Lecture 6

ANAPHORA AND CENTERING: FROM NL TO LOGIC OF CENTERING (LC)

I. Prominence-guided anaphora

- Pronouns and other *anaphors* refer to *discourse referents* introduced by their *antecedents* earlier in the discourse. If there are several referents, the choice is *guided by prominence*.
- We can model prominence-guided anaphora as changing *information-and-attention states* (IA-*states*). A state consists of a *world* (cand. reality) and two stacks of discourse referents (*drefs*): *top stack* (⊤) for the current center of attention, and *bottom* (□) for the periphery. On this view, a *context* is a set of IA-states, and speaking is like giving stage directions [dir].

Kalaallisut (Inuit Eskimo: Greenland): Topical **3SG** vs. plain 3SG[1]

0.	Initial state: cand. reality, no drefs	i = □w_i, □□, □□□	□ w_i in DL_0
1.	*Ilaanni anguti-qar-pu-q.*		
	once man-exist-IND.IV-3SG	i_1 = □w_i, □□, □□□□	[enter □ in *bck*]
	Once upon a time there was *a* man.	• □ is a man in w_i	[add info]
2.	*Angut taanna*		
	man **th**at	$i_{1□}$= □w_i, □□□, □□□□	[□ to **ctr**]
	[**Th**e man]	⋮	
	akira-qar-pu-q.	i_2 = □w_i, □□□, □@, □□□	[enter @ in *bck*]
	enemy.of-exist-IND.IV-**3SG**	⋮	
	had [*an* enemy].	• @ is an enemy of □ in w_i	[add info]
3.	*Ilaanni qajartur-lu-ni*	i_3 = □w_i, □□□, □@, …□□	
	once hunt.in.kayak-**ELA**-**3SG**	⋮	
	One day when **he** was hunting,	• □ is hunting in w_i	[add info]
	taku-va-a.	• □ sees @ in w_i	[add info]
	see-IND.TV-**3SG.**3SG		
	he saw *him.*		
4.	*Mali-lir-pa-a.*	i_4 = □w_i, □□□, □@, …□□	
	follow-begin-IND.TV-**3SG**.3SG	⋮	
	He began to follow *him.*	• □ begins to follow @ in w_i	[add info]
5.	*Mali-ta-ata*		
	follow-OBJ-3SG.ERG	$i_{4□}$= □w_i, □@, …□, □□, …□□	[@ to **ctr**,
	[**Th**e object of *his* pursuit]	⋮	□ to *bck*]
	taku-va-a tuqu-llu-gu=lu.	i_5 = □w_i, □@, …□, □□, …□□	
	see-IND.TV-**3SG.**3SG kill-**ELA**-3SG=and	• @ sees □ in w_i	[add info]
	saw *him* and killed *him.*	• @ kills □ in w_i	[add info]

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[1] **ELA** = topic elaboration, ERG = ergative, IND = indicative mood, IV = intransitive, TV = transitive, SG = singular.

II. *Logic of Centering* (LC): Syntax

D1.0 (LC types) The set of LC-*types* is the smallest set **Typ** such that:

b.	$t, e, \square, s \;\square\; \mathbf{Typ}$	(***t**ruth values*, ***e**ntities*, ***w**orlds*, IA-*states*)
f.	$(\square\square) \;\square\; \mathbf{Typ}$ if $\square, \square\square\; \mathbf{Typ}$	(functional types)

D1.1 (Basic LC terms)

c.	$\mathbf{Con}_{(se)}$	=	$\{e_{\top}, e_{\square}\}$	(*se*)-constants	(*demonstratives*)
	$\mathbf{Con}_{(s\square)}$	=	$\{r\}$	(*s*□)-constant	(*current reality*)
	$\mathbf{Con}_{(\square(et))}$	=	$\{E, hunt, \ldots, man, \ldots\}$	(□(*et*))-constants	(*property pred's*)
	$\mathbf{Con}_{(\square(e(et)))}$	=	$\{see, kill, \ldots, enemy, \ldots\}$	(□(*e*(*et*)))-constants	(*relational pred's*)
v.	$\mathbf{Var}_e$	=	$\{\mathbf{x}, y, y\square z, z\square$	*e*-variables	(var's over *entities*)
	$\mathbf{Var}_{\square}$	=	$\{w, w\square w''\}$	□-variables	(var's over *worlds*)
	$\mathbf{Var}_s$	=	$\{i, i\square j, j\square k, k\square h, h\square$	*s*-variables	(var's over IA-*states*)

