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Analysis of grouped data from field-failure reporting systems

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Abstract

Observed reliability data from fielded systems is highly desirable because they implicitly account for all actual usage and environmental stresses. Many companies and large organizations have instituted automated field-failure reporting systems to organize and disseminate these data. Despite these advantages, field data must be used with caution because they often lack sufficient detail. Specifically, the precise times-to-failure are often not recorded and only cumulative failure quantities and operating times are available. When only data of this type are available, it is difficult to determine whether component or system hazard function varies with time or is constant (i.e., exponential distribution). Analysts often use the exponential distribution to model time-to-failure because the distribution parameter can be estimated with just the merged data. However, this can be dangerous if the exponential distribution is not appropriate. An approach is presented in this paper for Type II censored data, with and without replacement, to evaluate this assumption even when individual times-to-failure are not available. A hypothesis test is presented to test the suitability of the exponential distribution for a particular data set composed of multiple merged data records. Two examples are presented to demonstrate the approach. The hypothesis test readily rejects an exponential distribution assumption when the data originate from a Weibull distribution. This is a very important result because it has generally been assumed that time-to-failure data were always required to evaluate the suitability of specific time-to-failure distributions. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Exponential distribution; Hypothesis test; Field failure data

Notation

θ exponential distribution parameter, $f(x) = (1/\theta)\,e^{-x/\theta}$
$\hat{\theta}$ an estimate of θ
$\hat{\theta}_r$ an estimate of θ made with exactly r observed failures
$\tilde{\theta}$ maximum likelihood estimate (MLE) of θ
r number of observed failures
X_i ith time-to-failure
T_r cumulative component operating time for a data record with r failures
N number of data records, $N = \sum_{i=1}^{m} n_i$
$\mathbf{n}$ $= (n_1, n_2, \ldots, n_m)$
n_r number of data records with r failures
m maximum number of failures for any data record within the data set $= \max\{r \mid n_r > 0\}$

1. Introduction

Reliability engineers and managers require timely, accurate data to assist in the analysis of system designs and to aid in decision making. Reliability data from components installed in fielded systems are considered to be very desirable because they inherently capture the appropriate usage and environmental stresses. Unfortunately, there are often missing details associated with field data. In particular, the actual times-to-failure are often not recorded and the data are grouped together and presented as a total number of failures and a cumulative number of operating hours. This lack of detail is often a result of maintenance record-keeping policies or a lack of sufficient elapsed time meters at all locations.

If time-to-failure is distributed according to an exponential distribution, then the grouped data can be used to estimate mean time to failure (MTTF) and the exponential distribution parameter. However, empirical evaluation or validation of the exponential distribution assumption has been problematic because of the missing detail in the data.

There has been other research concerned with the use of data with missing attributes. Dey [1] developed a simulation

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model to study the behavior of grouped data and test the exponential distribution assumption. Usher [2] and Lin et al. [3] have developed techniques to analyze time-to-failure data and determine Weibull distribution parameters when the time-to-failure is known but not all system failures have been sufficiently diagnosed to identify the failed component. Wang [4] developed a graphical approach for evaluating repairable system reliability when the data are grouped together and the populations are changing.

2. Attributes of field data

For the assessment of component reliability, field data have many distinct advantages. The principal advantage is that the operational and environmental stresses are those which are of most importance, i.e., the actual usage environment. In the field, the stresses are applied simultaneously and variable interactions are implicitly considered by any analysis using these data. Alternatively, even the most faithful and rigorous laboratory testing will fail to precisely simulate all field conditions. For this reason, there have been many dedicated company-specific and industry-specific field-data collection programs. There are examples of these large data systems in the defense, automotive, railroad and power utility industries. For example, the U.S. DoD Reliability Analysis Center (RAC) has empirical field reliability data on thousands of different parts [5].

For all of the advantages of field data, there are also disadvantages including incomplete or inaccurate data reporting, unconfirmed component failures and others. Several of these disadvantages are described in more detail by Coit et al. [6]. The disadvantage to be addressed in this paper is the observation that individual times-to-failure are often missing from field-data reporting systems. The data are most often available in the form of r collective failures observed in T_r cumulative hours with no further delineation or detail available. Analysts may have many of these merged data records available for the same component. The RAC database has data on thousands of components but the individual time-to-failures are almost always missing.

