Notes on Karttunen & Peters 1979 (Intensional System)
Extensional Fragment Recast in Ty_2

1. PRESUPPOSITIONAL NOUNS AND VERBS REVISITED

Abbreviatory convention: In what follows, for any curried function A from n arguments $B_1, \ldots B_n$ to a proposition (i.e., to a set of indices characterized by a function of type st), we write $A_i\ B_1 \ldots B_n$ for $((\ldots(A\ B_1)\ldots B_n)i)$. That is, for example, we write $male_i\ john$ for $((male\ john)i)$, and $like_i\ mary\ john$ for $(((like\ mary)\ john)\ i)$.

John likes Mary, 4
John | like Mary, 5
like | Mary

Basic meanings

Denotation of 'John' is ⟨**A**(John), **P**(John)⟩, where
A(John) = $\lambda P_{e(st)}\ P\ john$
P(John) = $\lambda P_{e(st)}\ \lambda i\ male_i\ john$

Denotation of 'Mary' is ⟨**A**(Mary), **P**(Mary)⟩, where
A(Mary) = $\lambda P_{e(st)}\ P\ mary$
P(Mary) = $\lambda P_{e(st)}\ \lambda i\ female_i\ mary$

Denotation of 'like' is ⟨**A**(like), **P**(like)⟩, where
A(like) = $\lambda Q_{(e(st))(st)}\ \lambda x_e\ (Q\ \lambda y_e\ \lambda i\ like_i\ yx)$
P(like) = $\lambda Q_{(e(st))(st)}\ \lambda x_e\ (Q\ \lambda y_e\ \lambda i\ acquainted\_with_i\ yx)$

Semantic composition

By RULE 5 (TV + T), 'like Mary' denotes ⟨**A**(like Mary), **P**(like Mary)⟩, where

A(like Mary) = **A**(like)**A**(Mary)
= $\lambda Q_{(e(st))(st)} \lambda x_e\ (Q\ \lambda y_e\ \lambda i\ like_i\ yx)\ \lambda P_{e(st)}\ P\ mary$
= $\lambda x_e\ \lambda i\ like_i\ mary\ x$

P(like Mary) = $\lambda x_e\ \lambda i\ (\mathbf{P}(\text{like})\mathbf{A}(\text{Mary})xi \wedge \exists P_{e(st)}\ \mathbf{P}(\text{Mary})Pi)$
= $\lambda x_e\ \lambda i\ ((\lambda Q_{(e(st))(st)} \lambda x_e\ (Q\ \lambda y_e\ \lambda i\ acquainted\_with_i\ yx)\ \lambda P_{e(st)}\ P\ mary)\ xi$
$\wedge\ \exists P_{e(st)}\ (\lambda P_{e(st)}\ \lambda i\ (female_i\ mary)\ Pi))$
= $\lambda x_e\ \lambda i\ (acquainted\_with_i\ mary\ x$
$\wedge\ female_i\ mary)$

By RULE 4 (T + IV), 'John likes Mary' denotes ⟨**A**(John likes Mary), **P**(John likes Mary)⟩, where

A(John likes Mary) = **A**(John)**A**(likes Mary)
= $\lambda P_{e(st)}\ (P\ john)\ \lambda x_e\ \lambda i\ like_i\ mary\ x$
= $\lambda i\ like_i\ mary\ john$

P(John likes Mary) = $\lambda i\ (\mathbf{P}(\text{John})\mathbf{A}(\text{like Mary})i \wedge \mathbf{A}(\text{John})\mathbf{P}(\text{like Mary})i)$
= $\lambda i\ ((\lambda P_{e(st)}\ \lambda i\ (male_i\ john)\ \lambda x_e\ \lambda i\ like_i\ mary\ x)i$
$\wedge\ (\lambda P_{e(st)}\ (P\ john)\ \lambda x_e\ \lambda i\ (acquainted\_with_i\ mary\ x \wedge female_i\ mary))i)$
= $\lambda i\ (male_i\ john$
$\wedge\ acquainted\_with_i\ mary\ john \wedge female_i\ mary)$

APPENDIX: EXTENSIONAL FRAGMENT OF KARTTUNEN & PETERS 1979 RECAST IN TY_2

Revise the *basic translations* as in the above sample derivation and, in each of the following *compositional rules*, leave the syntax unchanged while revising the translation as follows:

RULE 2 for $[\text{every } \zeta_{CN}]_T$, $[\text{a } \zeta_{CN}]_T$, $[\text{the } \zeta_{CN}]_T$:

$\mathbf{A}(\text{every } \zeta) = \lambda P_{e(st)} \lambda i\ \forall x_e(\mathbf{A}(\zeta)xi \rightarrow Pxi)$

$\mathbf{P}(\text{every } \zeta) = \lambda P_{e(st)} \lambda i\ \exists x_e(\mathbf{A}(\zeta)xi \wedge \mathbf{P}(\zeta)xi)$

$\mathbf{A}(\text{a } \zeta) = \lambda P_{e(st)} \lambda i\ \exists x_e(\mathbf{A}(\zeta)xi \wedge Pxi)$

$\mathbf{P}(\text{a } \zeta) = \lambda P_{e(st)} \lambda i\ \exists x_e\ \mathbf{P}(\zeta)xi$

$\mathbf{A}(\text{the } \zeta) = \lambda P_{e(st)} \lambda i\ \exists x_e(\mathbf{A}(\zeta)xi \wedge \forall z_e(\mathbf{A}(\zeta)xi \rightarrow z = x) \wedge Pxi)$

$\mathbf{P}(\text{the } \zeta) = \lambda P_{e(st)} \lambda i\ \exists x_e(\mathbf{A}(\zeta)xi \wedge \forall z_e(\mathbf{A}(\zeta)xi \rightarrow z = x) \wedge \mathbf{P}(\zeta)xi)$

RULE 4 for $[\alpha_T\ prs(\delta_{IV})]_S$:

