Research paper

Struct Multidisc Optim 26, 224–228 (2004)
DOI 10.1007/s00158-003-0346-z

Applications of regional strain energy in compliant structure design for energy absorption

H.C. Gea

Abstract Topology optimization of regional strain energy is studied in this paper. Unlike the conventional mean compliance formulation, this paper considers two main functions of structure: rigidity and compliance. For normal usages, rigidity is chosen as the design objective. For compliant design, a portion of the structure absorbs energy, while another part maintains the structural integrity. Therefore, we implemented a regional strain energy formulation for topology optimization. Sensitivity to regional strain energy is derived from the adjoint method. Numerical results from the proposed formulation are presented.

Key words topology optimization, compliant structure, energy absorption

1 Introduction

The traditional mechanical design of both structures and mechanisms is usually accomplished by using rigid links and rigid kinematic joints. However, in reality, nothing is ideally rigid, and this warrants detailed analysis to check for unwanted deformations. On the other hand, the lack of rigidity does not mean that the resulting designs will be less strong. Strength characterizes the resistance to breakage or failure, while stiffness is resistance to deformation. In this paper, topology optimization of regional strain energy is presented. Two main functions of structure are considered: rigidity and compliance. During its normal usage, the main function of a structure is to support external loadings. Under this condition, the commonly used mean compliance that produces the stiffest structure is chosen as the design objective. For compliant design, a portion of the structure is designed to absorb as much strain energy as possible; at the same time, another part of the structure needs to maintain the structure's structural integrity. To handle both situations, we divide the structure into two regions, designed for different functionalities: one for structure flexibility, and the other for structure integrity. Their formulations are completely different. In the compliant design zone, strain energy is to be maximized, and in the structural integrity zone, strain energy is to be minimized.

Recently, topology optimization has emerged as the focus of structural optimization, because it radically changes the structural design process. However, most of the research on topology optimization has been focused on improving the static and dynamic performance of structures. Little effort has been expended on researching the impact attenuation. Suzuki and Kikuchi (1991) considered shell structures and 3D linear elastics problems. Diaz and Kikuchi (1992) and Ma *et al.* (1993) studied vibration problems. Mayer *et al.* (1996) applied topology optimization to the crashworthiness problem. Luo and Gea (1998a, 1998b, 1999), and Yang *et al.* (1996) studied optimal stiffener design for shell structures. Luo and Gea extended topology optimization to noise reduction (1997) and nonlinear problems (2000). A microstructure-based design domain method (Gea 1996) is employed to formulate this topology optimization, and the Generalized Convex Approximation (Chickermane and Gea 1996) is used as the optimizer.

The sensitivity of regional strain energy is derived from the adjoint method. Results of two numerical examples from the conventional mean compliance design and the new proposed formulation are presented for comparison. Although this problem can be further complicated by the nonlinear responses resulting from collisions, we can only apply linear analysis here due to its complexity. Extension of this work to non-linear analysis is currently under development.

Received: 10 August 2001
Revised manuscript received: 11 November 2002
Published online: 16 January 2004

H.C. Gea

Department of Mechanical and Aerospace Engineering, Rutgers University, Piscataway, New Jersey 08854-8058, USA
e-mail: gea@rci.rutgers.edu

2 Problem formulation

Consider a linearly elastic structure subjected to the applied body force f_i^b, and the surface traction f_i^t, as shown in Fig. 1. Through the principle of virtual displacement, the weak formulation of this problem can be written as:

$$\int_\Omega \sigma_{ij}\delta\varepsilon_{ij}\,\mathrm{d}\Omega = \int_\Omega f_i^b \delta u_i\,\mathrm{d}\Omega + \int_\Gamma f_i^t\delta u_i\,\mathrm{d}\Gamma \qquad (1)$$

where σ_{ij} is stress tensor and δu_i represents the virtual displacement in a kinematically admissible displacement set.

For a linear elastic structure with linear material, we can express the linear strain-displacement relation and the material constitutive equation as the following:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (2)$$

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \qquad (3)$$

where C_{ijkl} is the elastic coefficient, which is a function of our design variables.