Field data generally involve censoring: Type I, Type II or random censoring. Type I censoring is when there is a predetermined maximum operating time. Data are unavailable after this time. Components still operating at this time are censored. Type II censoring is when there is a specified failure number and data collection is terminated at the arrival of this failure. An example of Type II data possibly with missing time-to-failure is an inventory control system of spare parts. Data may only become available when the inventory falls below some threshold and an order is placed. The difference between the existing inventory and the threshold is the Type II specified failure number.

There are well-known techniques to determine parameters for many parametric distributions and censoring types. Thorough descriptions of these techniques are presented in Mann et al. [7] and Barlow and Proschan [8]. However, the analysis of reliability data with missing times-to-failure is a non-standard problem and it is not addressed in these references.

Without individual time-to-failure data, it is impossible to fit the data to most popular distributions (e.g., Weibull, lognormal, gamma) using standard techniques such as maximum likelihood estimators (MLE) or regression analysis. However, the MLE for the exponential distribution (with any type of data censoring) only depends on the number of failures and the cumulative time. Therefore, r and T_r are sufficient to determine estimates of the exponential distribution parameter even without individual time-to-failure data.

The nature of the available field data and the simplicity of the exponential MLE have been used to justify or rationalize the exponential distribution for many components where it would seem to be a poor choice based on knowledge of the failure mechanisms. The constant hazard function associated with the exponential distribution does not intuitively seem appropriate for any failure mechanism that can be attributed to fracture, fatigue, corrosion and/or wear mechanisms.

Incorrect assumptions of the underlying distribution can have dire consequences. These should not be easily dismissed as mere mathematical details. For many fledgling companies, major decisions are made with limited data being used as rationale. When an incorrect distribution is assumed, particularly for reasons of convenience, it is particularly dangerous.

3. Distribution of estimated MTTF ($\hat{\theta}$) for Type II data

To study the applicability of the exponential distribution, it is necessary to first review the distribution of an estimated exponential distribution parameter. Then, empirical estimates can be compared with theoretical distributions to evaluate whether the exponential distribution is appropriate. The approach is to develop a probability density function for the estimated parameter, which only depends on r and T_r, rather than time-to-failure.

The type of data being considered is Type II censored data with s items in the field, and r failures observed in T_r cumulative hours. There may or may not be replacement of failed items in the field. Fig. 1 indicates these two scenarios. In Fig. 1(a), there is no replacement and the components have been arranged in accordance with ascending time-to-failure. In this case $s - r$ components did not fail. For the scenario depicted in Fig. 1(b), a failed component is replaced in the field with an as-new copy. There are s original components and a total of $s + r - 1$ unique components represented in the data set (i.e., the originals plus the replacements). In Fig. 1(b), d of the original s components failed where $d \leq r$. T_r is as follows for these two cases.

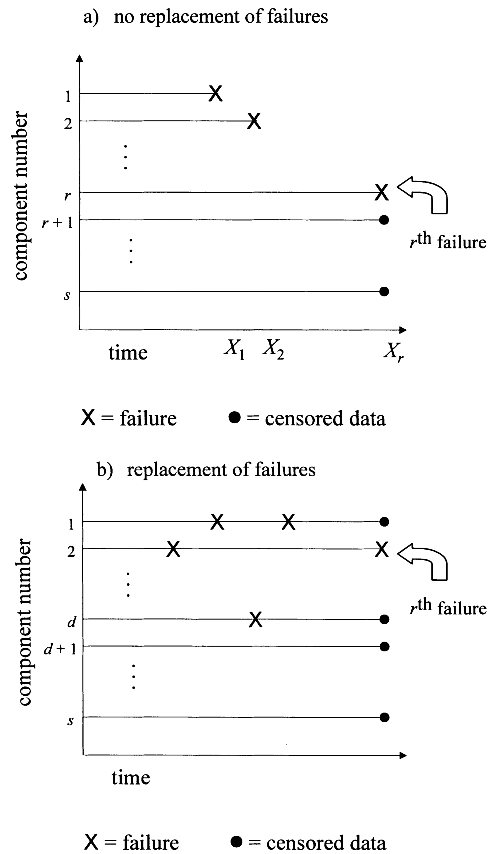

Fig. 1. Depiction of data sets.