D1.2 (LC syntax) For any □□ **Typ** the set of □-*terms* is the smallest set $\mathbf{Term}_{\square}$ such that:

b.	$\square \;\square\; \mathbf{Term}_{\square}$	if $\square \;\square\; \mathbf{Con}_{\square}$ or $\square \;\square\; \mathbf{Var}_{\square}$	[***b**asic term*]
¬.	$\neg\square \;\square\; \mathbf{Term}_t$	if $\square \;\square\; \mathbf{Term}_t$	[***n**egation*]
□.	$(\square \;\square\; \square) \;\square\; \mathbf{Term}_t$	if $\square, \square\square\; \mathbf{Term}_t$	[*conjunction*]
=.	$(\square = \square) \;\square\; \mathbf{Term}_t$	if $\square, \square\square\; \mathbf{Term}_{\square}$	[***i**dentity*]
a.	$\square\square \;\square\; \mathbf{Term}_{\square}$	if $\square \;\square\; \mathbf{Term}_{(\square\square)}$ and $\square \;\square\; \mathbf{Term}_{\square}$	[***a**pplication*]
□.	$\square u[\square] \;\square\; \mathbf{Term}_{(\square\square)}$	if $u \;\square\; \mathbf{Var}_{\square}$ and $\square \;\square\; \mathbf{Term}_{\square}$	[□*-abstraction*]
□.	$\square u\square \;\square\; \mathbf{Term}_t$	if $u \;\square\; \mathbf{Var}_{\square}$ and $\square \;\square\; \mathbf{Term}_t$	[□*-quantification*]
•.	$u \bullet \square \;\square\; \mathbf{Term}_s$	if $u \;\square\; \mathbf{Var}_e$ and $\square \;\square\; \mathbf{Term}_s$	[***s**tacking*]

N1 (Sugar coating). The following abbreviations may be used for LC terms:

().	( ) may be omitted if the result is unambiguous, e.g.,				
	$\square = \square$	:=	$(\square = \square)$		
□.	$\square_{\square}$	:=	$\square\square$	if this is a term	[*argument subscript*]
I.	$(\square_r \square)$	:=	$\square i[\square\; ri\; \square]$	"	[*property condition*]
		:=	$\square i[\square\; ri\; \square i]$	"	
T.	$(\square_r \square\; \square\square$	:=	$\square i[\square\; ri\; \square\; \square\square]$	"	[*relational condition*]
		:=	$\square i[\square\; ri\; \square\; \square\square i]$	"	
		:=	$\square i[\square\; ri\; \square i\; \square\square]$	"	
		:=	$\square i[\square\; ri\; \square i\; \square\square i]$	"	
□.	$(\square\; \square\; \square\square$	:=	$\square i[(\square = \square\square i)]$	"	[*identity condition*]
			$\square i[(\square i = \square\square i)]$	"	
[l.	$[\,\vert\, \square]$	:=	$\square i[\square j[((j = i) \;\square\; \square i)]]$	"	[*test box*]
[*u*.	$[u\vert\;]$	:=	$\square i[\square j[\square u(j = u \bullet i)]]$	"	[*recentering box*]
	$[u\vert\; \square]$	:=	$\square i[\square j[\square u((j = u \bullet i) \;\square\; \square i)]]$	"	
	$[u_1\; u_2\vert\; \square]$	:=	$\square i[\square j[\square u_1 \square u_2((j = u_1 \bullet u_2 \bullet i) \;\square\; \square i)]]$	"	
;.	$(\square_1\, ;\, \square_2)$	:=	$\square i[\square j[\square k(\square_1 ik \;\square\; \square_2 kj)]]$	"	[***s**equencing*]
;;.	$\square_1\, ;\, \square_2\, ;\, \square_3$	:=	$((\square_1\, ;\, \square_2)\, ;\, \square_3)$	"	[default assoc. to L]

III. <u>How to read definitions D1.0–D1.3:</u>

[*Typing* is used to ensure that whatever syntax generates, semantics can interpret. This is done by D1.0: defining a set of types □. D2.0–2.1: mapping basic □-terms to □-denotations. and D1.2–2.2: imposing type restrictions so that syntax only builds complex terms that semantics can interpret.]

D1.0 (LC types) The set of LC-*types* is the smallest set **Typ** such that:

b. t, e, □, s □ **Typ** (***t****ruth values*, ***e****ntities*, ***w****orlds*, IA-***s****tates*)
[Each of the symbols t, e, □, s is an LC-type]

f. (□□) □ **Typ** if □, □□ **Typ** (functional types)
[(□□) is an LC-type, if □ and □ are.
e.g., By ***b***: t, e, □ are LC-types (of *truth values*, *entities*, and *worlds*)
Hence by ***f***: (et) is an LC-type (of (et)-*functions*: from *entities* to *truth values*)
By ***f*** again: (□(et)) is an LC-type (of (□(et))-*functions*: from *worlds* to (et)-*functions*)

D1.1 (Basic LC terms)
[□-*constants* will be assigned fixed □-denotations by the *model*. □-*variables* will be assigned □-denotations that can vary, so that we can say, e.g., 'map any input □-denotation d to output $f(d)$'.]

