Composite Importance Measures for Multi-State Systems with Multi-State Components

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***Abstract*—This paper presents & evaluates composite importance measures (CIM) for multi-state systems with multi-state components (MSMC). Importance measures are important tools to evaluate & rank the impact of individual components within a system. For multi-state systems, previously developed measures do not meet all user needs. The major focus of the study is to distinguish between two types of importance measures which can be used for evaluating the criticality of components in MSMC with respect to multi-state system reliability. This paper presents Type 1 importance measures that are involved in measuring how a specific component affects multi-state system reliability. A Monte Carlo (MC) simulation methodology for estimating the reliability of a MSMC is used for computing the proposed CIM metrics. Previous approaches (Type 2) have focused on investigating how a particular component state or set of states affects multi-state system reliability. For some systems, it is not clear how to prioritize system component importance, collectively considering *all* of its states, using the previously developed importance measures. That detracts from those measures. Experimental results show that the proposed CIM can be used as an effective tool to assess component criticality for MSMC. Examples are used to illustrate & compare the proposed CIM with previous multi-state importance measures.**

ACRONYMS[1]

CIM	Composite Importance Measures
FV	Fussell-Vesely Importance
MAD	Mean Absolute Deviation
MMCV	Multi-state Minimal Cut Vector
MR_d	Multi-state Reliability at level d
MSMC	Multi-state System with Multi-state Components
RAW	Reliability Achievement Worth
RRW	Reliability Reduction Worth
SAD	Sum of Absolute Deviations
MMAW	Mean Multi-state Achievement Worth

NOTATION

φ	System structure function
$\mathbf{x}$	System state vector, $\mathbf{x} = (x_1, x_2, \ldots, x_{\vert\mathrm{A}\vert})$
x_i	Current state of component i
A	Set of system components
G	System Capacity
d	System demand
M_i	Maximum capacity for component i
ω_i	Number of states for component i
$\mathbf{b}_i$	State space vector for component i
I_i	Binary Birnbaum Importance Measure
p_{ij}	$\mathrm{P}(x_i = b_{ij})$

I. INTRODUCTION

IMPORTANCE measures quantify the criticality of a particular component within a system design. They have been widely used as tools for identifying system weaknesses, and to prioritize reliability improvement activities. They can also provide valuable information for the safety & operation of a system. Extensive research [1]–[4] for importance measures is available for the case when the system exhibits a binary functioning behavior. That is, the system & components are either fully functioning, or fully failed. Vasseur & Llory [5] reviews Reliability Achievement Worth (RAW), Reliability Reduction Worth (RRW), Fussell-Vesely (FV), and Birnbaum as the most valuable importance measures for binary systems. These importance measures assist in discriminating the most important component with respect to the overall system reliability. For the binary case, system components can be ranked with respect to the impact they have on system reliability based on a given importance measure.

Traditional reliability theory has been based on binary applications. System reliability in a binary-state context is the probability that the system properly provides the service for which it was intended, under the condition that the system & its components can be either fully working or failed (completely nonworking). However, some systems have more complicated behavior. Some components of the system may be operating in a degraded state causing the system to provide service at less than full capacity. However, the system may still be providing an acceptable level of service, or perhaps, a partial level of service. Multi-state reliability models have been proposed to describe such systems [6]–[10]. For systems such as water distribution, telecommunications, oil & gas supply, and power generation & transmission, multi-state reliability analysis is frequently the preferred approach. Generally, the elements of these systems degrade gradually, reducing their capacity, and the overall capability of the system.

In recent years, multi-state system reliability analysis has received considerable attention. Researchers have realized that for some systems, erroneous appraisal of system reliability could lead to 1) incorrect system modeling, 2) incorrect system reliability computation, and/or 3) incorrect conjectures regarding reliability dependant measures. Theoretical & applied

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[1] The singular and plural of an acronym are always spelled the same

studies have been devoted to the areas of multi-state system reliability, simulation, approximation methodologies, and optimization [11].

The theoretical aspects of importance measures in MSMC have been extensively investigated. El-Neweihi, *et al.* [12] analyzed the theoretical relationships between multi-state system reliability behavior, and multi-state component performance. Barlow & Wu [13] characterize component state criticality as a measure of how a particular component state affects a specific system state. Griffith [14] formalized the concept of multi-state system performance, and studied the impact of component improvement on the overall system reliability behavior. Moreover, Griffith [14] introduced the concept of reliability importance vector for each system component. Through this concept, a generalization of the binary Birnbaum importance measure can be extended to the multi-state case.

Levitin & Lisnianski [15] proposed importance & sensitivity measures for multi-state systems with binary capacitated components. These measures account for both the multi-state system performance, which is caused by the capacitated components, and stochastic system demand. Importance measures are obtained through the universal generating function. Zio & Podofillini [16] present multi-state extensions for RAW, RRW, FV, and Birnbaum for MSMC. Their results pertain to the importance of individual component state levels. Monte-Carlo simulation methods (MC) are used to imitate the stochastic nature of the multi-state components, and generate the proposed importance measures. Levitin, *et al.* [17] proposed similar importance measures as those presented in Zio & Podofillini [16]. Their evaluation method is performed via the universal generating function method. These approaches have proven to be valuable to the development of multi-state importance measures.

In summary, when considering MSMC, research efforts have been focused on generalizing frequently used binary importance measures to accommodate the multi-state behavior. These approaches characterize, for a given component, the most important component state with regard to its impact on system reliability. For many design & reliability problems, determination of the most critical component state is the primary concern, and existing multi-state importance measures are appropriate, and should be applied. However, there are other applications, where it is important to identify the most critical component. That is, the identification of critical components must be determined considering all of the prospective states. The most critical system component state may not necessarily correspond to the most critical system component. For example, for aging complex systems, it is desirable to prioritize components to assist in the selection of components to upgrade or replace. For these applications, currently available importance measures are not sufficient, and new measures are presented here. A measure that can discriminate among the different multi-state components, based on how they affect multi-state system reliability, is a valuable tool to gain insight on system criticality.

With the use of current multi-state importance measures [12]–[14], [16], [17], it can be difficult to directly determine or rank the most important components for complex MSMC. In this paper, composite importance measures (CIM) are presented with the aim of identifying & ranking particular components depending on their impact on the MSMC reliability behavior. Moreover, the paper presents insights into future CIM evaluation procedures based on the MC methodology presented in Ramirez-Marquez & Coit [18]. The approach provides a fast method to obtain an accurate approximation to the actual value of CIM.

The paper is organized as follows: Section II introduces concepts used in system reliability analysis for MSMC. Section III presents CIM mathematical expressions followed by a discussion on the physical insights of each measure. Section IV applies & compares the proposed measures for complex systems, and finally, Section V presents conclusions.

Assumptions:

1) Component states are statistically independent.
2) The structure function $\varphi(\mathbf{x})$ is coherent. That is, improvement of component states cannot damage the system.
3) Component states & associated probabilities are known.

II. MULTI-STATE SYSTEM RELIABILITY ANALYSIS

For systems where binary state analysis is insufficient, incorrect reliability assessment can lead to faulty decision-making regarding system performance. Unnecessary expenditures, incorrect maintenance scheduling, and reduction of safety standards can potentially be related to unsatisfactory reliability assessments. When considering MSMC, reliability can be understood as the probability that the system capacity can meet a required demand when the system components & demand follow a multi-state behavior.