- Type II without replacement:

$$T_r = \sum_{i=1}^{r} X_{(i)} + (s - r)X_{(r)}$$

- Type II with replacement:

$$T_r = \sum_{i=1}^{r} X_{(i)} + \sum_{i=1}^{s-1} L_{(i)}$$

where $X_{(i)} = i$th ordered time-to-failure and $L_{(i)} = i$th ordered censor time.

For these two scenarios, the estimated exponential distribution parameter, $\hat{\theta}_r$ (equal to the estimated MTTF), and T_r are distributed in accordance with related gamma distributions. This is a well-known result for Type II data without replacement (Fig. 1(a)). With replacement, T_r can be shown to be gamma distributed by considering the superposition of the s failure processes in Fig. 1(b) as a Poisson process with parameter s/θ. The time to the rth event of the superpositioned Poisson process is distributed as gamma with parameter θ/s and r. However, T_r is equal to s times the rth event time, and therefore T_r is gamma distributed with parameters θ/s and r.

More specifically, since r is always an integer, $\hat{\theta}_r$ and T_r are distributed in accordance with the k-Erlang distribution with k equal to r. These results are summarized as follows:

$$\begin{aligned} &X = \text{time to failure} \\ &X \sim \text{exponential}(\theta),\ E(X) = \theta,\ \text{Var}(X) = \theta^2 \\ &T_r \sim \text{gamma}(\theta, r) \\ &\hat{\theta}_r = \frac{T_r}{r},\ \hat{\theta}_r \sim \text{gamma}(\theta/r, r) \\ &\text{Var}\left(\hat{\theta}_r\right) = \left(\frac{\theta}{r}\right)^2 r = \frac{\theta^2}{r} = \frac{E(X)^2}{r} \end{aligned} \tag{1}$$

For many distributions with monotonically increasing hazard functions, the variance of T_r/r is smaller. Consider a time-to-failure random variable, Y, distributed according to a gamma distribution with a shape parameter (β) greater than one. The variance of the ratio of T_r/r is then given as follows:

$$Y = \text{time to failure}$$

$$Y \sim \text{gamma}(\alpha, \beta;\ \beta > 1),\ E(Y) = \alpha\beta,\ \text{Var}(Y) = \alpha^2\beta$$

$$T_r \sim \text{gamma}(\alpha, \beta r),\ E(T_r) = \alpha\beta r,\ \text{Var}(T_r) = \alpha^2\beta r$$

$$\text{Var}(T_r/T) = \frac{1}{r^2}\alpha^2\beta r = \frac{\alpha^2\beta}{r} = \frac{E(Y)^2}{\beta r}$$

If $E(X)$ and $E(Y)$ (i.e., MTTF) are equal, the variance of T_r/r will be lower for the gamma distribution for all r and for all β greater than one[1]. If the time-to-failure distribution is incorrectly assumed to be exponential when the gamma ($\beta > 1$) is actually correct, then T_r/r will exhibit a lower variance than would be anticipated. This can be detected even without individual time-to-failure data. A similar result can be demonstrated for the Weibull distribution with shape parameter greater than one.

$f\left(\hat{\theta}_r\right)$ is the density function of the estimated exponential parameter with exactly r failures. $f\left(\hat{\theta}_r\right)$ can be equivalently considered as a conditional distribution of $\hat{\theta}$, an estimate based on a random number of failures. If we impose the condition that exactly r failures were observed, then the conditional probability density function based on the gamma distribution is as follows:

$$f\left(\hat{\theta}_r\right) = f\left(\hat{\theta}|r\right) = \frac{(\theta/r)^{-r}\hat{\theta}^{r-1}\exp\left(-r\hat{\theta}/\theta\right)}{(r-1)!}, \qquad r > 0 \tag{2}$$

This is the density function for an estimate from a data set limited to Type II censored data with exactly r failures. In practice, a data set may include many data records with a

[1] When β equals one, the gamma distribution and the exponential distribution are equivalent.

variety of different failure quantities. It is important to consider all applicable data records in the evaluation of the exponential distribution. Therefore, a composite probability density function for mixed distributions is needed.

