

**Lecture 1**

## INFERENCE: WHAT IS PRESUPPOSED, ASSERTED, OR IMPLICATED

**I. Examples of linguistic inference**

- (1) a. John is competent.  
b. John is not brilliant.
- (2) a. John is incompetent.  
b. John is not brilliant.
- (3) a. Tom got a job at Harvard and moved to Cambridge.  
b. Tom got a job at Harvard and therefore moved to Cambridge.
- (4) a. Tom got a job at Harvard and moved to Cambridge.  
b. Tom got a job at Harvard.
- (5) a. There are almost no misprints in this book.  
b. There are misprints in this book.
- (6) a. John has solved this problem.  
b. This problem has been solved.
- (7) a. It is John who has solved this problem.  
b. This problem has been solved.
- (8) a. Fred has three children.  
b. Fred doesn't have four children.
- (9) a. My space ship broke down.  
b. I have a space ship.
- (10)a. John tried to cash the check.  
b. John didn't succeed in cashing the check.
- (11)a. John managed to cash the check.  
b. John cashed the check.
- (12)a. John called Mary a Republican and then she insulted him back.  
b. Calling someone a Republican is an insult.
- (13)a. Mary has photographed every unicorn in the Catskills.  
b. There are unicorns in the Catskills.
- (14)a. This thingy is in this set or it is in that set.  
b. This thingy is not in both sets.

## II. Presupposition, entailment, and implicature: Informal characterization

(At the informal level, all of the following working definitions are equally good.)

“*A presupposes B*” means roughly:

- Any speaker asserting *A* takes it for granted, or seems to take for granted, that *B*.
- In any situation where *B* is false, *A* is infelicitous/has no clear truth value.

“*A entails B*” means roughly:

- Any speaker asserting *A* is committed to *B*.
- In any situation where *A* is true so is *B*.
- The sentence “*A and ~B*” (where “*~B*” is a denial of *B*) is contradictory.

“*A implicates B*” means roughly:

- Any speaker asserting *A* is committed to *B*, unless he explicitly denies *B*.
- The sentence “*A and ~B*” (where “*~B*” is a denial of *B*) is not contradictory.

## III. Identifying implicature: Diagnostic CANCELLATION TEST

Implicatures can be cancelled by explicit denial. In a *suitable context* the test sentence is OK, as in (15). In failed cancellation tests, e.g., (16) and (17), “□” stands for “contradiction”.

- |  |  |
|--|--|
| (15)a. John dives or skates.                                     | <i>A</i>   |
| b. John doesn't do both.   | <i>B</i>   |
| CANCELLATION TESTS:  |  |
| John dives or skates, <i>in fact</i> , he does both.             | <i>A and ~B</i>  |
| John dives or skates, <i>as a matter of fact</i> , he does both. | <i>A and ~B</i>  |
| John dives or skates, <i>indeed</i> , he does both.              | <i>A and ~B</i>  |
| CONCLUSION:  | ✓, so <i>B</i> is an <i>implicature</i> of <i>A</i><br>(or <i>A implicates B</i> ) |
|  |  |
| (16)a. John dives and skates.                                    | <i>A</i>   |
| b. John dives.   | <i>B</i>   |
| CANCELLATION TESTS:  |  |
| John dives and skates, <i>in fact</i> , he doesn't dive.         | <i>A and ~B</i>  |
| John dives and skates, <i>but</i> he doesn't dive.               | <i>A and ~B</i>  |
| ⋮  | ⋮  |
| CONCLUSION:  | □, so <i>B</i> is not an implicature   |
|  |  |
| (17)a. John talked to Bill.                                      | <i>A</i>   |
| b. John talked to somebody.                                      | <i>B</i>   |
| CANCELLATION TEST:   |  |
| John talked to Bill, <i>but</i> he didn't speak to anybody.      | <i>A and ~B</i>  |
| CONCLUSION:  | □, so <i>B</i> is not an implicature   |

#### IV. Identifying presupposition: Diagnostic PROJECTION TESTS

- Certain sentential operators — e.g., *if*, *modals of possibility*, and *negation* — remove the commitment to the entailments of the embedded sentence, but not to its presuppositions.

We say that: (a) the embedded sentence, call it *A*, is in a *non-entailing context*, and (b) the presuppositions of *A* *project* out of that context. This is the basis of the projection tests.

Suppose that asserting *A* would normally commit the speaker to *B*. Try embedding *A* under:

- *Conditional antecedent* (If): “*If A*, then *C*”
- *Modal of possibility* (Poss): “It is *possible* that *A*”
- *Negation* (Neg): “It is *not* the case that *A*”

If *B* **projects** ( $\Rightarrow$ ) out of the test sentence — i.e., embedding *A* as in the test sentence still commits the speaker to *B* — then *A* **presupposes** *B*.

If *B* doesn’t project ( $\neq \Rightarrow$ ) — the commitment to *B* is gone — then *A* doesn’t presuppose *B*. Instead, *A* either implicates *B* (if *B* can be cancelled) or entails *B* (if *B* cannot be cancelled).

- (17)a. John talked to Bill. *A*  
 b. John talked to somebody. *B*

PROJECTION TESTS:

- If*: If [John talked to Bill], then I’ll be upset.  $(if\ A, \text{ then } C) \neq \Rightarrow B$   
*Poss*: It is possible that [John talked to Bill].  $(poss\ A) \neq \Rightarrow B$   
*Neg*: It is not the case that [John talked to Bill].  $(not\ A) \neq \Rightarrow B$

CONCLUSION: *B* does not project ( $\neq \Rightarrow$ ); so *A* does not presuppose *B*.

[Above we saw that *A* also does not implicate *B* (*B* can’t be cancelled); so *A* *entails* *B*.]

- (18)a. It was Bill that John talked to. *A*  
 b. John talked to somebody. *B*

PROJECTION TESTS:

- If*: If it was Bill that John talked to, then I’ll be upset.  $(if\ A, \text{ then } C) \Rightarrow B$   
*Poss*: It is possible that it was Bill that John talked to.  $(poss\ A) \Rightarrow B$   
*Neg*: It is not the case that it was Bill that John talked to.  $(not\ A) \Rightarrow B$

CONCLUSION: *B* projects ( $\Rightarrow$ ); so *A* presupposes *B*.

- (19)a. The pizzeria in the Vatican is closed. *A*  
 b. There is a pizzeria in the Vatican. *B*

PROJECTION TESTS:

- If*: If [the pizzeria in the Vatican is closed],  
 then I’ll be upset.  $(if\ A, \text{ then } C) \Rightarrow B$   
*Poss*: It is possible that  
 [the pizzeria in the Vatican is closed].  $(poss\ A) \Rightarrow B$   
*Neg*: It is not the case that  
 [the pizzeria in the Vatican is closed].  $(not\ A) \Rightarrow B$

CONCLUSION: *B* projects ( $\Rightarrow$ ), so *A* presupposes *B*.

V. Caution: Watch out for *metalinguistic negation* (Horn 1989)

• TRUTH-CONDITIONAL vs. META-LINGUISTIC NEGATION

Diagnostic tests for implicature and presupposition only work when negation is used to *deny the truth* of the embedded sentence (as in III-IV). This is **truth-conditional** negation.

In some contexts negation is instead used **meta-linguistically**, to *object to the use* of some word by the last speaker. The criticized word, or syllable, is then intonationally focused (SMALL CAPS). The meta-linguistic comment (in quotes ‘...’ below) does not deny truth. Instead, it objects to the last speaker’s use of the focused word(s) on other grounds. The *objection* (with the negation) is typically followed by a *correction*, as in these examples:

(20) ‘Wrong pronunciation’:

*Speaker 1*: So he called the [pólis].

*Speaker 2*: He didn’t call the [PÓLIS], he called the [POLÍS].

(21) ‘Faulty morphology’:

*Speaker 1*: So, you managed to trap two moongeese.

*Speaker 2*: I didn’t manage to trap two moonGEESE — I managed to trap two moonGOOSES.

(22) ‘Rude’:

*Speaker 1*: Grandpa’s feeling lousy.

*Speaker 2*: Grandpa isn’t FEELING LOUSY, Johnny, he’s just a tad INDISPOSED.

(23) ‘Racist/sexist/bigoted/...’:

*Speaker 1*: Lee is an uppity {nigger/broad/kike/wop}.

*Speaker 2*: I beg your pardon: Lee isn’t an uppity {NIGGER/BROAD/KIKE/WOP} — (s)he’s a strong, vibrant {BLACK/WOMAN/JEW/ITALIAN}.

(24) ‘Wrong focus’:

*Speaker 1*: ...

