

**Philosophical Issues in Quantified Modal Logic**  
Handout 9: Bennett's Two Axes of Actualism  
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First, we must give a little background about the semantics of 'actual' in a two-dimensional modal logic (this is not crucial, but it's important stuff to know anyway)

- (1) It is possible that everyone actually asleep would be awake.
- (2)  $\diamond \forall x(\text{Asleep}(x) \rightarrow \text{Awake}(x))$
- (3)  $\forall x(\text{Asleep}(x) \rightarrow \diamond(\text{Awake}(x)))$

(2) and (3) are inadequate translations of (1). (2) is inadequate because (1) could be true and (2) seems like it is most definitely false. (3) is inadequate, because it does not require, as a natural reading of (1) does, that there is a single possibility in which everyone actually asleep is awake in that possibility.

Solution: Introduce a distinguished world @, for the actual world, into every model. Define an operator 'A' (for 'actually'), with the following semantics:

'AΦ' is true at w if and only if Φ is true at @.

Consequences:

If Φ is true in the actual world, then '□AΦ' is true (Φ is true at every world).

Consequence: It is not a contingent matter what is actually the case. If Φ is actually true, then it is necessary that Φ is actually true.

Intuition: in some sense of "contingent", it is contingent what is actually the case.

Response: Modify the semantics a bit, and introduce an operator "F", for "fixedly". The truth of a formula is now taken with respect to two worlds – a world of evaluation, and a world of context (to use David Kaplan's terminology). In an ordered pair of worlds  $\langle w, w' \rangle$ , the first will be the world of evaluation and the second the world of context (the actual world). Then "F" has the following semantics:

'FΦ' is true at  $\langle w, w' \rangle$  if and only if for all  $w^*$ , Φ is true at  $\langle w, w^* \rangle$ .

Intuitively, 'FixedlyΦ' is true at a world if and only if Φ is true whatever one takes as the actual world.

Some results:

If Φ contains no occurrences of "A", then 'FΦ' is true at  $\langle w, w' \rangle$  if and only if Φ is true at  $\langle w, w' \rangle$ .

(If  $\Phi$  contains no occurrences of “A”, it won’t depend on its truth on the second world index)

With the two-dimensional semantic evaluation clauses in place, we can revise the rule for “actually”:

‘ $A\Phi$ ’ is true at  $\langle w, w' \rangle$  if and only if  $\Phi$  is true at  $\langle w', w' \rangle$ .

So the function of “actually” is to turn the evaluation world into the context world.

With the use of “F” and “A” we can introduce a new notion of necessity: ‘FA’, ‘Fixedly actual’. The following is a consequence of the semantic clause for ‘F’ together with the semantic clause for ‘A’:

‘ $FA\Phi$ ’ is true at  $\langle w, w' \rangle$  iff for all  $w^*$ ,  $\Phi$  is true at  $\langle w^*, w^* \rangle$

Notice that if  $\Phi$  contains no occurrences of ‘A’, ‘ $FA\Phi$ ’ is equivalent to ‘ $\Box\Phi$ ’. But if  $\Phi$  does contain an occurrence of ‘A’, the equivalence fails.

In particular, ‘ $A\Phi \rightarrow \Box A\Phi$ ’ is valid, but ‘ $A\Phi \rightarrow FA(A\Phi)$ ’ is not.

If  $\Phi$  is a contingent sentence like “Jason is wearing a shirt”, then “ $FA(\text{Jason is wearing a shirt})$ ” will be false, whereas “Necessarily Actually (Jason is wearing a shirt)” is true.

O.k., so that’s enough background on the semantics of “actual”.

What are the ‘two axes’ of actualism?

First axis: How should the actualist treat claims about things that merely could exist?

Second axis: How should one regard the modal status of actualism, and how should one think about the ‘actual’ in actualism?

As will become clear, I’m not so clear about the second axis.

Main important preliminary point: don’t understand the quantifier ‘everything’ in ‘everything is actual’ as a world-restricted quantifier, on pains of trivializing actualism.