When considering field data from multiple sources, there will likely be several (or many) data records with differing numbers of failures, r. For an underlying exponential time-to-failure distribution, estimates of θ can determined from each record even without knowledge of the individual times-to-failure, i.e., r and T_r are sufficient. If the time-to-failure are *iid*, the expected value for each estimate will be the same, but the variances will depend on the failure quantity. By considering the conditional density functions, a composite expression for the probability density function for $\hat{\theta}$ can be expressed as follows:

$$f\left(\hat{\theta}\right) = \sum_{r=1}^{\infty} f\left(\hat{\theta}|r\right) \Pr(R = r)$$

where $\Pr(R = r)$ = probability that a randomly selected data record includes exactly r failures.

When considering one specific data set with N data records, the problem becomes more accommodating. By conditioning on $\mathbf{n}$, the probability density function and cumulative distribution function for $\hat{\theta}$ is given by:

$$f\left(\hat{\theta}|\mathbf{n}\right) = \frac{1}{N}\sum_{r=1}^{m} n_r f\left(\hat{\theta}|r\right)$$

$$= \frac{1}{N}\sum_{r=1}^{m} \frac{n_r(\theta/r)^{-r}\hat{\theta}^{r-1}\exp\left(-r\hat{\theta}/\theta\right)}{(r-1)!}, \quad (3)$$

$\hat{\theta} > 0$

$$F\left(\hat{\theta}|\mathbf{n}\right) = 1 - \frac{1}{N}\sum_{r=1}^{m}\left(n_r \exp\left(-r\hat{\theta}/\theta\right)\sum_{l=0}^{r-1}\frac{(r\hat{\theta}/\theta)^l}{l!}\right), \quad (4)$$

$\hat{\theta} > 0$

θ is a fixed distribution parameter, but $\hat{\theta}$ is a random variable; it depends on the collected data. $f\left(\hat{\theta}|\mathbf{n}\right)$ and $F\left(\hat{\theta}|\mathbf{n}\right)$ describe the distribution of $\hat{\theta}$. They indicate the relative likelihood of different estimated exponential distribution parameter values for Type II censored data. For a particular data set, they are useful to explain the variability associated with estimates of θ for exponential time-to-failure.

4. Hypothesis test

By using standard goodness-of-fit testing methods, such as the chi-squared goodness-of-fit test, a hypothesis test can be constructed based on the observed data and Eq. (4). This hypothesis test is only appropriate for sets of field data meeting specific criteria. The data must conform to the following conditions.

1. The data set consists of N data records.
2. Each data record consists of known values of r and T_r (individual time-to-failures are not required).
3. Each data record is Type II censored.
4. Time-to-failure is independent and identically distributed (*iid*) for all data records.
5. Censoring is independent of the failure process.

Some of these assumptions require further scrutiny. In practice, censoring is often dependent on the failure process. When this is the case, these methods do not apply. Also, the *iid* condition may not be met in many field environments. This will be discussed in the conclusions to this paper.

There are no restrictions on left-censoring. Data collection can be initiated at any time with all component operating times set to zero. Any component history prior to the start of data collection is not considered as part of the analysis. The resulting analyses will still provide valid results because of the memoryless property of the exponential distribution. The remaining lifetimes after initiation of data collection will also be exponential with the same distribution parameter, θ.

The cumulative distribution function for $\hat{\theta}$ from a specific data set is $F\left(\hat{\theta}|\mathbf{n}\right)$. It is computed based only on the number of failures and the cumulative hours for each data record within the data set. If the time-to-failure distribution is gamma or Weibull with monotonically increasing hazard functions, the variance for $\hat{\theta}$ will be lower for all r, and $F\left(\hat{\theta}|\mathbf{n}\right)$ will not adequately describe the distribution of observed $\hat{\theta}$ values.

For *iid* data, the hypothesis test with null and alternative hypotheses is as follows.

$$\mathrm{H}_0: \; X \sim \text{exponential}(\theta)$$

$$\mathrm{H}_a: \; X \text{ is not exponential}$$

A test statistic based on the chi-squared test is demonstrated as follows. The chi-squared test is attractive because it readily accommodates the case where distribution parameters have been estimated initially using the same data set. Alternatively, the chi-squared test requires the selection of class intervals and has been observed to have low power in certain instances.