$\mathbf{A}(\alpha\ prs(\delta)) = \mathbf{A}(\alpha)\mathbf{A}(\delta)$

$\mathbf{P}(\alpha\ prs(\delta)) = \lambda i\ (\mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$

RULE 5 for $[\delta_{TV}\ acc(\beta_T)]_{IV}$:

$\mathbf{A}(\delta\ acc(\beta)) = \mathbf{A}(\delta)\mathbf{A}(\beta)$

$\mathbf{P}(\delta\ acc(\beta)) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge \exists P_{e(st)}\ \mathbf{P}(\beta)Pi)$

RULE 11 for $[\text{either } \phi_S \text{ or } \psi_S]_S$:

$\mathbf{A}(\text{either } \phi \text{ or } \psi) = \lambda i\ (\mathbf{A}(\phi)i \vee \mathbf{A}(\psi)i)$

$\mathbf{P}(\text{either } \phi \text{ or } \psi) = \lambda i\ ((\mathbf{P}(\phi)i \vee \mathbf{A}(\psi)i) \wedge (\mathbf{A}(\phi)i \vee \mathbf{P}(\psi)i))$

RULE 14.n for $[\phi_S[\alpha_T/\text{he}_n]]_S$:

$\mathbf{A}(\phi[\alpha/\text{he}_n]) = \mathbf{A}(\alpha)\ \lambda x_n\ \mathbf{A}(\phi)$

$\mathbf{P}(\phi[\alpha/\text{he}_n]) = \lambda i\ ((\mathbf{P}(\alpha)\ \lambda x_n\ \mathbf{A}(\phi))i \wedge (\mathbf{A}(\alpha)\ \lambda x_n\ \mathbf{P}(\phi))i)$

TOO RULE$_n$ for $[\phi_S[\alpha_T/\text{he}_n] \text{ too}]_S$:

$\mathbf{A}(\phi[\alpha/\text{he}_n] \text{ too}) = \mathbf{A}(\alpha)\ \lambda x_n\ \mathbf{A}(\phi)$

$\mathbf{P}(\phi[\alpha/\text{he}_n] \text{ too}) = \lambda i\ ((\mathbf{P}(\alpha)\ \lambda x_n\ \mathbf{A}(\phi))i \wedge (\mathbf{A}(\alpha)\ \lambda x_n\ \mathbf{P}(\phi))i$
$\wedge\ (\mathbf{A}(\alpha)\ \lambda y_e\ \lambda i\ \exists z_e(\neg\ z = y \wedge (\lambda x_n\ \mathbf{A}(\phi)\ z)i))i)$

RULE 17 for $[\alpha_T\ neg(\delta_{IV})]_S$, $[\alpha_T\ NEG(\delta_{IV})]_S$:

$\mathbf{A}(\alpha\ neg(\delta)) = \neg\mathbf{A}(\alpha)\mathbf{A}(\beta)$ *ordinary negation*

$\mathbf{P}(\alpha\ neg(\delta)) = \lambda i\ (\mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$

$\mathbf{A}(\alpha\ NEG(\delta)) = \lambda i\ \neg(\mathbf{A}(\alpha)\mathbf{A}(\beta)i \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$ *contradiction negation*

$\mathbf{P}(\alpha\ NEG(\delta)) = \lambda i\ \top$

More Notes on Karttunen & Peters 1979 (Intensional System)
Full Intensional Theory Recast in Ty_2

1. ITERATIVE ADVERBS REVISITED

the unicorn is pregnant too, TOO_7

the unicorn, 2 he$_7$ is pregnant, 4

unicorn he$_7$ be_pregnant

Basic meanings

The denotation of 'unicorn' is $\langle \mathbf{A}(\text{unicorn}), \mathbf{P}(\text{unicorn})\rangle$, where

$\mathbf{A}(\text{unicorn}) = \lambda y_e\, \lambda i\, unicorn_i\, y$

$\mathbf{P}(\text{unicorn}) = \lambda y_e\, \lambda i\, \top$

The denotation of 'be_pregnant' is $\langle \mathbf{A}(\text{be\_pregnant}), \mathbf{P}(\text{be\_pregnant})\rangle$, where

$\mathbf{A}(\text{be\_pregnant}) = \lambda y_e\, \lambda i\, pregnant_i\, y$

$\mathbf{P}(\text{be\_pregnant}) = \lambda y_e\, \lambda i\, female_i\, y$

The denotation of 'he$_7$' is $\langle \mathbf{A}(\text{he}_7), \mathbf{P}(\text{he}_7)\rangle$, where

$\mathbf{A}(\text{he}_7) = \lambda P_{e(st)}\, P x_7$

$\mathbf{P}(\text{he}_7) = \lambda P_{e(st)}\, \lambda i\, \top$

Semantic composition

By RULE 2 (D + CN), 'the unicorn' denotes $\langle \mathbf{A}(\text{the unicorn}), \mathbf{P}(\text{the unicorn})\rangle$, where

$$\begin{aligned}\mathbf{A}(\text{the unicorn}) &= \lambda P_{e(st)}\, \lambda i\, \exists y_e(\mathbf{A}(\text{unicorn})yi \wedge \forall z_e(\mathbf{A}(\text{unicorn})zi \rightarrow z = y) \wedge Pyi)\\ &= \lambda P_{e(st)}\, \lambda i\, \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge Pyi)\end{aligned}$$

$$\begin{aligned}\mathbf{P}(\text{the unicorn}) &= \lambda P_{e(st)}\, \lambda i\, \exists y_e(\mathbf{A}(\text{unicorn})yi \wedge \forall z_e(\mathbf{A}(\text{unicorn})zi \rightarrow z = y) \wedge \mathbf{P}(\text{unicorn})yi)\\ &= \lambda P_{e(st)}\, \lambda i\, \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y))\end{aligned}$$

