Topic 3 - Random Events And Logic

Statistics for Managers

June 3, 1999

Basic Experiments for Illustrative Purposes

NH -"Number Heads" Experiment:

Toss a balanced coin 10 times, observe the number of heads.

Model applies to number of machines that breakdown in a month out of 10 machines.

NT - "Number Trials" Experiment:

Toss a balanced coin until it comes up heads, observe the number of tosses.

Model applies to number days a machine runs until it needs repair.

PD - "Prisoner's Dilemma" Experiment:

Two suspects in a crime may choose to cooperate with each other by remaining silent or ratting on the other by confessing. If they both remain silent, they both will get 30 days for a lessor crime. If both confess, they both get 2 years; but if one remains silent while the other rats, the silent one gets 40 years, and the confessing prisoner gets off for providing evidence for the state. Observe the outcome of the pre-trial negotiations.

A classical model for studying cooperation and conflict in business.

CC - "Competing Contractors" Experiment:

Two contractors submit bids for each of two jobs. If the bid is lower than an amount unknown to both bidders, the lowest bid for a particular job wins the job. Observe the number of contracts each bidder wins.

ER - "Measurement Errors in Astronomy":

The true position of a planet is measured. The outcome is the measured orbital position. The error in measurement is unknown however. Due to the precision of the astronomical instrument (or lack thereof) the observed measurement represents a small continuous interval of values.

A fundamental model for observing with errors.

Note. Each of the experiments will be denoted by their respective leading letters as listed above.

Some Basic Definitions

Outcomes

An experiment will result in certain possibilities or outcomes.

NH: one head is an outcome.

NT: 25 trials is an outcome, although an unlikely one.

Sample Space

A sample space is the collection of all possible outcomes; it is denoted by S.

NH: S = {0, 1, 2, 3, …, 10}

NT: S = {1, 2, 3, …, infinity}

PD: S = {(s, s), (s, r), (r, s), (r, r)} where "s" denotes silent, and "r" denotes "rats" for the first and second prisoner.

CC: S = {(0,0),(0,1),(0,2),(1,0),(1,1),(2,0)}

Elementary Outcomes

Sample spaces are best represented by the elementary outcomes, called elements or points.

NH: {2} i.e., "2 heads" is a point.

NT: {3} i.e. "3 trials or flips" is a point.

PD: {(s, r)} i.e. "#1 is silent, and #2 rats" is a point.

CC: {(0,0)} i.e. "neither contractor gets a job" is a point.

Finite And Infinite Sample Spaces

There are finite and infinite sample spaces. Some infinite sample spaces have only discrete outcomes; these are called countably infinite sample spaces. Some infinite sample spaces have continuous intervals as outcomes; these are called continuous sample spaces. Some sample spaces have the properties of both countably infinite and continuos sample spaces.

NH: finite.

NT: countably infinite.

PD: finite.

CC: finite

ER: continuous

Event

An event is a collection of outcomes from the sample space, each outcome is said to be favorable to the event; an event will denoted by the capital letters A, B, etc.

NH: A = {0, 2, 4, 6, 8, 10} “number of heads is an even number”

NT: B = {2, 3, …} “head on trial 2 or later”

PD: C = {(s, s), (s, r)} “#1 is silent”

CC: D = {(1,0),(0,1)} “only one job is won”

ER: E = {X: -1 < X – m < 1} “The error between the observed (X) and true (m) orbital position is less than one degree”

Empty Event

The special event that contains no outcomes is the empty event; it is denoted by the special symbol ∅.

Mutually Exclusive

If two events C and E have no elements in common, they are said to be mutually exclusive.

NH: C = {even # heads}, E = {odd # heads}

NT: C = {2 trials}; E = {at least 5 trials}

PD: C = {#1 rats}; E = {#1 is silent}

CC: C = {no jobs won}; E = {#2 gets 2 jobs}

Note: All elementary outcomes are mutually exclusive.

NH: {0}, {1}, {2}, {3}, …, {10}

NT: {1}, {2}, {3}, …, {infinity}

PD: {(s, s)}, {(s, r)}, {(r, s)}, {(r, r)}

CC: {(0,0)}, {(0,1)}, {(0,2)}, {(1,0)}, {(1,1)}, {(2,0)}

Unions, Intersections, And Complements

Some events can be expressed in terms of two or more events by forming unions, intersections, and complements.

This will be especially useful for probability calculations later.

Union

The union of two events X and Y, denoted by $X \cup Y$, is the event which consists of all the elements either in X or in Y.

NH: $\{0,1,9,10\} \cup \{\text{even \# heads}\} = \{0, 1, 2, 4, 6, 8, 9, 10\}$

Intersection

The intersection of two events X and Y, denoted by $X \cap Y$, is the event which consists of all the elements in both X and Y.

NT: $\{\text{5 or less trials}\} \cap \{\text{2 or more trials}\} = \{2, 3, 4, 5\}$

Mutually Exclusive

Note: Any two mutually exclusive events, X and Y, have an empty intersection: $X \cap Y = \varnothing$

CC: $\{\text{\#1 gets 2 jobs}\} \cap \{\text{\#2 gets 2 jobs}\} = \varnothing$

Complement

The complement of the event X, denoted by X' or $X^{\neg}$, is the event which consists of all the elements not in X.

