75 770
8	145 683	8	103 210	6	37 169	6	78 720
9	37 607	9	93 010	6	94 708	6	102 790
				8	45 844	8	105 040
				8	77 176	8	111 010
				8	153 207	8	82 460
				8	107 193	8	122 650
				9	122 848	9	162 500
				9	108 142	9	118 100

$$\psi(rk) = \ln(rk) - \frac{1}{2rk} - \frac{1}{12(rk)^2} + \frac{1}{120(rk)^4} - \frac{1}{252(rk)^6} + \cdots \quad (8)$$

Therefore, k can now be readily solved, from Equations (7) and (8), to determine $\hat{k}$ using a Newton–Raphson search or simply a bisection search. $\hat{k}$ is then substituted into Equation (6) to determine $\hat{\lambda}$. In summary,

$$\hat{k} = \left\{ k \left| \sum_{r=1}^{m} rn_r\psi(rk) - M \ln k = K' \right. \right\}$$

where $K' = \sum_{r=1}^{m} \sum_{j=1}^{n_r} r \ln T_{rj} - M \ln \bar{t}$ and $\hat{\lambda} = \hat{k}/\bar{t}$

The resulting gamma estimates should then be formally tested for goodness-of-fit using standard methods such as the Chi-squared test, observed T_{rj} data and Equation (2).

5. Examples

The MLEs developed in the previous section were demonstrated and evaluated on five examples. The first example was the airplane indicator light example presented in Table 1. This data is from the RAC database [1]. For this example, $m = 9$, $N = 6$, $M = 38$ and $\mathbf{n} = (0,1,0,0,1,1,0,2,1)$. The remaining examples use simulated data. Simulated data is useful to evaluate statistical estimators because the underlying distribution parameters are known.

The simulated data sets are presented in Table 2. To generate a simulated data set, gamma distribution parameters and an **n** vector are required. For the four simulated examples, gamma shape parameters of one and three were selected. **n** and 2**n** were chosen using **n** from Example 1 as typical values for field data, and $\lambda = k/\bar{t}$ was computed using the average failure time from Example 1.

The MLEs are presented in Table 3. The estimates were efficiently found using a quasi-Newton search on a Personal Computer using the data solver routine on a readily available spreadsheet program.

To evaluate the MLEs, the simulation was repeated for a total of 1000 iterations for Examples 2 through 5. For each iteration, $\hat{k}$ and $\hat{\lambda}$ were calculated. For a small percentage of the simulated cases ($<0.3\%$), the quasi-Newton search would not converge to a solution. For the remaining simulation cases, the results are presented in Table 4. The table presents average parameter values and standard errors. Also presented in Table 4 are the theoretical standard error estimates [12] from an analogous hypothetical data set with no censoring.

As can be seen from Table 4, the parameter estimates are biased. It is not unusual for MLEs to be biased with limited data. As can be observed from the results in the table, the bias is reduced as N increases. Recommendations to reduce bias for gamma distribution MLEs are presented in Bain and Engelhardt [12] and Lee [13].

The Table 4 results also show that the standard error is much lower for the case with M uncensored data records compared to N merged data sets. This demonstrates that there is a significant penalty when time-to-failure data is missing. These new MLE estimators provide additional

Table 3. Example results

Example number	$\hat{k}$	$\hat{\lambda}$
1	0.70	0.000 0484
2	0.70	0.000 0609
3	3.43	0.000 2483
4	1.06	0.000 0870
5	3.44	0.000 2479

Table 4. MLE evaluation results (1000 iterations)

k	λ	**n**/2**n**	$\hat{k}$				$\hat{\lambda}$		
			Median	*Mean*	*Standard error*	*Standard error uncensored**	*Median*	*Mean*	*Standard error*
1.0	0.000 068 79	**n**	1.32	1.77	1.319	0.232	0.000 0951	0.000 124	0.000 0972
3.0	0.000 2064	**n**	3.50	4.34	2.656	0.761	0.000 243	0.000 301	0.000 189
1.0	0.000 068 79	2**n**	1.13	1.38	0.654	0.152	0.000 0773	0.000 0875	0.000 0445
3.0	0.000 2064	2**n**	3.25	3.61	1.652	0.497	0.000 224	0.000 252	0.000 118

* this is a theoretical standard error based on unbiased estimates of k and λ and M uncensored data records; for Examples 1 and 2, $M = 38$; for examples 3 and 4, $M = 76$

capabilities to analyze data sets that previously could not be used to determine distribution parameters. However, the best approach to this problem is to avoid it. Improvement of data collection practices is a better solution to this problem than the use of additional analytical techniques.

6. Conclusions

When failure times are missing, practitioners sometimes assume that the exponential distribution is appropriate so that they can use the data. A preferable and more rigorous approach has been described involving the derivation of maximum likelihood estimators for the gamma distribution. These results are important because the gamma distribution can model diverse time-to-failure behavior. This provides a particularly useful tool for data sets that may otherwise not be satisfactorily analyzed.

It should be emphasized that an analyst should never indiscriminately choose a distribution without some physical, theoretical or empirical rationale. However, there are several observations that can be made. The gamma distribution is capable of modeling a variety of different probability density functions (and hazard functions).

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References

[1] Anon (1991) *Non-electronic Parts Reliability Data*, RAC Report NPRD, Reliability Analysis Center, Rome, NY.

[2] Anon (1997) *Electronic Parts Reliability Data*, RAC Report EPRD, Reliability Analysis Center, Rome, NY.

[3] Dey, K.A. (1982) "Statistical analysis of noisy and incomplete failure data, in *Proceedings Annual Reliability and Maintainability Symposium* (RAMS), IEEE, Piscataway, NJ. pp. 93–97.

[4] Coit, D.W. and Dey, K.A. (1999) Analysis of grouped data from field-failure reporting systems. *Reliability Engineering and System Safety*, **65**, 95–101.

[5] Usher, J.S. (1996) Weibull component reliability-prediction in the presence of masked data. *IEEE Transactions on Reliability*, **45**, 229–232.

[6] Lin, D.K.J., Usher, J.S. and Guess, F.M. (1996) Bayes estimation of component from masked system-life data. *IEEE Transactions on Reliability*, **45**, 233–237.

[7] Wang, C.J. (1991) Graphical analysis of ill-collected interval data for a repairable system in vehicles, *Proceedings Annual Reliability and Maintainability Symposium* (RAMS), IEEE, Piscataway, NJ. pp. 93–97.

[8] Coit, D.W., Dey, K.A. and Turkowski, W. (1986) Practical reliability data and analysis. *Reliability Engineering*, **14**, 1–18.

[9] Mann, N.R., Shafer, R.E. and Singpurwalla, N.D. (1974) *Methods for Statistical Analysis of Reliability and Life Data*, John Wiley and Sons, New York, NY.

[10] Barlow, R.E. and Proschan, F. (1965) *Mathematical Theory of Reliability*, John Wiley and Sons, New York, NY.

[11] Lawless, J.F. (1982) *Statistical Models and Methods for Lifetime Data*, John Wiley and Sons, New York, NY.

[12] Bain, L.J. and Engelhardt, M. (1991) *Statistical Analysis of Reliability and Life Testing Models: Theory*, Marcel Dekker, New York, NY.

[13] Lee, E. (1992) *Statistical Methods for Survival Data Analysis*, John Wiley and Sons, New York, NY.

Biographies

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