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New insights on multi-state component criticality and importance

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Abstract

In this paper, new importance measures for multi-state systems with multi-state components are introduced and evaluated. These new measures complement and enhance current work done in the area of multi-state reliability. In general, importance measures are used to evaluate and rank the criticality of component or component states with respect to system reliability. The focus of the study is to provide intuitive and clear importance measures that can be used to enhance system reliability from two perspectives: (1) how a specific *component* affects multi-state system reliability and (2) how a particular component *state* or set of states affects multi-state system reliability. The first measure unsatisfied demand index, provides insight regarding a component or component state contribution to unsatisfied demand. The second measure multi-state failure frequency index, elaborates on an approach that quantifies the contribution of a particular component or component state to system failure. Finally, the multi-state redundancy importance identifies where to allocate component redundancy as to improve system reliability. The findings of this study indicate that both perspectives can be used to complement each other and as an effective tool to assess component criticality. Examples illustrate and compare the proposed measures with previous multi-state importance measures.

Keywords: Reliability; Multi-state system; Component criticality; Importance measures

1. Introduction

Traditionally, system reliability has been analyzed from a binary perspective assuming the system and its components can be in either of two states: completely functioning or failed. However, many systems that provide basic services, such as telecommunications, gas and oil production, transportation and electric power distribution, operate at various levels of performance as opposed to the binary perspective. These types of systems may provide a service or function at degraded component performance levels. Therefore, it is essential to model and analyze them accordingly. For these systems, multi-state system reliability methods have been proposed as a more appropriate modeling and computational approach. Currently, most reliability work on multi-state systems has focused on two cases: (1) multi-state systems with binary capacitated components [1–4] where in general, the system has to fulfill a number of different demands during a specified time interval assuming the components can either work at a nominal capacity level or not work at all, and (2) multi-state systems with multi-state components (MSMC) where in general, the system has to fulfill a known demand based on the different component performance states [5–13]. This paper deals with systems following the second case.

In reliability theory, importance measures (IM) have been recognized to provide critical information regarding the impact components have in system reliability [14–22]. IM are essential in determining and explaining the effects of components on the overall reliability of a system. For systems exhibiting binary behavior, IM have enabled engineers to determine system configuration improvements and ultimately cost effective methods to maintain high levels of system reliability. For the binary case, a variety of IM have been proposed and are in existence today. Amongst these, Vasseur and Llory [21] recognize reliability achievement worth (RAW), reliability reduction worth (RRW), Fussell–Veseley (FV) and Birnbaum as the most widely used in industry. For the binary case, system components can be ranked with respect to the impact they have on system reliability based on a given importance measure.

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Most research in the area of multi-state IM has been confined to the case of identifying the most important component performance level or set of levels with respect to multi-state reliability [23–27]. However, there may be cases when it is necessary to identify the most important component in a MSMC. In this respect, Ramirez-Marquez and Coit [28] have proposed that for multi-state systems, IM can be divided into two main categories:

Type 1 Quantify the impact of a component as a whole on system reliability [28– 31].
Type 2 Quantify how a particular component state or set of states affect system reliability [23– 27].

The purpose of this research is to further advance the knowledge of IM for MSMC by proposing new methods to quantify component criticality. Each of the proposed IM captures two perspectives of criticality as discussed in Ramirez-Marquez and Coit [28]. The first of these new measures, unsatisfied demand index (UDI), identifies the impact of a component or component state in terms of unsatisfied demand for the multi-state system. The second IM, multi-state failure frequency index (MFFI), quantifies what percentage of system failure can be attributed to a specific component or component state. Finally, the multi-state redundancy importance (MRI) quantifies the increase in reliability when a redundant component is added to the system. Following the categorization of criticality measures based on their application insights presented in Ramirez-Marquez and Coit [28], the first two of these new IM can be thought of as risk-averse measures while the remaining measure can be thought of as a reliability-potential measure. While most studies have focused on extending frequently used binary IM to the multi-state case, this paper introduces three new measures that complement the tools available to assess component criticality in MSMC.

The remainder of the paper is organized as follows: Section 2 describes the general MSMC reliability problem and presents previous work regarding IM for multi-state systems, Section 3 elaborates on the new concepts proposed for calculating the criticality of components in multi-state systems and Section 4 shows the results obtained by using the proposed importance measures in two test examples. Finally, Section 5 provides conclusions on the findings.

Assumptions

(i) Components and component states are statistically independent.
(ii) Improvements of component states cannot damage the system.
(iii) Component states and associated probabilities are known.