In the proposed new formulation, our design objectives are to minimize the total strain energy for the first loading condition in order to find the stiffest structure, and to maximize the regional strain energy for the second loading condition so that the structure can absorb as much energy as possible. A weight constraint is always imposed. These two objectives are combined as:

$$\text{Minimizing} \qquad \frac{E_T^1}{E_V^2} \qquad (4)$$

where E_T^1 is the total strain energy under the first load case and E_V^2 is the regional strain energy under the second load case. Since linear analysis is used for the current study, only one analysis is needed to evaluate them. To solve this problem iteratively using any mathematical programming, we need to evaluate the design sensitivities of mean compliance and regional strain energy. Since the derivation of the sensitivity of mean compliance can be easily found in the literature, we will skip it. In the next section, we will derive the sensitivity of the regional strain energy.

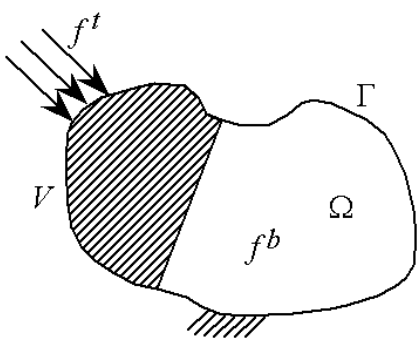

Fig. 1 Elastic structure under loading

3 Sensitivity of regional strain energy

For the compliant design region, a portion of the structure is designed to absorb as much strain energy as possible while the rest of the structure is designed to absorb as little as possible. Therefore, we need to derive the formulation of strain energy for a portion of the structure.

Consider a portion of the structure, V, as being under consideration (the shaded region in Fig. 1). The functional of regional strain energy, H, of V is defined as:

$$H = \frac{1}{2}\int_V u_i k_{ij} u_j\,\mathrm{d}\Omega = \int_V h(u,p)\,\mathrm{d}\Omega \qquad (5)$$

where h is a displacement functional and p represents the design parameter.

To derive the sensitivity of regional strain energy, we apply the adjoint method. First, we define a new functional H^* by including additional terms: replacing the virtual displacement in (1) with the adjoint variable u^a, and multiplying (2) and (3) with appropriate adjoint variables. The new functional H^* can be expressed as:

$$H^* = H + \int_\Omega f_i^b u_i^a\,\mathrm{d}\Omega + \int_\Gamma f_i^t u_i^a\,\mathrm{d}\Gamma - \frac{1}{2}\int_\Omega \sigma_{ij}\left(\frac{\partial u_i^a}{\partial x_j} + \frac{\partial u_j^a}{\partial x_i}\right)\mathrm{d}\Omega + \int_\Omega \sigma_{ij}^a\left(\varepsilon_{ij} - \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)\mathrm{d}\Omega + \int_\Omega \varepsilon_{ij}^a\left(\sigma_{ij} - C_{ijkl}\varepsilon_{kl}\right)\mathrm{d}\Omega \qquad (6)$$

where σ_{ij}^a and ε_{ij}^a are adjoint variables to be determined. It is relatively simple to prove that H^* is the same as H. Taking derivatives of (6) with respect to the design variable p, we have:

$$\frac{\mathrm{d}H^*}{\mathrm{d}p} = \frac{\mathrm{d}H}{\mathrm{d}p} = \int_\Omega h_{,p}\,\mathrm{d}\Omega + \int_\Omega h_{,u}u_{,p}\,\mathrm{d}\Omega - \frac{1}{2}\int_\Omega \sigma_{ij,p}\left(\frac{\partial u_i^a}{\partial x_j} + \frac{\partial u_j^a}{\partial x_i}\right)\mathrm{d}\Omega + \int_\Omega \sigma_{ij}^a\left(\varepsilon_{ij,p} - \frac{1}{2}\left(\frac{\partial u_{i,p}}{\partial x_j} + \frac{\partial u_{j,p}}{\partial x_i}\right)\right)\mathrm{d}\Omega + \int_\Omega \varepsilon_{ij}^a\left(\sigma_{ij,p} - C_{ijkl,p}\varepsilon_{kl} - C_{ijkl}\varepsilon_{kl,p}\right)\mathrm{d}\Omega \qquad (7)$$