By RULE 4 (T + IV), 'he$_7$ is pregnant denotes $\langle \mathbf{A}(\text{he}_7 \text{ is pregnant}), \mathbf{P}(\text{he}_7 \text{ is pregnant})\rangle$, where

$$\begin{aligned}\mathbf{A}(\text{he}_7 \text{ is pregnant}) &= \mathbf{A}(\text{he}_7)\mathbf{A}(\text{be\_pregnant})\\ &= \lambda P_{e(st)}\,(Px_7)\; \lambda y_e\, \lambda i\,(pregnant_i\, y)\\ &= \lambda y_e\, \lambda i\,(pregnant_i\, y)\, x_7\\ &= \lambda i\, pregnant_i\, x_7\end{aligned}$$

$$\begin{aligned}\mathbf{P}(\text{he}_7 \text{ is pregnant}) &= \lambda i\,(\mathbf{P}(\text{he}_7)\mathbf{A}(\text{be\_pregnant})i \wedge \mathbf{A}(\text{he}_7)\mathbf{P}(\text{be\_pregnant})i)\\ &= \lambda i\,((\lambda\, P_{e(st)}\, \lambda i\, \text{¡}\; \lambda y_e\, \lambda i\,(pregnant_i\, y))i \wedge (\lambda P_{e(st)}\,(Px_7)\; \lambda y_e\, \lambda i\, female_i\, y)i)\\ &= \lambda i\,(\text{¡} \wedge female_i\, x_7 i)\\ &= \lambda i\, female_i\, x_7\end{aligned}$$

By TOO RULE$_7$ (analogous to the EVEN RULE on p. 52 of K&P 79), 'the unicorn is pregnant too' denotes $\langle \mathbf{A}(\text{the unicorn is pregnant too}), \mathbf{P}(\text{the unicorn is pregnant too})\rangle$, where

$$\begin{aligned}\mathbf{A}(\text{the unicorn is pregnant too}) &= \mathbf{A}(\text{the unicorn})\; \lambda x_7\, \mathbf{A}(\text{he}_7 \text{ is pregnant})\\ &= \lambda P_{e(st)}\, \lambda i\, \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge Pyi)\\ &\qquad \lambda x_7\, \lambda i\, pregnant_i\, x_7\\ &= \lambda i\, \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge pregnant_i\, y)\end{aligned}$$

$$\begin{aligned}\mathbf{P}(\text{the unicorn is pregnant too}) &= \lambda i\,((\mathbf{P}(\text{the unicorn})\; \lambda x_7\, \mathbf{A}(\text{he}_7 \text{ is pregnant}))i\\ &\qquad \wedge (\mathbf{A}(\text{the unicorn})\; \lambda x_7\, \mathbf{P}(\text{he}_7 \text{ is pregnant}))i\\ &\qquad \wedge (\mathbf{A}(\text{the unicorn})\, \lambda y_e\, \lambda i\, \exists z_e(\neg\, z = y \wedge (\lambda x_7\, \mathbf{A}(\text{he}_7 \text{ is pregnant})z)i))i)\\ &= \lambda i\,(\exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y)\\ &\qquad \wedge \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge female_i\, y)\\ &\qquad \wedge \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge \exists z_e(\neg\, z = y \wedge pregnant_i\, z)))\\ &= \lambda i\, \exists y_e(unicorn_i\, y \wedge \forall z_e(unicorn_i\, z \rightarrow z = y) \wedge female_i\, y\\ &\qquad \wedge \exists x_e(\neg\, x = y \wedge pregnant_i\, x))\end{aligned}$$

Note. In what follows we use the following abbreviations:

S_0	:=	the unicorn is pregnant	
uni	:=	*unicorn*	
prg	:=	*pregnant*	
fem	:=	*female*	
bel	:=	*believe*	
$\beta_j(\alpha_2, \alpha_1)$	:=	$(((\beta\alpha_1)\alpha_2)j)$	for any β of type $a_1(a_2(st))$, α_1 of type a_1, α_2 of type a_2, j of type s

2. KNOW-TYPE VERBS ('HOLES')

Kim knows that the unicorn is pregnant too, 4

Kim   know that the unicorn is pregnant too, 7.KNOW

know   the unicorn is pregnant too, PART 1

<u>Basic meanings</u>

The denotation of 'know' is $\langle \mathbf{A}(\text{know}), \mathbf{P}(\text{know})\rangle$, where

$$\mathbf{A}(\text{know}) = \lambda p_{st}\, \lambda x_e\, \lambda i\; know_i(x, p)$$
$$\mathbf{P}(\text{know}) = \lambda p_{st}\, \lambda x_e\, \lambda i\; pi$$

The denotation of 'Kim' is $\langle \mathbf{A}(\text{Kim}), \mathbf{P}(\text{Kim})\rangle$, where

$$\mathbf{A}(\text{Kim}) = \lambda P_{e(st)}\, P\; kim$$
$$\mathbf{P}(\text{Kim}) = \lambda P_{e(st)}\, \lambda i\; \top$$

<u>Semantic composition</u>

By PART 1 above, S_0 denotes $\langle \mathbf{A}(S_0), \mathbf{P}(S_0)\rangle$, where

$$\mathbf{A}(S_0) = \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y)$$
$$\mathbf{P}(S_0) = \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))$$

So by RULE 7.KNOW (IV/S that S), 'know that S_0' denotes $\langle \mathbf{A}(\text{know that } S_0), \mathbf{P}(\text{know that } S_0)\rangle$, where

$$\begin{aligned}\mathbf{A}(\text{know that } S_0) &= \mathbf{A}(\text{know})\mathbf{A}(S_0)\\ &= \lambda p_{st}\, \lambda x_e\, \lambda i\; know_i(x, p)\; \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y)\\ &= \lambda x_e\, \lambda i\; know_i(x, \lambda j\; \exists y_e(uni_j\, y \wedge \forall z_e(uni_j\, z \to z = y) \wedge prg_j\, y))\end{aligned}$$

$$\begin{aligned}\mathbf{P}(\text{know that } S_0) &= \lambda x_e\, \lambda i\; (\mathbf{P}(\text{know})\mathbf{A}(S_0)xi \wedge \mathbf{P}(S_0)i)\\ &= \lambda x_e\, \lambda i\; ((\lambda p_{st}\, \lambda x_e\, \lambda i\; (pi)\; \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y)))xi\\ &\qquad \wedge \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))i)\\ &= \lambda x_e\, \lambda i\; (\exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y\\ &\qquad \wedge \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x)))\\ &= \lambda x_e\, \lambda i\; (\exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y\\ &\qquad \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))\end{aligned}$$