Notation

ϕ system structure function
X system state vector, $\mathbf{x}=(x_1, x_2,\ldots,x_{|A|})$
x_i current state of component i
A set of system components
d system demand
M_i maximum capacity for component i
ω_i number of states for component i
$\mathbf{b}_i$ state space vector for component i
I_i binary Birnbaum Importance Measure
p_{ij} $P(x_i=b_{ij})$
U unsatisfied demand, a random variable
ρV_h density of MMCV_h

Acronyms

MAD mean absolute deviation
IM importance measures
MMFV multi-state mean Fussell– Vesely
MMCV multi-state minimal cut vector
MMPV multi-state minimal path vector
MR_d multi-state reliability at level d
MSMC multi-state system with multi-state components
MMAW mean multi-state achievement worth
MFFI multi-state failure frequency index
UDI unsatisfied demand index
MRI multi-state redundancy index
RAW reliability achievement worth
RRW reliability reduction worth
FV Fussell– Vesely

2. Background

2.1. Multi-state system reliability modeling

Let $A=\{1,\ldots,|A|\}$ represent the set of components for a stochastic capacitated system. The current state (capacity) of component i, is represented by $x_i \varepsilon \mathbf{b}_i$ where $\mathbf{b}_i$ represents component i state space vector. Therefore, x_i, takes values $b_{i1}=0, b_{i2},\ldots,b_{i\omega i}=M_i$, where $b_i\in\Re^+$, M_i equals the nominal capacity of component i and ω_i equals its number of states. The vector $\mathbf{p}_i$ represents the probability associated to each of the values taken by x_i. The system state vector $\mathbf{x}=(x_1, x_2,\ldots,x_{|A|})$ denotes the current state of all the components in the system. The function ϕ: $\Re^{|A|}\rightarrow\Re^+$ is the multi-state structure function. It maps the system state vector into a system state. That is, $\phi(\mathbf{x})$ is the system capacity under system state vector **x**. For MSMC with constant demand, d, multi-state reliability is given by,

$$\text{MR}_d = P\big(\varphi(\mathbf{x})\geq d\big)$$

Although this definition of reliability for MSMC will be used throughout the paper, it is important to mention that in some cases reliability may be measured based on $\varphi(\mathbf{x})\leq d$. As an example consider the case where a MSMC has to process some information (through multi-state components) within some time limit. For this case, $\text{MR}_d = P\big(\varphi(\mathbf{x})\leq d\big)$. Thus, in general, for MSMC reliability can be defined as

$\text{MR}_d = P\big(f(\varphi(\mathbf{x}), d) \geq 0\big)$, where $f\big(\varphi(\mathbf{x}), d\big)$expresses the desired relationship between system performance and demand.

2.2. Multi-state reliability evaluation

IM are developed by assuming specific cases of component reliability behavior and then quantifying the impact of this behavior at the system level. For example, RAW assumes a system component is perfectly reliable and then quantifies the increase in system reliability that such an assumption provides. That is, for the computation of IM it is necessary to obtain system reliability through efficient computational approaches.

The last decade has seen the development of methods for the exact computation of MSMC reliability. These methods can be roughly divided into four major computational techniques: MMPV, MMCV, rule generation methods and universal generating function.

The first technique focuses on identifying the system's multi-state minimal path vectors; the multi-state equivalent of minimal path sets. Lin [7] proposed a method that is based on the a priori knowledge of the system minimal path sets. These sets are used to develop a set of inequalities that are solved to provide the MMPV. The approach is only valid for systems where component states are consecutive (i.e. of the form $b_{i,j+1} = b_{ij} + 1$). Ramirez-Marquez et al. [13] presented a method for computing MMPV without the restrictions of consecutive component states nor a priori knowledge of the system binary path sets. This method provides MMPV by iteratively analyzing system component successors and inheriting potential component states that guarantee system success. The method uses a network decomposition approach similar to the one presented in Ramirez-Marquez and Coit [9].

The second technique includes methods that compute multi-state minimal cut vectors; the multi-state equivalent of minimal cut sets. Lin [8] and Yeh [10] have made significant contributions by developing reduced implicit enumeration approaches for finding MMCV. Both methods depend on the a priori knowledge of the system minimal cut sets and can only be applied to systems where the components have consecutive states. This research encouraged Ramirez-Marquez et al. [12] to develop an information sharing approach that significantly reduces the number of implicit enumerations necessary to obtain the MMCV. The rationale of this approach is that since all MMCV can be obtained from the set containing all minimal cuts [8,10], a select number of MMCV called offspring cuts can inherit information from a select number of MMCV called parent cuts, therefore reducing the number of implicit enumerations.

The third category includes simulation approaches developed to generate decision rules. In this area, Rocco and Muselli [32] presented a machine-learning-based method to develop a new algorithm, based on the procedure of Hamming Clustering, which is capable of considering multi-state systems and any success criterion. The main idea is to employ a classification technique, trained on a restricted subset of data, to produce an estimate of the reliability expression, which provides reasonably accurate values of the reliability and the MMCV.

