Topic 4 - Basic Rules of Probability

Statistics for Managers

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First Postulate of Probability

The probability of any event is a real positive number or zero; symbolically, $P(A) \geq 0$ for any event *A*.

Example 1. Classical Probability

If an experiment can occur in n equally likely ways, and A occurs in s of them, then

$P(A) = s/n \geq 0$

Example 2. American Roulette

$P(1) = 1/37 \geq 0$

Second Postulate of Probability

The probability of any sample space is equal to 1; symbolically, $P(S) = 1$ for any sample space *S*.

Example 3. Number of Heads

$P(S) = P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10) = 1$

Third Postulate of Probability

If two events are mutually exclusive, then the probability, that one or the other will occur equals the sum of their probabilities. Symbolically, for any two mutually exclusive events *A* and *B*:

$$A \cap B = \varnothing$$
$$P(A \cup B) = P(A) + P(B)$$

Example 4. Competing Contractors

Assume the following probabilities:

P(0,0) = 0.30

P(0,1) = 0.25

P(0,2) = 0.15

P(1,0) = 0.15

P(1,1) = 0.10

P(2,0) = 0.05

Find P(A ∪ B) where A and B are defined as follows:

A = {#1 gets 2 jobs}

B = {#2 gets 2 jobs}

A ∪ B = {one contractor gets both jobs}

Note A and B are mutually exclusive:

A ∩ B = ∅

P(A ∪ B) = 0.05 + 0.15 = 0.20

More Rules of Probability

$$P(A) \le 1$$
$$P(\emptyset) = 0$$
$$P(A) + P(A') = 1$$
$$P(A') = 1 - P(A)$$

Example 5. Classical Probability

P{A} = s/n ≤ n/n = 1

Example 6. Competing Contractors

A = {#1 gets 2 jobs}

A' = {#1 gets 0 or 1 job}

P(A') = 1 - P(A) = 1 - 0.05 = 0.95

Generalized Rule for Mutually Exclusive Events

If *k* events are mutually exclusive, the probability that at least one of them will occur equals the sum of their individual probabilities; symbolically

$$P\left(A_1 \cup A_2 \cup \cdots \cup A_k\right) =$$
$$P\left(A_1\right) + P\left(A_2\right) + \cdots + P\left(A_k\right)$$

For mutually exclusive events $A_1, A_2, \ldots A_k$.

Homework 1 Coin Tossing

Toss three fair coins. What is the probability of tossing at least one tail? What is the probability of tossing exactly three heads or exactly three tails?

Rule for Calculating the Probability of an Event

The probability of any event A is most easily given by the sum of the probabilities of the elementary outcomes comprising *A*.

Example 7. American Roulette

{Even Number} = {2} ∪ {4} ∪ … ∪ {36}

P(Even Number) = P{2} + P{4} + … + P{36} = 1/37 + 1/37 + … + 1/37 = 18/37

General Addition Rule

For any two events *A* and *B*, not necessarily mutually exclusive,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 8. American Roulette

A = {Even Number}

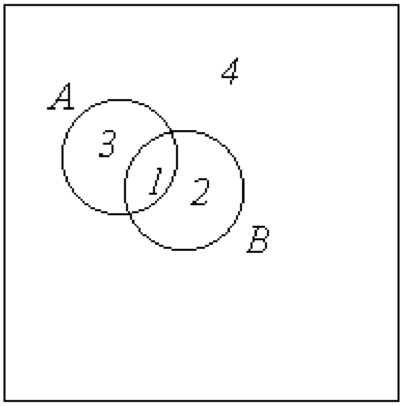
B = {Red Number}

A∩B = {2, 6, 10, 14, 18, 22, 26, 30, 34}

P(Even Or Red) = 18/38 + 18/38 - 9/38 = 27/38

Calculations of Probability by Using the Mutually Exclusive Events

Basic Venn diagram for two events



A Contingency Table Representation of the Events

	B	B'	Margin
A	(1) = $A \cap B$	(3) = $A \cap B'$	A = (1) $\cup$ (3)
A'	(2) = $A' \cap B$	(4) = $A' \cap B'$	A' = (2) $\cup$ (4)
Margin	B = (1) $\cup$ (2)	B' = (3) $\cup$ (4)	S=(1) $\cup$ (2) $\cup$ (3) $\cup$ (4)

A Contingency Table Representation of the Probabilities

	B	B'	Margin
A	$P(A \cap B)$	$P(A \cap B')$	$P(A) = P(A \cap B) + P(A \cap B')$
A'	$P(A' \cap B)$	$P(A' \cap B')$	$P(A') = P(A' \cap B) + P(A' \cap B')$
Margin	$P(B) = P(A \cap B) + P(A' \cap B)$	$P(B') = P(A \cap B') + P(A' \cap B')$	1

Example 9. Contingency table solution

Suppose P(A) = 0.6, P(B') = 0.2, and P(A $\cap$ B) = 0.4, show that A' and B' are mutually exclusive. (This is generally a hard question for beginner students. However, it is quite easy using the contingency table approach.)

Solution

Cells (a), (b) and (c) are given in the problem statement. Cell (d) is determined by values in cells (a) and (c). Cell (e) is determined by cells (b) and (d). Since cell (e) is zero, A and B must be mutually exclusive.

	B	B'	Margin
A	(a) .4	(d) .2	(c) .6
A'	.4	(e) 0	.4
Margin	.8	(b) .2	1

Note that once *any* three values are given, then all cells are completely determined.

Example 10. Ambiguous statements

Show that it is impossible that P(A) = 0.6, P(B) = 0.3, and P(A $\cap$ B) = P(A' $\cap$ B') = 0.3.

Solution

	B	B'	Margin
A	.3	Ambiguous .3 or .4	.6
A'	Ambiguous 0 or .1	.3	.4
Margin	.3	.7	1

Homework 2 Double Agent

The CIA suspects one of its agents is a double agent. Over the course of time, the CIA-agent has visited Paris, France 25 out of 100 weeks. By tracking the travel schedule of the CIA agent and his suspected enemy contact, the CIA has determined the following estimates of probability. The probability that CIA-agent and the enemy contact are in Paris in the same week is 0.20. The probability that neither one are in Paris in the same week is 0.25. What is the probability that in the same week the CIA agent is in Paris and the enemy contact is not?