*Speaker 2*: Ben Ward is not a black Police Commissioner but a Police Commissioner who is black. (*New York Times* editorial, 8 Jan. 1983)

(25) ‘Wrong connotations’:

*Speaker 1*: Who was the lady I saw you with last night?

*Speaker 2*: That was no LADY, that was my WIFE!

Crucially, the possible grounds for objection include ‘false implicature’ (26) or ‘false presupposition’ (27), so meta-linguistic negation is not suited for diagnostic tests:

(26) ‘Wrong implicature’:

- a. They didn’t have a BABY and get MARRIED, they got MARRIED and had a BABY.
- b. John isn’t COMPETENT, he’s BRILLIANT.

(27) ‘Wrong presupposition’:

I didn’t STOP beating my wife, I never BEAT her in the first place.

## Lecture 2

### FORMAL THEORY OF INFERENCE: *Dynamic Propositional Logic (DL<sub>0</sub>)*

#### I. Basic ideas informally

- *Possible worlds: Representing partial information*

Many kinds of conversation can be modelled as talk about how the world might be. The participants take for granted—or behave as if they did—certain information: e.g., that they are now at Rutgers, that it's a winter day in January, that Rutgers is a university, etc.

The facts may happen to be that way, but we could easily imagine things being otherwise. Formally, we can represent the various possible states of the world we can imagine and talk about as (*possible*) *worlds*:  $w_0, w_1, w_2$ , etc. NOTE: Talking does not imply existence, so this way of formalizing *how we talk* does **not** amount to a claim that there are 'parallel realities'.

- *Context: Set of worlds considered 'live candidates' for reality*

If you have no information whatsoever, then for all you know you could be in any possible world. The set of all logically possible worlds is called the *logical space*:  $W = \{w_0, w_1, \dots\}$ .

More realistically, at different stages in the conversation the participants all agree that they inhabit such-and-such a non-empty subset—current *common ground*—of  $W$ . For now, we identify the *context* with the common ground. The goal of the conversation is to gain information, by eliminating 'candidate realities', but still have some 'live options' at the end.

- *Proposition: Set of worlds where so-and-so is true. Entailment as superset.*

Suppose the speaker says: 'I am tired and hungry.' If the speaker is John, then he has thereby expressed a proposition which is true in just those (logically possible) worlds where he (John) is tired and hungry at the time when he says this. Likewise for other declarative sentences, which intuitively can be true or false (in contrast to questions: 'Are you hungry?').

We say that when a speaker utters a declarative sentence he expresses a *proposition*—formally, he introduces for consideration the set of worlds where so-and-so is true.

We say that one proposition *entails* another just in case the truth of the former (in context  $c$ ) guarantees the truth of the latter (in  $c$ ): e.g., if a person says (in  $c$ ) 'I am tired and hungry', he expresses a proposition that entails the one he would express if he said (in  $c$ ) 'I am tired.' Formally, a proposition  $p_1$  *entails* proposition  $p_2$ , just in case  $p_1$  is a *subset* of  $p_2$ .

- *Test & eliminative update: What is presupposed, asserted, or implicated*

*Presupposition* submits the input context to a *test*: is it *felicitous* to utter the sentence in the first place? For instance, suppose Sue says: 'I passed'. This will be felicitous only if all of the live candidates for reality in the input common ground are worlds  $w$  such that Sue took exam so-and-so in  $w$ —i.e., her taking the exam must be part of the shared information in the input.

If the input context satisfies this test, then by *asserting* 'I passed' the speaker (Sue) proposes to *update* the input context by *eliminating* those worlds where the proposition expressed (that Sue passed) is false. If this proposal is accepted by the other participants, the assertion is *successful*: in the updated context the asserted proposition is true—since all the candidate worlds where it was false have been eliminated—and so are all of its *entailments*.

*Implicatures* (e.g., 'I didn't ace this test') are *default extra* updates, licensed unless the speaker defeats this default by explicit assertion to the contrary (e.g., 'I even aced it.')

## II. Toward a formally-precise theory

The above ideas can be made *formally precise*—i.e., precise enough to program on a computer if one wanted to—by means of a suitable formal language: *Dynamic Propositional Logic* ( $DL_0$ ). To ensure this level of precision, we will use an explicit definition with the customary three parts:

- **Syntax**  
Syntax defines what is a *well-formed term* (aka *expression*) of the language, including a class of terms—we'll call them *t-terms*—to which **Semantics** will assign a *truth value*.
- **Model theory**  
Model theory defines what constitutes a possible *model* for the language—intuitively, the contextual information required to determine the interpretation of each well-formed term.
- **Semantics**  
Semantics defines the *interpretation* (aka *denotation*, or *semantic value*) of each well-formed term in each model. It also defines: (i) *truth* in a model, and (ii) *entailment*.

## III. Syntax of Dynamic Propositional Logic ( $DL_0$ )

- Definitions:

DEFINITION 1.1 (Basic  $DL_0$ -terms)

$$\mathbf{Con}_t = \{A, B, C, \dots, Z\} \quad [t\text{-constants}]$$

DEFINITION 1.2 ( $DL_0$  syntax) The set of *t-terms* of  $DL_0$  is the smallest set  $\mathbf{Term}_t$  such that:

- b.*  $\square \square \mathbf{Term}_t$  if  $\square \square \mathbf{Con}_t$  [*basic*]
- n.*  $\neg \square \square \mathbf{Term}_t$  if  $\square \square \mathbf{Term}_t$  [*negation*]
- p.*  $\partial \square \square \mathbf{Term}_t$  if  $\square \square \mathbf{Term}_t$  [*presup.*]
- s.*  $(\square ; \square) \square \mathbf{Term}_t$  if  $\square, \square \square \mathbf{Term}_t$  [*sequencing*]

- Sample *t-terms* of  $DL_0$  (Abbreviations: D = DEFINITION, F = FACT):

FACT 1.  $T \square \mathbf{Term}_t$

PROOF:

1.  $T \square \mathbf{Con}_t$  D1.1
2.  $T \square \mathbf{Term}_t$  1, D1.2.b

□

FACT 2.  $(\partial T ; P) \square \mathbf{Term}_t$

PROOF:

1.  $\partial T \square \mathbf{Term}_t$  F1, D1.2.p
2.  $P \square \mathbf{Con}_t$  D1.1
3.  $P \square \mathbf{Term}_t$  2, D1.2.b
4.  $(\partial T ; P) \square \mathbf{Term}_t$  1, 3, D1.2.s

□

#### IV. Model theory for *Dynamic Propositional Logic* (DL<sub>0</sub>)

- Definitions:

DEFINITION 2.0 (DL<sub>0</sub>-frames). A DL<sub>0</sub>-frame based on a non-empty set  $W$  is the set  $\{p: p \sqsubseteq W\}$ .

DEFINITION 2.1 (DL<sub>0</sub>-models). A DL<sub>0</sub>-model is a structure  $M = \langle D_t^M, \llbracket \cdot \rrbracket^M \rangle$  such that:

- (i)  $D_t^M$  is a DL<sub>0</sub>-frame (*t*-domain in  $M$ )
- (ii) for any  $\square \sqsubseteq \mathbf{Con}$ ,  $\llbracket \square \rrbracket^M \sqsubseteq D_t^M$  ( $M$ -denotation of  $\square$ ).

- Sample DL<sub>0</sub>-frames:

$W \neq \{\}$ ( <i>set of worlds</i> )	DL <sub>0</sub> -frame based on $W$ : $\{p: p \sqsubseteq W\}$
$\{w_1\}$	$\{\{\}, \{w_1\}\}$
$\{w_1, w_2\}$	$\{\{\}, \{w_1\}, \{w_2\}, \{w_1, w_2\}\}$
$\{w_1, w_2, w_3\}$	$\{\{\}, \{w_1\}, \{w_2\}, \{w_3\}, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_3\}, \{w_1, w_2, w_3\}\}$

Sample (partially specified) DL<sub>0</sub>-model, with ‘ $T$ ’ representing the proposition that Sue took the test, and ‘ $P$ ’, that she passed it:

$$M_3 = \langle D_t^{M_3}, \llbracket \cdot \rrbracket^{M_3} \rangle$$

where  $D_t^{M_3} = \{\{\}, \{w_1\}, \{w_2\}, \{w_3\}, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_3\}, \{w_1, w_2, w_3\}\}$

$$\llbracket T \rrbracket^{M_3} = \{w_1, w_2\}$$

$$\llbracket P \rrbracket^{M_3} = \{w_1\}$$

#### V. Semantics of *Dynamic Propositional Logic* (DL<sub>0</sub>)

- Definitions:

