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Optimal fastener pattern design considering bearing loads

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Abstract In many engineering structures, failure occurs either at the connection itself or in the component at the point of attachment of the connection. To extend the service life of the structure it is important to ensure that the loads borne by the connections are distributed as uniformly as possible. This would also minimize the possibility of localized high stress regions within the component. In this work a topology optimization based approach has been developed to incorporate fastener load constraints into a problem formulated for optimal location of fasteners. The computational results indicate that it is effective in reducing the maximum fastener loads without compromising on the overall stiffness of the structure.

1 Introduction

Structural failure is a major concern for engineers at the initial design phase and during testing, field trials and actual use. Factors like wear and tear, corrosion are often contributors to structural failure, but since they depend on the actual working conditions the structure is subjected to, they rarely play an important role in the design process. Some of the possible causes of structural failure that are influenced by the actual design of the structure are:

- poor adaptability of the structure to functional requirements;
- Inadequate stiffness: for example, machine tools should be able to hold tolerances and prevent chattering, or certain components have undesirable natural frequencies;
- failure of structural connections.

In many engineering structures, failure occurs either at the connection itself or in the component at the point of attachment of the connection. This is usually a result of high stresses at the connection and stress concentrations in the component at fastener locations, whenever the structure is subjected to loading. One factor that contributes to this is improper location of the connections, that is certain areas of the joint may require more connections than others. Also the design of the individual components itself determines which connections take a greater proportion of the load. In order to extend the service life of the structure it is important to ensure that the loads borne by the connections are distributed as uniformly as possible. This would also minimize the possibility of localized high stress regions within the component.

2 Topology optimization-based framework

The past decade has witnessed many significant developments in the area of application of topology optimization methods to structural design. However, the focus has been on single structural components. To optimize the performance of a larger system comprising of a number of structural components and connections, the entire system design must be performed simultaneously. The subject of optimal design of structural connections has been gaining interest from researchers (Jiang and Chirehdast 1996; Johanson *et al.* 1994; Yang *et al.* 1996) as it has considerable influence on the structural performance and on the manufacturing cost. The work presented here is part of a larger effort to extend the scope of topology optimization methods to the design of multicomponent structural systems. In earlier work (Chickermane and Gea 1995), a method was developed for the optimal location of structural connections and to perform component and connection level topology optimization simultaneously. A crucial component of this is the modelling of the interconnections and definition of design domains for them. Within these design domains the problem of optimal fastener location is posed in a manner similar to that of a material redistribution problem. This method is applicable to many different types of connections like rivets, fasteners, spots welds and adhesive patches.

The focus of this paper is on addressing the issue of failure of structural connections by ensuring that this is considered early in the design process. A two-pronged strategy of reducing the maximum fastener loads by determining optimal locations for the fasteners and ensuring a more even distribution of loads among the fasteners has been employed.

2.1 Formulation of an optimization problem

While the optimal layout designs obtained using a mean compliance formulation provide the maximum structural stiffness, certain connections are subjected to very high stresses. These are prime candidates for failure by various modes, for instance by shear failure of the connection, tensile failure, compressive or crushing (bearing) failure or shearing failure of the plate. To tackle this issue, it is necessary to consider the distribution of loads among the connections while determining their optimal locations. The approach employed can be summarized as follows.

- Identification of the most likely mode of failure. The connection may fail either due to high tensile loads or high shear loads or a combination of both.
- Performing a structural analysis to find the connections most likely to fail, that is, the fasteners with the highest loads are found.
- These connections are targeted by placing constraints on the loads being transmitted by them within the optimization problem.
- During the optimization process, fasteners are redistributed accordingly, thereby ensuring that the fastener loads in the final design are within limits.
- This would also lead to a more even load distribution among the connections and minimize the possibility of localized high stress regions in the components.