In the last category, Levitin et al. [1,2] proposed a procedure based on the universal generating function to compute MR_d. This approach has been used in multi-state systems with binary capacitated components for optimization purposes. It requires relatively small computation time yet no information regarding MMCV or MMPV can be obtained. The universal generating function method is detailed by Levitin [33,34].

2.3. Importance measures in MSMC

As proposed by Ramirez-Marquez and Coit [28] multi-state IM can be classified into two major categories: Type 1 and Type 2 IM. It must be noted that Type 1 and Type 2 measures are fundamentally different and can be used for different reasons and different applications.

The initial focus in multi-state IM research has been the analysis of Type 2 measures. These IM evaluate how a particular state or set of states of a specific component affect multi-state reliability. They can identify critical states enabling engineers to allocate resources properly. However, given that Type 2 mainly pertains to the effects of individual states, one cannot conclusively or simply deduct the criticality of the component as a whole in the system. For some scenarios existing Type 2 measures may very well be the most appropriate. For example, when system components wear out and gradual degradation happens it may be more important for such systems to find out the effect of a certain state of a component (Type 2) on system reliability than knowing which component is more critical via a more general measure that considers all possible states at once (Type 1).

Among the different Type 2 IM proposed, El-Neweihi et al. [23] presented theoretical foundations between multi-state system reliability behavior and multi-state component performance. Barlow and Wu [24] characterized component state criticality as a measure of how a particular component state affects a specific system state. Zio and Podofillini [26] used Monte Carlo simulation to present multi-state extensions for RAW, RRW, FV and Birnbaum. Similarly, Levitin et al. [27] proposed extensions of binary IM via the universal generating function method.

Although Type 2 measures can provide useful information regarding multi-state system reliability, there are other multi-state applications and problems where Type 1 measures are preferable. Some existing multi-state systems are old and complex, yet budgets to upgrade and replace old components are significantly limited. These systems require importance measures to prioritize components (for an electrical power distribution system these may be lines, buses, transformers, etc.) that have the most impact on system reliability. Type 1 measures meet this need. In general, Type 1 IM assess how a specific component affects multi-state system reliability. Thus, they can be particularly useful in instances where one is solely interested in which component has the most significant impact on the system reliability. As noted by Wu and Chan [30], from a multi-state system configuration viewpoint, IM are essential to quantify the impact of a system component as a whole.

With respect to Type 1 measures, Ramirez-Marquez and Coit [28] developed two sets of extensions of binary IM to the multi-state case. The first set included direct extensions of Birnbaum, RAW and FV. This set applies to multi-state systems but these extensions only consider the possible state levels and not the probability of a component being in that state. Thus, the second set recognized the need to incorporate state probabilities into the computation of multi-state reliability importance. Similarly, Aven and Ostebo [31] developed two Type 1 IM that can assist in quantifying: (1) the difference between system reliability assuming a component has infinite capacity and system reliability considering the component's true capacities, and (2) the level of degradation a component suffers before the system demand is not met. Finally, Wu and Chan [30] extended Griffith's [25] importance vector to a Type 1 measure. The extension follows the definition of a utility function that can differentiate which components contribute to multi-state system reliability.

3. IM development

Current multi-state IM [23–31] fall into one of three different categories: (1) reliability-potential, (2) risk-averse, and (3) risk-neutral measures. The purpose of the proposed IM is to complement the different categories so that regardless of the application, component impact can be assessed and system performance improved.

3.1. Unsatisfied demand index (UDI)

Aven and Ostebo [31] developed two Type 1 IM focused on identifying component criticality from a capacity perspective. Both of these measures assist in cases where the emphasis is on improving the system. Thus, they can be included in the category of reliability-potential measures. UDI has been developed following the opposite rationale of Aven and Ostebo [31]. In general, UDI is concerned with assigning component responsibility for unsatisfied demand. Such a measure is important in MSMC because for these systems, reliability is regarded as a mixture of system connectivity and capacity. Thus, for some applications it may be of interest to know how a component or the states of a component affect demand satisfaction. If a Type 2 measure is considered, UDI provides the expected unsatisfied demand due to state b_{ij}. The extension to a Type 1 IM can be explained as the average expected unsatisfied demand due to component i. It is important to note that for both types of IM, UDI is specific to each system demand

From a general perspective UDI can be considered in the category of risk averse measures. It must be noted that for both Type 1 and 2 measures, UDI can be linked to an estimate of lost revenue caused by a component or a particular state. The mathematical expression for this measure is as follows:

Type 1:

$$\mathrm{UDI}_k = \frac{1}{\omega_k}\sum_j \mathrm{UDI}_{kj} \tag{1}$$

Type 2:

$$\mathrm{UDI}_{kj} = E[U|x_k = b_{kj}] \tag{2}$$

U in Eq. (2) is defined as unsatisfied demand. The rhs of this expression can be re-written as:

$$\mathrm{UDI}_{kj} = E[U|x_k = b_{kj}] = \sum_{u>0}^{d} uP(U = u|x_k = b_{kj})$$
$$= \sum_{w\in W}(d-w)P\big(\varphi(x) = w|x_k = b_{kj}\big)$$

where the set W is given by: $W=\{w|\varphi(\mathbf{x})=w \wedge w \leq d\}$

UDI may require extensive computational effort because of the need to obtain $P\big(\varphi(x)=d-u|x_k=b_{kj}\big)$ for all different values of u.