NH: {at least one head}' = {0}

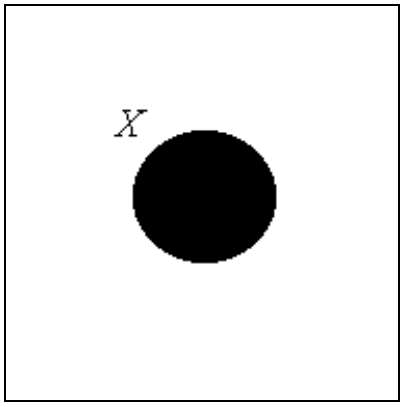
Homework 1 Coin Tossing

Imagine tossing three coins. List all elementary outcomes of the sample space. Write the event that there are two heads in terms of the appropriate elementary events. What is the complement of the event that there is at least one head?

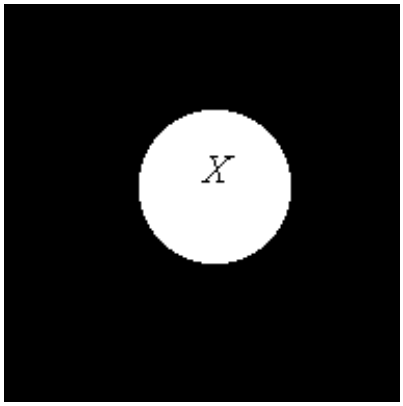
Venn Diagrams

Sample spaces and events are often pictured by Venn diagrams.

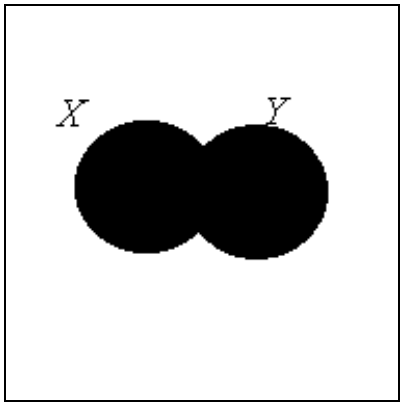
Venn Diagram of X



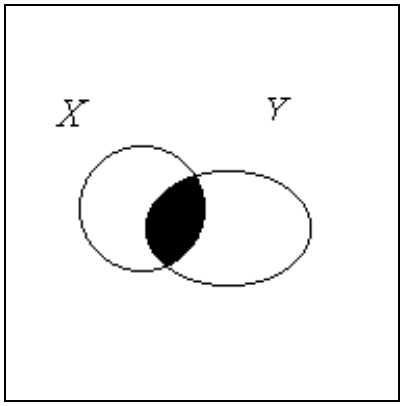
Venn Diagram of X'



Venn Diagram of X ∪ Y

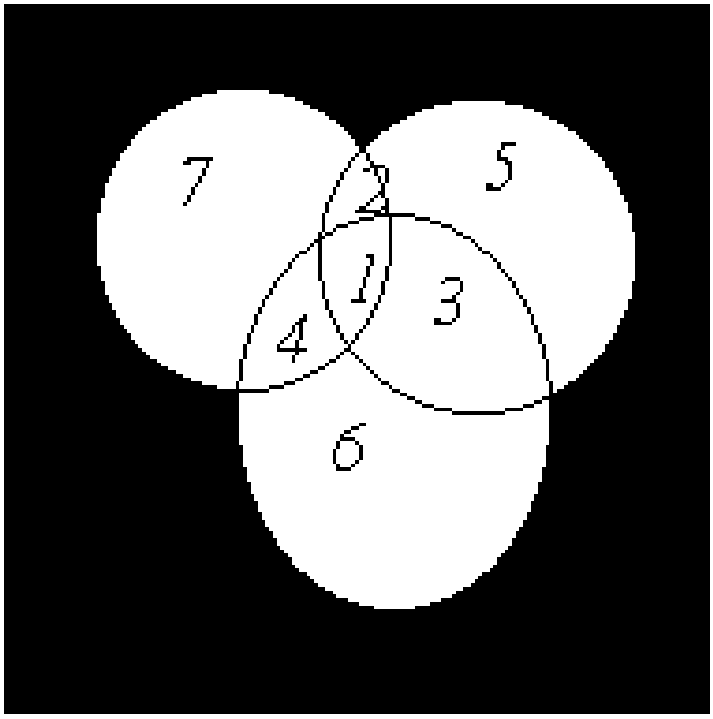


Venn Diagram of X ∩ Y



256 Different Events from 3 Events

John Venn discovered that 256 different events can be obtained from taking the unions of all the combinations of the eight events and their complements described in this figure:



∅

{1} ∪ {2} ∪ {3} ∪ {4} ∪ {5} ∪ {6} ∪ {7} ∪ {8}

{1} ∪ {2} ∪ {3} ∪ {4} ∪ {5} ∪ {6} ∪ {7} ∪ {8}'

Etc.