1. Select c class intervals for $\hat{\theta}$: $[a_0,a_1)$, $[a_1,a_2)$, …, $[a_{c-1},a_c)$ with $a_0 = 0$ and $a_c = \infty$. Intervals should be selected such that the expected number of observations will be greater than or equal to five for each interval.
2. Compute $\hat{\theta}$ for each data record as T_r/r.
3. Determine the number of observed $\hat{\theta}$ within each interval and define this quantity as o_i.
4. Compute the expected number of observations (under H_0) for each interval as

$$e_i = N(F(a_i|\mathbf{n}) - F(a_{i-1}|\mathbf{n}))$$

Table 1
Exponential test set

No.	Failures (r)	Time (T)	No.	Failures (r)	Time (T)
1	1	88	11	3	290
2	1	24	12	3	193
3	1	118	13	3	102
4	2	283	14	3	594
5	2	285	15	3	51
6	2	188	16	4	285
7	2	278	17	4	127
8	2	250	18	4	580
9	2	475	19	5	658
10	2	412	20	6	583

5. Compute X^2 statistic as

$$X^2 = \sum_{i=1}^{c} \frac{(o_i - e_i)^2}{e_i}$$

If θ was determined or specified uniquely from H_0, then the limiting distribution of X^2 is χ^2_{c-1}, and it is appropriate to compare the test statistic to $\chi^2_{\alpha,c-1}$. Often, θ must be determined from the same data being used to conduct the hypothesis test. If the maximum likelihood estimate of θ is computed from the unmerged data set, then the limiting distribution of X^2 is bounded by χ^2_{c-1} and χ^2_{c-2} (for one estimated distribution parameter, θ) as described by Kendall and Stuart [9]. For this hypothesis test, percentiles of χ^2_{c-2} should be used.

Other test statistics that can be used when the theoretical distribution is not completely defined are the Cramer–von Mises and Anderson–Darling goodness-of-fit statistics. They have been observed to be powerful in the presence of unknown parameters [10].

For large m, the equation for $F\left(\hat{\theta}|\mathbf{n}\right)$ can become cumbersome. However, the k-Erlang distribution can be closely approximated by the normal distribution for large k. Therefore an approximate equation for the conditional cumulative density function can be stated as follows. r' is a selected integer where the r'-Erlang distribution is considered to be sufficiently close to a normal distribution ($r' \approx 10$):

$$F\left(\hat{\theta}|\mathbf{n}\right) = 1 - \frac{1}{N}\sum_{r=1}^{r'}\left(n_r \exp\left(-r\hat{\theta}/\theta\right)\sum_{l=0}^{r-1}\frac{(r\hat{\theta}/\theta)^l}{l!}\right) - \frac{1}{N}\sum_{r=r'}^{m} n_r \Phi\left(\sqrt{n_r}\left(1-\hat{\theta}/\theta\right)\right), \quad (5)$$

$1 < r' < m$

where

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{2\pi}\, \mathrm{e}^{-\frac{1}{2}u^2}\, \mathrm{d}u$$

(from any standardized normal distribution table).

Table 2
Weibull test set

No.	Failures (r)	Time (T)	No.	Failures (r)	Time (T)
1	1	49	11	3	421
2	1	92	12	3	304
3	1	98	13	3	342
4	2	209	14	3	292
5	2	78	15	3	342
6	2	169	16	4	304
7	2	167	17	4	279
8	2	120	18	4	337
9	2	155	19	5	466
10	2	212	20	6	535

5. Examples

To demonstrate the hypothesis test, two examples are provided. The data sets are presented in Tables 1 and 2. For both data sets, there are 20 data records ($N = 20$) and the only information available is the number of failures and the cumulative time recorded on the components. The data in Table 1 were simulated with the exponential distribution (θ = 100 h) as the underlying time-to-failure distribution. The data in Table 2 were simulated with the Weibull distribution as the underlying distribution with a scale parameter of 100 h and a shape parameter of 3. Once the data were simulated, they were grouped together and the individual failure times were considered to be lost. This simulates the case where only grouped data is available.

Each data set was analyzed twice. First, the data were tested using the theoretical MTTF from the simulation as the exponential distribution parameter, i.e., $\theta = E(X)$. Then, the data were analyzed again assuming no prior knowledge of the underlying time-to-failure distribution. The MLE for the exponential parameter ($\tilde{\theta}$) was first computed from the data available in Tables 1 and 2. The hypothesis test was then conducted as if $\tilde{\theta}$ was the actual (but unknown) distribution parameter, i.e., $\theta = \tilde{\theta}$.