3.2. Multistate failure frequency index (MFFI)

The purpose of this IM is to explain system failure, $\phi(\mathbf{x})<d$, in terms of component degradation. From a general perspective, the value of MFFI provides an indication of how likely a component is to cause system failure. In this respect, MFFI can be regarded as a risk averse measure. Currently, risk averse importance measures for MSMC make assumptions regarding component behavior in order to obtain a value of component criticality. For example, in the multi-state case RRW is computed by looking at a function of MSMC reliability under the assumption that a specific system component is degraded. These indices provide valuable information on how reliability degrades as components degrade. However, they do not provide a direct measure of how the stochastic nature of the component interacts with system failure.

For the binary state case, Wang et al. [22] proposed a component failure frequency index that is generated by simulating the availability of a system and assessing how failures of components impacted system failure. Likewise, Butler [15,16] developed an IM based on cut set information that generates a lexicographic ordering of components based on the number of cut sets that include a specific component. Thus, following these ideas MFFI uses information regarding the system design and the stochastic nature of component behavior to develop a component failure frequency index that captures system failure information in MSMC.

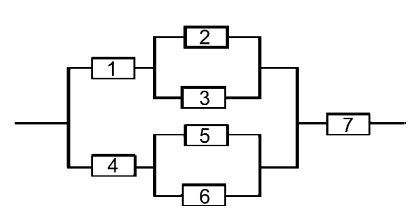

Fig. 1. Levitin et al. [27] multi-state system.

Table 1
Component states and associated probabilities

I	States					State occupancy probabilities				
	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	p_{i1}	p_{i2}	p_{i3}	p_{i4}	p_{i5}
1	0	1	2	3	4	0.1	0.05	0.15	0.35	0.35
2	0	1	2			0.1	0.05	0.85		
3	0	1	2			0.1	0.05	0.85		
4	0	1	2	3		0.2	0.1	0.45	0.25	
5	0	1	2			0.1	0.05	0.85		
6	0	1	2			0.1	0.05	0.85		
7	0	1	2	3	4	0.15	0.15	0.05	0.45	0.2

Table 2
Example 1 MMCV at level 3

MMCV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	MMCV	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	4	2	2	3	2	2	2	14	0	2	2	3	0	2	4
2	4	2	0	3	0	0	4	15	1	2	2	3	1	0	4
3	4	0	2	3	0	0	4	16	1	2	2	3	0	1	4
4	4	0	0	3	2	0	4	17	0	2	2	3	1	1	4
5	4	0	0	3	0	2	4	18	4	2	0	0	2	2	4
6	4	1	1	3	0	0	4	19	4	0	2	0	2	2	4
7	4	1	0	3	1	0	4	20	4	0	0	2	2	2	4
8	4	1	0	3	0	1	4	21	4	1	0	1	2	2	4
9	4	0	1	3	1	0	4	22	4	0	1	1	2	2	4
10	4	0	1	3	0	1	4	23	4	1	1	0	2	2	4
11	4	0	0	3	1	1	4	24	2	2	2	0	2	2	4
12	2	2	2	3	0	0	4	25	1	2	2	1	2	2	4
13	0	2	2	3	2	0	4	26	0	2	2	2	2	2	4

MFFI is constructed by noting that the MMCV at demand level d provide a concise summary of the MSMC design. These MMCV have innate information regarding how component behavior affects system failure. Moreover, if the component state probabilities are known then the MMCV can be used to provide the reliability of the MSMC. It should be noted that for both Type 1 and Type 2 IM, MFFI depends on the a priori computation of MMCV. Lin [8], Yeh [10] and, Ramirez-Marquez et al. [12] have developed algorithms that provide these vectors. However, it should be mentioned that these approaches rely on the knowledge of the system minimal cut sets, which is an NP-hard problem.