Every fastener is modelled by rigid springs in six directions to simulate the effect of the axial and shear stiffness. The spring acting in the direction of the external loading is responsible for the resistance of the fastener in that direction. The force acting on the fastener is equal to the force exerted by the fastener on the primary and secondary components. For a particular fastener i, the load transferred in direction j is

$$F_{ij} = k_{ij}(u_{1ij} - u_{2ij}), \tag{1}$$

where u_{1ij} and u_{2ij} are the primary and secondary structure displacements at the point of attachment, respectively, and k_{ij} is the fastener stiffness. Fasteners having the maximum loads are targeted and appropriate limits are placed on them based on results obtained from previous designs. The fastener load constraints are then introduced into the optimization problem as constraints. The problem is defined as

minimize mean compliance of the system,

subject to

constraint on the number of fasteners used,

constraints on maximum fastener loads.

The design variables in the optimization problem defined above are *densities* of the fasteners. This is a way of converting a problem of optimal location of fasteners, which is a discrete optimization problem into one with continuous variables. Design variables are assigned to all possible fastener locations, and to begin with all the fasteners are uniformly distributed among them. During the optimization process the fasteners are redistributed in way that minimizes an energy criterion.

A density function derived from a microstructure-based model for optimal layout design of structures by Gea (1996) has been used here. The discretized elements are made of a *composite* material which consists of spherical micro-inclusions (Fig. 1) embedded in solid material (referred to as the matrix).

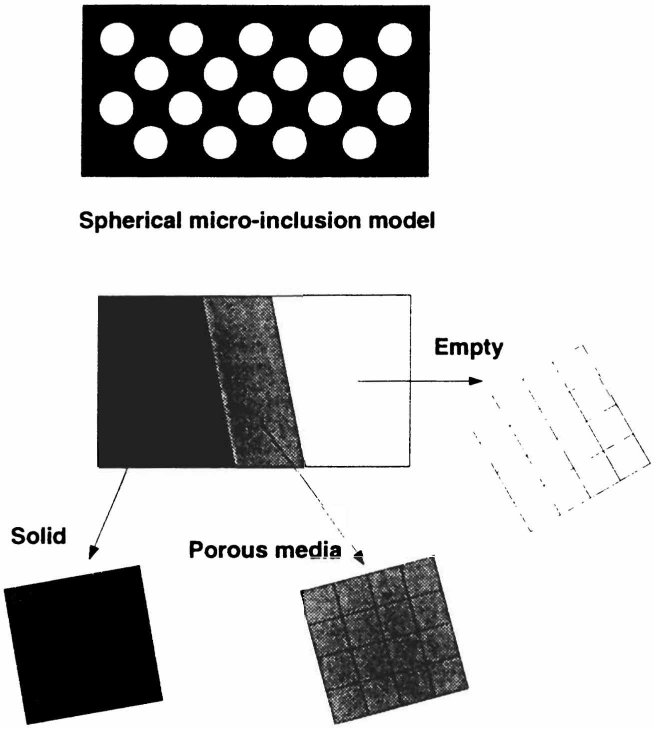

Fig. 1. Microstructure-based model

The choice of spherical inclusions in an isotropic matrix ensures that the shape and orientation of the inclusion do not enter the problem formulation. The derivation of the density function is based on the work by Weng (1984) In this paper, the effective material properties are obtained using the Mori-Tanaka mean field theory (Mori and Tanaka 1973) in conjunction with Eshelby's equivalence principle and his solution for an ellipsoidal inclusion. This model is applicable for finite void concentrations unlike other schemes for dilute concentrations based solely on Eshelby's solution (Eshelby 1957) for spherical inclusions. For a composite material with a volume fraction c_0 of the matrix (or phase 0) and c_1 of the spherical inclusion (referred to as phase 1), the effective material properties of the composite are

$$\frac{\kappa}{\kappa_0} = 1 + \frac{(1-c_0)(\kappa_1-\kappa_0)}{c_0\alpha_0(\kappa_1-\kappa_0)+\kappa_0}, \tag{2}$$

$$\frac{\mu}{\mu_0} = 1 + \frac{(1-c_0)(\mu_1-\mu_0)}{c_0\beta_0(\mu_1-\mu_0)+\mu_0}, \tag{3}$$

with

$$\alpha_0 = \frac{1}{3}\frac{1+\nu_0}{1-\nu_0}, \tag{4}$$

$$\beta_0 = \frac{2}{15}\frac{4-5\nu_0}{1-\nu_0}, \tag{5}$$

here ν_0 is the Poisson's ratio of the matrix. Since the inclusions are voids κ_1 and μ_1 are set to zero.