To further simplify (7), we rearrange the right hand side as follows:

$$\frac{\mathrm{d}H^*}{\mathrm{d}p} = \frac{\mathrm{d}H}{\mathrm{d}p} = \int_\Omega h_{,p}\,\mathrm{d}\Omega - \int_\Omega \varepsilon^a\, C_{ijkl,p}\varepsilon_{kl}\,\mathrm{d}\Omega + \int_\Omega \sigma_{ij,p}\left(\varepsilon^a_{ij} - \frac{1}{2}\left(\frac{\partial u^a_i}{\partial x_j} + \frac{\partial u^a_j}{\partial x_i}\right)\right)\mathrm{d}\Omega + \int_\Omega \varepsilon_{ij,p}(\sigma^a_{ij} - C_{ijkl}\varepsilon^a)\,\mathrm{d}\Omega + \int_\Omega h_{,u}u_{,p}\,\mathrm{d}\Omega - \int_\Omega \sigma^a_{ij}\frac{1}{2}\left(\frac{\partial u_{i,p}}{\partial x_j} + \frac{\partial u_{j,p}}{\partial x_i}\right)\mathrm{d}\Omega\,. \tag{8}$$

When the adjoint field is defined as:

$$\varepsilon^a_{ij} = \frac{1}{2}\left(\frac{\partial u^a_i}{\partial x_j} + \frac{\partial u^a_j}{\partial x_i}\right) \tag{9}$$

$$\sigma^a_{ij} = C_{ijkl}\varepsilon^a_{kl} \tag{10}$$

$$\int_\Omega h_{,u}u_{,p}\,\mathrm{d}\Omega = \int_\Omega \sigma^a_{ij}\frac{1}{2}\left(\frac{\partial u_{i,p}}{\partial x_j} + \frac{\partial u_{j,p}}{\partial x_i}\right)\mathrm{d}\Omega \tag{11}$$

(8) can be further reduced to the first two terms only as:

$$\frac{\mathrm{d}H^*}{\mathrm{d}p} = \frac{\mathrm{d}H}{\mathrm{d}p} = \int_\Omega h_{,p}\,\mathrm{d}\Omega - \int_\Omega \varepsilon^a_{ij}\, C_{ijkl,p}\varepsilon_{kl}\,\mathrm{d}\Omega \tag{12}$$

To evaluate (12), we must consider two different cases: (1) the design variable is outside the region V; (2) the design variable is inside the region V. In the first case, the first term in the right hand side of (12) is zero and the sensitivity becomes:

$$\frac{\mathrm{d}H}{\mathrm{d}p} = -\int_\Omega \varepsilon^a_{ij}\, C_{ijkl,p}\varepsilon_{kl}\,\mathrm{d}\Omega \tag{13}$$

For the second case, the sensitivity of regional strain energy is:

$$\frac{\mathrm{d}H}{\mathrm{d}p} = \frac{1}{2}\int_\Omega u_i k_{ij,p} u_j\,\mathrm{d}\Omega - \int_\Omega \varepsilon^a_{ij}\, C_{ijkl,p}\varepsilon_{kl}\,\mathrm{d}\Omega \tag{14}$$

The above equations can be further simplified using the closed form relation between material properties and design variable, as shown in Gea (1996). The only unknown is the strain field of the adjoint field defined in (9), (10) and (11). It can be obtained by applying the adjoint load $h_{,u}$ to the structure. After the design sensitivity of regional strain energy is calculated, the problem is ready to be solved.

4 Numerical examples

In this section, results of two numerical examples from the conventional mean compliance design and the new proposed formulation are presented for comparison. For easier illustration, both examples are two-dimensional examples. However, the proposed new formulation is not restricted to two dimensions; it can be used in three-dimensional applications too.

4.1 Example 1

In the first example, a square design domain is fixed at the left hand side and two concentrated forces are applied to its right side. This design domain is further divided into two regions, as shown in Fig. 2. The shaded region on the right hand side is designed to absorb as much energy as possible, while the whole structure is required to be the stiffest one under a given weight constraint.