Finally, by RULE 4 (T *prs*(IV)), 'Kim knows that S_0' denotes
$\langle \mathbf{A}(\text{Kim knows that } S_0), \mathbf{P}(\text{Kim knows that } S_0)\rangle$, where

$$\begin{aligned}\mathbf{A}(\text{Kim knows that } S_0) &= \mathbf{A}(\text{Kim})\mathbf{A}(\text{know that } S_0)\\ &= \lambda P_{e(st)}\, (P\; kim)\; \lambda x_e\, \lambda i\; know_i(x, \lambda j\; \exists y_e(uni_j\, y \wedge \forall z_e(uni_j\, z \to z = y) \wedge prg_j\, y))\\ &= \lambda i\; know_i(kim, \lambda j\; \exists y_e(uni_j\, y \wedge \forall z_e(uni_j\, z \to z = y) \wedge prg_j\, y))\end{aligned}$$

$$\begin{aligned}\mathbf{P}(\text{Kim knows that } S_0) &= \lambda i\; (\mathbf{P}(\text{Kim})\mathbf{A}(\text{know that } S_0)i \wedge \mathbf{A}(\text{Kim})\mathbf{P}(\text{know that } S_0)i)\\ &= \lambda i\; ((\lambda P_{e(st)}\, \lambda i\; ¡\; \lambda x_e\, \lambda i\; know_i(x, \lambda j\; \exists y_e(uni_j\, y \wedge \forall z_e(uni_j\, z \to z = y) \wedge prg_j\, y)))i\\ &\qquad \wedge (\lambda P_{e(st)}\, (P\; kim)\; \lambda x_e\, \lambda i\; (\exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y\\ &\qquad\qquad \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x)))i)\\ &= \lambda i\; (\top \wedge \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y\\ &\qquad\qquad \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x)))\\ &= \lambda i\; \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))\end{aligned}$$

3. CLAIM-TYPE VERBS ('PLUGS')

Kim claims that the unicorn is pregnant too, 4

Kim claim that the unicorn is pregnant too, 7.CLAIM

claim the unicorn is pregnant too, PART 1

Basic meanings

The denotation of 'claim' is $\langle \mathbf{A}(\text{claim}), \mathbf{P}(\text{claim})\rangle$, where

$\mathbf{A}(\text{claim}) = \lambda p_{st}\, \lambda x_e\, \lambda i\ claim_i(x, p)$

$\mathbf{P}(\text{claim}) = \lambda p_{st}\, \lambda x_e\, \lambda i\ \top$

The denotation of 'Kim' is as in PART 2, $\langle \mathbf{A}(\text{Kim}), \mathbf{P}(\text{Kim})\rangle$, where

$\mathbf{A}(\text{Kim}) = \lambda P_{e(st)}\, P\ kim$

$\mathbf{P}(\text{Kim}) = \lambda P_{e(st)}\, \lambda i\ \top$

Semantic composition

By PART 1 above, S_0 denotes $\langle \mathbf{A}(S_0), \mathbf{P}(S_0)\rangle$, where

$\mathbf{A}(S_0) = \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)$

$\mathbf{P}(S_0) = \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge fem_i\ y \wedge \exists x_e(\neg\, x = y \wedge prg_i\ x))$

So by RULE 7.CLAIM (IV//S that S), 'claim that S_0' denotes $\langle \mathbf{A}(\text{claim that } S_0), \mathbf{P}(\text{claim that } S_0)\rangle$, where

$\mathbf{A}(\text{claim that } S_0) = \mathbf{A}(\text{claim})\mathbf{A}(S_0)$
$= \lambda p_{st}\, \lambda x_e\, \lambda i\ claim_i(x, p)$
$\quad \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)$
$= \lambda x_e\, \lambda i\ claim_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$

$\mathbf{P}(\text{claim that } S_0) = \lambda x_e\, \lambda i\ \mathbf{P}(\text{claim})\mathbf{A}(S_0)xi$
$= \lambda x_e\, \lambda i\ (\lambda p_{st}\, \lambda x_e\, \lambda i\ ¡\ \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y))xi$
$= \lambda x_e\, \lambda i\ \top$

Finally, by RULE 4 (T *prs*(IV)), 'Kim claims that S_0' denotes
$\langle \mathbf{A}(\text{Kim claims that } S_0), \mathbf{P}(\text{Kim claims that } S_0)\rangle$, where

$\mathbf{A}(\text{Kim claims that } S_0) = \mathbf{A}(\text{Kim})\mathbf{A}(\text{claim that } S_0)$
$= \lambda P_{e(st)}\, (P\ kim)\ \lambda x_e\, \lambda i\ claim_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$
$= \lambda i\ claim_i(kim, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$

$\mathbf{P}(\text{Kim claims that } S_0) = \lambda i\ (\mathbf{P}(\text{Kim})\mathbf{A}(\text{claim that } S_0)i \wedge \mathbf{A}(\text{Kim})\mathbf{P}(\text{claim that } S_0)i)$
$= \lambda i\ ((\lambda P_{e(st)}\, \lambda i\ ¡\ \lambda x_e\, \lambda i\ claim_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y)))i$
$\quad \wedge (\lambda P_{e(st)}\, (P\ kim)\ \lambda x_e\, \lambda i\ ¡)i)$
$= \lambda i\ (\top \wedge \top)$
$= \lambda i\ \top$