The advantage of this method is that it is derived from the rigorous formulation of the theory of composite materials. A closed-form expression for the density function can be derived which is relatively simple to use. The stiffness of a fastener i in direction j, $(j = 1, \ldots, 6)$ is related to its density ρ_i using the density function $f(\rho_i)$ as

$$k_i = k_0 f(\rho_i) = k_0 \frac{\rho_i}{2 - \rho_i}\,, \tag{6}$$

where k_0 is the fastener stiffness coefficient. This provides a convenient way of mapping the fastener densities in the optimization problem to the strength of the fasteners in the structural model. Mathematically, the problem becomes

$$\text{minimize} \quad \ell(u) = \int_{\Gamma} \mathbf{f}\mathbf{u}\, \mathrm{d}\Gamma\,, \tag{7}$$

subject to

$$\int \rho\, \mathrm{d}\Omega \leq N\,, \tag{8}$$

$$F_{ij} \leq \overline{F}\,, \tag{9}$$

where ℓ is the mean compliance of the structural system subjected to a traction $\mathbf{f}$ on its boundary Γ and $\mathbf{u}$ are the nodal displacements; N and $\overline{F}$ are the upper limits on fastener density and bearing loads, respectively.

2.2 Design sensitivity analysis

The objective is to evaluate the effect of the change in strength of any fastener ℓ on the load F_{ij} transferred by fastener i in direction j,

$$\frac{\partial F_{ij}}{\partial x_\ell} = \frac{\partial k_{ij}}{\partial x_\ell}(u_{1ij} - u_{2ij}) + k_{ij}\left(\frac{\partial u_{1ij}}{\partial x_\ell} - \frac{\partial u_{2ij}}{\partial x_\ell}\right)\,, \tag{10}$$

$$\frac{\partial k_{ij}}{\partial x_\ell} = 0\,, \quad \text{if} \quad i \neq \ell\,. \tag{11}$$

The variables x are the design variables, that is ρ. The static equilibrium of the structural system is expressed by

$$[K]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = [F]\,. \tag{12}$$

Differentiating with respect to x_ℓ and rearranging

$$\frac{\partial}{\partial x_\ell}[K]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + [K]\begin{bmatrix} \dfrac{\partial u_1}{\partial x_\ell} \\ \dfrac{\partial u_2}{\partial x_\ell} \end{bmatrix} = 0\,, \tag{13}$$

$$\begin{bmatrix} \dfrac{\partial u_1}{\partial x_\ell} \\ \dfrac{\partial u_2}{\partial x_\ell} \end{bmatrix} = -[K]^{-1}\frac{\partial}{\partial x_\ell}[K]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\,. \tag{14}$$

To extract the sensitivities of u_1 and u_2 corresponding to a particular fastener load F_{ij}, premultiply by a matrix with unit vectors in both rows,

$$\begin{bmatrix} e_1^{ij\,T} \\ e_2^{ij\,T} \end{bmatrix}\begin{bmatrix} \dfrac{\partial u_1}{\partial x_\ell} \\ \dfrac{\partial u_2}{\partial x_\ell} \end{bmatrix} = - \begin{bmatrix} e_1^{ij\,T} \\ e_2^{ij\,T} \end{bmatrix}[K]^{-1}\frac{\partial}{\partial x_\ell}[K]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\,. \tag{15}$$