Each component in the MSMC can be characterized by x_i. If all MMCV at some demand level d are known then MSMC reliability can be completely characterized by Eq. (3):

$$\mathrm{MR}_d = 1 - \sum_{h=1}^{L} P(\mathbf{x} \le \mathrm{MMCV}_h) + \sum_{h<k} P(\mathbf{x} \le \mathrm{MMCV}_h \wedge \mathbf{x} \le \mathrm{MMCV}_k) + \ldots + (-1)^L P(\mathbf{x} \le \mathrm{MMCV}_h \wedge \mathbf{x} \le \mathrm{MMCV}_k \wedge \ldots \wedge \mathrm{MMCV}_L) \quad (3)$$

where, L = number of MMCV, MMCV_h is the h-th MMCV and $P(\mathbf{x} \le \mathrm{MMCV}_h) = \prod_{i=1}^{m} P(x_i \le y_i) \forall \; x_i$ (the i-th entry of vector $\mathbf{x}$) and y_i (the i-th entry of vector $\mathbf{y}$), $\mathbf{y} = \mathrm{MMCV}_h$.

By following the rationale of Butler [15,16] and Wang et al. [22] the density of each MMCV can be obtained. This density provides an approximation of the proportion of failures generated by a specific MMCV. That is, the density for MMCV_h, called ρV_h, can be calculated as follows:

Table 3
Example 1 Type 1 IM and rankings

Rank	i	MFFI	i	UDI		i	MRI	
				Average	Variance[a]		Average	Variance[a]
1	7	0.55902	7	1.31011	1.04663	7	0.20586	1.89641
2	4	0.37294	1	1.09668	8.2080	1	0.06361	3.09878
3	1	0.33501	2	1.00036	11.9529	4	0.06264	2.42150
4	2	0.07243	3	1.00002	11.6138	3	0.01090	2.77777
5	3	0.07243	4	0.99392	10.4301	2	0.01084	2.56988
6	5	0.03849	6	0.97039	12.6968	6	0.00331	1.60824
7	6	0.03849	5	0.97039	14.1908	5	0.00324	1.51716

[a] E−06.

Table 4
Example 1 Type 2 IM and rankings

i	State	MFFI	Rank	UDI	Rank	MRI	Rank
1	0	0.15533	3	1.71074	3	0	20
	1	0.07906	7	1.14842	5	0.04801	10
	2	0.10062	5	0.94123	18	0.05986	9
	3	0	16	0.83304	22	0.07072	6
	4	0		0.83304	22	0.07072	6
2	0	0.05698	8	1.09647	7	0	20
	1	0.01545	12	0.95676	12	0.00927	15
	2	0		0.93297	19	0.01024	13
3	0	0.05698	8	1.09653	6	0	20
	1	0.01545	13	0.9557	13	0.00955	14
	2	0		0.93263	20	0.0106	12
4	0	0.16691	2	1.28703	4	0	20
	1	0.08625	6	0.98227	11	0.06222	8
	2	0.13177	4	0.87182	21	0.07503	5
	3	0		0.8153	24	0.08584	4
5	0	0.03023	10	0.99528	10	0	20
	1	0.00827	14	0.95282	15	0.00209	18
	2	0		0.94472	17	0.00233	16
6	0	0.03023	10	0.99695	9	0	20
	1	0.00827	14	0.95374	14	0.00178	19
	2	0		0.94482	16	0.00219	17
7	0	0	16	3	1	0	20
	1	0	16	2.02192	2	0.04243	11
	2	0.55902	1	1.06668	8	0.16744	3
	3	0	16	0.22561	25	0.29445	1
	4	0		0.22561	25	0.29445	1

$$\rho V_h = \frac{P(\mathbf{x} \leq \mathrm{MMCV}_h)}{\sum_i P(\mathbf{x} \leq \mathrm{MMCV}_i)} \quad (4)$$

Note that in Eq. (4): $\sum_i P(\mathbf{x} \leq \mathrm{MMCV}_i) \neq 1 - \mathrm{MR}_d$

Once the density of each MMCV is calculated, Type 1 and 2 IM can be constructed. As a Type 1 IM, MFFI_k can be used to approximate the system failure frequency due to failure of a particular component. Alternatively as a Type 2 IM, MFFI_{kj} provides information about the component state that causes the majority of system failures. The expression for Type 1 and Type 2 MFFI are given as follows:

Type 1:

$$\mathrm{MFFI}_k = \sum_h \rho V_h \; \forall \; h \text{ such that } x_k < \max\{b_{kj}\} \text{in } \mathrm{MMCV}_h \quad (5)$$

Type 2:

$$\mathrm{MFFI}_{kj} = \begin{cases} \sum_h \rho V_h \; \forall h \text{ such that } x_k = b_{kj} \text{ in } \mathrm{MMCV}_h \text{ and } j \neq \omega_k \\ 0 \qquad \text{otherwise} \end{cases} \quad (6)$$

It is important to note that ρV_h is itself a criticality index although not in the traditional form of IM. This index provides information regarding the set of component states that have the highest impact in system failure. Moreover, the index can be regarded as composite index of components and used as a guide for 'safeguarding' the system from failures.