A. *Multi-State System Reliability Modeling*

Let $\mathrm{A} = \{1, \ldots, |\mathrm{A}|\}$ represent the set of components for a stochastic capacitated system. The random current state (capacity) of component i is represented by $x_i \in \mathbf{b}_i$, where $\mathbf{b}_i$ represents the component i state space vector. Therefore, x_i takes values $b_{i1} = 0, b_{i2}, \ldots, b_{i\omega i} = M_i$, where $b_{ij} \in \Re^+$. The vector $\mathbf{p}_i$ represents the probability associated with each of the values taken by x_i. The system state vector $\mathbf{x} = (x_1, x_2, \ldots, x_{|\mathrm{A}|})$ denotes the current state of all the components of the system. The function φ: $\Re^{|\mathrm{A}|} \rightarrow \Re^+$ is the multi-state structure function. It maps the system state vector into a system state. That is, $\varphi(\mathbf{x})$ is the available system capacity under system state vector $\mathbf{x}$.

The system is required to supply a demand. The demand d may be fixed, or it may vary depending on the conditions in different operating periods. If demand is constant, then multi-state reliability is given by

$$MR_d = P\left(\varphi(\mathbf{x}) \geq d\right)$$

For other systems, demand may vary. The loss of load probability (LOLP) index is commonly used as a system reliability metric for capacitated multi-state systems [[19]. This index is understood as the probability that the system cannot supply a given demand load for an operating period that is divided into k operating intervals. The vectors $\mathbf{T} = (T_1, \ldots, T_k)$, and $\mathbf{d} =$

$(d_1, \ldots, d_k)$ define the duration T_w, and demand level d_w of interval w. Lisnianski, *et al.* [19] defines LOLP as

$$\mathrm{LOLP} = \frac{\sum_{w=1}^{k} P(G < d_w) T_w}{\sum_{w=1}^{k} T_w}$$

For these systems, multi-state reliability (MR_d) can be understood as the probability that during the total time interval, the capacity of the system can meet a demand of d_w (for $w = 1, \ldots, k$) through the multi-state components. Thus

$$MR_d = \frac{\sum_{w=1}^{k} P\left(\varphi(\mathbf{x}) \geq d_w\right) T_w}{\sum_{w=1}^{k} T_w} \tag{1}$$

The probability that total system capacity, $G = \varphi(\mathbf{x})$, is greater than or equal to a specific demand level d_w is given by $P(G \geq d_w) = P(\varphi(\mathbf{x}) \geq d_w)$.

B. Multi-State Reliability Evaluation Methodologies

Given that importance measures are dependent on the computation of multi-state system reliability, methodologies that compute MR_d efficiently are extremely important. Even for relatively small systems, identifying all component state vectors $\mathbf{x}$ such that $\varphi(\mathbf{x}) \geq d_w$ can be very inefficient.

Lin [20], and Ramirez-Marquez & Coit [21] have developed approaches for computing the exact MR_d by introducing the multi-state version of minimal path sets. The algorithm presented by Lin [20] considers weakly homogeneous components. That is, components can have a different number of states, yet for any two components h & k with $|\mathbf{b}_h| = l_h$ & $|\mathbf{b}_k| = l_k$, and $l_h > l_k$, the first l_k component states must be equal. Thus, for systems where components are heterogeneous, the methodology may not be suitable. The methodology described by Ramirez-Marquez & Coit [21] relies on a network reduction technique to obtain all possible multi-state minimal path sets. It is interesting to note that this methodology, when considering binary components, reduces to the approach developed by Kuo, *et al.* [22] for the binary case. Both of the approaches [20], [21] have been applied to relatively small networks.

Lin [23], and Yeh [24] presented approaches to identify the multi-state version of minimal binary cut sets. Both approaches compute minimal cut vectors (MMCV) in a similar form. For each binary cut, they enumerate all different state combinations of the components in the cut to obtain the multi-state levels that guarantee a MMCV. These methods consider weakly homogeneous components with the added constraint that the capacity of component i must be an integer-valued random variable.

The algorithm presented by Ramirez-Marquez, *et al.* [25] reduces the computational burden inherent in previous approaches [23], [24]. The approach is based on two ideas: first, that all MMCV can be obtained from the set containing all minimal cuts [23], [24], and second, that a select number of MMCV called offspring cuts inherit information from a select number of MMCV called parent cuts. This information sharing approach significantly reduces the number of vector enumerations needed to obtain all MMCV.

Levitin, *et al.* [8] use a procedure based on the concept of the universal generating function for computing MR_d. This technique has proven to be a valuable & efficient tool for relatively complex systems. It requires relatively small computational resources for evaluating multi-state reliability indices. A detailed description of the approach is presented in [11].

Reliability estimation for multi-state systems generally requires that system states are independent. This assumption can be met for many mature systems that have been operated & maintained for an extended period. However, there are other systems (newly installed systems with all new components) where this assumption can not be reasonable met, and reliability analysis methods requiring independent system states are not appropriate. In this respect, an analytical approach for analyzing multi-state systems with dependent components has been suggested by Levitin [31]. For these cases, an analogous approach based on discrete event simulation can be applied. This is introduced in Section V-A, Future Research.

III. COMPOSITE IMPORTANCE MEASURES

When considering MSMC, importance measures can be associated with one of the following types:

1. Type 1: Measures how a specific *component* affects multi-state system reliability.
2. Type 2: Measures how a particular *state* or set of *states* of a specific component affects multi-state system reliability.

Previous approaches [14], [16], [17], [27] have focused on developing Type 2 importance measures. For many user needs, Type 2 measures are very useful, and existing measures should be applied. However there are other design problems where it is necessary to identify the most critical component, inclusive of all its states. For example, as part of a system upgrade, it may be necessary to identify & prioritize components that should be replaced with newer, more reliable replacements.

For some systems, deciding which of the system components has the overall highest impact on multi-state system reliability, based on Type 2 importance measures, is not clear, nor directly evident. Both of the approaches presented by Zio & Podofillini [16], and Levitin, *et al.* [17] include a step for reassigning component state occupancy probabilities for evaluating the importance measures. In this paper, we present new Type 1 importance measures for prioritizing components. These Type 1 measures are not necessarily better than Type 2 measures; they simply provide metrics appropriate for different types of analyses.

Direct computation of Type 1 importance measures has been considered by Aven & Ostebo [29]. This work has provided two CIM that can provide 1) indication of component potential improvement on system reliability, and 2) identification of components with unused capacity. The first measure quantifies the difference between the system reliability assuming a component has infinite capacity, and the system reliability considering the component's true capacities. The second CIM indicates the level of degradation a component can suffer before the system demand is not met. These measures have only been applied to small binary capacitated systems. For complex configurations, the problem of computing MMCV when considering each component to have infinite capacity may prove exhausting. Recently, Wu & Chan [28] have extended Griffith's [14] importance vector of a specific component to the composite case. The extension is based on the definition of a utility function that can differentiate which components significantly contribute to multi-state system reliability.

A. Type 1, and Type 2 IM Considerations

For many scenarios & design analyses, Type 2 measures can provide more useful information than Type 1 measures. For example, it is of interest to know how an increase or a decrease in the state of a component impacts system reliability. However, many researchers have indicated the need for the development of new multi-state IM so that multi-state components can be distinguished as a whole based on their contribution to MR_d. For example, Wu & Chang [28] note that from a system architecture perspective, the process of identifying how components impact system reliability is of utmost importance. Once this process is completed, the results can be used to identify system weaknesses, and allocate resources or engineering effort to increase system reliability [30].