The product of the first two matrices on the right-hand sides is the displacements due to a unit load applied at the nodes in consideration; $[K]$ contains the contributions of structural elements and fasteners. Therefore, $[K]$ is a function of fastener densities only, and

$$\frac{\partial}{\partial x_\ell}[K] = \frac{\partial f(x_\ell)}{\partial x_\ell}k_0^e\begin{bmatrix} I & -I \\ -I & I \end{bmatrix}\begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}\,, \tag{16}$$

where $[u_1^e,\, u_2^e]^T$ is the nodal displacement vector for fastener ℓ and k_0^e is its stiffness,

$$\begin{bmatrix} \dfrac{\partial u_{1ij}}{\partial x_\ell} \\ \dfrac{\partial u_{2ij}}{\partial x_\ell} \end{bmatrix} = - \begin{bmatrix} \lambda_{11}^{ij\,T} & \lambda_{12}^{ij\,T} \\ \lambda_{21}^{ij\,T} & \lambda_{22}^{ij\,T} \end{bmatrix}\frac{\partial f(x_\ell)}{\partial x_\ell}k_0^e\begin{bmatrix} I & -I \\ -I & I \end{bmatrix}\begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix} = \tag{17}$$

$$-\frac{\partial f(x_\ell)}{\partial x_\ell}k_0^e\begin{bmatrix} (\lambda_{11}^{ij\,T} - \lambda_{12}^{ij\,T})(u_1^e - u_2^e) \\ (\lambda_{21}^{ij\,T} - \lambda_{22}^{ij\,T})(u_1^e - u_2^e) \end{bmatrix}\,, \tag{18}$$

where λ_{11}^{ij} and λ_{12}^{ij} are primary and secondary displacements due to a unit dummy load on the primary structure, and λ_{21}^{ij} and λ_{22}^{ij} are primary and secondary displacements due to a unit dummy load on the secondary structure.

For any general loading condition, the sensitivity of the mean compliance is found in the paper by Chickermane and Gea (1997). If the fasteners connect the component to a ground structure (that is absolutely rigid) then in all the above equations u_2 can be taken as zero. Once the sensitivities of the objective function and constraints have been computed, the generalized convex approximation (Chickermane and Gea 1996) is used to formulate the optimization problem.

The overall implementation of the method can be summarized as follows.

1. Set up a finite element model of the structure and analyse.
2. Use nodal displacements to compute mean compliance, sensitivity of mean compliance and fastener loads.
3. Determine the fasteners having the maximum loads.

4. Apply unit dummy loads at all fastener locations on primary and secondary and perform an analysis.
5. Compute sensitivity of the fastener loads to fastener strengths.
6. Formulate the approximated problem using the generalized convex approximation and solve by mathematical programming.

3 Examples

3.1 Square plate

The square plate in Fig. 2 is to be fixed to a support structure using fasteners. The applied loading shown is such that the fasteners are subjected to in-plane shear loads due to P (P_y =1000 N) in addition to a bending moment M (M_x =2000 N.mm). The size of the plate is 200 mm × 200 mm. The starting design in Fig. 3 consists of a total of 7 fasteners distributed uniformly throughout the design domain consisting of a total of 225 possible locations. The optimal location of fasteners obtained without fastener load constraints is shown in Fig. 4. From the analysis results it is observed that the fastener shear loads transmitted in the y direction dominates in comparison to the loads in the other directions.

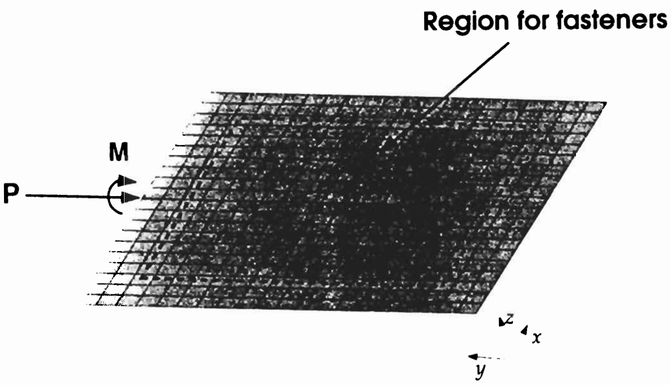

Fig. 2. 3-D view of the plate

The optimization problem is modified to include the constraints on the fastener loads. The objective is to minimize the mean compliance of the plate with constraints on the number of fasteners and load constraints. However, since only a few of these constraints would be active, the number of load constraints is equal to the total number of fasteners. In this case since there are a total of 7 fasteners available for distribution, 7 load constraints are introduced into the optimization problem. The upper limit chosen for the constraint is about 30% less than the maximum fastener load observed in the old design. In the optimal design obtained for this problem, shown in Fig. 5, the fasteners in the lowermost row in the previous design have moved down one more row. From a comparison of the fastener loads in these two situations seen in Table 1, the following observations can be made.