The theoretical and empirical distributions for the cumulative distribution functions are presented in Figs. 2 and 3 for the case where $\theta = E(X)$. Even without any statistical testing, it is readily apparent from the figures that the theoretical distribution provides a relatively good fit to the empirical distribution of $\hat{\theta}$ for the exponential test set (Fig. 2), but not for the Weibull test set (Fig. 3).

The results of chi-squared tests are presented in Tables 3 and 4. For α levels of 0.1 and 0.01, the chi-squared test fails to reject the null hypothesis when the merged data were simulated from an exponential distribution. The test does not indicate that the exponential distribution is inappropriate and a user could merge these data together to compute a singular exponential distribution parameter. It must be emphasized that failure to reject the null hypothesis should never be interpreted to mean that the null hypothesis is true.

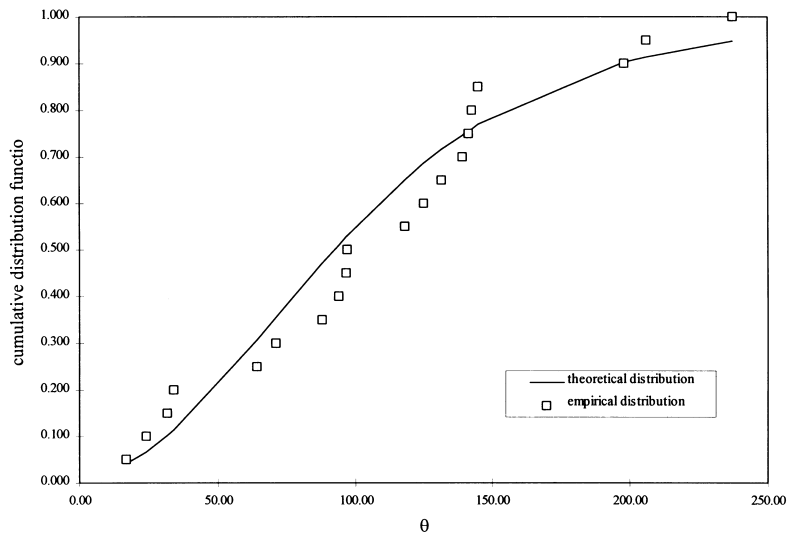

Fig. 2. Results from exponential test set.

The hypothesis test resulted in a reject decision when the Weibull distribution was the underlying distribution in all cases. For this case, a user should not assume the exponential distribution. These are very important results because many analysts have contended that there is no method to test the exponential distribution without individual time-to-failure data.

6. Conclusions

A hypothesis test was developed to test the suitability of the exponential distribution when only merged data records are available from a data set with Type II censoring and an *iid* assumption. For Type II data, this test should be applied each time the exponential distribution is being considered for a component with only merged data available. If the null hypothesis is rejected, the exponential distribution should not be used. In this case, an exponential assumption could provide incorrect or misleading results.

Without the availability of this hypothesis test, analysts may inappropriately select the exponential distribution when it is not suitable because the exponential distribution MLE can accommodate missing time-to-failure data. They may have cited ease-of-use and the existence of no contradictory empirical evidence. Now, more intelligent decisions can be made because of the availability of the hypothesis test developed and demonstrated in this paper. Future research is necessary to extend these results to any data type.

The hypothesis test described and demonstrated here is based on an *iid* assumption for component time-to-failure. If *iid*, this hypothesis test is valuable to test the suitability of the exponential distribution. However, the *iid* assumption is often violated in practice. Different plants or geographical