For MSMC, the problem related to multi-state system reliability improvement is still evolving. To solve this problem, methods dependent on the information obtained from multistate IM can be developed for efficient resource allocation. In this respect, Type 2 measures can be used to improve MR_d by allocating resources so that the most important component state has high occupancy probability. In contrast, Type 1 measures can be used as a guide to provide redundancy so that system reliability is increased.

From a reliability perspective, at the design stage system designers are highly interested in knowing the impact of each potential system component on the system design. Thus, measures that can differentiate such an impact are highly desirable. However, research related to the investigation of how a multi-state component as a whole impacts multi-state reliability has been limited. That is, current measures fail to provide information regarding which component has the greatest impact on system functioning. In this paper, research efforts have been focused on developing Type 1 importance measures, called CIM. Two types of CIM have been developed. The first set is a generalization or extension of frequently used binary importance measures as reviewed by Vasseur & Llory [5]. The second set consists of newly developed alternative CIM that capture additional important information regarding the components multi-state reliability behavior.

B. General CIM

General CIM have been developed by reformulating existing binary importance measures into a multi-state context. This is similar to the approach of Zio & Podofillini [15], except that the new measures pertain to the component itself, and not just to a component state.

Birnbaum or Average of the Sum of Absolute Deviations (SAD) CIM: Birnbaum importance of a given component is defined as the probability that such component is critical to the functioning of the system [4]. For binary system behavior, it can be mathematically expressed as

$$I_i = P\left(\varphi(\mathbf{x}) = 1 | x_i = 1\right) - P\left(\varphi(\mathbf{x}) = 1 | x_i = 0\right)$$

Birnbaum importance measures can be considered in a multistate context by considering the binary case as a special case multi-state problem with $\mathbf{b}_i = (0, 1)$, and $d = 1$. To facilitate extension to the multi-state case, I_i can be rewritten as

$$\begin{aligned} I_i &= P\left(\varphi(\mathbf{x}) = 1 | x_i = 1\right) - P\left(\varphi(\mathbf{x}) = 1\right) \\ &\quad + P\left(\varphi(\mathbf{x}) = 1\right) - P\left(\varphi(\mathbf{x}) = 1 | x_i = 0\right) \\ &= \left|P\left(\varphi(\mathbf{x}) = 1 | x_i = 1\right) - P\left(\varphi(\mathbf{x}) = 1\right)\right| \\ &\quad + \left|P\left(\varphi(\mathbf{x}) = 1 | x_i = 0\right) - P\left(\varphi(\mathbf{x}) = 1\right)\right| \\ &= \sum_{j=0}^{1} \left|P\left(\varphi(\mathbf{x}) = 1 | x_i = b_{ij}\right) - P\left(\varphi(\mathbf{x}) = 1\right)\right| \\ &= \frac{\sum_{j=0}^{1} \left|P\left(\varphi(\mathbf{x}) = 1 | x_i = b_{ij}\right) - P\left(\varphi(\mathbf{x}) = 1\right)\right|}{|\mathbf{b}_i| - 1}. \end{aligned} \tag{2}$$

In (2), note that the summation for j is from 0 to 1 because the binary case can be thought of as a special case of a more general multi-state reliability problem. For the binary case, $b_{i0} = 0$, and $b_{i1} = 1$.

The CIM generalization for I_i can be expressed, for constant demand d as

$$MI_i^{SAD} = \frac{\sum_{j=1}^{\omega_i} \left|P\left(\varphi(\mathbf{x}) \geq d | x_i = b_{ij}\right) - P\left(\varphi(\mathbf{x}) \geq d\right)\right|}{\omega_i - 1} \tag{3}$$

and for varying demand d_w as

$$MI_i^{SAD} = \frac{\sum_{j=1}^{\omega_i} \sum_{w} \left|P(\varphi(\mathbf{x}) \geq d_w | x_i = b_{ij}) - P(\varphi(\mathbf{x}) \geq d_w)\right| T'_w}{\omega_i - 1} \tag{4}$$

where $\omega_i = |\mathbf{b}_i|$ = number of states for component i, and T'_w is given by

$$T'_w = \frac{T_w}{\sum_{i=1}^{k} T_i}.$$

Reliability Achievement Worth CIM: The RAW measure quantifies the maximum percentage increase in system reliability generated by a particular component. From a binary perspective, with $\mathbf{b}_i = (0, 1)$, it is defined as

$$\begin{aligned} \mathrm{RAW}_i &= \frac{P\left(\varphi(\mathbf{x}) = 1 | x_i = 1\right)}{P\left(\varphi(\mathbf{x}) = 1\right)} \\ &= 1 + \max\left(0, \frac{P\left(\varphi(\mathbf{x}) = 1 | x_i = 1\right)}{P\left(\varphi(\mathbf{x}) = 1\right)} - 1\right) \\ &= 1 + \sum_{j=0}^{1} \max\left(0, \frac{P\left(\varphi(\mathbf{x}) = 1 | x_i = b_{ij}\right)}{P\left(\varphi(\mathbf{x}) = 1\right)} - 1\right). \end{aligned}$$

RAW_i can be extended to the multi-state case as follows. For a constant demand d

$$\mathrm{MRAW}_i = 1 + \frac{1}{\omega_i - 1} \sum_{j=1}^{\omega_i} \max(0, \beta_{ij}) \tag{5}$$

and for a varying demand d_w

$$\mathrm{MRAW}_i = 1 + \frac{1}{\omega_i - 1}\sum_{j=1}^{\omega_i}\sum_{w} \max\left(0, \beta_{ij}^{w}\right) \tag{6}$$

where

$$\beta_{ij} = \frac{P\left(\varphi(\mathbf{x}) \geq d | x_i = b_{ij}\right) - P\left(\varphi(\mathbf{x}) \geq d\right)}{P\left(\varphi(\mathbf{x}) \geq d\right)}$$

$$\beta_{ij}^{w} = \frac{\sum_{w} P\left(\varphi(\mathbf{x}) \geq d_w | x_i = b_{ij}\right) T'_w - P\left(\varphi(\mathbf{x}) \geq d\right) T'_w}{\sum_{w} P\left(\varphi(\mathbf{x}) \geq d_w\right) T'_w}.$$

Fussell-Vesely CIM: The Fussell-Vesely importance measure quantifies the maximum decrement in system reliability caused by a particular component. The binary expression is

$$\begin{aligned}\mathrm{FV}_i &= \frac{P\left(\varphi(\mathbf{x})=1\right) - P\left(\varphi(\mathbf{x})=1|x_i=0\right)}{P\left(\varphi(\mathbf{x})=1\right)} \\ &= \max\left(0, \frac{P\left(\varphi(\mathbf{x})=1\right) - P\left(\varphi(\mathbf{x})=1|x_i=0\right)}{P\left(\varphi(\mathbf{x})=1\right)}\right) \\ &= \sum_{j=0}^{1} \max\left(0, \frac{P\left(\varphi(\mathbf{x})=1\right) - P\left(\varphi(\mathbf{x})=1|x_i=b_{ij}\right)}{P\left(\varphi(\mathbf{x})=1\right)}\right).\end{aligned} \tag{7}$$

In (7), note that the summation for j is from 0 to 1 because the binary case can be thought of as a special case of a more general multi-state reliability problem. For the binary case, $b_{i0} = 0$, and $b_{i1} = 1$.