4. HOPE-TYPE VERBS

Kim hopes that the unicorn is pregnant too, 4

Kim   hope that the unicorn is pregnant too, 7.HOPE

hope   the unicorn is pregnant too, PART 1

<u>Basic meanings</u>

The denotation of 'hope' is ⟨**A**(hope), **P**(hope)⟩, where

A(hope) $= \lambda p_{st}\, \lambda x_e\, \lambda i\ hope_i(x, p)$

P(hope) $= \lambda p_{st}\, \lambda x_e\, \lambda i\ \top$

The denotation of 'Kim' is as in PART 2, ⟨**A**(Kim), **P**(Kim)⟩, where

A(Kim) $= \lambda P_{e(st)}\, P\ kim$

P(Kim) $= \lambda P_{e(st)}\, \lambda i\ \top$

<u>Semantic composition</u>

By PART 1 above, S_0 denotes ⟨**A**(S_0), **P**(S_0)⟩, where

A(S_0) $= \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)$

P(S_0) $= \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge fem_i\ y \wedge \exists x_e(\neg\ x = y \wedge prg_i\ x))$

So by RULE 7.HOPE (IV///S that S), 'hope that S_0' denotes ⟨**A**(hope that S_0), **P**(hope that S_0)⟩, where

A(hope that S_0)
$= \mathbf{A}(\text{hope})\mathbf{A}(S_0)$
$= \lambda p_{st}\, \lambda x_e\, \lambda i\ hope_i(x, p)$
$\quad \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)$
$= \lambda x_e\, \lambda i\ hope_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$

P(hope that S_0)
$= \lambda x_e\, \lambda i\ (\mathbf{P}(\text{hope})\mathbf{A}(S_0)xi \wedge believe_i(x, \mathbf{P}(S_0)))$
$= \lambda x_e\, \lambda i\ (\lambda p_{st}\, \lambda x_e\, \lambda i\ ¡\ \lambda i\ \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y))xi$
$\quad \wedge bel_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x))))$
$= \lambda x_e\, \lambda i\ (¡$
$\quad \wedge bel_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x))))$
$= \lambda x_e\, \lambda i\ bel_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x)))$

Finally, by RULE 4 (T *prs*(IV)), 'Kim hopes that S_0' denotes
⟨**A**(Kim hopes that S_0), **P**(Kim hopes that S_0)⟩, where

A(Kim hopes that S_0)
$= \mathbf{A}(\text{Kim})\mathbf{A}(\text{hope that } S_0)$
$= \lambda P_{e(st)}\, (P\ kim)\ \lambda x_e\, \lambda i\ hope_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$
$= \lambda i\ hope_i(kim, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y))$

P(Kim hopes that S_0)
$= \lambda i\ (\mathbf{P}(\text{Kim})\mathbf{A}(\text{hope that } S_0)i \wedge \mathbf{A}(\text{Kim})\mathbf{P}(\text{hope that } S_0)i)$
$= \lambda i\ ((\lambda P_{e(st)}\, \lambda i\ ¡\ \lambda x_e\, \lambda i\ hope_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge prg_j\ y)))i$
$\quad \wedge (\lambda P_{e(st)}\, (P\ kim)\ \lambda x_e\, \lambda i\ bel_i(x, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y$
$\quad \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x)))i)$
$= \lambda i\ (¡ \wedge bel_i(kim,$
$\quad \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x)))$
$= \lambda i\ bel_i(kim, \lambda j\ \exists y_e(uni_j\ y \wedge \forall z_e(uni_j\ z \rightarrow z = y) \wedge fem_j\ y \wedge \exists x_e(\neg\ x = y \wedge prg_j\ x)))$

5. MODALS

perhaps the unicorn is pregnant too, 9

perhaps the unicorn is pregnant too, PART 1

Basic meanings

A(perhaps) $= \lambda p_{st}\, \lambda i\, \exists j(v_{0,\, s(st)}ij \wedge pj)$
P(perhaps) $= \lambda p_{st}\, \lambda i\, \top$

Semantic composition

By PART 1 above, S_0 denotes $\langle \mathbf{A}(S_0), \mathbf{P}(S_0)\rangle$, where

A(S_0) $= \lambda i\, \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y)$
P(S_0) $= \lambda i\, \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))$

So by RULE 9 (S/S S), 'perhaps S_0' denotes $\langle \mathbf{A}(\text{perhaps } S_0), \mathbf{P}(\text{perhaps } S_0)\rangle$, where

A(perhaps S_0) $= \mathbf{A}(\text{perhaps})\mathbf{A}(S_0)$
$= \lambda p_{st}\, \lambda i\, \exists j(v_{0,\, s(st)}ij \wedge pj)$
$\quad \lambda i\, \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y)$
$= \lambda i\, \exists j(v_{0,\, s(st)}ij \wedge \exists y_e(uni_j\, y \wedge \forall z_e(uni_j\, z \to z = y) \wedge prg_j\, y)$

P(perhaps S_0) $= \lambda i\, (\mathbf{P}(\text{perhaps})\mathbf{A}(S_0)i \wedge \mathbf{P}(S_0)i)$
$= \lambda i\, (\top \wedge \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x)))$
$= \lambda i\, \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))$

6. CONDITIONALS

Abbreviations: S_0 = the unicorn is pregnant too, S_1 = Kim is right, S_{-1} = the centaur is pregnant, *ctr* = *centaur*

- if the unicorn is pregnant too then Kim is right, 11 *projection from antecedent*

the unicorn is pregnant too, PART 1 Kim is right, 4

Kim be_right

Basic meanings

A(Kim) $= \lambda P_{e(st)}\, P\ kim$
P(Kim) $= \lambda P_{e(st)}\, \lambda i\, \top$

A(be_right) $= \lambda y_e\, \lambda i\, be\_right_i\, y$
P(be_right) $= \lambda y_e\, \lambda i\, \top$

Semantic composition

By RULE 4 (T *prs*(IV)), 'Kim is right' denotes $\langle \mathbf{A}(S_1), \mathbf{P}(S_1)\rangle$, where

A(S_1) $= \mathbf{A}(\text{Kim})\mathbf{A}(\text{be\_right})$
$= \lambda i\, be\_right_i\ kim$

