Statistics 553: Categorical Data Analysis
Homework Set 1: Fall - 2018

This homework set will be periodically updated.

1. Chapter 1, pages 16-20
   Basic: 1.1, 1.2, 1.3, 1.4, 1.6, 1.8, 1.9, 1.10
   More Challenging: 1.7, 1.12, 1.13*, 1.14*, 1.15, 1.16, 1.17a*, 1.18*
   Problems with * should be attempted after lecture 2.

2. (Advanced.) Using the appropriate expansions, verify that the sum of the geometric probabilities, the sum of the binomial probabilities, and the sum of the Poisson probabilities all add to 1.

3. If \((n_1, \ldots, n_k)\) have independent Poisson distributions with mean parameters \((\mu_1, \ldots, \mu_k)\) respectively, then show that given \(n = \sum_{j=1}^{k} n_j\) the distribution of \((n_1, \ldots, n_k)\) is multinomial. Also find the parameters of this multinomial distribution.

4. (Advanced.) Show that for fixed \(0 < \pi_o < 1\), the function,
   \[
   \Lambda(p) = \left(\frac{\pi_o}{p}\right)^{np} \left(\frac{1 - \pi_o}{1 - p}\right)^{n(1-p)}, \quad 0 \leq p \leq 1,
   \]
   is increasing for \(p > \pi\) and decreasing for \(p \leq \pi\).

5. Construct an \(\alpha = 0.05\) rejection region for testing
   \[
   H_o : \pi = \pi_o \quad \text{vs.} \quad H_a : \pi > \pi_o
   \]
   (a) when \(n = 100, \pi_o = .10\),
   (b) when \(n = 10, \pi_o = .10\), and
   (c) when \(n = 100, \pi_o = .01\).

6. Generate the formula for a \((1 - \alpha)\) confidence interval for a population proportion \(\pi\) based upon the approximate standard normal distribution of
   \[
   Z = \frac{p - \pi}{\sqrt{\pi(1 - \pi)/n}}.
   \]

7. Find a 90% confidence upper bound for a population proportion \(\pi\) when one observes no successes in 5 independent Bernoulli trials.
8. (Advanced.) Given the approximate normality of \( p - \pi \), use the delta method to show \( \hat{\gamma} - \gamma \approx \text{Normal}(0, \sigma_{\hat{\gamma}}^2) \), where \( \gamma = \log\left(\frac{\pi}{1-\pi}\right) \) and 
\[
\sigma_{\hat{\gamma}}^2 = \frac{1}{n} \left( \frac{1}{\pi} + \frac{1}{1-\pi} \right)
\]

9. Derive the approximate \( (1 - \alpha) \) confidence interval for \( \pi \) based on the approximate \( (1 - \alpha) \) confidence interval for \( \gamma = \log\left(\frac{\pi}{1-\pi}\right) \).