The CIM generalization for FV_i can be expressed as, for a constant demand d

$$\mathrm{MFV}_i = \frac{1}{\omega_i - 1}\sum_{j=1}^{\omega_i} \max(0, -\beta_{ij}) \tag{8}$$

and for a varying demand d_w,

$$\mathrm{MFV}_i = \frac{1}{\omega_i - 1}\sum_{j=1}^{\omega_i}\sum_{w} \max\left(0, -\beta_{ij}^{w}\right). \tag{9}$$

Reliability Reduction Worth CIM: Levitin, *et al.* [17] defined RRW as the index measuring the potential damage caused to the system by a particular component. The binary expression of RRW is given as

$$\begin{aligned}\mathrm{RRW}_i &= \frac{P\left(\varphi(\mathbf{x}) = 1\right)}{P\left(\varphi(\mathbf{x}) = 1 | x_i = 0\right)} \\ &= \frac{1}{\frac{P(\varphi(\mathbf{x})=1|x_i=0)}{P(\varphi(\mathbf{x})=1)}} \\ &= \frac{1}{1 - \frac{P(\varphi(\mathbf{x})=1) - P(\varphi(\mathbf{x})=1|x_i=0)}{P(\varphi(\mathbf{x})=1)}} \\ &= \frac{1}{1 - \mathrm{FV}_i}.\end{aligned} \tag{10}$$

The CIM generalization for RRW_i can be expressed as

$$\mathrm{MRRW}_i = \frac{1}{1 - \mathrm{MFV}_i}. \tag{11}$$

C. Alternative CIM

The general CIM apply to multi-state systems, but they only consider the possible state levels, and not the probability of a component being in that state. Birnbaum CIM considers the absolute deviation that a particular component causes to the overall system reliability. The remaining general CIM provides a measure of relative improvement or decrement in multi-state system reliability generated by a particular component. However, the probability that the deviation, improvement, or decrement occur is not considered by the general CIM. Alternative CIM have been developed because of the need to incorporate state probabilities into the computation of multi-state reliability importance.

The first alternative CIM is called mean absolute deviation (MAD). It is the expected absolute deviation between $P(\varphi(\mathbf{x}) \geq d|x_i)$, and $P(\varphi(\mathbf{x}) \geq d)$. It measures the expected absolute deviation in the reliability of a MSMC caused by a particular component's different performance levels, and associated probabilities. It should be recognized that $P(\varphi(\mathbf{x}) \geq d|x_i)$ is a random variable because x_i is a random variable. Its mathematical expression is analogous to the Birnbaum CIM, and is given as follows.

Mean Absolute Deviation (MAD): Under a constant demand d

$$\begin{aligned}\mathrm{MAD}_i &= E\left[\left|P\left(\varphi(\mathbf{x}) \geq d|x_i\right) - P\left(\varphi(\mathbf{x}) \geq d\right)\right|\right] \\ &= \sum_{j} p_{ij}\left|P\left(\varphi(\mathbf{x}) \geq d|x_i = b_{ij}\right)\right. \\ &\qquad \left. - P\left(\varphi(\mathbf{x}) \geq d\right)\right|.\end{aligned} \tag{12}$$

Under varying demand d_w

$$\begin{aligned}\mathrm{MAD}_i &= E\left[\sum_{w}\left|P\left(\varphi(\mathbf{x}) \geq d_w|x_i = b_{ij}\right)\right.\right. \\ &\qquad \left.\left. - P\left(\varphi(\mathbf{x}) \geq d_w\right)\right| T'_w\right] \\ &= \sum_{j} p_{ij}\sum_{w}\left|P\left(\varphi(\mathbf{x}) \geq d_w|x_i = b_{ij}\right)\right. \\ &\qquad \left. - P\left(\varphi(\mathbf{x}) \geq d_w\right)\right| T'_w.\end{aligned} \tag{13}$$

The same rationale has been applied to the remaining general CIM; namely, MRAW, and MFV. MRRW has not been extended because all information that can be obtained from this IM is already provided by MFV.

Mean Multi-State Reliability Achievement Worth (MMAW): Under constant demand d

$$\mathrm{MMAW}_i = 1 + E\left[\max(0, \beta_{ij})\right] = 1 + \sum_{j} p_{ij}\max(0, \beta_{ij}) \tag{14}$$

and under varying demand d_w

$$\begin{aligned}\mathrm{MMAW}_i &= 1 + E\left[\sum_{w}\max\left(0, \beta_{ij}^{w}\right)\right] \\ &= 1 + \sum_{j} p_{ij}\sum_{w}\max\left(0, \beta_{ij}^{w}\right).\end{aligned} \tag{15}$$

Mean Multi-State Fussell-Vesely (MMFV): Under constant demand d

$$\begin{aligned}\mathrm{MMFV}_i &= E\left[\max(0, -\beta_{ij})\right] \\ &= \sum_{j} p_{ij}\max(0, -\beta_{ij})\end{aligned} \tag{16}$$

and under varying demand d_w

$$\begin{aligned}\mathrm{MMFV}_i &= E\left[\sum_{w}\max\left(0, -\beta_{ij}^{w}\right)\right] \\ &= \sum_{j} p_{ij}\sum_{w}\max\left(0, -\beta_{ij}^{w}\right).\end{aligned} \tag{17}$$

D. Discussion of CIM

The main purpose of Type 1 importance measures is to quantitatively discern which components have the greatest impact on multi-state system reliability. From a general perspective, there is a major difference between the General, and the Alternative CIM. Alternative CIM account for both the impact a given component has on system reliability, when system reliability is perturbed by changes in a component state, and for the probability that such changes take place. On the other hand, General CIM, which are extensions of commonly used binary importance measures [5], are exclusively concerned with quantifying the average impact of a given component on system reliability, when system reliability is perturbed by changes in a component state. For the binary case, and from a physical perspective, Vesely [32] identified this distinction as "occurring events," and "existing conditions". An example of an occurring event is the failure of a component, whereas an example of an existing condition is a component being faulty. The importance of an occurring event depends on both the effect of the event on the system, and on the probability of occurrence of that event. On the other hand, the importance of an existing condition is related to the fallbacks of that condition on the system, regardless of its occurrence probability.

Based on Vesely's [32] perspective, General CIM may be considered as part of the 'existing condition' perspective, whereas Alternative CIM may be considered as part of the 'occurring event' perspective. This distinction has a rather high impact on the physical interpretation of the measures. Consider for example the RAW-related measures. The MRAW adopts the existing condition perspective, and indicates which component is likely to improve the system performance the most, after it has been replaced by a better performing component. Note that the existing condition in this case would be the replacement of the component. The MMAW adopts the occurring event perspective, and indicates which component, in the current system configuration, "is tending" to improve the system performance the most. It does not mean that improving such component will have a high benefit to the system.

It is also important to note that these measures can distinctly differ when computation & ranking is performed. Thus, it is important to provide physical interpretation about their insights so that they can be employed in the correct context. The rankings differ because they focus on different implications of reliability improvement or decrement. In general, the CIM are metrics describing 1) the potential for reliability improvement, 2) the implications of worst-case scenarios, or 3) a combination or hybrid approach. These three different categories can be thought of as reliability-potential, risk-adverse, and risk-neutral measures, respectively.

The choice of CIM may depend on the particular application. For a fixed, existing system, the risk-averse measures may be appropriate to select components to inspect or perform preventive maintenance. Alternatively, the reliability-potential metrics would be more appropriate in selecting components to re-design to improve the reliability. Finally, if the desire is to develop a robust system, that is not overly sensitive to any component, then the risk-neutral metrics would be advisable.