3.3. Multistate redundancy importance (MRI)

This measure provides information regarding component potential for improvement. It can be viewed as an indirect

Table 5
Ramirez-Marquez and Coit [28] alternative Type 1 IM rankings

Rank	Comp.	MAD	Comp.	MMAW	Comp.	MMFV
1	7	0.3822655	7	1.347437511	7	0.35
2	1	0.1029915	1	1.093039591	1	0.094866813
3	4	0.072286	4	1.06534574	4	0.066538953
4	3	0.021842	3	1.019695311	3	0.020155081
5	2	0.0214055	2	1.01912516	2	0.019928845
6	5	0.0060245	5	1.005503558	6	0.005561941
7	6	0.0059725	6	1.005334793	5	0.00548805

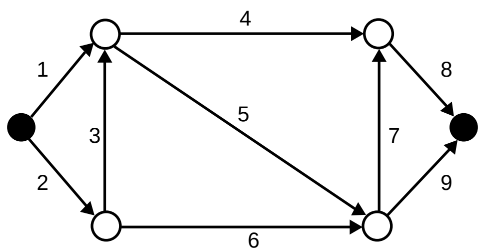

Fig. 2. Two-terminal multi-state network.

multi-state extension of RAW in binary systems. As such, it belongs to the category of reliability potential measures. MRI has been defined using the rationale followed by Boland et al. [17]. For constructing this IM it is assumed that a spare component is at hand for each component in the MSMC. Moreover, this spare has the same reliability and states as its copy in the system. Based on these assumptions, MRI measures the improvement in MSMC reliability that is achieved by allocating redundancy in component i.

Similar to UDI, for both Type 1 and 2, MRI can be linked to an estimate of increased profit caused by adding redundancy to a component or improving a particular state. The mathematical expression for this measure is as follows:

Type 1:

$$\mathrm{MRI}_k = P\big(\varphi(\mathbf{x}, x_k^+) \geq d\big) - P\big(\varphi(\mathbf{x}) \geq d\big) \tag{7}$$

$P(\varphi(\mathbf{x}, x_k^+) \geq d)$ is defined as the probability associated with the event that the capacity of the system is greater than or equal to demand d when a copy of component k has been added to the system design.

Type 2:

$$\mathrm{MRI}_{kj} = P\big(\varphi(\mathbf{x}, x_k^+) \geq d | x_k = b_{kj}\big) - P\big(\varphi(\mathbf{x}) \geq d | x_k = b_{kj}\big) \tag{8}$$

$\mathrm{MRI}_k = P(\varphi(\mathbf{x}, x_k^+) \geq d | x_k = b_{kj})$is defined as the probability associated with the event that the capacity of the system is greater than or equal to demand d when component k^+ has been added to the system design and the current state of component k equals b_{kj}, $x_k = b_{kj}$. Notice that only component k is fixed to state b_{kj} while component k^+ preserves its stochastic behavior.

If either the MMPV or MMCV of the MSMC are readily at hand then they can be transformed in a relatively simple form to allow for the computation of $P(\varphi(\mathbf{x}, x_k^+) \geq d)$or $P(\varphi(\mathbf{x}, x_k^+) \geq d | x_k = b_{kj})$. For every MMCV/MMPV that includes component k not at its highest/lowest state, the transformed vectors will include the state of component k^+ dictated as a function of the current state of component k. The remaining MMCV/MMPV (component k at its highest/lowest state) will include component k^+ at it highest/lowest state since the state of component k is irrelevant in MSMC failure/success.

4. Computational examples

4.1. Example 1

Levitin et al. [27] analyzed the system depicted in Fig. 1 with the universal function method. For this example, the system is assumed to be working if the capacity of the network between source and sink nodes is greater than or equal to a demand of 3 units. Table 1 presents component states and their respective state occupancy probabilities. Table 2 presents MMCV associated to the network. Based on the data presented in these tables the approach presented in Ramirez-Marquez and Coit [11] was used to obtain an approximation of system reliability. The approximation yielded a value of $\mathrm{MR}_d =$ 0.5481. Moreover, the simulation approach was implemented to approximate the value of the different measures presented in Section 3. Table 3 presents the ranking relevant to each proposed IM from a Type 1 perspective. Given that the simulation approach introduces variability, Table 3 presents the average and associated variance for both UDI and MRI obtained from 10 different experiments with 100,000 runs each. Table 4 introduces component state rankings based on Type 2 measures. Finally, Table 5 presents component rankings obtained with the criticality measures obtained by Ramirez-Marquez and Coit [28] applied to this example.