P(S_1) $= \lambda i\, (\mathbf{P}(\text{Kim})\mathbf{A}(\text{be\_right})i \wedge \mathbf{A}(\text{Kim})\mathbf{P}(\text{be\_right})i)$
$= \lambda i\, \top$

So by PART 1 (for S_0) and RULE 11, 'if S_0 then S_1' denotes $\langle \mathbf{A}(\text{if } S_0 \text{ then } S_1), \mathbf{P}(\text{if } S_0 \text{ then } S_1)\rangle$, where

A(if S_0 then S_1) $= \lambda i\, (\mathbf{A}(S_0)i \to \mathbf{A}(S_1)i)$
$= \lambda i\, (\exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge prg_i\, y) \to be\_right_i\ kim)$

P(if S_0 then S_1) $= \lambda i\, (\mathbf{P}(S_0)i \wedge (\mathbf{A}(S_0)i \to \mathbf{P}(S_1)i))$
$= \lambda i\, (\exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))$
$\quad \wedge (\mathbf{A}(S_0)i \to \top))$
$= \lambda i\, \exists y_e(uni_i\, y \wedge \forall z_e(uni_i\, z \to z = y) \wedge fem_i\, y \wedge \exists x_e(\neg\, x = y \wedge prg_i\, x))$

- if Kim is right then the unicorn is pregnant too, 11 *projection from consequent*

  Kim is right, 4 the unicorn is pregnant too, PART 1

  Kim be_right

  <u>Semantic composition</u> (key point)

  $$\mathbf{A}(\text{if } S_1 \text{ then } S_0) = \lambda i\, (\mathbf{A}(S_1)i \rightarrow \mathbf{A}(S_0)i)$$
  $$= \lambda i\, (be\_right_i\ kim \rightarrow \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)))$$

  $$\mathbf{P}(\text{if } S_1 \text{ then } S_0) = \lambda i\, (\mathbf{P}(S_1)i \wedge (\mathbf{A}(S_1)i \rightarrow \mathbf{P}(S_0)i))$$
  $$= \lambda i\, (¡ \wedge (be\_right_i\ kim \rightarrow \mathbf{P}(S_1)i))$$
  $$= \lambda i\, (be\_right_i\ kim \rightarrow \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge fem_i\ y \wedge \exists x_e(\neg\, x = y \wedge prg_i\ x)))$$

- if the centaur is pregnant then the unicorn is pregnant too, 11 *presup. of consequent filtered by antecedent*

  the centaur is pregnant the unicorn is pregnant too, PART 1

  <u>Semantic composition</u> (key point)

  $$\mathbf{A}(\text{if } S_{-1} \text{ then } S_0) = \lambda i\, (\mathbf{A}(S_{-1})i \rightarrow \mathbf{A}(S_0)i)$$
  $$= \lambda i\, (\exists y_e(ctr_i\ y \wedge \forall z_e(ctr_i\ z \rightarrow z = y) \wedge prg_i\ y)) \rightarrow \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge prg_i\ y)))$$

  $$\mathbf{P}(\text{if } S_{-1} \text{ then } S_0) = \lambda i\, (\mathbf{P}(S_{-1})i \wedge (\mathbf{A}(S_{-1})i \rightarrow \mathbf{P}(S_0)i))$$
  $$= \lambda i\, (\exists y_e(ctr_i\ y \wedge \forall z_e(ctr_i\ z \rightarrow z = y) \wedge fem_i\ y) \wedge (\exists y_e(ctr_i\ y \wedge \forall z_e(ctr_i\ z \rightarrow z = y) \wedge prg_i\ y)) \rightarrow \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge fem_i\ y \wedge \exists x_e(\neg\, x = y \wedge prg_i\ x))))$$
  $$= \lambda i\, \exists y_e(ctr_i\ y \wedge \forall z_e(ctr_i\ z \rightarrow z = y) \wedge fem_i\ y \wedge (prg_i\ y \rightarrow \exists y_e(uni_i\ y \wedge \forall z_e(uni_i\ z \rightarrow z = y) \wedge fem_i\ y))))$$

APPENDIX: KARTTUNEN & PETERS 1979 RECAST IN TY_2

Revise the *basic translations* as in the above sample derivations and, in each of the following *compositional rules*, leave the syntax unchanged while revising the translation as follows:

RULE 2 for $[\text{every } \zeta_{CN}]_T$, $[\text{a } \zeta_{CN}]_T$, $[\text{the } \zeta_{CN}]_T$:

$$\mathbf{A}(\text{every } \zeta) = \lambda P_{e(st)}\, \lambda i\, \forall x_e(\mathbf{A}(\zeta)xi \rightarrow Pxi)$$
$$\mathbf{P}(\text{every } \zeta) = \lambda P_{e(st)}\, \lambda i\, \exists x_e(\mathbf{A}(\zeta)xi \wedge \mathbf{P}(\zeta)xi)$$

$$\mathbf{A}(\text{a } \zeta) = \lambda P_{e(st)}\, \lambda i\, \exists x_e(\mathbf{A}(\zeta)xi \wedge Pxi)$$
$$\mathbf{P}(\text{a } \zeta) = \lambda P_{e(st)}\, \lambda i\, \exists x_e\, \mathbf{P}(\zeta)xi$$

$$\mathbf{A}(\text{the } \zeta) = \lambda P_{e(st)}\, \lambda i\, \exists x_e(\mathbf{A}(\zeta)xi \wedge \forall z_e(\mathbf{A}(\zeta)zi \rightarrow z = x) \wedge Pxi)$$
$$\mathbf{P}(\text{the } \zeta) = \lambda P_{e(st)}\, \lambda i\, \exists x_e(\mathbf{A}(\zeta)xi \wedge \forall z_e(\mathbf{A}(\zeta)zi \rightarrow z = x) \wedge \mathbf{P}(\zeta)xi)$$