First, we applied the conventional mean compliance formulation which will produce the stiffest structure under a given weight constraint. A simple two-bar truss

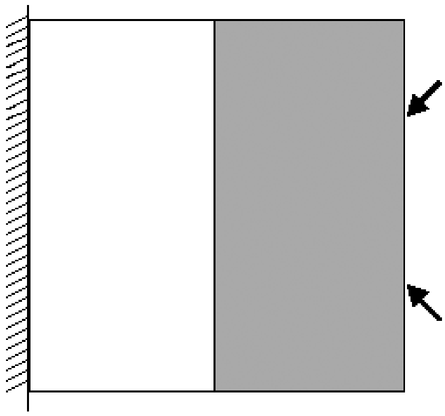

Fig. 2 A square design domain is fixed at the left hand side and two forces are applied to its right side

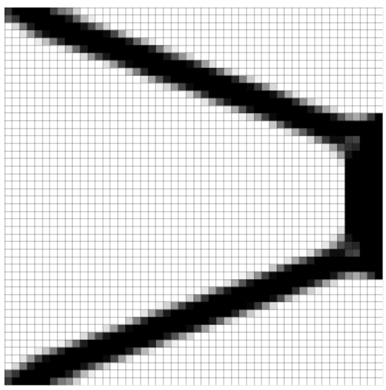

Fig. 3 The optimal design generated from the conventional topology optimization

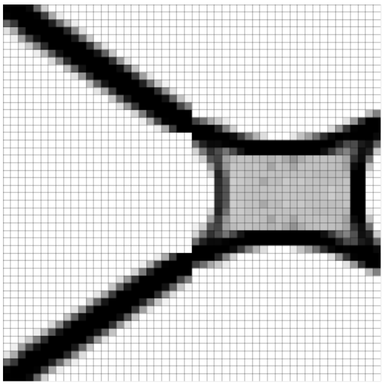

Fig. 4 The optimal design generated from the proposed new formulation

design is generated to support the loadings, as shown in Fig. 3. It is obvious that this design has little energy absorption function.

The same problem is solved again using the proposed new formulation, and the result is shown in Fig. 4. Although the left side resembles the two-bar truss design, the configuration of the right hand side is changed completely. A box type of structure is formed to absorb energy on the right side, and an arch connects the box to the two-bar truss at its left side. We found that this design is not as stiff as the design from the conventional mean compliance formulation, but the deformation is mainly in the box. Therefore, the structural integrity of the left side is protected very well.

4.2 Example 2

In the second example, we change the ratio of the design domain from 1 : 1 to 2 : 3 as shown in Fig. 5. The bottom

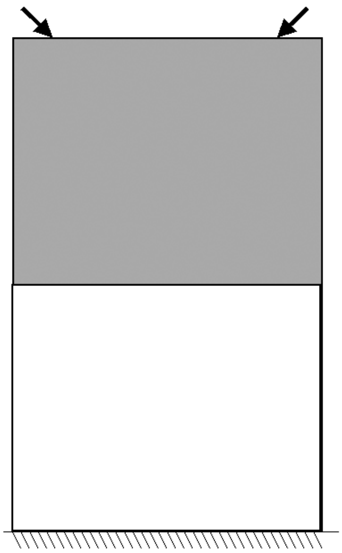

Fig. 5 The bottom of the structure is fixed and loadings are applied from the top

of the structure is fixed and loadings are applied from the top. Similar to the first example, two zones are considered here. The shaded zone is to absorb energy and the other zone is to simulate the passenger compartment where the structural integrity is the major concern.