DEFINITION 2.2 (DL<sub>0</sub>-semantics). Suppose  $M = \langle D_t^M, \llbracket \cdot \rrbracket^M \rangle$  is a DL<sub>0</sub>-model,  $c \sqsubseteq D_t^M$  ( $M$ -context), and  $\top D_t^M$  (*set of worlds*) is the maximum of  $D_t^M$  (i.e.,  $\top D_t^M \sqsubseteq D_t^M$  & for all  $p \sqsubseteq D_t^M$ ,  $p \sqsubseteq \top D_t^M$ ). Then  $c \llbracket \square \rrbracket^M$  ( $M$ -context resulting from updating  $c$  with  $\square$  in  $M$ ) is defined as follows:

- b.**  $c \llbracket \square \rrbracket^M = \{w \sqsubseteq \top D_t^M \mid w \sqsubseteq c \ \& \ w \sqsubseteq \llbracket \square \rrbracket^M\}$  if  $\square \sqsubseteq \mathbf{Con}_t$  [*basic*]
- n.**  $c \llbracket \neg \square \rrbracket^M = (c - c \llbracket \square \rrbracket^M)$  if  $\square \sqsubseteq \mathbf{Term}_t$  [*negation*]
- p.**  $c \llbracket \partial \square \rrbracket^M = c$  if  $\square \sqsubseteq \mathbf{Term}_t$  &  $c \llbracket \square \rrbracket^M = c$  [*presup.*]
- $= \{\}$  if  $\square \sqsubseteq \mathbf{Term}_t$  &  $c \llbracket \square \rrbracket^M \neq c$
- s.**  $c \llbracket (\square ; \square) \rrbracket^M = c \llbracket \square \rrbracket^M \llbracket \square \rrbracket^M$  if  $\square, \square \sqsubseteq \mathbf{Term}_t$  [*sequencing*]

DEFINITION 3 (Felicity, truth, entailment).

✓. Let a DL<sub>0</sub>-model  $M = \langle D_t^M, \llbracket \cdot \rrbracket^M \rangle$  and  $M$ -context  $c \sqsubseteq D_t^M$ , be given.

- $\square$  is *felicitous* in  $c$  relative to  $M$  (written  $\checkmark_{M, c} \square$ ), iff  $\{\} \sqsubseteq c \llbracket \square \rrbracket^M \sqsubseteq c$ .

Let a DL<sub>0</sub>-model  $M = \langle D_t^M, \llbracket \cdot \rrbracket^M \rangle$   $M$ -context  $c \sqsubseteq D_t^M$ , and world  $w \sqsubseteq c$ , be given.

- + •  $\square$  is *true* in world  $w$  of  $c$  relative to  $M$  (written  $\models_{M, w \sqsubseteq c} \square$ ), iff  $w \sqsubseteq c \llbracket \square \rrbracket^M$
- •  $\square$  is *false* in world  $w$  of  $c$  relative to  $M$  (written  $\not\models_{M, w \sqsubseteq c} \square$ ), iff  $w \not\sqsubseteq c \llbracket \square \rrbracket^M$

□.  $\square$  *entails*  $\square$  in DL<sub>0</sub> (written  $\square \models_{DL_0} \square$ ),

iff, for every DL<sub>0</sub>-model  $M$ ,  $M$ -context  $c$ , and  $w \sqsubseteq c$ , if  $\models_{M, w \sqsubseteq c} \square$  then  $\models_{M, w \sqsubseteq c} (\square ; \square)$ .

- Sample analysis of *contextual felicity & truth value*

CONTEXT: Sue took a test. John knows it & Sue knows that he does. He doesn't know the result.

SPEECH ACT: Sue to John: 'I passed.'

INTUITION: Felicitous, possibly true, possibly false.

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_3$  on p. 7  
 representing *input context*:  $\{w_1, w_2\}$  (shared  $T$ , not shared  $P$ )  
 representing *what was said*:  $(\partial T ; P)$  (presuppose  $T$ , assert  $P$ )
- *predicted* output context:  $\{w_1, w_2\}[(\partial T ; P)]^{M_3}$   
 $= \{w_1\}$  F4 below
- predicted* (in)felicity: *Felicitous*:  $\checkmark_{M_3, \{w_1, w_2\}} (\partial T ; P)$  D3.✓  
 $\{\} \sqcap \{w_1\} \sqcap \{w_1, w_2\}$
- predicted* truth values: Of input  $\{w_1, w_2\}$ ,  
*true* in  $w_1$  (survives this update) D3.+  
*false* in  $w_2$  (doesn't survive) D3.-

FACT 3: Given  $M_3$  on p. 7:

$$\{w_1, w_2\}[(\partial T)]^{M_3} = \{w_1, w_2\}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_2\}[(T)]^{M_3} \\ = & \{w \sqcap \top D_t^{M_3} | w \sqcap \{w_1, w_2\} \ \& \ w \sqcap [(T)]^{M_3}\} & \text{D2.2.b} \\ = & \{w \sqcap \{w_1, w_2, w_3\} | w \sqcap \{w_1, w_2\} \ \& \ w \sqcap \{w_1, w_2\}\} & \text{D2.2.2nd line, df. } M_3 \text{ on p. 7} \\ = & \{w_1, w_2\} & \text{df. } \{\dots | \dots\} \end{aligned}$$

Hence:

$$\begin{aligned} & \{w_1, w_2\}[(\partial T)]^{M_3} \\ = & \{w_1, w_2\} & \text{D2.2.p} \end{aligned}$$

□

FACT 4: Given  $M_3$  on p. 7:

$$\{w_1, w_2\}[(\partial T ; P)]^{M_3} = \{w_1\} \quad \text{(test input, update)}$$

PROOF:

$$\begin{aligned} & \{w_1, w_2\}[(\partial T ; P)]^{M_3} \\ = & \{w_1, w_2\}[(\partial T)]^{M_3}[[P]]^{M_3} & \text{D2.2.s} \\ = & \{w_1, w_2\}[[P]]^{M_3} & \text{F3} \\ = & \{w \sqcap \top D_t^{M_3} | w \sqcap \{w_1, w_2\} \ \& \ w \sqcap [[P]]^{M_3}\} & \text{D2.2.b} \\ = & \{w \sqcap \{w_1, w_2, w_3\} | w \sqcap \{w_1, w_2\} \ \& \ w \sqcap \{w_1\}\} & \text{D2.2.2nd line, df. } M_3 \text{ on p. 7} \\ = & \{w_1\} & \text{df. } \{\dots | \dots\} \end{aligned}$$

□

**Homework 1**

CONTEXT: Sue took a test. John doesn't know it & Sue knows that he doesn't.

SPEECH ACT: Sue to John: 'I passed.'

INTUITION: Infelicitous.

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_3$  on p. 7  
 representing *input context*:  $\{w_1, w_2, w_3\}$  (T not shared)  
 representing *what was said*:  $(\partial T ; P)$  (presuppose T, assert P)
- *predicted* output context:  $\{w_1, w_2, w_3\} \llbracket (\partial T ; P) \rrbracket^{M_3}$   
 = \_\_\_\_\_ F6 below
- predicted* (in)felicity: \_\_\_\_\_ D3.✓  
 because \_\_\_\_\_

FACT 5: Given  $M_3$  on p. 7:

$$\{w_1, w_2, w_3\} \llbracket \partial T \rrbracket^{M_3} = \underline{\hspace{10em}}$$

PROOF: We first note that:

$$\{w_1, w_2, w_3\} \llbracket T \rrbracket^{M_3}$$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

D2.2.b

D2.2.2nd line, df.  $M_3$  on p. 7

df. {...|...}

Hence:

$$\{w_1, w_2, w_3\} \llbracket \partial T \rrbracket^{M_3}$$

= \_\_\_\_\_

D2.2.p

□

FACT 6: Given  $M_3$  on p. 7:

$$\{w_1, w_2, w_3\} \llbracket (\partial T ; P) \rrbracket^{M_3} = \underline{\hspace{10em}}$$

(test, update)

PROOF:

$$\{w_1, w_2, w_3\} \llbracket (\partial T ; P) \rrbracket^{M_3}$$

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

= \_\_\_\_\_

D2.2.s

F5

D2.2.b

D2.2.2nd line, df.  $M_3$  on p. 7

df. {...|...}

□

### Solution to Homework 1

CONTEXT: Sue took a test. John doesn't know it & Sue knows that he doesn't.

SPEECH ACT: Sue to John: 'I passed.'

INTUITION: Infelicitous.