It should be mentioned that the proposed IM have intuitive and straightforward insights. For example, if the values associated to each component MFFI were normalized then one could roughly regard this measure as an approximation to the percentage of system failures caused by a particular component. Following this rationale, from the results presented

Table 6
Component states and associated probability

	States				State occupancy probabilities			
I	b_{i1}	b_{i2}	b_{i3}	b_{i4}	p_{i1}	p_{i2}	p_{i3}	p_{i4}
1	0	1	2	3	0.005	0.005	0.01	0.98
2	0	1	2	3	0.02	0.01	0.015	0.955
3	0	1			0.02	0.98		
4	0	1	2		0.01	0.015	0.975	
5	0	1			0.02	0.98		
6	0	1	2		0.005	0.02	0.975	
7	0	1			0.01	0.99		
8	0	1	2	3	0.01	0.015	0.005	0.97
9	0	1	2	3	0.02	0.01	0.01	0.96

Table 7
Example 2 MMCV

MMCV	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	MMCV	x_1	x_2	X_3	x_4	x_5	x_6	x_7	X_8	x_9
1	3	3	1	2	1	2	1	0	3	13	3	3	1	2	0	1	1	3	3
2	3	3	1	2	1	2	1	1	2	14	3	3	1	1	1	1	1	3	3
3	3	3	1	2	1	2	1	2	1	15	3	1	1	2	1	2	1	1	3
4	3	3	1	2	1	2	1	3	0	16	1	3	0	2	1	2	1	3	3
5	3	3	1	0	1	2	1	3	3	17	3	1	1	2	0	2	1	3	3
6	3	3	1	1	1	2	0	3	2	18	3	1	1	1	1	2	1	3	3
7	3	3	1	2	0	2	1	1	3	19	3	0	1	2	1	2	1	3	3
8	3	3	1	2	1	1	1	1	3	20	0	3	1	2	1	2	1	3	3
9	3	3	1	2	1	2	0	3	1	21	2	1	1	2	1	2	1	3	3
10	3	3	1	1	1	2	1	3	1	22	1	2	1	2	1	2	1	3	3
11	3	3	1	1	0	2	1	3	3	23	2	3	0	2	1	1	1	3	3
12	3	3	1	2	1	0	1	3	3	24	1	3	1	2	1	1	1	3	3
										25	1	3	1	2	1	2	0	3	2

in Table 3, it can be said that almost 37.54% of system failures can be associated with component 7 while components 4 and 1 together account for approximately 47.55% and the remaining components accounting for roughly 15%. These results are intuitively more understandable that those obtained from MMFV in Table 5. As discussed by Ramirez-Marquez and Coit [28], this measure can help in identifying the multi-state component that provides the largest decrease on multi-state system reliability. Although the rankings for both measures agree (as is expected since, they have similar intuitive explanation), MMFV cannot assist in providing a rough estimate of system failure due to a particular component unless such component is connected in series within the system structure (component 7 in this example). In such cases, MMFV provides the exact value of a component's contribution to system failure.

A similar interpretation can be provided for MRI where the value obtained shows the actual increase in reliability when enhancing the system with an identical copy of component *i*. In this sense, Table 3 shows that by adding a redundant copy of component 7, system reliability would be improved by 0.20586 (i.e. an increase of 37.55% in reliability). On the other hand, when considering MMAW in Table 5, the explanation is that an expected 34.74% increase in reliability would be obtained if component 7 only resided in states that improve the current value of reliability. Intuitively, MMAW [28] identifies the multi-state component that has the greatest potential to improve multi-state system reliability when residing in its 'good' states.

Finally, UDI provides dissimilar rankings as those obtained with the other IM. In fact, it is interesting to note that this UDI provides evidence that the ranking obtained with Type 2 measures may not identify or agree with the most important component obtained with Type 1 measures. Consider component 4 in Table 4; when this component is failed (state 0) the expected unsatisfied demand equals 1.28703, and thus, based on this value it could be intuitively ranked as the 3rd most critical component after component 3 and 1. However, when considering UDI as a Type 1 measure, component 4 ranks in the 5th place in terms of criticality. As a Type 2 measure, UDI identifies states that may be tolerated in case a component degrades. Consider component 6, as degradation occurs UDI Type 2 remains roughly constant. However, for component 7 system degradation rapidly occurs as the states of the component change. Thus one would be more interested in maintaining high performance levels for component 7.

4.2. Example 2

The second example considers the network analyzed by Ramirez-Marquez et al. [12] depicted in Fig. 2. This network must supply a demand of 3 units from source to sink. Component states and associated probabilities are displayed in Table 6. For this component data $MR_d = 0.92911$. To obtain multi-state reliability the MMCV at a demand level $d = 3$ units, shown in Table 7, were obtained, and then, used with the Monte-Carlo simulation approach presented by Ramirez-Marquez and Coit [11]. Tables 8 and 9 present the results

Table 8
Type 1 IM and rankings

Rank	Arc	MFFI	Arc	UDI	Arc	MRI
1	2	0.29449	4	0.45849	2	0.01990
2	9	0.29213	6	0.43914	9	0.01909
3	8	0.17523	8	0.38578	8	0.01045
4	4	0.16073	1	0.37962	4	0.01027
5	6	0.08917	9	0.34682	1	0.00644
6	1	0.08286	2	0.34651	6	0.00590
7	5	0.02671	5	0.12429	5	0.00148
8	7	0.00399	7	0.09217	3	0.00036
9	3	0.00267	3	0.08255	7	0.00025