Birnbaum & MAD CIM: These CIM account for the absolute deviation of each component state from the actual value of multi-state system reliability. From a general perspective, these CIM provide an answer to the question of which multi-state component most significantly impacts, positively or negatively, multi-state system reliability. A positive impact is considered when a specific component state improves MR_d from its actual value. Conversely, a negative impact is considered when a specific component state degrades MR_d from its true value. Because Birnbaum CIM & MAD account for both positive & negative impacts, they can be regarded as risk-neutral CIM. From a particular perspective, a low value of Birnbaum or MAD indicates that system reliability is not affected by changes in the states of a component. On the other hand, high values of this measure indicate that reliability is highly sensitive to perturbations in the state of a component.

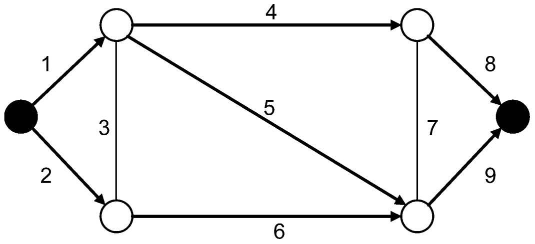

Fig. 1. ARPA network.

MRAW & MMAW CIM: MRAW & MMAW CIM are reliability-potential measures. Both of these CIM quantify how changes in component states positively contribute to multi-state system reliability. From a general perspective, these CIM provide an answer to the problem of identifying the multi-state component that has the greatest potential to improve multi-state system reliability. The mathematical expression of this CIM only considers values that improve multi-state system reliability. From a particular perspective, low values of MRAW & MMAW are a strong indication that as the states of a specific component improve multi-state system reliability will not be affected beneficially. In contrast, high values can provide a clear indication that, as the states of specific component improve, multi-state system reliability is positively impacted.

MFV & MMFV CIM: MFV & MMFV account for the average change in multi-state system reliability when component states negatively contribute to multi-state system reliability. From a general perspective, these measures can help in identifying the multi-state component that can provide the largest decrease on multi-state system reliability. These are the risk-averse measures. The mathematical expression for MFV considers only negative contributions to the overall system reliability, and thus, it can be regarded as a pessimistic measure. From a particular perspective, low values of this CIM provide a suitable indication that the different component states have no negative impact on the behavior of the system. On the contrary, high values for MFV are indicative of multi-state system reliability deterioration as the states of a specific component degrade.

IV. EXAMPLES/COMPARISON OF CIM

Three examples are presented to illustrate how the different CIM can effectively assist in obtaining the criticality of multi-state components regarding the reliability of the system.

TABLE I
ARPA NETWORK DATA

Arc	Transmission Capacity				State Probability				$MR_d = 0.91622$			
									$P(\varphi(\mathbf{x})\geq d \mid x_{ij} = b_{ij})$			
1	0	3	4	8	0.005	0.005	0.010	0.980	0.00000	0.00000	0.90286	0.92632
2	0	3	4	6	0.020	0.010	0.015	0.955	0.00000	0.92610	0.92632	0.93532
3	0	3			0.020	0.980			0.91596	0.91624		
4	0	3	4		0.010	0.015	0.975		0.00000	0.88844	0.92628	
5	0	3			0.020	0.980			0.88528	0.91686		
6	0	3	6		0.005	0.020	0.975		0.00000	0.88920	0.92158	
7	0	3			0.010	0.990			0.90640	0.91634		
8	0	3	4	6	0.010	0.015	0.005	0.970	0.00000	0.91562	0.91594	0.92548
9	0	3	4	8	0.020	0.010	0.010	0.960	0.00000	0.00000	0.91638	0.94440

TABLE II
MEAN & VARIANCE FOR GENERAL CIM

	Birnbaum			MRAW			MFV			MRRW	
j	Mean	Variance	*j*	Mean	Variance	*j*	Mean	Variance	*j*	Mean	Variance
1	0.61876	4.484E-08	1	1.00348	5.078E-09	1	0.67190	1.809E-08	1	3.04782	1.558E-06
2	0.31804	3.207E-08	2	1.01381	4.655E-08	2	0.33333	3.081E-33	2	1.50000	4.437E-32
3	0.00032	2.832E-10	3	1.00001	2.465E-11	3	0.00034	4.828E-10	3	1.00034	4.834E-10
4	0.47643	5.167E-08	4	1.00537	5.913E-09	4	0.51466	4.124E-08	4	2.06039	7.440E-07
5	0.03190	2.767E-08	5	1.00072	5.458E-10	5	0.03410	2.965E-08	5	1.03530	3.405E-08
6	0.47417	2.644E-08	6	1.00288	9.073E-10	6	0.51469	7.979E-09	6	2.06054	1.439E-07
7	0.00986	5.228E-09	7	1.00011	1.919E-10	7	0.01065	5.502E-09	7	1.01077	5.742E-09
8	0.30855	2.531E-07	8	1.00326	2.988E-07	8	0.33352	5.712E-09	8	1.50043	2.895E-08
9	0.62042	2.826E-08	9	1.01053	1.987E-08	9	0.66667	1.220E-11	9	3.00001	9.880E-10

TABLE III
MEAN & VARIANCE FOR ALTERNATIVE CIM

	MAD			MMAW			MMFV	
j	Mean	Variance	*j*	Mean	Variance	*j*	Mean	Variance
1	0.01869	3.504E-08	1	1.01024	4.390E-08	1	0.01016	1.628E-11
2	0.03673	1.198E-08	2	1.02009	1.620E-08	2	0.02000	1.204E-35
3	0.00001	1.724E-11	3	1.00001	2.367E-11	3	0.00001	1.931E-13
4	0.01916	1.900E-08	4	1.01047	2.248E-08	4	0.01044	3.712E-11
5	0.00127	4.937E-10	5	1.00071	5.242E-10	5	0.00068	1.186E-11
6	0.01026	2.768E-09	6	1.00561	3.450E-09	6	0.00559	1.277E-11
7	0.00020	1.608E-10	7	1.00011	1.881E-10	7	0.00011	5.502E-13
8	0.01783	2.091E-06	8	1.00946	2.495E-06	8	0.01001	6.179E-12
9	0.05495	3.878E-08	9	1.02998	5.043E-08	9	0.03000	1.098E-14

The multi-state reliability of the systems analyzed has been obtained with the method presented by Ramirez-Marquez, *et al.* [25]. The evaluation of CIM & MR_d has been performed with the MC simulation method described in Ramirez-Marquez & Coit [18].