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_3$  on p. 7
- representing *input context*:  $\{w_1, w_2, w_3\}$  ( $T$  not shared)
- representing *what was said*:  $(\partial T; P)$  (presuppose  $T$ , assert  $P$ )
- *predicted* output context:  $\{w_1, w_2, w_3\} \llbracket (\partial T; P) \rrbracket^{M_3}$
- =  $\{\}$  F6 below
- predicted* (in)felicity: **Infelicitous** because D3.✓
- not**:  $\{\} \sqcap \{w_1, w_2, w_3\} \llbracket (\partial T; P) \rrbracket^{M_3}$

NOT PART OF **H1**, BUT WORTH NOTING:

- predicted* truth values: Of input  $\{w_1, w_2, w_3\}$ ,
- true* in none of these worlds D3.+
- false* in each of  $w_1, w_2, w_3$  D3.–

FACT 5: Given  $M_3$  on p. 7:

$$\{w_1, w_2, w_3\} \llbracket \partial T \rrbracket^{M_3} = \{\}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_2, w_3\} \llbracket T \rrbracket^{M_3} \\ = & \{w \sqcap \top D_i^{M_3} \mid w \sqcap \{w_1, w_2, w_3\} \ \& \ w \sqcap \llbracket T \rrbracket^{M_3}\} && \text{D2.2.b} \\ = & \{w \sqcap \{w_1, w_2, w_3\} \mid w \sqcap \{w_1, w_2, w_3\} \ \& \ w \sqcap \{w_1, w_2\}\} && \text{D2.2.2nd line, df. } M_3 \text{ on p. 7} \\ = & \{w_1, w_2\} && \text{df. } \{\dots \mid \dots\} \end{aligned}$$

Hence:

$$\begin{aligned} & \{w_1, w_2, w_3\} \llbracket \partial T \rrbracket^{M_3} \\ = & \{\} && \text{D2.2.p} \end{aligned}$$

□

FACT 6: Given  $M_3$  on p. 7:

$$\{w_1, w_2, w_3\} \llbracket (\partial T; P) \rrbracket^{M_3} = \{\} \quad (\text{test input, update})$$

PROOF:

$$\begin{aligned} & \{w_1, w_2, w_3\} \llbracket (\partial T; P) \rrbracket^{M_3} \\ = & \{w_1, w_2, w_3\} \llbracket \partial T \rrbracket^{M_3} \llbracket P \rrbracket^{M_3} && \text{D2.2.s} \\ = & \{\} \llbracket P \rrbracket^{M_3} && \text{F5} \\ = & \{w \sqcap \top D_i^{M_3} \mid w \sqcap \{\} \ \& \ w \sqcap \llbracket P \rrbracket^{M_3}\} && \text{D2.2.b} \\ = & \{w \sqcap \{w_1, w_2, w_3\} \mid w \sqcap \{\} \ \& \ w \sqcap \{w_1\}\} && \text{D2.2.2nd line, df. } M_3 \text{ on p. 7} \\ = & \{\} && \text{df. } \{\dots \mid \dots\} \end{aligned}$$

□

**Lecture 3**  
APPENDICES AND SOME REFERENCES

Appendix A: Greek alphabet

Letter	Name
□	Alpha
□	Beta
□	Gamma
□	Delta
□	Epsilon
□	Zeta
□	Eta
□	Theta
□	Iota
□	Kappa
□	Lambda
□	Mu
□	Nu
□	Xi
o	Omicron
□	Pi
□	Rho
□	Sigma
□	Tau
□	Upsilon
□	Phi
□	Chi
□	Psi
□	Omega

Appendix B: Some logical symbols

Symbol (read)	Name
¬	(‘not’) Negation symbol
□	(‘and’) Conjunction symbol
□	(‘or’) Disjunction symbol
□	(‘if...then’) Conditional symbol
□	(‘if and only if’) Biconditional symbol
iff	(‘if and only if’)
=	(‘is’) Identity symbol
⊨ □	(‘□ is true’)
⊭ □	(‘□ isn’t true’)
□ ⊨ □	(‘□ entails □’)
□ ⊭ □	(‘□ doesn’t entail □’)
(...)	Parentheses
{...}	Braces
[...]	Brackets
□..□	Angled brackets
[[...]]	Denotation brackets
[[□]]	(‘denotation of □’)

Appendix C: Mathematical English 101

A	B	(A or B)	(if A then B)	(A iff B)
true	true	true	true	true
true	false	true	false	false
false	true	true	true	false
false	false	false	true	true

Appendix D: Set theory 101

- <sub>1</sub> The terms *set* (aka *class*) are used interchangeably. A set is completely specified by saying either (i) that it’s empty, or (ii) that it consists of such-and-such *elements* (aka *members*).
- <sub>2</sub> One way to specify a set is to list its elements, if any, separate them by commas, and enclose the whole *list* in braces ( $\{\dots\}$  indicate that the *order* and any *repetitions* don’t matter). E.g.,
 

$\{\}$	<i>empty set</i> (aka <i>null set</i> )
$\{w_1, w_2, w_3\}$	set consisting of $w_1, w_2,$ and $w_3$
$= \{w_2, w_1, w_3, w_1\}$	(NOTE: Same set because the elements listed are the same)
$\{\{\}, \{w_1\}, \{w_2\}, \{w_1, w_2\}\}$	set consisting of $\{\}$ ( <i>empty set</i> ), $\{w_1\}$ ( <i>singleton</i> of $w_1$ ), $\{w_2\}$ ( <i>singleton</i> of $w_2$ ), $\{w_1, w_2\}$ ( <i>set</i> consisting of $w_1, w_2$ )

•<sub>3</sub> Some *set-theoretic relations*:

Written	Read	True iff	Examples
$x \sqsubseteq y$	$x$ is an <i>element</i> of (set) $y$	$x$ is on the list of elements of $y$	$w_1 \sqsubseteq \{w_1, \dots\}$ $\{w_1\} \sqsubseteq \{\dots, \{w_1\}, \dots\}$ $\{\} \sqsubseteq \{\dots, \{\}\}$
$x \not\sqsubseteq y$	$x$ is <i>not</i> an <i>element</i> of (set) $y$	' $x \sqsubseteq y$ ' is false	$\{\} \not\sqsubseteq \{w_1, w_2, w_3\}$ $w_1 \not\sqsubseteq \{\{\}, \{w_1\}, \{w_2\}\}$ $\{\} \not\sqsubseteq \{\}$
$x \sqsubseteq y$	(set) $x$ is a <i>subset</i> of (set) $y$	for all $z$ , if $z \sqsubseteq x$ then $z \sqsubseteq y$	$\{w_1\} \sqsubseteq \{w_1, \dots\}$ $\{\{\}, \{w_1\}\} \sqsubseteq \{\{w_1\}, \{\}\}$ $\{\} \sqsubseteq \{\}$
$x \not\subseteq y$	(set) $x$ is <i>not</i> a <i>subset</i> of (set) $y$	' $x \sqsubseteq y$ ' is false (i.e., for some $z$ , $z \sqsubseteq x$ & $z \not\sqsubseteq y$ )	$\{w_1\} \not\subseteq \{\{w_1\}, \{w_2\}\}$ $\{\{\}, \{w_1\}\} \not\subseteq \{\{w_1\}\}$ $\{\{\}, w_1\} \not\subseteq \{\{w_1\}, \{\}\}$
$x \sqsubset y$	(set) $x$ is a <i>proper subset</i> of (set) $y$	$x \sqsubseteq y$ & $x \neq y$	$\{w_1\} \sqsubset \{w_1, \{\}\}$ $\{\} \sqsubset \{\{\}\}$
$x \not\sqsubset y$	(set) $x$ is <i>not</i> a <i>proper subset</i> of (set) $y$	' $x \sqsubseteq y$ ' is false (i.e. $x \not\subseteq y$ or $x = y$ )	$\{\{\}, w_1\} \not\sqsubset \{w_1, \{\}\}$ $\{\} \not\sqsubset \{\}$

•<sub>4</sub> *Set-building abstraction*: set of objects such that so-and-so (alternative way to specify a set).