- Maximum fastener loads have reduced by nearly 30%. There is an increase of only 0.53% in the mean compliance even with all the fastener loads kept within limits.

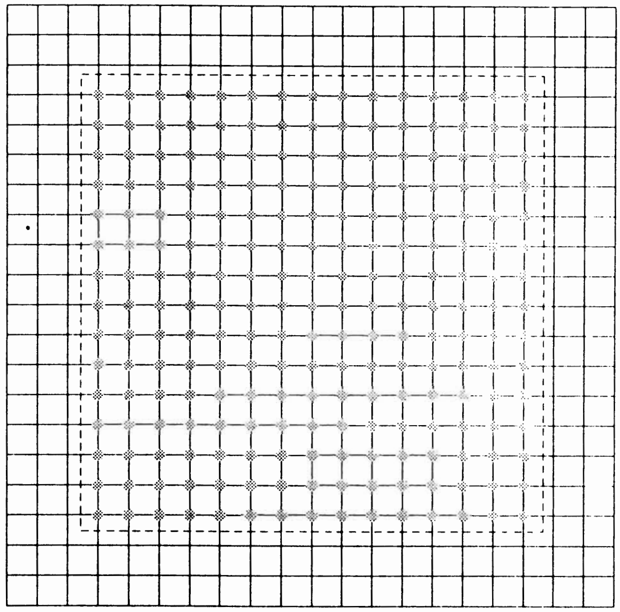

Fig. 3. Initial design

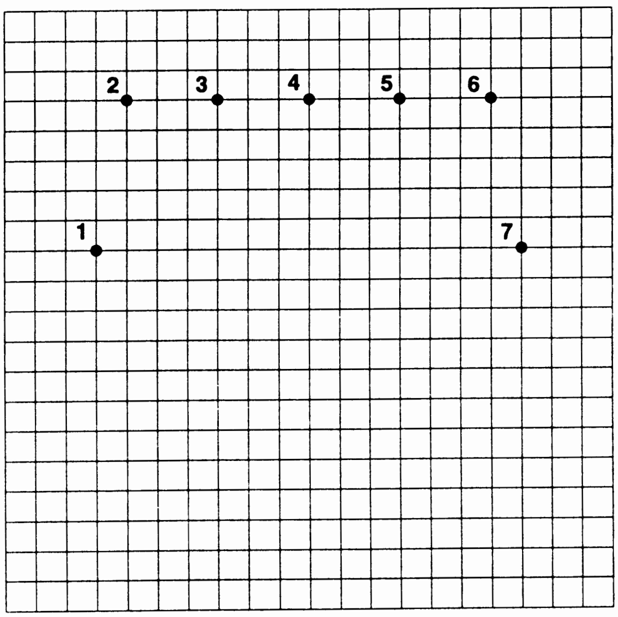

Fig. 4. Location of fasteners on the plate *without* fastener load constraints

- A more even load distribution between the fasteners has been obtained. This is because the difference between the highest and lowest fastener loads has decreased.

In the final design, two fasteners of intermediate density are observed between fasteners 1 and 7. One reason for this is that this is the only way the optimizer can satisfy the upper limit on the fastener load. One possible way of eliminating them would be to allow some slight relaxation in the upper bound on the loads.