Example 1: For the ARPA network depicted in Fig. 1, it is desired to obtain the probability that a demand of $d = 10$ units can be supplied from source to sink. Table I presents problem data considered for analyzing MR_d, assuming different arc state probabilities. Moreover, this table contains the results of applying the MC simulation method described in Ramirez-Marquez & Coit [18] for obtaining MR_d, and $P(\varphi(\mathbf{x}) \geq d \mid x_{ij} = b_{ij})$. The values presented in Table I (probabilities of arcs' states & corresponding transmission capacities) allow for the computation of each of the proposed CIM. However, to provide a measure of statistical robustness regarding the estimated values of the proposed CIM, ten independent simulation experiments have been performed. Each of these experiments consisted of obtaining an estimate for each CIM based on 1 000 000 randomly generated system state vectors $\mathbf{x}$. Tables II, and III present the average & associated variance for General, and Alternative CIM computed from the independent experiments, respectively. Finally, Table IV presents the component rankings with respect to the different

TABLE IV
TYPE 1 IM COMPONENTS RANKINGS

Rank	Birnbaum		MRAW		MFV		MRRW		MAD		MMAW		MMFV	
	j	Mean	*j*	Mean	*j*	Mean	*j*	Mean	*j*	Mean	*j*	Mean	*j*	Mean
1	9	0.62042	2	1.01381	1	0.67190	1	3.04782	9	0.05495	9	1.02998	9	0.03000
2	1	0.61876	9	1.01053	9	0.66667	9	3.00001	2	0.03673	2	1.02009	2	0.02000
3	4	0.47643	4	1.00537	6	0.51469	6	2.06054	4	0.01916	4	1.01047	4	0.01044
4	6	0.47417	1	1.00348	4	0.51466	4	2.06039	1	0.01869	1	1.01024	1	0.01016
5	2	0.31804	8	1.00326	8	0.33352	8	1.50043	8	0.01783	8	1.00946	8	0.01001
6	8	0.30855	6	1.00288	2	0.33333	2	1.50000	6	0.01026	6	1.00561	6	0.00559
7	5	0.03190	5	1.00072	5	0.03410	5	1.03530	5	0.00127	5	1.00071	5	0.00068
8	7	0.00986	7	1.00011	7	0.01065	7	1.01077	7	0.00020	7	1.00011	7	0.00011
9	3	0.00032	3	1.00001	3	0.00034	3	1.00034	3	0.00001	3	1.00001	3	0.00001

TABLE V
NETWORK 2 DATA, AND CONDITIONAL MR

Arc	Transmission Capacity			State Probability			$\mathrm{MR}_d = 0.94618$		
							$P(\varphi(\mathbf{x})\geq d \mid x_{ij}=b_{ij})$		
1	0	3	5	0.1163	0.0616	0.8221	0.9371	0.9447	0.9476
2	0	3	5	0.1624	0.1224	0.7152	0.9273	0.9434	0.9502
3	0	3	5	0.2014	0.0900	0.7086	0.9294	0.9451	0.9506
4	0	3	5	0.0689	0.1155	0.8156	0.8944	0.9332	0.9528
5	0	3	5	0.1863	0.1366	0.6771	0.9281	0.9443	0.9521
6	0	3	5	0.2244	0.0214	0.7542	0.9458	0.9463	0.9464
7	0	3	5	0.2220	0.1334	0.6445	0.9388	0.9464	0.9483
8	0	3	5	0.1265	0.0762	0.7973	0.9427	0.9459	0.9468
9	0	3	5	0.2993	0.0343	0.6664	0.9423	0.9469	0.9477
10	0	3	5	0.3016	0.0813	0.6171	0.9437	0.9466	0.9467
11	0	3	5	0.2385	0.0785	0.6830	0.9425	0.9462	0.9474
12	0	3	5	0.3460	0.0269	0.6272	0.9426	0.9468	0.9479
13	0	3	5	0.3512	0.0441	0.6048	0.9345	0.9486	0.9531
14	0	3	5	0.0326	0.0182	0.9492	0.8563	0.9222	0.9497
15	0	3	5	0.0231	0.1268	0.8501	0.9378	0.9445	0.9466
16	0	3	5	0.0373	0.0830	0.8797	0.9001	0.9291	0.9494
17	0	3	5	0.0222	0.0192	0.9586	0.0000	0.0000	0.9868
18	0	3	5	0.0052	0.0411	0.9537	0.8994	0.9281	0.9470
19	0	3	5	0.3935	0.0625	0.5440	0.9460	0.9461	0.9463
20	0	3	5	0.0651	0.0457	0.8893	0.9402	0.9457	0.9467
21	0	3	5	0.1260	0.0495	0.8245	0.9265	0.9436	0.9502

Type 1 importance measures. As can be observed from the tables, the variances are consistently low, and the measures can be considered to be robust.

For this example, the ranking of components based on General CIM do not agree. This behavior can be related to the physical insights of the CIM. The results obtained for Birnbaum CIM indicate that, on the average, multi-state system reliability is mostly perturbed by changes in the states of component 9. However, changes on component states that most positively impact MR_d can be attributed to component 2. Finally, as the states of components degrade, multi-state system reliability is impacted mostly by component 1.

Contrary to the General CIM, Alternative CIM provide consistent rankings for each of the different measures. The values & rankings of MAD present a better approach to measure "robustness" in a component than Birnbaum CIM. This is because MAD provides the expected value of the absolute deviations from MR_d when changes in component states occur. Similarly, MMAW (MMFV) provide a better measure than MRWA (MFV) because it takes into account the probability of an increase (decrease) in reliability when component states increase (decrease). Although for this case Alternative CIM rankings agree, this agreement may not always be the case. If rankings were to disagree, the perspective & needs of the application should be considered in order to decide which measure to use.

Finally, these new importance measures (Type 1) present a consistent analytical approach to identify & rank critical

TABLE VI
CIM NETWORK 2

Arc	Birnbaum	MRAW	MFV	MRRW	MAD	MMAW	MMFV
1	0.00599	1.00075	0.00558	1.00561	0.00231	1.00123	0.00121
2	0.01284	1.00212	0.01145	1.01158	0.00628	1.00304	0.00360
3	0.01114	1.00234	0.00944	1.00953	0.00661	1.00331	0.00367
4	0.03569	1.00350	0.03422	1.03543	0.01047	1.00571	0.00536
5	0.01294	1.00313	0.01055	1.01066	0.00763	1.00424	0.00383
6	0.00036	1.00018	0.00020	1.00020	0.00025	1.00018	0.00009
7	0.00486	1.00124	0.00390	1.00392	0.00303	1.00148	0.00173
8	0.00219	1.00033	0.00199	1.00199	0.00096	1.00052	0.00049
9	0.00306	1.00118	0.00205	1.00205	0.00220	1.00110	0.00123
10	0.00171	1.00050	0.00131	1.00131	0.00110	1.00038	0.00079
11	0.00246	1.00066	0.00194	1.00195	0.00171	1.00088	0.00093
12	0.00296	1.00124	0.00189	1.00190	0.00233	1.00116	0.00131
13	0.01051	1.00494	0.00617	1.00621	0.00839	1.00454	0.00434
14	0.05869	1.00186	0.06017	1.06402	0.00671	1.00353	0.00356
15	0.00524	1.00022	0.00532	1.00534	0.00076	1.00038	0.00043
16	0.03319	1.00170	0.03338	1.03453	0.00597	1.00299	0.00331
17	0.96649	1.02147	1	0	0.07811	1.04115	0.04140
18	0.03284	1.00043	0.03427	1.03549	0.00177	1.00083	0.00104
19	0.00019	1.00006	0.00014	1.00014	0.00014	1.00007	0.00008
20	0.00349	1.00027	0.00341	1.00343	0.00087	1.00049	0.00043
21	0.01314	1.00212	0.01176	1.01190	0.00592	1.00350	0.00276

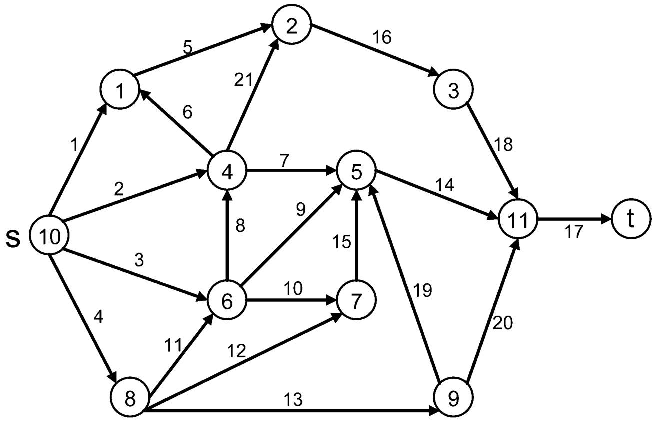

Fig. 2. Network 2.

components based on how they affect multi-state system reliability. For some applications, existing Type 2 measures are already available & appropriate. However, for some applications, Type 2 importance measures may be less useful for component ranking because they pertain to specific component states. These new Type 1 measures will be more appropriate in these cases.