Written	Read	Definition (...iff...)	Example
$\{x: \square\}$	the set of all objects $x$ that satisfy $\square$	$z \sqsubseteq \{x: \square\}$ iff $[z/x]\square$	$\{\} \sqsubseteq \{p: p \sqsubseteq \{w_1, w_2\}\}$ because: $\{\} \sqsubseteq \{w_1, w_2\}$
$\{x \sqsubseteq y: \square\}$	the set of all elements $x$ of $y$ that satisfy $\square$	$z \sqsubseteq \{x \sqsubseteq y: \square\}$ iff $z \sqsubseteq y$ & $[z/x]\square$	$\square \sqsubseteq \{x \sqsubseteq A: x \text{ is a cat}\}$ iff $(\square \sqsubseteq A$ & $\square$ is a cat)

•<sub>5</sub> Some other *set-theoretic operations* (with sets as input and output):

Written	Read	Definition (...=...)	Example(s)
$(x \sqcap y)$	<i>intersection</i> of $x$ and $y$	$= \{z: z \sqsubseteq x \text{ \& } z \sqsubseteq y\}$	$(\{\{\}\} \sqcap \{w_1\}) = \{\}$ $(\{1, 2\} \sqcap \{2\}) = \{2\}$
$(x \sqcup y)$	<i>union</i> of $x$ and $y$	$= \{z: z \sqsubseteq x \text{ or } z \sqsubseteq y\}$	$(\{\{\}\} \sqcup \{w_1\}) = \{\{\}, w_1\}$ $(\{1, 2\} \sqcup \{2\}) = \{1, 2\}$
$(x - y)$	$x$ <i>minus</i> $y$	$= \{z: z \sqsubseteq x \text{ \& } z \not\sqsubseteq y\}$	$(\{\{\}\} - \{w_1\}) = \{\{\}\}$ $(\{1, 2\} - \{2\}) = \{1\}$

Since the subset relation ( $\sqsubseteq$ ) ranks sets by size, we can also define, for a *class*  $x$  of sets:

$\top x$	<i>top</i> element of (class) $x$	$y = \top x$ , iff (i) $y \sqsubseteq x$ , (ii) for all $z \sqsubseteq x: z \sqsubseteq y$	$\top \{p \mid p \sqsubseteq \{w_1, w_2, w_3\}\}$ $= \{w_1, w_2, w_3\}$
$\sqcap x$	<i>bottom</i> element of (class) $x$	$y = \sqcap x$ , iff (i) $y \sqsubseteq x$ , (ii) for all $z \sqsubseteq x: y \sqsubseteq z$	$\sqcap \{p \mid p \sqsubseteq \{w_1, w_2, w_3\}\}$ $= \{\}$

## References

The EMPIRICAL DIAGNOSTICS in **Lecture 1** are based on work by many people, including:

- Karttunen 1973 on *presupposition projection*
- Grice 1975 on *cancelable implicatures*
- Horn 1985 on *metalinguistic negation*

The FORMAL THEORY presented in **Lecture 2** implements ideas due to:

- Stalnaker 1974, Karttunen 1974 on *presupposition*
- Stalnaker 1978 on *assertion*
- Gazdar 1979 on *implicature*

along the lines of:

- Veltman 1986 *update semantics*
- Beaver 2001 *presupposition operator*

If you would like to read some of these papers, mostly classics, here are the references, ‘graded’ as follows: **A** = *informal*, should be intelligible; **B** = *semi-formal*, read for the gist; **C** = *formal*, probably too difficult for now. (Beaver starts off at **B**, ends at **C**.)

- B?** Beaver, D. 2001. *Presupposition and Assertion in Dynamic Semantics*. CSLI, Stanford.
- C** Gazdar, G. 1979. ‘A Solution to the Projection Problem’. In Ch. Oh and D. Dineen, eds., *Syntax and Semantics* 11, 57–89. Academic Press, New York.
- A** Grice, H. 1975. ‘Logic and Conversation’. In P. Cole and J. Morgan, eds., *Syntax and Semantics* 3, 41–58. Academic Press, New York.
- A** Horn, L. 1985. ‘Metalinguistic Negation and Pragmatic Ambiguity’. *Language* **61**:121–174.
- B** Karttunen, L. 1973 ‘Presuppositions of Compound Sentences’. *Linguistic Inquiry* **4**:167–193.
- A** Karttunen, L. 1974. ‘Presupposition and Linguistic Context’. *Theoretical Linguistics* **1**:181–194.
- A** Stalnaker, R. 1974. ‘Pragmatic Presuppositions’. In M. Munitz and P. Unger (eds.) *Semantics and Philosophy*. New York University Press.
- B** Stalnaker, R. 1978. ‘Assertion’. In P. Cole, ed., *Syntax and Semantics* 9, 315–332. Academic Press, New York.
- C** Veltman, F. 1986. ‘Data Semantics and the Pragmatics of Indicative Conditionals’. In E. Traugott *et al*, eds., *On Conditionals*, Cambridge University Press, Cambridge.

I also recommend a beautiful paper by Lewis—as insightful as it is entertaining—which outlines many other potential applications of this theoretical approach (e.g., *accommodation*, context repair, which we may talk about later). The excellent reader edited by Davis includes Lewis’s paper and many of the above classics, among others (mostly **A**’s, a few **B**’s):

- A** Davis, S., ed. 1991. *Pragmatics: A Reader*. Oxford University Press, Oxford.
- A** Lewis, D. 1979. ‘Scorekeeping in a Language Game’. *Journal of Philosophical Logic* **8**:339–359.

### Lecture 4

#### NEGATION REVISITED: ENTAILMENT VS. PRESUPPOSITION

#### I. Felicitous negation reverses truth value and, therefore, entailment

CONTEXT: Sue took a test. John knows it & Sue knows that he does. He doesn't know the result.

SPEECH ACT 1: Sue to John: 'I passed.'

SPEECH ACT 2: Sue to John: 'I didn't pass.'

INTUITION: Both felicitous, if 1 true, then 2 is false, and vice versa.

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_3$  on p. 7
  - representing *input context*:  $\{w_1, w_2\}$  (shared  $T$ , not shared  $P$ )
  - representing *what was said* in 1:  $(\partial T; P)$  (presuppose  $T$ , assert  $P$ )
  - representing *what was said* in 2:  $\neg(\partial T; P)$  (negation of the above)
  
- *predicted* output context for 1:  $\{w_1, w_2\} \llbracket (\partial T; P) \rrbracket^{M_3}$ 
  - $= \{w_1\}$  F4
- predicted* (in)felicity for 1: *Felicitous*:  $\checkmark_{M_3, \{w_1, w_2\}} (\partial T; P)$  D3.✓
  - $\{ \} \sqcap \{w_1\} \sqcap \{w_1, w_2\}$
- predicted* truth values for 1: Of input  $\{w_1, w_2\}$ ,
  - true* in  $w_1$  (survives this update) D3.+
  - false* in  $w_2$  (doesn't survive) D3.–
  
- *predicted* output context for 2:  $\{w_1, w_2\} \llbracket \neg(\partial T; P) \rrbracket^{M_3}$ 
  - $= (\{w_1, w_2\} - \{w_1, w_2\}) \llbracket (\partial T; P) \rrbracket^{M_3}$  D2.2.n
  - $= (\{w_1, w_2\} - \{w_1\})$  F4 (as above)
  - $= \{w_2\}$  df.  $(x - y)$  for sets
- predicted* (in)felicity for 2: *Felicitous*:  $\checkmark_{M_3, \{w_1, w_2\}} \neg(\partial T; P)$  D3.✓
  - $\{ \} \sqcap \{w_2\} \sqcap \{w_1, w_2\}$
- predicted* truth values for 2: Of input  $\{w_1, w_2\}$ ,
  - false* in  $w_1$  (doesn't survive) D3.+
  - true* in  $w_2$  (survives this update) D3.–

GENERAL RESULTS (for any model  $M$  and  $M$ -context  $c \sqcap \top D_i^M$ ):

- Suppose the proposition expressed by  $T$  in  $M$  is *shared* information in the input context  $c$ :
  - (1)  $c \sqcap \llbracket T \rrbracket^M$
- Then:
  - (2)  $c \llbracket T \rrbracket^M = \{w \sqcap \top D_i^M : w \sqcap c \ \& \ w \sqcap \llbracket T \rrbracket^M\}$  D2.2.b
    - $= c$  df.  $\{\dots\dots\}$ , (1)
- And so:
  - (3)  $c \llbracket \partial T \rrbracket^M = c$  D2.2.p
  - (4)  $c \llbracket (\partial T; P) \rrbracket^M = (c \sqcap \llbracket P \rrbracket^M)$  D2.2.s, (3), App. D
  - (5)  $c \llbracket \neg(\partial T; P) \rrbracket^M = (c - \llbracket P \rrbracket^M)$  D2.2.n, (4), App. D

## II. Negation preserves commitment to presupposition, on the pain of infelicity

CONTEXT: Sue took a test. John doesn't know it & Sue knows that he doesn't.

SPEECH ACT 1: Sue to John: 'I passed.'