RULE 3 for $[\zeta_{CN} \text{ such that } \phi^{(n)}{}_S]_{CN}$:

$$\mathbf{A}(\zeta \text{ such that } \phi^{(n)}) = \lambda x_e\, \lambda i\, (\mathbf{A}(\zeta)xi \wedge (\lambda x_n\, \mathbf{A}(\phi))xi)$$
$$\mathbf{P}(\zeta \text{ such that } \phi^{(n)}) = \lambda x_e\, \lambda i\, (\mathbf{P}(\zeta)xi \wedge (\lambda x_n\, \mathbf{P}(\phi))xi)$$

RULE 4 for $[\alpha_T\ prs(\delta_{IV})]_S$:

$$\mathbf{A}(\alpha\ prs(\delta)) = \mathbf{A}(\alpha)\mathbf{A}(\delta)$$
$$\mathbf{P}(\alpha\ prs(\delta)) = \lambda i\ (\mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$$

RULE 5 for $[\delta_{TV}\ acc(\beta_T)]_{IV}$:

$$\mathbf{A}(\delta\ acc(\beta)) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta\ acc(\beta)) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge \exists P_{e(st)}\ \mathbf{P}(\beta)Pi)$$

RULE 6 for $[\delta_{IAV/T}\ acc(\beta_T)]_{IAV}$:

$$\mathbf{A}(\delta\ acc(\beta)) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta\ acc(\beta)) = \lambda P_{e(st)}\ \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)Pxi \wedge \exists P_{e(st)}\ \mathbf{P}(\beta)Pi)$$

RULE 7.KNOW for $[\delta_{IV/S}$ that $\beta_S]_{IV}$:

$$\mathbf{A}(\delta_{IV/S} \text{ that } \beta_S) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta_{IV/S} \text{ that } \beta_S) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge \mathbf{P}(\beta)i)$$

RULE 7.CLAIM for $[\delta_{IV//S}$ that $\beta_S]_{IV}$:

$$\mathbf{A}(\delta_{IV//S} \text{ that } \beta_S) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta_{IV//S} \text{ that } \beta_S) = \lambda x_e\ \lambda i\ \mathbf{P}(\delta)\mathbf{A}(\beta)xi$$

RULE 7.HOPE for $[\delta_{IV///S}$ that $\beta_S]_{IV}$:

$$\mathbf{A}(\delta_{IV///S} \text{ that } \beta_S) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta_{IV///S} \text{ that } \beta_S) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge believe_i\ \mathbf{P}(\beta)x)$$

RULE 8 for $[\delta_{IV//IV}$ to $\beta_{IV}]_{IV}$:

$$\mathbf{A}(\delta_{IV//IV} \text{ to } \beta_{IV}) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta_{IV//IV} \text{ to } \beta_{IV}) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge \mathbf{P}(\beta)xi)$$

RULE 9 for $[\delta_{S/S}\ \beta_S]_S$:

$$\mathbf{A}(\delta_{S/S}\ \beta_S) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\delta_{S/S}\ \beta_S) = \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)i \wedge \mathbf{P}(\beta)i)$$

RULE 10 for $[\beta_{IV}\ \delta_{IV/IV}]_{IV}$:

$$\mathbf{A}(\beta_{IV}\ \delta_{IV/IV}) = \mathbf{A}(\delta)\mathbf{A}(\beta)$$
$$\mathbf{P}(\beta_{IV}\ \delta_{IV/IV}) = \lambda x_e\ \lambda i\ (\mathbf{P}(\delta)\mathbf{A}(\beta)xi \wedge \mathbf{P}(\beta)xi)$$

RULE 11 for [if ϕ_S then ψ_S], $[\phi_S$ and $\psi_S]_S$, [either ϕ_S or $\psi_S]_S$:

$$\mathbf{A}(\text{if } \phi \text{ then } \psi) = \lambda i\ (\mathbf{A}(\phi)i \rightarrow \mathbf{A}(\psi)i)$$
$$\mathbf{P}(\text{if } \phi \text{ then } \psi) = \lambda i\ (\mathbf{P}(\phi)i \wedge (\mathbf{A}(\phi)i \rightarrow \mathbf{P}(\psi)i))$$ (simplified from K&P's (54))

$$\mathbf{A}(\phi \text{ and } \psi) = \lambda i\ (\mathbf{A}(\phi)i \wedge \mathbf{A}(\psi)i)$$
$$\mathbf{P}(\phi \text{ and } \psi) = \lambda i\ (\mathbf{P}(\phi)i \wedge (\mathbf{A}(\phi)i \rightarrow \mathbf{P}(\psi)i))$$

$$\mathbf{A}(\text{either } \phi \text{ or } \psi) = \lambda i\ (\mathbf{A}(\phi)i \vee \mathbf{A}(\psi)i)$$
$$\mathbf{P}(\text{either } \phi \text{ or } \psi) = \lambda i\ ((\mathbf{P}(\phi)i \vee \mathbf{A}(\psi)i) \wedge (\mathbf{A}(\phi)i \vee \mathbf{P}(\psi)i))$$

RULE 12 for $[\gamma_{IV} \text{ and } \delta_{IV}]_{IV}$, $[\text{either } \gamma_{IV} \text{ or } \delta_{IV}]_{IV}$:

$\mathbf{A}(\gamma \text{ and } \delta) \;=\; \lambda x_e\, \lambda i\, (\mathbf{A}(\gamma)xi \wedge \mathbf{A}(\delta)xi)$

$\mathbf{P}(\gamma \text{ and } \delta) \;=\; \lambda x_e\, \lambda i\, (\mathbf{P}(\gamma)xi \wedge (\mathbf{A}(\gamma)xi \rightarrow \mathbf{P}(\delta)xi))$

$\mathbf{A}(\text{either } \gamma \text{ or } \delta) \;=\; \lambda x_e\, \lambda i\, (\mathbf{A}(\gamma)xi \vee \mathbf{A}(\delta)xi)$