In many design situations we are interested in using the minimum number of connections while we want to avoid the

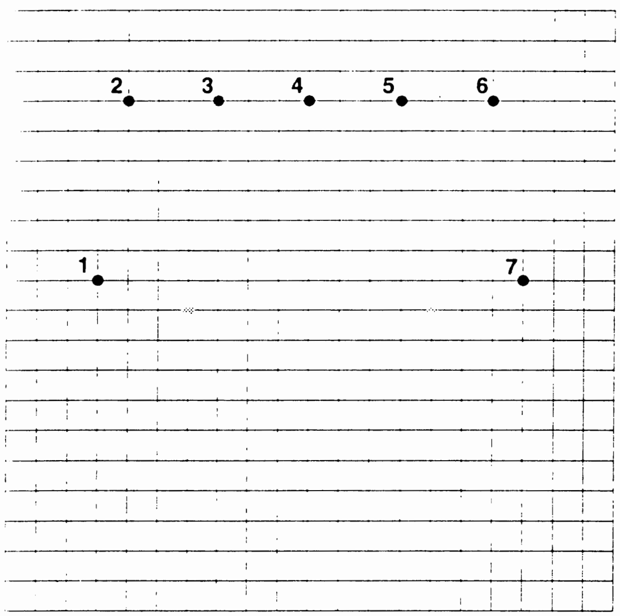

Fig. 5. Location of fasteners *using* fastener load constraints

Table 1. Comparison of fastener shear loads in the final designs

Fastener location	Fastener shear loads (N) without load constraints	Fastener shear loads (N) with load constraints
3, 5	238.29	180.00
2,6	166.91	179.99
1,7	86.29	112.62
4	16.98	54.39
Mean compliance	5815.65 N.mm	5846.97 N.mm

failure of the connection due to high stresses. The problem formulation would be

minimize total number of fasteners,

subject to

$$F_{ij} \leq \overline{F}, \quad j = 1, \ldots, m,$$

where F_{ij} are the fastener load constraints placed on a total of m fasteners and $\overline{F}$ is the maximum permissible loads.

However, this method does not work well without an additional constraint on the mean compliance of the system. This is because while reducing fastener densities would lead to large nodal displacements, as the fastener stiffnesses near zero, the fastener loads are very small and well within limits. On trying this formulation, we confirmed that the total number of fasteners go to zero in the final design. With the addition of a constraint on the maximum permissible mean compliance the problem formulation becomes

minimize total number of fasteners,

subject to

mean compliance $\ell \leq \overline{\ell}, \quad F_{ij} \leq \overline{F}, \quad j = 1, \ldots, m.$

Here $\overline{\ell}$ is the upper limit on the mean compliance. Two things need to be ensured before using this formulation. Firstly, the starting solution must have a large number of fasteners uniformly distributed to ensure a feasible starting design. Secondly, the upper bound $\overline{\ell}$ for the mean compliance must be carefully chosen in order to ensure feasible designs are available.

For this problem, the value of $\overline{\ell}$ was taken to be 5850 N.mm based on the optimal design obtained from the first formulation. Figure 6 shows the iteration history of the objective function: total number of fasteners and the constraints: mean compliance and fastener loads. The starting design chosen has 10 fasteners but this design is infeasible as the mean compliance and the fastener loads are above the upper bounds. In the very next iteration, the number of fasteners increases to bring the design into the feasible design space. Corresponding to this, substantial drops can be seen in the mean compliance and the maximum fastener load. Thereafter both the mean compliance and the fastener loads remain within bounds. After 10 iterations, the process was stopped and in this design the total fastener density is 6.5. The reason for this is that, the objective is to minimize the number of fasteners and during the optimization it is not possible to ensure that this value is an integral value. Consequently in Fig. 7 there are more than 7 fasteners many of which have intermediate densities.

3.2 Automotive bracket

An automotive bracket is used as the second example for this study. The largest component of the applied load, P (P_x = 17527.0 N, P_y = -1406.0 N, P_z = -66.275 N) is parallel to the bottom surface as shown in Fig. 8. The design domain for fastener locations is on the flanges of the bracket within the region enclosed by the dashed lines. In the first run, the problem formulation used was to minimize the mean compliance of the structure with a constraint only on the number of fasteners used (specified as 4). There are a total of 208 possible fastener locations. The optimal location of the fasteners is shown in the Fig. 8. The nature of the applied force is such that the x component is the largest. The problem formulation was modified to include constraints on the shear load on the fasteners to reduce the possibility of shear failure at those locations. The resulting design is shown in Fig. 9. A comparison of the shear loads before and after the introduction of fastener load constraints is shown in Table 2. The actual locations of the fasteners are different in the two designs, so they are numbered in order of decreasing shear loads. As can be observed in the second case, there is considerable redistribution of shear loads among the fasteners as the difference between the minimum and maximum fastener shear loads has decreased.