SPEECH ACT 2: Sue to John: 'I didn't pass.' (= *Neg* projection test: colloquial version)

INTUITION: Both **infelicitous**.

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_3$  on p. 7
- representing *input context*:  $\{w_1, w_2, w_3\}$  ( $T$  not shared)
- representing *what was said* in 1:  $(\partial T; P)$  (presuppose  $T$ , assert  $P$ )
- representing *what was said* in 2:  $\neg(\partial T; P)$  (negation of the above)
  
- *predicted* output context for 1:  $\{w_1, w_2, w_3\}[(\partial T; P)]^{M_3}$   
 $= \{\}$  F6 (in Homework 1)
- predicted* (in)felicity for 1: **Infelicitous** because D3.✓  
**not**  $\{\}$   $\square$   $\{w_1, w_2, w_3\}$
  
- *predicted* output context for 2:  $\{w_1, w_2, w_3\}[\neg(\partial T; P)]^{M_3}$   
 $= (\{w_1, w_2, w_3\} - \{w_1, w_2, w_3\}[(\partial T; P)]^{M_3})$  D2.2.n  
 $= (\{w_1, w_2, w_3\} - \{\})$  F6 (as above)  
 $= \{w_1, w_2, w_3\}$  df.  $(x - y)$  for sets
- predicted* (in)felicity for 2: **Infelicitous** because D3.✓  
**not**  $\{\}$   $\square$   $\{w_1, w_2, w_3\}$   $\square$   $\{w_1, w_2, w_3\}$

GENERAL RESULTS (for any model  $M$  and  $M$ -context  $c \square \top D_i^M$ ):

- Suppose the proposition expressed by  $T$  in  $M$  is *not shared* information in the input context  $c$ :  
(1)  $c \square \llbracket T \rrbracket^M$
- Then:  
(2)  $c \llbracket T \rrbracket^M = \{w \square \top D_i^M: w \square c \ \& \ w \square \llbracket T \rrbracket^M\}$  D2.2.b  
 $\neq c$  df.  $\{\dots\dots\}$ , (1)
- Therefore:  
(3)  $c \llbracket \partial T \rrbracket^M = \{\}$  D2.2.p  
(4)  $c \llbracket (\partial T; P) \rrbracket^M = \{\}$  D2.2.s, (3), App. D  
(5)  $c \llbracket \neg(\partial T; P) \rrbracket^M = c$  D2.2.n, (4), App. D

So both  $(\partial T; P)$  and its negation  $\neg(\partial T; P)$  are **infelicitous**, given  $c$  and  $M$ , because of *failure to add any information* to  $c$ . More specifically:

- (6)  $(\partial T; P)$  is **infelicitous**, given  $c$  and  $M$ , because *no input worlds survive* this update:  
 $\{\} \square \{\} = c \llbracket (\partial T; P) \rrbracket^M$
- (7)  $\neg(\partial T; P)$  is **infelicitous**, given  $c$  and  $M$ , because it *fails to eliminate any input worlds*:  
 $c \llbracket \neg(\partial T; P) \rrbracket^M = c \square c$

**Homework 2**

CONTEXT: Sue took a test for a course. John knows about the course, but not about the test.

SPEECH ACT: Sue to John: ‘If I passed, I’ll pass this course.’

INTUITION: Infelicitous

FORMAL ACCOUNT:

- Suitable DL<sub>0</sub>-model:  $M_6 = \langle D_t^{M_6}, \llbracket \cdot \rrbracket^{M_6} \rangle$ 
  - where  $D_t^{M_6} = \{p \mid \{w_1, w_2, w_3, w_4, w_5, w_6\}$
  - $\llbracket T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$  (Sue takes the test)
  - $\llbracket P \rrbracket^{M_6} = \{w_1, w_3\}$  (Sue passes the test)
  - $\llbracket C \rrbracket^{M_6} = \{w_1, w_2, w_5\}$  (Sue passes the course)
  - representing *input context*:  $\{w_1, w_2, w_3, w_4, w_5\}$  (Sue takes the course)
  - representing *what was said*:  $\neg((\partial T; P); \neg C)$  (a la Math. Eng: App C)

FACT 7: Given  $M_6$  above:

$$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \partial T \rrbracket^{M_6} = \underline{\hspace{10em}}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket T \rrbracket^{M_6} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \end{aligned}$$

D2.2.b

df.  $M_6$

df.  $\{\dots\}$

Hence:

$$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \partial T \rrbracket^{M_6} = \underline{\hspace{10em}}$$

D2.2.p

□

FACT 8: Given  $M_6$  above:

$$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg((\partial T; P); \neg C) \rrbracket^{M_6} = \underline{\hspace{10em}}$$

PROOF:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg((\partial T; P); \neg C) \rrbracket^{M_6} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \\ = & \underline{\hspace{10em}} \end{aligned}$$

D2.2.n

D2.2.s

D2.2.s

F7

D2.2.b

df.  $\{\dots\}$

D2.2.n

df.  $(x - y)$

df.  $(x - y)$

□

## Solution to Homework 2

CONTEXT: Sue took a test for a course. John knows about the course, but not about the test.

SPEECH ACT: Sue to John: ‘If I passed, I’ll pass this course.’

INTUITION: Infelicitous

FORMAL ACCOUNT:

- Suitable  $DL_0$ -model:  $M_6 = \langle D_t^{M_6}, \llbracket \cdot \rrbracket^{M_6} \rangle$ 
  - where  $D_t^{M_6} = \{p \mid \{w_1, w_2, w_3, w_4, w_5, w_6\}\}$
  - $\llbracket T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$  (Sue takes the test)
  - $\llbracket P \rrbracket^{M_6} = \{w_1, w_3\}$  (Sue passes the test)
  - $\llbracket C \rrbracket^{M_6} = \{w_1, w_2, w_5\}$  (Sue passes the course)
  - representing *input context*:  $\{w_1, w_2, w_3, w_4, w_5\}$  (Sue takes the course)
  - representing *what was said*:  $\neg((\partial T; P); \neg C)$  (a la Math. Eng: App C)

FACT 7: Given  $M_6$  above:

$$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \partial T \rrbracket^{M_6} = \{\}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket T \rrbracket^{M_6} \\ = & \{w \mid \top D_t^{M_6} \mid w \mid \{w_1, w_2, w_3, w_4, w_5\} \ \& \ w \mid \llbracket T \rrbracket^{M_6}\} && \text{D2.2.b} \\ = & \{w \mid \top D_t^{M_6} \mid w \mid \{w_1, w_2, w_3, w_4, w_5\} \ \& \ w \mid \{w_1, w_2, w_3, w_4\}\} && \text{df. } M_6 \\ = & \{w_1, w_2, w_3, w_4\} && \text{df. } \{\dots \mid \dots\} \end{aligned}$$

Hence:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket \partial T \rrbracket^{M_6} \\ = & \{\} && \text{D2.2.p} \quad \square \end{aligned}$$

FACT 8: Given  $M_6$  above:

$$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg((\partial T; P); \neg C) \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4, w_5\}$$

(*predict infelicity*:  
 $c \llbracket \square \rrbracket^M \mid c$ )

PROOF:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg((\partial T; P); \neg C) \rrbracket^{M_6} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4, w_5\} \llbracket ((\partial T; P); \neg C) \rrbracket^{M_6}) && \text{D2.2.n} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4, w_5\} \llbracket (\partial T; P) \rrbracket^{M_6} \llbracket \neg C \rrbracket^{M_6}) && \text{D2.2.s} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4, w_5\} \llbracket \partial T \rrbracket^{M_6} \llbracket P \rrbracket^{M_6} \llbracket \neg C \rrbracket^{M_6}) && \text{D2.2.s} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{\} \llbracket P \rrbracket^{M_6} \llbracket \neg C \rrbracket^{M_6}) && \text{F7} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w \mid \top D_t^{M_6} \mid w \mid \{\} \ \& \ w \mid \llbracket P \rrbracket^{M_6}\} \llbracket \neg C \rrbracket^{M_6}) && \text{D2.2.b} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{\} \llbracket \neg C \rrbracket^{M_6}) && \text{df. } \{\dots \mid \dots\} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - (\{\} - \{\} \llbracket C \rrbracket^{M_6})) && \text{D2.2.n} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{\}) && \text{df. } (x - y) \\ = & \{w_1, w_2, w_3, w_4, w_5\} && \text{df. } (x - y) \end{aligned}$$

□

### Quiz 1

Key to abbreviations:

- OK  $A$              $A$  is felicitous & true in a suitable context  
 $\square A$              $A$  is contradictory in every context  
 $A \Rightarrow B$         saying  $A$  commits the speaker to  $B$   
 $A \not\Rightarrow B$         saying  $A$  does not commit the speaker to  $B$

1. (5 pts) Fill in the *relevant blanks* and underline the *relevant & correct options* in each { }.