Example 2: Fig. 2 shows a relatively large network, with 110 binary minimal cut sets. The original problem was to find the probability that the capacity of the network from source to sink is greater or equal to five units. All arcs have the same three states, corresponding to transmission capacities 0, 3, and 5. Table V presents problem data with assumed different arc state probabilities, and the multi-state reliability conditioned on a component state. Table VI presents the values of different CIM, while Table VII presents the ranking of components according to the computed values.

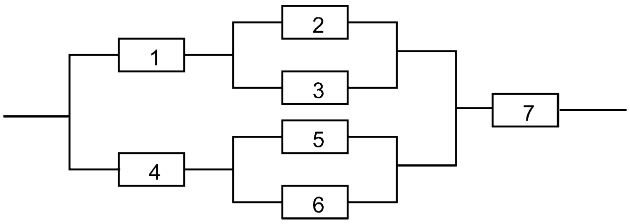

Fig. 3. Levitin *et al.* [17] system.

As anticipated, all CIM identify component 17 as the most important system component from each criticality perspective. Moreover, similar to the rankings of Example 1, for this example, General CIM rankings distinctly disagree depending on the measure employed. However, the rankings obtained though the Alternative CIM are in general consistent within each other. For example, the first & last four rankings belong to the same components for each criticality criterion. The disagreement in the remaining components can be associated to the physical insights of each CIM, as discussed in Section III-D.

Example 3: Levitin, *et al.* [17] analyzed the system depicted in Fig. 3, with component reliability data shown in Table VIII. In their study, they consider different demand levels, and graphically analyze component Type 2 importance measures. For the present analysis, a demand of 5 units is assumed for obtaining MR.

There are two important observations from this example. First, it illustrates two points: how CIM can be used to compare

TABLE VII
COMPONENT RANKINGS FOR NETWORK 2 BASED ON PROPOSED CIM

Rank	*j*	Birnbaum	*j*	MRAW	*j*	MFV	*j*	MRRW	*j*	MAD	*j*	MMAW	*J*	MMFV
1	17	0.96649	17	1.02147	17	1.00000	17	∞	17	0.07811	17	1.04115	17	0.04140
2	14	0.05869	13	1.00494	14	0.06017	14	1.06402	4	0.01047	4	1.00571	4	0.00536
3	4	0.03569	4	1.00350	18	0.03427	18	1.03549	13	0.00839	13	1.00454	13	0.00434
4	16	0.03319	5	1.00313	4	0.03422	4	1.03543	5	0.00763	5	1.00424	5	0.00383
5	18	0.03284	3	1.00234	16	0.03338	16	1.03453	14	0.00671	14	1.00353	3	0.00367
6	21	0.01314	2	1.00212	21	0.01176	21	1.01190	3	0.00661	21	1.00350	2	0.00360
7	5	0.01294	21	1.00212	2	0.01145	2	1.01158	2	0.00628	3	1.00331	14	0.00356
8	2	0.01284	14	1.00186	5	0.01055	5	1.01066	16	0.00597	2	1.00304	16	0.00331
9	3	0.01114	16	1.00170	3	0.00944	3	1.00953	21	0.00592	16	1.00299	21	0.00276
10	13	0.01051	7	1.00124	13	0.00617	13	1.00621	7	0.00303	7	1.00148	7	0.00173
11	1	0.00599	12	1.00124	1	0.00558	1	1.00561	12	0.00233	1	1.00123	12	0.00131
12	15	0.00524	9	1.00118	15	0.00532	15	1.00534	1	0.00231	12	1.00116	9	0.00123
13	7	0.00486	1	1.00075	7	0.00390	7	1.00392	9	0.00220	9	1.00110	1	0.00121
14	20	0.00349	11	1.00066	20	0.00341	20	1.00343	18	0.00177	11	1.00088	18	0.00104
15	9	0.00306	10	1.00050	9	0.00205	9	1.00205	11	0.00171	18	1.00083	11	0.00093
16	12	0.00296	18	1.00043	8	0.00199	8	1.00199	10	0.00110	8	1.00052	10	0.00079
17	11	0.00246	8	1.00033	11	0.00194	11	1.00195	8	0.00096	20	1.00049	8	0.00049
18	8	0.00219	20	1.00027	12	0.00189	12	1.00190	20	0.00087	15	1.00038	20	0.00043
19	10	0.00171	15	1.00022	10	0.00131	10	1.00131	15	0.00076	10	1.00038	15	0.00043
20	6	0.00036	6	1.00018	6	0.00020	6	1.00020	6	0.00025	6	1.00018	6	0.00009
21	19	0.00019	19	1.00006	19	0.00014	19	1.00014	19	0.00014	19	1.00007	19	0.00008

TABLE VIII
LEVITIN, *et al.* [17] SYSTEM DATA, AND CONDITIONAL MR

Component	Transmission Capacity					State Probability					$\mathrm{MR}_d = 0.74542$				
											$P(\varphi(\mathbf{x}) \geq d \mid x_{ij} = b_{ij})$				
1	0	1	3	5	7	0.10	0.05	0.15	0.35	0.35	0.4802	0.5833	0.6716	0.8132	0.8132
2	0	2	4	0	0	0.10	0.05	0.85	0.00	0.00	0.6288	0.7453	0.7597	0.7454	0.7454
3	0	2	4	0	0	0.10	0.05	0.85	0.00	0.00	0.6288	0.7453	0.7597	0.7454	0.7454
4	0	2	6	8	0	0.20	0.10	0.45	0.25	0.00	0.4784	0.7054	0.8280	0.8280	0.7454
5	0	2	4	0	0	0.10	0.05	0.85	0.00	0.00	0.6713	0.7471	0.7540	0.7454	0.7454
6	0	2	4	0	0	0.10	0.05	0.85	0.00	0.00	0.6713	0.7471	0.7540	0.7454	0.7454
7	0	6	10	14	18	0.15	0.15	0.05	0.45	0.20	0.0000	0.8765	0.8765	0.8765	0.8765

the different types of importance measures (Types 1 & 2), and also, how CIM can adequately rank components based on the different criticality criterion. In this sense, Table IX presents CIM for each of the components, while Table X illustrates its associated rankings. Second, it is used to provide insights about the proposed measures when considering the binary case (i.e., components have only two sates: working, or failed). Data for the binary case is given in Table XI, while results associated to the computation of each component IM, and associated component rankings are given in Tables XII, and XIII respectively.

When multi-state components are used, General & Alternative CIM provide almost the same rankings. The only difference in these rankings is given by component 4. This component is next-to-most critical when considering an improvement criterion for the General CIM. It is interesting to note that when the Alternative measures are used, the rankings agree with those presented for MRAW. From an overall perspective, the proposed CIM agree with the rankings of Levitin, *et al.* [17]. In their study, components 5 & 6 were selected as more important than components 2 & 3 because they have a stronger impact on MR_d. However, this may also be explained by the choice of state probabilities that Levitin, *et al.* [17] used for the computation of Type 2 importance measures.