$\mathbf{P}(\text{either } \gamma \text{ or } \delta) \;=\; \lambda x_e\, \lambda i\, ((\mathbf{P}(\gamma)xi \vee \mathbf{A}(\delta)xi) \wedge (\mathbf{A}(\gamma)xi \vee \mathbf{P}(\delta)xi))$

RULE 13 for $[\alpha_T \text{ or } \beta_T]_T$:

$\mathbf{A}(\text{either } \alpha \text{ or } \beta) \;=\; \lambda P_{e(st)}\, \lambda i\, (\mathbf{A}(\alpha)Pi \vee \mathbf{A}(\beta)Pi)$

$\mathbf{P}(\text{either } \alpha \text{ or } \beta) \;=\; \lambda P_{e(st)}\, \lambda i\, ((\mathbf{P}(\alpha)Pi \vee \mathbf{A}(\beta)Pi) \wedge (\mathbf{A}(\alpha)Pi \vee \mathbf{P}(\beta)Pi))$

RULE 14.n for $[\phi_S[\alpha_T/\text{he}_n]]_S$:

$\mathbf{A}(\phi[\alpha/\text{he}_n]) \;=\; \mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{A}(\phi)$

$\mathbf{P}(\phi[\alpha/\text{he}_n]) \;=\; \lambda i\, ((\mathbf{P}(\alpha)\, \lambda x_n\, \mathbf{A}(\phi))i \wedge (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{P}(\phi))i)$

RULE 15.n for $[\zeta_{CN}[\alpha_T/\text{he}_n]]_{CN}$:

$\mathbf{A}(\zeta[\alpha/\text{he}_n]) \;=\; \lambda y_e\, (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{A}(\zeta)y)$

$\mathbf{P}(\zeta[\alpha/\text{he}_n]) \;=\; \lambda y_e\, \lambda i\, ((\mathbf{P}(\alpha)\, \lambda x_n\, \mathbf{A}(\zeta)y)i \wedge (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{P}(\zeta)y)i)$

RULE 16.n for $[\delta_{IV}[\alpha_T/\text{he}_n]]_{IV}$:

$\mathbf{A}(\delta[\alpha/\text{he}_n]) \;=\; \lambda y_e\, (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{A}(\delta)y)$

$\mathbf{P}(\delta[\alpha/\text{he}_n]) \;=\; \lambda y_e\, \lambda i\, ((\mathbf{P}(\alpha)\, \lambda x_n\, \mathbf{A}(\delta)y)i \wedge (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{P}(\delta)y)i)$

RULE 17 for $[\alpha_T\, neg(\delta_{IV})]_S$, $[\alpha_T\, NEG(\delta_{IV})]_S$, $[\alpha_T\, fut(\delta_{IV})]_S$, $[\alpha_T\, negfut(\delta_{IV})]_S$: $[\alpha_T\, prf(\delta_{IV})]_S$, $[\alpha_T\, negprf(\delta_{IV})]_S$:

$\mathbf{A}(\alpha\; neg(\delta)) \;=\; \neg\mathbf{A}(\alpha)\mathbf{A}(\beta)$ *ordinary negation*

$\mathbf{P}(\alpha\; neg(\delta)) \;=\; \lambda i\, (\mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$

$\mathbf{A}(\alpha\; NEG(\delta)) \;=\; \lambda i\, \neg(\mathbf{A}(\alpha)\mathbf{A}(\beta)i \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)i \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)i)$ *contradiction negation*

$\mathbf{P}(\alpha\; NEG(\delta)) \;=\; \lambda i\, \top$

$\mathbf{A}(\alpha\; fut(\delta)) \;=\; \lambda i\, \exists j(i \approx j \wedge i < j \wedge \mathbf{A}(\alpha)\mathbf{A}(\beta)j)$

$\mathbf{P}(\alpha\; fut(\delta)) \;=\; \lambda i\, \exists j(i \approx j \wedge i < j \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)j \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)j)$

$\mathbf{A}(\alpha\; negfut(\delta)) \;=\; \lambda i\, \neg\exists j(i \approx j \wedge i < j \wedge \mathbf{A}(\alpha)\mathbf{A}(\beta)j)$

$\mathbf{P}(\alpha\; negfut(\delta)) \;=\; \lambda i\, \neg\exists j(i \approx j \wedge i < j \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)j \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)j)$

$\mathbf{A}(\alpha\; prf(\delta)) \;=\; \lambda i\, \exists j(i \approx j \wedge i > j \wedge \mathbf{A}(\alpha)\mathbf{A}(\beta)j)$

$\mathbf{P}(\alpha\; prf(\delta)) \;=\; \lambda i\, \exists j(i \approx j \wedge i > j \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)j \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)j)$

$\mathbf{A}(\alpha\; negprf(\delta)) \;=\; \lambda i\, \neg\exists j(i \approx j \wedge i > j \wedge \mathbf{A}(\alpha)\mathbf{A}(\beta)j)$

$\mathbf{P}(\alpha\; negprf(\delta)) \;=\; \lambda i\, \neg\exists j(i \approx j \wedge i > j \wedge \mathbf{P}(\alpha)\mathbf{A}(\delta)j \wedge \mathbf{A}(\alpha)\mathbf{P}(\delta)j)$

TOO RULE$_n$ for $[\phi_S[\alpha_T/\text{he}_n] \text{ too}]_S$:

$\mathbf{A}(\phi[\alpha/\text{he}_n] \text{ too}) \;=\; \mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{A}(\phi)$

$$\mathbf{P}(\phi[\alpha/\text{he}_n] \text{ too}) \;=\; \lambda i\, ((\mathbf{P}(\alpha)\, \lambda x_n\, \mathbf{A}(\phi))i \wedge (\mathbf{A}(\alpha)\, \lambda x_n\, \mathbf{P}(\phi))i \wedge (\mathbf{A}(\alpha)\, \lambda y_e\, \lambda i\, \exists z_e(\neg\, z = y \wedge (\lambda x_n\, \mathbf{A}(\phi)\, z)i))i)$$