INFERENCE

- (1) Mary hit John back.  
 (2) John has hit Mary.

CANCELLATION TEST:

(3) \_\_\_\_\_

JUDGMENT

OK \_\_,  $\square$  \_\_,  
 \_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 1:

- (1) {does, does not} {presuppose, entail, implicate} (2)

*Neg* PROJECTION TEST:

(4) \_\_\_\_\_

JUDGMENT

OK \_\_,  $\square$  \_\_,  
 \_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 2:

- (1) {does, does not} {entail, implicate, presuppose} (2)

2. (5 pts) Fill in the *relevant blanks* and underline the *relevant & correct options* in each { }.

INFERENCE

- (1) John has solved this problem.  
 (2) This problem has been solved.

CANCELLATION TEST:

(3) \_\_\_\_\_

JUDGMENT

OK \_\_,  $\square$  \_\_,  
 \_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 1:

- (1) {does, does not} {presuppose, entail, implicate} (2)

*If* PROJECTION TEST:

(4) \_\_\_\_\_

JUDGMENT

OK \_\_,  $\square$  \_\_,  
 \_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 2:

- (1) {does, does not} {entail, implicate, presuppose} (2)

In parts **3** and **4** below use the following  $DL_0$ -model:

(May be used to model)

$$M_6 = \langle \mathcal{D}_t^{M_6}, \llbracket \cdot \rrbracket^{M_6} \rangle$$

where  $\mathcal{D}_t^{M_6} = \{p : p \sqsubseteq \{w_1, w_2, w_3, w_4, w_5, w_6\}\}$

(relevant alternatives)

$$\llbracket T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$$

(Sue takes the test)

$$\llbracket P \rrbracket^{M_6} = \{w_1, w_3\}$$

(Sue passes the test)

$$\llbracket C \rrbracket^{M_6} = \{w_1, w_2, w_5\}$$

(Sue passes the course)

**3.** (4 pts). Complete the following facts and proofs.

FACT 9:  $\{w_1, w_3\} \llbracket (\partial T ; P) \rrbracket^{M_6} = \underline{\hspace{2cm}}$

PROOF:

$$\{w_1, w_3\} \llbracket (\partial T ; P) \rrbracket^{M_6}$$

=  $\underline{\hspace{2cm}}$

D2.2.s

=  $\underline{\hspace{2cm}}$

F10 below

=  $\underline{\hspace{2cm}}$

D2.2.b

=  $\underline{\hspace{2cm}}$

df.  $M_6, \{\dots\}$

□

FACT 10:  $\underline{\hspace{2cm}} \llbracket \partial T \rrbracket^{M_6} = \underline{\hspace{2cm}}$

PROOF: We first note that:

$$\underline{\hspace{2cm}} \llbracket T \rrbracket^{M_6}$$

=  $\underline{\hspace{2cm}}$

D2.2.b

=  $\underline{\hspace{2cm}}$

df.  $M_6, \{\dots\}$

Hence:

$$\underline{\hspace{2cm}} \llbracket \partial T \rrbracket^{M_6}$$

=  $\underline{\hspace{2cm}}$

D2.2.p

□

4. (6 pts). Complete the following facts and proofs.

FACT 11:  $\{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg(T; \neg(\partial T; P)) \rrbracket^{M_6} = \underline{\hspace{2cm}}$

PROOF:

$\{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg(T; \neg(\partial T; P)) \rrbracket^{M_6}$	
= <hr/>	D2.2.n
= <hr/>	D2.2.s
= <hr/>	
= <hr/>	D2.2.b
= <hr/>	df. $M_6, \{..l..\}$
= <hr/>	D2.2.n
= <hr/>	D2.2.s
= <hr/>	F12 below
= <hr/>	
= <hr/>	D2.2.b
= <hr/>	df. $M_6, \{..l..\}$
= <hr/>	df. $(x - y)$
= <hr/>	df. $(x - y)$

□

FACT 12:  $\underline{\hspace{2cm}} \llbracket \partial T \rrbracket^{M_6} = \underline{\hspace{2cm}}$

PROOF: We first note that:

$\underline{\hspace{2cm}} \llbracket T \rrbracket^{M_6}$	
= <hr/>	D2.2.b
= <hr/>	df. $M_6, \{..l..\}$

Hence:

$\underline{\hspace{2cm}} \llbracket \partial T \rrbracket^{M_6}$	
= <hr/>	D2.2.p

□

**Solution to Quiz 1**

Key to abbreviations:

- OK  $A$              $A$  is felicitous & true in a suitable context  
 $\square A$              $A$  is contradictory in every context  
 $A \Rightarrow B$         saying  $A$  commits the speaker to  $B$   
 $A \not\Rightarrow B$         saying  $A$  does not commit the speaker to  $B$

1. (5 pts) Fill in the *relevant blanks* and underline the *relevant & correct options* in each { }.

INFERENCE

- (1) Mary hit John back.  
 (2) John has hit Mary.

CANCELLATION TEST:

- (3) Mary hit John back; in fact, he hadn't hit her (when she hit him back).

JUDGMENT

OK \_\_,  $\square$  (3),\_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 1:

- (1) {does, does not} {presuppose, entail, implicate} (2)

*Neg* PROJECTION TEST:

- (4) Mary didn't hit John back

JUDGMENT

OK \_\_,  $\square$  \_\_,(4)  $\Rightarrow$  (2), \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 2:

- (1) {does, does not} {entail, implicate, presuppose} (2)

2. (5 pts) Fill in the *relevant blanks* and underline the *relevant & correct options* in each { }.

INFERENCE

- (1) John has solved this problem.  
 (2) This problem has been solved.

CANCELLATION TEST:

- (3) John has solved this problem; in fact, it hasn't been solved yet.

JUDGMENT

OK \_\_,  $\square$  (3),\_\_  $\Rightarrow$  \_\_, \_\_  $\not\Rightarrow$  \_\_

CONCLUSION 1:

- (1) {does, does not} {presuppose, entail, implicate} (2)

*If* PROJECTION TEST:

- (4) If John has solved this problem, he'll get the Nobel Prize.

JUDGMENT

OK \_\_,  $\square$  \_\_,\_\_  $\Rightarrow$  \_\_, (4)  $\not\Rightarrow$  (2)

CONCLUSION 2:

- (1) {does, does not} {entail, implicate, presuppose} (2)

CONCLUSION 1 + 2:

- (1) {does, does not} {entail, implicate, presuppose} (2),

In parts **3** and **4** below use the following  $DL_0$ -model:

$$M_6 = \langle D_t^{M_6}, \llbracket \cdot \rrbracket^{M_6} \rangle$$

where  $D_t^{M_6} = \{p: p \sqsubseteq \{w_1, w_2, w_3, w_4, w_5, w_6\}\}$

$$\llbracket T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$$

$$\llbracket P \rrbracket^{M_6} = \{w_1, w_3\}$$

$$\llbracket C \rrbracket^{M_6} = \{w_1, w_2, w_5\}$$

(May be used to model)

(relevant alternatives)

(Sue takes the test)

(Sue passes the test)

(Sue passes the course)

**3.** (4 pts). Complete the following facts and proofs.

CONTEXT: Sue and John both know she took a test and passed.

SPEECH ACT: Sue to John: ‘I passed.’