For the binary case, RAW significantly differs from the other General IM measures. RAW naturally varies because it has a different perspective & interpretation of criticality. The choice of the most important component for each of the IM can be considered from three perspectives.

1. The highest improvement on reliability is given by component 4, and it is measured by RAW.
2. The greatest negative impact, measured by FV, on system reliability is given by component 7.

TABLE IX
LEVITIN, *et al.* [17] SYSTEM CIM

Arc	Birnbaum	MRAW	MFV	MRRW	MAD	MMAW	MMFV
1	0.15917	1.04546	0.16807	1.20203	0.09314	1.06364	0.06130
2	0.06548	1.00957	0.07828	1.08493	0.02379	1.01626	0.01565
3	0.06548	1.00957	0.07828	1.08493	0.02379	1.01626	0.01565
4	0.15737	1.07382	0.13730	1.15915	0.11518	1.07751	0.07701
5	0.04219	1.00687	0.04973	1.05233	0.01481	1.00992	0.00995
6	0.04219	1.00687	0.04973	1.05233	0.01481	1.00992	0.00995
7	0.31746	1.17587	0.25000	1.33333	0.22325	1.14949	0.15000

TABLE X
LEVITIN, *et al.* [17] SYSTEM COMPONENT RANKINGS

Rank	*j*	Birnbaum	*j*	MRAW	*j*	MFV	*j*	MRRW	*j*	MAD	*j*	MMAW	*j*	MMFV
1	7	0.31746	7	1.17587	7	0.25000	7	1.33333	7	0.22325	7	1.14949	7	0.15000
2	1	0.15917	4	1.07382	1	0.16807	1	1.20203	4	0.11518	4	1.07751	4	0.07701
3	4	0.15737	1	1.04546	4	0.13730	4	1.15915	1	0.09314	1	1.06364	1	0.06130
4	2	0.06548	2	1.00957	2	0.07828	2	1.08493	2	0.02379	2	1.01626	2	0.01565
5	3	0.06548	3	1.00957	3	0.07828	3	1.08493	3	0.02379	3	1.01626	3	0.01565
6	5	0.04219	5	1.00687	5	0.04973	5	1.05233	5	0.01481	5	1.00992	5	0.00995
7	6	0.04219	6	1.00687	6	0.04973	6	1.05233	6	0.01481	6	1.00992	6	0.00995

3. Overall, the component that mostly contributes to deviations from reliability is component 7, and the measure of deviation is given by Birnbaum.

Finally, for the binary case, the Alternative CIM provide an extra level of information. That is, they quantify the expected value associated with the criticality criterion. For example, based on MRAW, the expected improvement in reliability associated with component 4 is approximately 1.6%, while based on RAW the average improvement is approximately 2.1%. These are important considerations when decision making is based on component criticality.

TABLE XI
DATA FOR BINARY CASE

Component	States		State Probability		$R = 0.968989$	
					$P(\varphi(\mathbf{x})\geq d\mid x_{ij}= b_{ij})$	
1	0	1	0.07	0.93	0.739716	0.986246
2	0	1	0.15	0.85	0.949204	0.97248
3	0	1	0.1	0.9	0.937565	0.97248
4	0	1	0.25	0.75	0.90689	0.989688
5	0	1	0.025	0.975	0.959872	0.969222
6	0	1	0.15	0.85	0.967664	0.969222
7	0	1	0.01	0.99	0	0.978776

V. CONCLUSIONS

In this paper, the CIM for establishing the criticality of components in MSMC have been developed. CIM belong to the multi-state Type 1 importance measures that characterize how a specific component affects multi-state system reliability. For some systems, assessing criticality with Type 2 importance measures is not directly obvious or straightforward. Two sets of CIM have been developed. The first set extends frequently used binary importance measures to the multi-state case, while the second set is introduced to consider state probabilities. Experimental results for different system complexities show that these new CIM can effectively estimate the criticality of components with respect to multi-state system reliability. For relatively complex MSMC, CIM present a computationally efficient, and analytically sound approach for measuring component criticality. Based on experimental results, Alternate CIM provide more consistent rankings.

A. Future Research

Recently, Wang *et al.* [26] developed an alternate approach to obtain importance measures for complex systems. Wang *et al.* [26] observe that a "failure criticality index" can be constructed by considering the percentage of system failure events induced by a particular component. Discrete event simulation is used to obtain an index for each component. Mathematically, the "failure criticality index" is given as

$$I_i^{FCI} = \frac{\text{Number of system failures due to component } i}{\text{Number of system failures}}$$

The information obtained by Type 2 importance measures can be used to develop a multi-state interpretation of the "failure criticality index." This index could be approximated through a methodology similar to the MC approach of Ramirez-Marquez & Coit [18] to analyze reliability in MSMC.

Consider the conditional MR_d presented in Table I. It is evident that whenever the capacity of arc 1 equals zero or three,

TABLE XII
BINARY CASE COMPONENT IMPORTANCE MEASURES

Component	Birnbaum	MRAW	MFV	MRRW	MAD	MMAW	MFV
1	0.24653	1.01781	0.23661	1.30995	0.032098	1.016563	0.016563
2	0.023276	1.003603	0.020418	1.02084	0.005935	1.003063	0.003063
3	0.034915	1.003603	0.032429	1.03352	0.006285	1.003243	0.003243
4	0.082798	1.021362	0.064086	1.06847	0.031049	1.016021	0.016022
5	0.00935	1.000241	0.009408	1.00950	0.000455	1.000235	0.000235
6	0.001558	1.000241	0.001367	1.00137	0.000397	1.000205	0.000205
7	0.978776	1.010101	1	∞	0.019379	1.01	0.01

TABLE XIII
BINARY CASE COMPONENT RANKINGS

Rank	*j*	Birnbaum	*j*	RAW	*j*	FV	*J*	MAD	*j*	MMAW	*j*	MFV
1	7	0.978776	4	1.021362	7	1	1	0.032098	1	1.016563	1	0.016563
2	1	0.24653	1	1.01781	1	0.23661	4	0.031049	4	1.016021	4	0.016022
3	4	0.082798	7	1.010101	4	0.064086	7	0.019379	7	1.01	7	0.01
4	3	0.034915	2	1.003603	3	0.032429	3	0.006285	3	1.003243	3	0.003243
5	2	0.023276	3	1.003603	2	0.020418	2	0.005935	2	1.003063	2	0.003063
6	5	0.00935	5	1.000241	5	0.009408	5	0.000455	5	1.000235	5	0.000235
7	6	0.001558	6	1.000241	6	0.001367	6	0.000397	6	1.000205	6	0.000205

TABLE XIV
PRELIMINARY RESULTS FOR ARPA NETWORK

Rank	Arc	Simulation
1	9	0.34877
2	2	0.24731
3	4	0.12103
4	1	0.11769
5	8	0.11721
6	6	0.06254

the system fails. The MC approach of Ramirez-Marquez & Coit [18] can be used to develop a "multi-state failure index." This index can be approximated by a subroutine that counts the number of system vectors that are generated with $x_i \leq 3$. The main issue that arises for the application of this index is related to the computation of component-induced failures when it is not evident that such component induces system failures. Preliminary results of this research have been obtained for the ARPA network depicted in Fig. 1 by considering the data in Table I. Table XIV presents the results of the "multi-state failure index" for the six most critical arcs from example 1, Table IV. Notice that the ranking of the components agrees with the rankings obtained through CIM illustrated in Tables II & III.

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