First axis of actualism:

Domain inclusion actualism vs. non-domain inclusion actualism

The domain inclusion actualist thinks that the domain of every world is included in our world. Linksy and Zalta are obvious examples of domain inclusion actualists; on one reading of him, Williamson is another. ‘Merely possible’ objects exist in this world, but are somehow lacking in the kind of reality you and I possess. Perhaps (as Linksy and Zalta would have it) they are abstract rather than concrete here.

“...domain inclusion actualism requires either flat-out rejecting the possibility of aliens, or else adopting some version of the proxy strategy.”

Non-domain inclusion actualism: things that merely possibly exist do not exist at all.

Ersatzist line: Worlds are actually existing representational devices. “...while actualism requires that everything that is a literal constituent of any world – everything that does the representing—actually exist, it does not require that everything that exists according to any world actually exist.”

Bennett proceeds to deny that Kripke semantics with varying domains is inconsistent with actualism (section 5, “Interlude: Actualism and Kripke Semantics”).

An argument that Kripke semantics is inconsistent with actualism (Bennett, section 5):

- (a) Some  $D(w)$  contains things that are not contained in  $D(@)$ .
- (b) Each  $D(w)$  is a subset of  $D$ .
- (c) The universal quantifier in the slogan “everything is actual” ranges over  $D$ .

Bennett suggests that the actualist (the non-domain inclusion actualist) can deny either (b) or (c).

I don’t understand her brief motivation for denying (b). One problem with denying (b) is that it would complicate the semantics. I’m not quite sure right now how bad the complications would be. In the standard variable-domain Kripke semantics, the assignment functions assign members of  $D$  to variables. If the different domains are not subsets of  $D$ , then we would get the truth-conditions of modal formulae wrong. So we would have to relativize assignment functions to worlds. This seems to result in complications – there doesn’t seem to be a straightforward way of doing this.

Suppose our definition of truth-in-a-world was ‘ $\Phi$  is true at  $w$  if and only if  $\Phi$  is satisfied by all sequences  $s$  of  $w$ ’, where the sequences at a world only assign objects in that world to variables. Now suppose that there is nothing in this world that could be  $F$ , though there could have been an  $F$ . It’s not clear to me how to write the satisfaction clause for ‘ $s$  satisfies ‘ $\diamond\exists xFx$ ’ at  $w$ ’ in such a way as to make this true. Perhaps the modal operator also shifts assignments – but how does it shift the assignment  $s$ ? To what assignment? It can’t just be to an arbitrary assignment. For suppose Dan is childless. It is true that ‘There is someone such that there could have been someone who was the child of Dan and her’. So the assignment one switches to must agree on the original assignment on what it assigns to the pronoun ‘her’. Anyway, there are difficulties here.

Bennett gives a nod to something like this – at least she recognizes the need to complicate the formal semantics. She writes: “Taking this approach requires complicating the formal semantics by adding some sort of representation function from  $D$  onto  $D(w)$ ”. But I’m not sure what she means here, and it’s not the problem of relativizing assignment functions to worlds.

I just don't understand what Bennett has in mind when she talks of D and the D(w)s being associated by "some sort of representational function".

Another problem Bennett mentions with the strategy of denying (b) is that "It also requires being comfortable with the thought that all the various D(w)s are sets of nonexistent things."

But I'm not sure why Bennett thinks this is an option. All actualists, even Plantinga, think that a set only exists if its members exist. So there is no actualist I'm aware of who could be comfortable with this thought. That seems to be a very serious objection against this approach.

The second option Bennett suggests involves denying (c). She suggests two different ways to deny (c). The first way would be to "interpret D as the set of things that any world represents as existing, claim that the various D(w)s are indeed subsets of it—and instead deny that 'everything' in the slogan ["everything is actual"] ranges over it."

But this again sticks us with the problem of a set existing which contains non-existent members.

Bennett favors a second way of denying (c). It is to say that the slogan 'everything is actual' cannot be stated in the semantics.

I am not happy about this. This would be a substantial restriction on the semantics, akin to the objections to Tarskian approaches to the Liar Paradox.

Bennett concludes that "...there are several ways the non-domain-inclusion actualist can tweak the standard Kripke semantics to suit her purposes. Where exactly she chooses to fiddle is, I think, largely a matter of bookkeeping."