Table 9
Type 2 IM and rankings

Arc	State	MFFI	Rank	UDI	Rank	MRI	Rank
1	0	0.06361	5	1.16305	2	0	20
	1	0.01150	14	0.18331	8	0.00606	14
	2	0.00776	16	0.10033	15	0.0064	13
	3	0.00000		0.07180	22	0.00644	12
2	0	0.25442	1	1.12139	4	0	20
	1	0.03435	8	0.14571	11	0.01884	5
	2	0.00572	17	0.06411	26	0.02016	2
	3	0.00000		0.05484	28	0.02025	1
3	0	0.00267	19	0.08736	17	0	20
	1	0.00000		0.07774	19	0.00037	18
4	0	0.12721	3	1.14711	3	0	20
	1	0.03352	9	0.16251	10	0.00989	10
	2	0.00000		0.06586	24	0.01036	9
5	0	0.02671	10	0.17265	9	0	20
	1	0.00000		0.07593	21	0.0015	17
6	0	0.06361	5	1.10313	6	0	20
	1	0.02557	11	0.14287	12	0.00565	16
	2	0.00000		0.07143	23	0.00593	15
7	0	0.00399	18	0.10680	14	0	20
	1	0.00000		0.07754	20	0.00025	19
8	0	0.12721	3	1.18569	1	0	20
	1	0.03657	7	0.19951	7	0.00971	11
	2	0.01145	15	0.09325	16	0.01043	8
	3	0.00000		0.06466	25	0.01051	7
9	0	0.25442	1	1.11357	5	0	20
	1	0.02481	12	0.13895	13	0.01844	6
	2	0.01290	13	0.07889	18	0.0191	4
	3	0.00000		0.05585	27	0.01945	3

obtained when obtaining Type 1 and Type 2 IM, respectively. Finally, Table 10 provides criticality measures with the alternative IM presented in Ramirez-Marquez and Coit [28].

This example provides evidence regarding the complementary nature of Type 1 and 2 measures. When a Type 2 measure is considered for MFFI there is a tie between components 2 and 9 for the most important component state. In this case one would like to know which of these two components to improve, and thus, Type 1 measures would aid by illustrating that from a system-wide perspective component 2 has a higher contribution to system failure.

For these components, the same rationale can also be applied to MRI. Special attention should be given to the results obtained for UDI since the rankings greatly differ with other measures. As a Type 1 measure, UDI shows that on average, the states of component 4 contribute to unsatisfied demand in the most significant way. However, from a Type 2 perspective, the highest index is associated with complete failure of component 8. Notice that it may also be of interest to obtain information regarding the component state that has the smallest UDI. For this case if component 9 is at its highest state, the expected demand not supplied equals 0.05585.

5. Conclusion

This paper introduced three new multi-state IM that complement existing work done in the area. The expressions introduced, assist in analyzing MSMC from two perspectives: (1) obtaining the most important component, and (2) obtaining the most important state. As evidenced by the results the most important state may not always correspond with the most important component. Moreover, it is shown that both perspectives can be used to complement each other, and thus, the necessity to provide both Type 1 and Type 2 IM

Table 10
Ramirez-Marquez and Coit [28] alternative Type 1 IM rankings

Rank	Arc	MAD	Arc	MMAW	Arc	MMFV
1	2	0.0383712	2	1.020896019	2	0.020402859
2	9	0.0377054	9	1.020357439	9	0.020224839
3	8	0.0201336	8	1.010831226	8	0.010838544
4	4	0.0197446	4	1.010556877	4	0.010694213
5	6	0.01088015	6	1.00579264	6	0.005917652
6	1	0.0095615	1	1.004957432	1	0.005333599
7	5	0.0020146	5	1.001054773	5	0.001113539
8	7	0.0002229	7	1.00013852	3	0.000102248
9	3	0.0001146	3	1.000021095	7	0.000101387

perspectives. The proposed measures are intuitive and straightforward. The first measure UDI, provides insight regarding a component or component state contribution to unsatisfied demand. Originally defined as a risk-averse measure, the results show that it also has the potential to be used from a reliability potential perspective. Although computationally complex, it provides useful information about both the capacity and connectivity of the system. The second measure has been developed based on the rationale of Butler [15,16] and Wang et al. [22]. MFFI quantifies the contribution of a particular component or component state to MSMC failure. The last IM, MRI extends to the multi-state case the redundancy index presented by Boland [17] for binary systems. MRI helps in identifying where to allocate component redundancy as to improve system reliability. Finally, both MRI and UDI can be directly linked to economic implications of system failure or improvement.

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