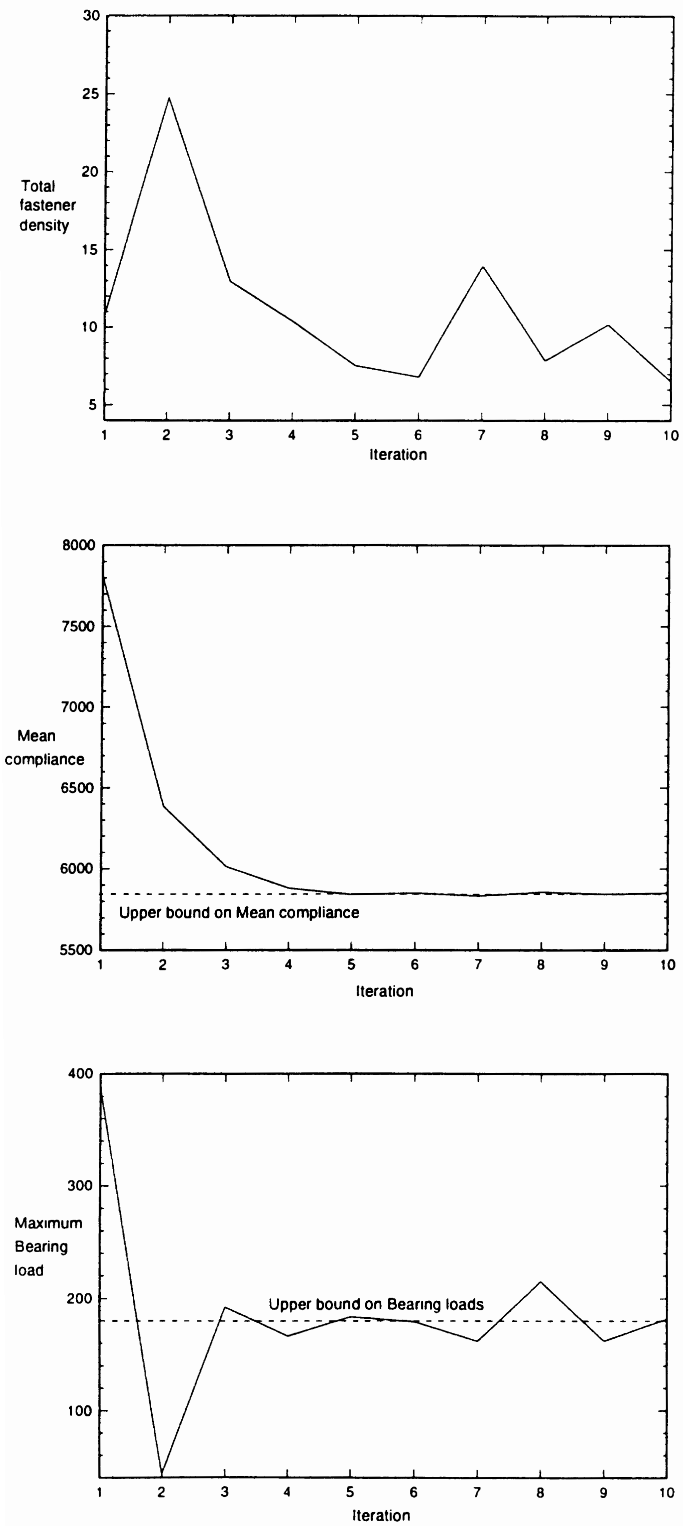

Fig. 6. Iteration history plots for the total fastener density, mean compliance and maximum bearing load