INTUITION: ‘Yeah, and what else is new?’

$$\text{FACT 9: } \{w_1, w_3\} \llbracket (\partial T; P) \rrbracket^{M_6} = \{w_1, w_3\}$$

PROOF:

$$\begin{aligned} & \{w_1, w_3\} \llbracket (\partial T; P) \rrbracket^{M_6} \\ = & \{w_1, w_3\} \llbracket \partial T \rrbracket^{M_6} \llbracket P \rrbracket^{M_6} \\ = & \{w_1, w_3\} \llbracket P \rrbracket^{M_6} \\ = & \{w \sqsubseteq \top D_t^{M_6} \mid w \sqsubseteq \{w_1, w_3\} \ \& \ w \sqsubseteq \llbracket P \rrbracket^{M_6}\} \\ = & \{w_1, w_3\} \end{aligned}$$

D2.2.s

F10 below

D2.2.b

df.  $M_6, \{\dots\}$

□

$$\text{FACT 10: } \{w_1, w_3\} \llbracket \partial T \rrbracket^{M_6} = \{w_1, w_3\}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_3\} \llbracket T \rrbracket^{M_6} \\ = & \{w \sqsubseteq \top D_t^{M_6} \mid w \sqsubseteq \{w_1, w_3\} \ \& \ w \sqsubseteq \llbracket T \rrbracket^{M_6}\} \\ = & \{w_1, w_3\} \end{aligned}$$

D2.2.b

df.  $M_6, \{\dots\}$

Hence:

$$\begin{aligned} & \{w_1, w_3\} \llbracket \partial T \rrbracket^{M_6} \\ = & \{w_1, w_3\} \end{aligned}$$

D2.2.p

□

Still the same model:

$$M_6 = \langle \mathcal{D}_t^{M_6}, \llbracket \cdot \rrbracket^{M_6} \rangle$$

where $\mathcal{D}_t^{M_6} = \{p: p \sqsubseteq \{w_1, w_2, w_3, w_4, w_5, w_6\}\}$	(relevant alternatives)
$\llbracket T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$	(Sue takes the test)
$\llbracket P \rrbracket^{M_6} = \{w_1, w_3\}$	(Sue passes the test)
$\llbracket C \rrbracket^{M_6} = \{w_1, w_2, w_5\}$	(Sue passes the course)

4. (5 pts). Complete the following facts and proofs.

CONTEXT: John and Bill know Sue's taking a tough class.

They know the class had a test but not whether Sue took it.

SPEECH ACT: John to Bill: 'If she took this test, she passed it.'

INTUITION: Felicitous. False if she in fact took the test & failed.

$$c_1 = \{w_1, w_2, w_3, w_4, w_5\}$$

$$c_2 = c_1 \llbracket \neg(T; \neg(\partial T; P)) \rrbracket^{M_6} \\ = \{w_1, w_3, w_5\}$$

$$\text{FACT 11: } \{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg(T; \neg(\partial T; P)) \rrbracket^{M_6} = \{w_1, w_3, w_5\}$$

PROOF:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4, w_5\} \llbracket \neg(T; \neg(\partial T; P)) \rrbracket^{M_6} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4, w_5\} \llbracket (T; \neg(\partial T; P)) \rrbracket^{M_6}) && \text{D2.2.n} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4, w_5\} \llbracket T \rrbracket^{M_6} \llbracket \neg(\partial T; P) \rrbracket^{M_6}) && \text{D2.2.s} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} \\ & - \{w \sqsubseteq \top \mathcal{D}_t^{M_6} \mid w \sqsubseteq \{w_1, w_2, w_3, w_4, w_5\} \ \& \ w \sqsubseteq \llbracket T \rrbracket^{M_6}\} \llbracket \neg(\partial T; P) \rrbracket^{M_6}) && \text{D2.2.b} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} \\ & - \{w \sqsubseteq \top \mathcal{D}_t^{M_6} \mid w \sqsubseteq \{w_1, w_2, w_3, w_4, w_5\} \ \& \ w \sqsubseteq \{w_1, w_2, w_3, w_4\}\} \llbracket \neg(\partial T; P) \rrbracket^{M_6}) && \text{df. } M_6 \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_1, w_2, w_3, w_4\} \llbracket \neg(\partial T; P) \rrbracket^{M_6}) && \{\dots\} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - (\{w_1, w_2, w_3, w_4\} - \{w_1, w_2, w_3, w_4\} \llbracket (\partial T; P) \rrbracket^{M_6})) && \text{D2.2.n} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - (\{w_1, w_2, w_3, w_4\} - \{w_1, w_2, w_3, w_4\} \llbracket \partial T \rrbracket^{M_6} \llbracket P \rrbracket^{M_6})) && \text{D2.2.s} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - (\{w_1, w_2, w_3, w_4\} - \{w_1, w_2, w_3, w_4\} \llbracket P \rrbracket^{M_6})) && \text{F12 below} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} \\ & - (\{w_1, w_2, w_3, w_4\} - \{w \sqsubseteq \top \mathcal{D}_t^{M_6} \mid w \sqsubseteq \{w_1, w_2, w_3, w_4\} \ \& \ w \sqsubseteq \llbracket P \rrbracket^{M_6}\})) && \text{D2.2.b} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} \\ & - (\{w_1, w_2, w_3, w_4\} - \{w \sqsubseteq \top \mathcal{D}_t^{M_6} \mid w \sqsubseteq \{w_1, w_2, w_3, w_4\} \ \& \ w \sqsubseteq \{w_1, w_3\}\})) && \text{df. } M_6 \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - (\{w_1, w_2, w_3, w_4\} - \{w_1, w_3\})) && \text{df. } \{\dots\} \\ = & (\{w_1, w_2, w_3, w_4, w_5\} - \{w_2, w_4\}) && \text{df. } (x - y) \\ = & \{w_1, w_3, w_5\} && \text{df. } (x - y) \end{aligned}$$

□

$$\text{FACT 12: } \{w_1, w_2, w_3, w_4\} \llbracket \partial T \rrbracket^{M_6} = \{w_1, w_2, w_3, w_4\}$$

PROOF: We first note that:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4\} \llbracket T \rrbracket^{M_6} \\ = & \{w \sqsubseteq \top \mathcal{D}_t^{M_6} \mid w \sqsubseteq \{w_1, w_2, w_3, w_4\} \ \& \ w \sqsubseteq \llbracket T \rrbracket^{M_6}\} && \text{D2.2.b} \\ = & \{w_1, w_2, w_3, w_4\} && \text{df. } M_6, \{\dots\} \end{aligned}$$

Hence:

$$\begin{aligned} & \{w_1, w_2, w_3, w_4\} \llbracket \partial T \rrbracket^{M_6} \\ = & \{w_1, w_2, w_3, w_4\} && \text{D2.2.p} \end{aligned}$$

□

## Lecture 5

### LOOKING BACK AND AHEAD

#### I. Some good results

- (1) It is *infelicitous* to:
- *presuppose* a proposition your addressee doesn't know [**H1**]
  - *assert* a proposition your addressee already knows [**Q1**: part 3]
- (2) • Felicitous assertion is *true* in some of the worlds of the input context and *false* in others.  
 • If this assertion is accepted, the former worlds are kept, while the latter are eliminated.  
 In this sense successful assertion *updates* the information of the participants. [**L2**, p. 8]
- (3) • Felicitous *negation*  $\neg(\partial B; A)$  reverses the truth value, i.e., it eliminates what  $(\partial B; A)$  would keep. [**L4**, p. 14].  
 • If  $(\partial B; A)$  is infelicitous (in context  $c$  of  $M$ ), so is the negation  $\neg(\partial B; A)$ . [**L4**, p. 15].
- (4) • Felicitous *conditional*  $\neg((\partial B; A); \neg C)$  — i.e., presupposing  $B$ , if  $A$  then  $C$  — eliminates those worlds where the *antecedent*  $A$  is true and the *consequent*  $C$  is false. [**Q1**: part 4].  
 • *Antecedent presuppositions project*:  
 If  $(\partial B; A)$  is infelicitous (in context  $c$  of  $M$ ), so is the conditional  $\neg((\partial B; A); \neg C)$ . [**H2**]  
 • *Consequent presuppositions do not project*, **if** the antecedent guarantees satisfaction, e.g., even if  $(\partial B; A)$  is infelicitous (in  $c$  of  $M$ ) the conditional  $\neg(B; \neg(\partial B; A))$  is ok. [**Q1**: part 4].

#### II. Problematic result

- (5) DL<sub>0</sub>-theory conflates two intuitively different kinds of infelicity:
- *what*-infelicity (e.g., ‘What test? What are you talking about?’)
  - *what's-new*-infelicity (e.g., ‘Yeah, and so? What else is new?’)

#### III. Toward a solution: Anaphora and presupposition

- (6) The DL<sub>0</sub>-theory is a so-called *satisfaction theory*: it treats presupposition as a test on the current input context. On this view presupposition failure is failure to pass this test.  
 An alternative, developed by van der Sandt 1992 and others, is an *anaphoric theory*.  
 (7a) is a paradigm example of *anaphora*: we say that the pronouns ‘she’ and ‘it’ are *anaphoric*, or *refer back*, to their *antecedents* — ‘Sue’ and ‘a test’, respectively.

- (7) a. Sue took a test. *She* passed *it*.  
 b. I passed [*it*].

According to the anaphoric theory, the presupposition of the verb ‘passed’ in (7b) is like an anaphoric pronoun: if the addressee cannot find any antecedent, the intuition is ‘What test?’, i.e., *what*-infelicity.

Beginning with TOPIC 2, we'll enrich our theory of context change with a theory of anaphora which can spell out this idea, among others.