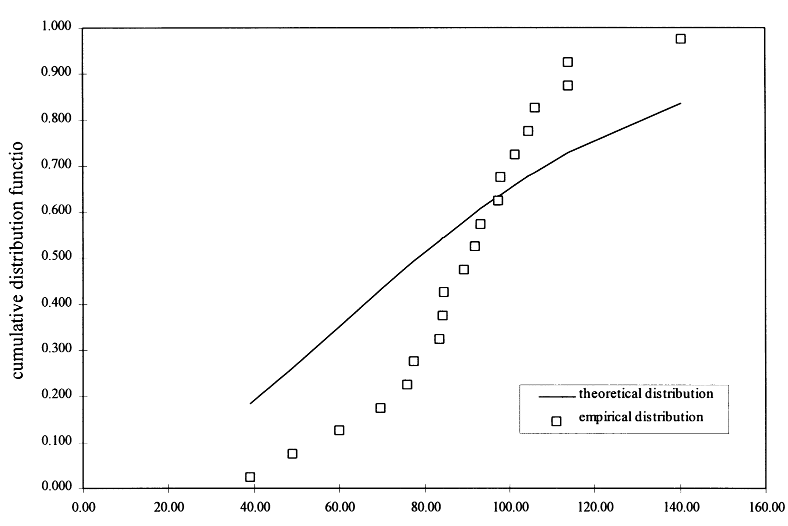

Fig. 3. Results from Weibull test set.

Table 3
Exponential test set hypothesis test results

$[a_{i-1},a_i)$	o_i	$\theta = E(X)$		$\theta = \tilde{\theta}$	
		e_i	$\frac{(o_i - e_i)^2}{e_i}$	e_i	$\frac{(o_i - e_i)^2}{e_i}$
[0,55)	4	5.328	0.331	4.834	0.144
[55,110)	6	7.604	0.338	7.307	0.234
[110,∞)	10	7.068	1.216	7.859	0.583
		$X^2 = 1.885$		$X^2 = 0.961$	
		$\chi^2_{0.01,2} = 9.210$		$\chi^2_{0.01,1} = 6.637$	
		$\chi^2_{0.1,2} = 4.605$		$\chi^2_{0.1,1} = 2.706$	

Table 4
Weibull test set hypothesis test results

$[a_{i-1},a_i)$	o_i	$\theta = E(X)$		$\theta = \tilde{\theta}$	
		e_i	$\frac{(o_i - e_i)^2}{e_i}$	e_i	$\frac{(o_i - e_i)^2}{e_i}$
[0,55)	2	6.304	2.939	6.19	2.840
[55,110)	15	7.982	6.171	7.952	6.247
[110,∞)	3	5.714	1.289	5.854	1.391
		$X^2 = 10.399$		$X^2 = 10.478$	
		$\chi^2_{0.01,2} = 9.210$		$\chi^2_{0.01,1} = 6.637$	
		$\chi^2_{0.1,2} = 4.605$		$\chi^2_{0.1,1} = 2.706$	

locations may result in unique distributions that are not identically distributed. In this case, the data conditions imposed in this paper cannot be met.

The hypothesis test could have alternatively been formulated to test the *iid* condition with an underlying exponential distribution assumption. The test would then be conducted as described. In practice, a user may be unclear about the appropriateness of these assumptions. However, any time there is a reject decision from this hypothesis test, the user must not compute a combined exponential distribution parameter. It may not be known which specific condition or assumption is violated but this still represents an important tool for analysts with multiple merged data records.

References

[1] Dey KA. Statistical analysis of noisy and incomplete failure data. In: Proceedings Annual Reliability and Maintainability Symposium (RAMS). New York: IEEE, 1982:93–97.

[2] Usher JS. Weibull component reliability-prediction in the presence of masked data. IEEE Transactions on Reliability 1996;45:229–232.

[3] Lin DKJ, Usher JS, Guess FM. Bayes estimation of component from masked system-life data. IEEE Transactions on Reliability 1996;45:233–237.

[4] Wang CJ. Graphical analysis of ill-collected interval data for a repairable system in vehicles. In: Proceedings Annual Reliability and Maintainability Symposium (RAMS). New York: IEEE, January 1991:93–97.

[5] Reliability Analysis Center. Electronic parts reliability data. RAC Report EPRD, Rome, NY. 1997.

[6] Coit DW, Dey KA, Turkowski W. Practical reliability data and analysis. Reliability Engineering 1986;14(1):1–18.

[7] Mann NR, Shafer RE, Singpurwalla ND. Methods for statistical analysis of reliability and life data. New York; John Wiley and Sons, 1974.

[8] Barlow RE, Proschan F. Mathematical theory of reliability. New York: John Wiley and Sons, 1965.

[9] Kendall MG, Stuart A. The advanced theory of statistics, vol. 2. London: Griffen, 1967:Ch. 30.

[10] Lawless JF. Statistical models and methods for lifetime data. New York: John Wiley and Sons, 1982.