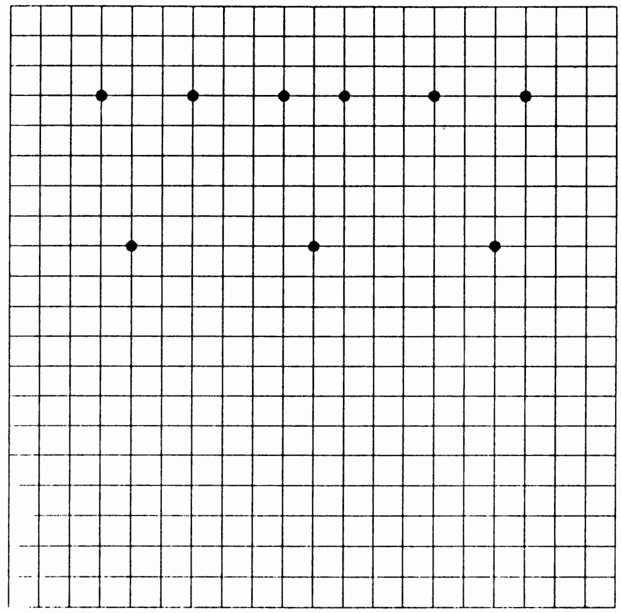

Fig. 7. Design obtained using the second formulation for fastener loads

Table 2. Comparison of fastener shear loads in the final designs

Fastener number	Fastener shear loads (N) without load constraints	with load constraints
1	9336.96	5584.09
2	4902.26	4487.27
3	4026.39	4369.29
4	717.72	3155.32

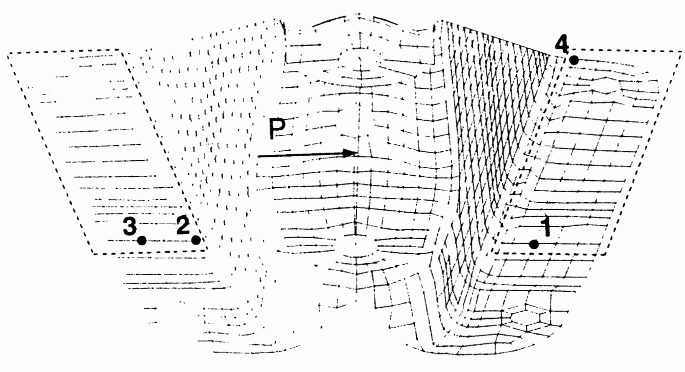

Fig. 8. Optimal design obtained without considering load constraints; the black dots represent the locations of fasteners and the design domain for fasteners is enclosed by dashed lines

4 Concluding remarks

This paper presents a method for the incorporation of fastener load constraints into a methodology for the optimal topology design of structural systems. The overall stiffness of the structural system is not compromised due to the decrease in the fastener loads. It is observed that the difference between the maximum and minimum loads is reduced due to a redistribution of fastener loads. This leads to a more uniform load distribution among the fasteners and minimizes the possibility of localized high stress regions in the components and failure of the fasteners.

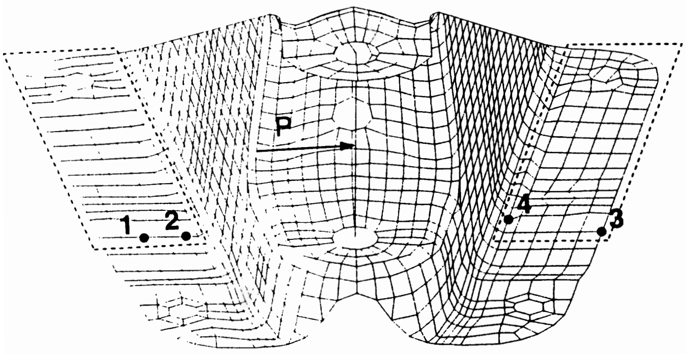

Fig. 9. Optimal design obtained using load constraints; the black dots represent the locations of fasteners and the design domain for fasteners is enclosed by dashed lines

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