Topic 5 - Conditional Probabilities

Statistics for Managers

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Odds to Probability Formula

If the probability of an event is p, the odds for its occurrence are a to b, where a and b are positive values such that

$$\frac{a}{b} = \frac{p}{1-p}$$

Example 1. One balanced coin toss

Odds of a head is a=1 to b=1:

$$\frac{1}{1} = \frac{1/2}{1-1/2}$$

Example 2. Two balanced coin tosses

Odds of two heads is a=1 to b=3:

$$\frac{1}{3} = \frac{1/4}{1-1/4}$$

Example 3. Unvaccinated child getting polio

The probability of an unvaccinated child getting polio is .05. The odds are a=5 to b=95:

$$\frac{5}{95} = \frac{.05}{1-.05}$$

Probability to Odds Formula

If the odds are a to b that an event will occur, the probability of its occurrence is

$$p = \frac{a}{a+b}$$

Example 4. Odds of a certain horse winning

The odds of a certain horse winning are 5 to 2. The probability is 5/7:

$$\frac{5}{7} = \frac{5}{5+2}$$

Example 5. Long shot's odds

A long shot's odds are 1 to 49. The probability is .02:

$$.02 = \frac{1}{1+49}$$

Conditional Probability

If P(B) is not equal to zero, then the conditional probability of A relative to B, namely, the probability of A given B, is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example 6. Having disease if the test is positive

	Disease (D)	Disease (D')
Test Positive (P)	48	2
Test Positive (P')	10	40

What is the probability of having the disease if the test is positive. Obviously, the odds are 48 to 2, so the probability is 0.96.

$$P(D \cap P) = \frac{48}{100}$$

$$P(P) = \frac{50}{100}$$

$$P(D \mid P) = \frac{P(D \cap P)}{P(P)} = \frac{\frac{48}{100}}{\frac{50}{100}} = \frac{48}{50} = 0.96$$

Example 7. Positive test given that a person has the disease

The event of a positive test given that a person has the disease has odds 48 to 10, so the probability is 48/58 = 0.83

$$P(P \mid D) = \frac{P(P \cap D)}{P(D)} = \frac{\frac{48}{100}}{\frac{58}{100}} = \frac{48}{58}$$

Example 8. Positive test given that a person does not have the disease

The event of a positive test given that a person does not have the disease has odds 2 to 40, so the probability is 2/42 = 0.05. This is the probability of a "false positive" or in other words, it is the probability of falsely declaring that the person has the disease.

$$P(P \mid D') = \frac{P(P \cap D')}{P(D')} = \frac{\frac{2}{100}}{\frac{42}{100}} = \frac{2}{42}$$

Example 9. Negative test given that a person does have the disease

The event of a negative test given that a person does have the disease has odds 10 to 48, so the probability is 10/58 = 0.17. This is the probability of a "false negative" or in other words, it is the probability of falsely declaring that the person does not have the disease.

$$P(P' \mid D) = \frac{P(P \cap D)}{P(D)} = \frac{\frac{10}{100}}{\frac{58}{100}} = \frac{10}{58}$$

Independence

Two events, A and B, are said to be independent if P(A|B)=P(A) or if P(B|A)=P(B). In other words, if the probability of one event does not depend on the occurrence of the other event, then the two events are independent.

Example 10. Carrots

	Likes Carrots (B)	Doesn't Like Carrots (B')	
Boy (A)	10	40	50
Girl (A')	10	40	50
	20	80	100

A boy's odds of liking carrots (B|A) is 10 to 40: P(B|A) = 10/50.

Overall, odds of liking carrots is 20 to 80: P(B) = 20/100 = 10/50.

So P(B|A) = P(B), likewise, P(A|B) = P(A). Thus A and B are independent events; in other words, gender and liking carrots are independent random variables.

General Multiplication Rule

$$P(A \cap B) = P(B) \times P(A \mid B)$$
$$P(A \cap B) = P(A) \times P(B \mid A)$$

Special Multiplication Rule

A and B are independent events if and only if:

$$P(A \cap B) = P(A) \times P(B)$$

Example 11. Trucks

	Likes Trucks (B)	Doesn't Like Trucks (B')	
Male (A)	10	40	50
Female (A')	2	48	50
	12	88	100

The odds of a male child liking trucks is 10 to 40 so P(B|A) = 10/50 = 1/5.

The odds of liking trucks is 12 to 88 so P(B) = 12/100 = 3/25

So, being male and liking trucks are dependent events. Gender and truck liking are dependent random variables.

Example 12. Coin Toss Twice

	Toss 2: Head (B)	Toss 2: Tails (B')	
Toss 1: Head(A)	1	1	2
Toss 1: Tails(A')	1	1	2
	2	2	4

Coin Toss Twice: The odds of A∩B is 1 to 3 so P(A∩B) = 1/4.

The odds of A is 1 to 1 so P(A) = ½.

The odds of B is 1 to 1 so P(B) = ½.

Therefore, P(A) X P(B) = (½)(½) = ¼.

Therefore, A and B are independent events.

Toss 1 and toss 2 outcomes are independent random variables.

Homework 1 Double Agent

The CIA suspects one of its agents is a double agent. Over the course of time, the CIA-agent has visited Paris, France 25 out of 100 weeks. Through tracking the travel schedule of the CIA agent and his suspected enemy contact, the CIA has determined the following estimates of probability. The probability that CIA-agent and the enemy contact are in Paris in the same week is 0.20. The probability that neither one are in Paris in the same week is 0.25. What is the probability that the CIA-agent is in Paris given that the enemy contact is in Paris? Are the two agents appearing in Paris in the same week independent events?