The conventional mean compliance design is shown in Fig. 6. The design is very similar to the previous one, except an extra light member is generated in the top portion of the structure due to two design domains having different aspect ratios. The proposed formulation maximizes the energy absorption of the shaded region as well as minimizing the total strain energy. This gives a rather different solution, as shown in Fig. 7. A very strong arch combined with a two-truss design is generated to maintain the structural integrity of the bottom portion, and a box with narrow base at the top can absorb much en-

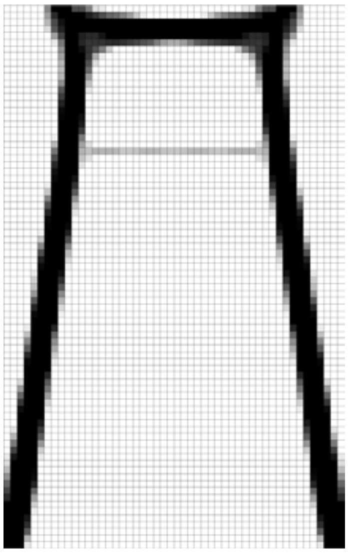

Fig. 6 The optimal design generated from the conventional topology optimization

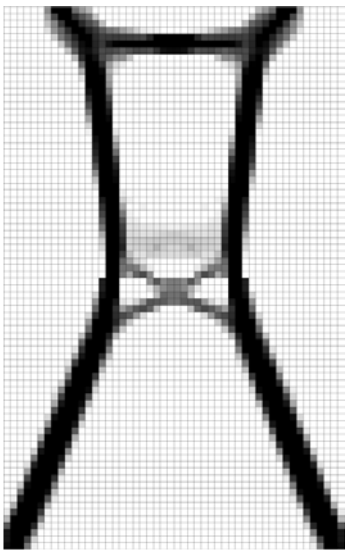

Fig. 7 The optimal design generated from the proposed new formulation

ergy. A cross member is found to connect two regions. Again, we found that the design from the mean compliance formulation is stiffer than that from the new formulation. At the same time, the bottom portion of the design from the new formulation is almost intact under loading.

5 Conclusion

Instead of a single mean compliance objective function, two design objectives are implemented in the formulation. Minimizing total mean compliance is used for ensuring structural rigidity, and maximizing regional strain energy is used for compliant design. In order to handle both situations, we divide the structure into two regions and they are designed for different functionalities. The sensitivity analysis of regional strain energy is derived from the adjoint method. Numerical results show the new proposed formulation produces some reasonable solutions. One drawback of the current study is that linear analysis is used while nonlinear responses are expected from collisions, so extension to non-linear analysis is currently under investigation.

References

Chickermane, H.; Gea, H.C. 1996: A new local function approximation method for structural optimization problems. *Int. J. Numer. Meth. Eng.* **39**, 829–846

Diaz, A.R.; Kikuchi, N. 1992: Solutions to shape and topology eigenvalue optimization problems using a homogenization method. *Int. J. Numer. Meth. Eng.* **35**, 1487–1502

Gea, H.C. 1996: Topology optimization: a new micro-structure bsed design domain method. *Comput. Struct.* **61**, 781–788

Gea, H.C.; Luo, J.H. 1999: Automated optimal stiffener pattern design. *Mech. Struct. Mach.* **27**, 275–292

Luo, J.H.; Gea, H.C. 1997: Modal sensitivity analysis of coupled acoustic-structural systems. *J. Vib. Acoust.* **119**, 545–550

Luo, J.H.; Gea, H.C. 1998: Optimal bead orientation of 3D shell/plate structures. *Finite Elem. Anal. Des.* **31**, 55–71

Luo, J.H.; Gea, H.C. 1998: A systematic topology optimization approach for optimal stiffener design. *Struct. Optimization* **16**, 280–288

Luo, J.H.; Gea, H.C. 2000: Topology optimization of structures with geometrical nonlinearities. *ASME 2000 Design Engineering Technical Conferences*, DETC00/DAC-14266

Ma, Z.D.; Kikuchi, N.; Hagiwara, I. 1993: Structural topology and shape optimization for a frequency response problem. *Comput. Mech.* **13**, 157–174

Mayer, R.R.; Kikuchi, N.; Scott, R.A. 1996: Application of topological optimization techniques to structural crashworthiness, *Int. J. Numer. Meth. Eng.*. **39**, 1383–1403

Suzuki, K; Kikuchi, N. 1991: A homogenization method for shape and topology optimization. *Computer Methods Appl. M.* **93**, 291–318

Yang, R.J.; Chen, C.J.; Lee, C.H. 1996: Bead pattern optimization. *Struct. Optimization* **12**, 217–221