I am not happy with this conclusion. Bennett has suggested three ways to challenge the argument that Kripke semantics is inconsistent with possibilism – actually, she has more like gestured at the possibility of some ways of denying the argument. None of them seem promising. Only one of them (denying (b)) involves "tweaking" the standard Kripke semantics, and it's completely unclear how to tweak. I am therefore befuddled about the comment about "largely a matter of bookkeeping".

Bennett concludes that the "domain inclusion understanding...*cannot be stated within Kripke semantics*. Either the semantics must be modified by the addition of a representation function, or else we must accept that the slogan is a piece of presemantic stage direction."

All of this is puzzling to me. Perhaps my main confusion here concerns Bennett's suggestion that "the semantics must be modified by the addition of a representation function." What is it to modify a semantics by the addition of a representation function? Is this a trivial thing to do? Difficult to do? Impossible to do?

And what is meant by ‘presemantic stage direction’? How are we to take this? Perhaps the actualist slogan is some kind of Wittgensteinian ‘nonsense’ claim?

I must confess that I am one of these people who do not “see these moves”. Discussion?

Onto the second axis:

Is actualism true, or is actualism fixedly true?

w-ism: the thesis of actualism, whatever it is, is true whichever world w is actual.

@-ism: the thesis of actualism, whatever it is, is true given that this world @ is the actual world.

World-hopping objection to actualism (one version of this is Lewis’s objection to linguistic ersatzism in section 3.2 of On the Plurality of Worlds). Consider a world w in which Jeff King does not exist. Relative to that world, it should be possible that Jeff King exists (since Jeff King exists in our world). But there can’t be de re possibilities about non-existent things, so it is not possible in that world that Jeff King exists.

One response: deny symmetry. It is not possible in that world that Jeff King exists, because although that world is accessible from our world, our world is not accessible from that world.

Another response (presumably this would be Jeff King’s method to rescue himself from necessary non-existence): Make a distinction between two senses of “possible”. In one sense of “possible” (the one defined in terms of truth-in), it is not possible that Jeff King exists in w. In another sense of “possible” (the one defined in terms of truth-at), it is possible that Jeff King exists in w.

Bennett doesn’t consider this.

Another response: Deny the analogy. Relative to w, it is possible that Jeff King exists, because @ is *special*.

This is an application of @-ism. Perhaps it is also a version of the truth-in/truth-at distinction.

Note that it is exactly the conclusion that we reached when we discussed section 3.2 of On the Plurality of Worlds.

So I think Bennett is correct that this is an advantage of @-ism over w-ism.

Onto worries with @-ism.

Two worries with @-ism, which Bennett thinks show that it is “hopeless”

(a) Possibility is not the same as possible actuality.

I’m not feeling this worry. Why should possibility be synonymous with possible actuality? What definition of “actual” and “possible” should lead us to accept this?

“Actual” as an intensifier vs. “actual” as a modal operator.

Second objection:

“@-ism undercuts the motivation for an otherwise very attractive line about [the McMichael problem]”

(1)  $\diamond \exists x(Fx \ \& \ \diamond Gx)$

The problem: Non-domain inclusion actualism cannot account for genuinely de re modal truths about things that do not exist at all.

Bennett’s purportedly attractive line: All such statements are false.

Bennett’s motivation: sentences like (1) are genuinely de re modal claims about non-existents. No such statements should be true in the actual world.

But this seems odd to me. (1) feels like a general statement, rather than a singular proposition. The variable in the scope of the second quantifier is bound, albeit by a quantifier outside its scope. I’m not feeling the motivation for Bennett’s purportedly attractive line [develop this in class – make Will Starr say something about it.]

Anyway, Bennett’s purportedly attractive line on the McMichael problem is not supposed to be consistent with @-ism, because @-ism responds to the world-hopping problem by allowing there to be true de re modal claims in worlds in which the objects don’t exist. So the @-ist is precluded from responding in this way to the McMichael problem by her response to the world-hopping objection.

I’m not compelled by this, because I’m not compelled by “the otherwise very attractive line” about the McMichael problem. But I have other worries besides. As Elizabeth Miller has pointed out to me, it’s not clear it’s consistent with Bennett’s defense of the consistency of Kripke semantics with modal actualism. If you have Kripke semantics in place, then you can make the McMichael sentences come out true.