Topic 2 - Basic Concepts of Probability

Statistics for Managers

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Basic Definitions

An Event is the random outcome of an experience (covering anything from a designed experiment, like tossing a coin, to observational phenomena, such as the number of points gains in the Dow-Jones Average.)

Probabilities are proportions that denote how likely the events are.

Example 1. Tossing of a balanced coin

In the tossing of a balanced coin, there are two events: “Head” and “Tail”

Intuitively, the probability of a head is 0.5.

Classical Probability Concept

Classical probability is a useful method where the experiment has many equally likely possibilities, such as in games of chance.

Example 2. Roulette

In roulette, the ball falls on either 38 (American version) or 37 (European versions) possible numbers. All numbers equally likely. Eighteen numbers are red and 18 are black. Therefore, the probability of a ball falling on a red number is 18/38 US.

Classical Probability Formula

If there are n equally likely possibilities, one of which must occur, and s are regarded as favorable, or as a “success,” then the probability of a “success” is given by the ratio $P = \frac{s}{n}$

Example 3. Applications of classical probability

In a well-shuffled deck of 52 cards, the probability of one draw of a card resulting in an ace is 4/52. All cards are equally likely. Four possibilities in favor of drawing an ace are the four different aces.

Manufacturing application: Sampling machine components from a production run to determine rate of defectives.

Classical Probability is good for insuring a sample is representative of a population.

Problems with the classical approach: Not all events can be cast in terms of a game of chance or there is no situation with equally likely possibilities.

Homework 1 Coin Tossing

Toss four coins. What is the probability that there will be exactly four heads?

The Frequency Interpretation of Probability

Sometime called the Empirical Approach: Based on analysis of data randomly sampled from population of values.

The probability of an event (happening or outcome) is the proportion of time that events of the same kind will occur in the long run.

Example 4. Airplanes

Airplanes arrive on time in 89% of their flights. So we estimate the probability of arriving on time is 0.89.

How good are the estimates?

Law of Large Numbers

If a situation, trial, or experiment is repeated repeatedly, the proportion of successes will tend to approach the probability that any one outcome will be a success.

Example 5. Computer simulation

We will illustrate the Law of Large Numbers with a simulation: To simulate the flipping of a balance coin: Generate a random number between 0 and 1, U. If $U < 0.5$, then a "head" occurred, otherwise a "tail" occurred.

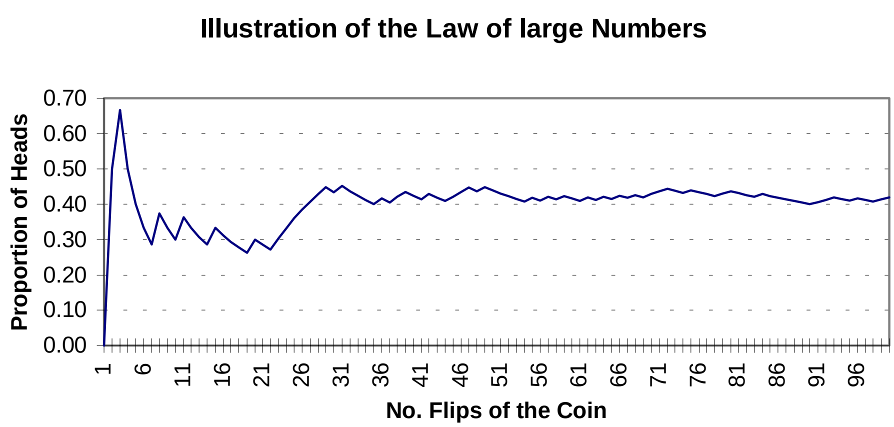
Determine the proportion of heads as the number of tosses increases.

The Law of Large Numbers predicts that this proportion will get arbitrarily close to 0.5.

Simulation results

Flip Number	Random Number	Event Outcome	Proportion of Heads (1)
1	0.680066286	0	0.00
2	0.142323069	1	0.50
3	0.966183109	0	0.33
4	0.09564372	1	0.50
5	0.528299137	0	0.40
6	0.637146786	0	0.33
7	0.629910131	0	0.29
8	0.558724429	0	0.25
9	0.443939341	1	0.33
10	0.13959071	1	0.40
11	0.131690048	1	0.45
12	0.539287436	0	0.42
13	0.553232844	0	0.38
14	0.511087505	0	0.36
15	0.601716911	0	0.33
16	0.673114783	0	0.31
17	0.871614495	0	0.29
18	0.337283812	1	0.33
19	0.675644216	0	0.32

Sequence plot of the proportion of heads



Homework 2 Computer simulation

Obtain three random devices, such as three coins, dice, deck of cards or any three random devices that can be repeatedly manipulated. You will perform several repeated experiments with the three devices. Choose to observe some outcome, i.e. event, about the three devices that can occur in the experiment, such as at least two heads as with coins. Repeat the experiment 100 times, each time recording that the event did or did not occur. Plot the proportion of experiments in which the event occurred as in the above example. What probability does your event have as estimated from the Law of Large Numbers?

Personal or Subjective Probability

The frequency approach has its limitations. In many situations, it is not possible to repeat the experiment. For example, in the predicting a presidential election, it is impossible to repeat the election. A particular female does not receive promotion in a large firm; it is impossible to repeat the situation leading to the rejection or to change the sex of the applicant.

Subjective approach

The subjective approach is to base the probability of an event on a person's perceived probability of the event. The probability can be elicited by exposing the person to a betting situation where he/she would win if the event in question occurred.

Bayesian methods

Bayesian methods use subjective probabilities and likelihoods of data configurations. For example, a prior probability that a machine manufactures a defective part would be updated after data on the number of defective parts is observed. This prior probability would be based on the machine operator's subjective probability.

Decision theory

Decision theory uses subjective probabilities to determine the best course of action that will result in maximum expected profit (or minimum expected cost). For example, two competitors must decide in which markets to place their products, each one's profits depends on the actions of the other. So each competitor subjectively estimates the probability of particular actions the competitor will take.

Utility

Utility is a subjective value of the outcome of an experiment. It is useful when the probability of an event can determined objectively, but different people still act differently. For example, some people will not risk travel by airplane, but will travel by automobile. They have a high utility for remaining safe-and-sound in case of air travel, although the probability of a crash is much lower than travel by automobile.