D1.2 (LC syntax) For any □□ **Typ** the set of □-*terms* is the smallest set **Term**$_□$ such that:

b. □ □ **Term**$_□$ if □ □ **Con**$_□$ or □ □ **Var**$_□$ [***b****asic term*]
[□ is a □-term, if □ is a □-constant or a □-variable]

¬. ¬□ □ **Term**$_t$ if □ □ **Term**$_t$ [***n****egation*]
[¬□ ('*not* □') is a *t-term*, if the *scope* □ is a *t-term*]
Sem preview: □ is false.

□. (□ □ □) □ **Term**$_t$ if □, □□ **Term**$_t$ [***c****onjunction*]
[(□ □ □) ('□ *and* □') is a t-term, if the *conjuncts* □ and □ are both t-terms]
Sem preview: □ and □ are true.

=. (□ = □) □ **Term**$_t$ if □, □□ **Term**$_□$ [***i****dentity*]
[(□ = □) ('□ *is* □') is a t-term, if, for some type □, the *arguments* □ and □ are both □-terms]
Sem preview: □ and □ co-denote.

a. □□ □ **Term**$_□$ if □ □ **Term**$_{(□□)}$ and □ □ **Term**$_□$ [***a****pplication*]
[□□ ('□ *of* □') is a □-term if the *functor* □ is a (□□)-term & the *argument* □, a □-term]
Sem preview: the value of the □-function at the □-argument.

□. □u[□] □ **Term**$_{(□□)}$ if u □ **Var**$_□$ and □ □ **Term**$_□$ [***□****-abstract*]
[□u[□] ('*map any u to its* □') is a (□□)-term if u is a □-variable & the *scope* □, a □-term]
Sem preview: the function that maps any u-object to its □-value.

□. □u□ □ **Term**$_t$ if u □ **Var**$_□$ and □ □ **Term**$_t$ [□-*quantification*]
[□u□ ('*there is a u such that* □') is a t-term if u is a □-variable & the *scope* □, a t-term.
Sem preview: For some u-object, its □-value is true.

•. u • □ □ **Term**$_s$ if u □ **Var**$_e$ and □ □ **Term**$_s$ [*stacking*]
[u • □ ('*u plus* □') is an s-term, if u is an e-variable & □ is an s-term]
Sem preview: Output of adding the u-entity to the designated stack of the input IA.state □.

IV. From Natural Language (NL) syntax to LC syntax

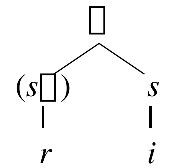
EXAMPLE 0 (Sugar coating). Each abbreviation is exemplified at •, except for a few left for **H3**.

(). ( ) may be omitted if the result is *unambiguous*, e.g.,

	□ = □	:=	(□ = □)	
•	*i* = *j*	:=	(*i* = *j*)	*unambiguous* even w/o ( ), so **ok** to omit ( )
•	¬□ = □		(¬□ = □) or ¬(□ = □)	*ambiguous* w/o ( ), so **can't** omit ( )

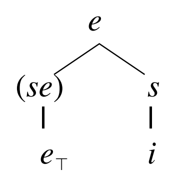
□. □$_□$:= □□ **if this is a term** [*argument subscript*]

• r_i := *ri*



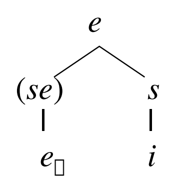
Sem preview: *i-reality*, w_i, of state □w_i, ⊤$_i$, □$_i$□ assigned to *i* (written □(*i*))

• $e_{\top i}$:= $e_\top i$



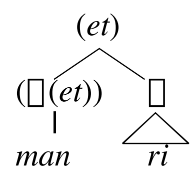
Sem preview: 1*st entity* on ⊤$_i$ if ⊤$_i$ ≠ □□, *nonexistent entity* (i.e. error), otherwise

• $e_{□i}$:= $e_□ i$



Sem preview: 1*st entity* on □$_i$ if □$_i$ ≠ □□, *nonexistent entity* (i.e. error) otherwise

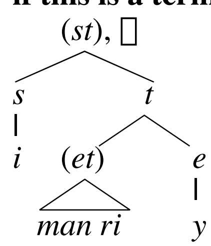
• man_{ri} := *man ri*



Sem preview: function from entities *d* to truth values s.t.: *d* □ 1 if *d* is a w_i-man □ 0 otherwise

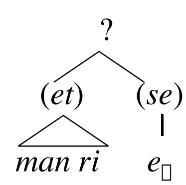
I. (□$_r$ □) := □*i*[□ *ri* □] **if this is a term** [*property condition*]

• (man_r *y*) := □*i*[*man ri y*]



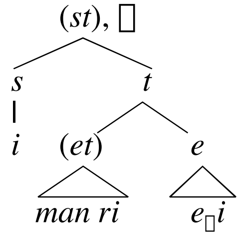
Sem preview: function from states to truth values s.t.: □w_i, ⊤$_i$, □$_i$□□ 1, if □(*y*) is a w_i-man □ 0, otherwise

• (man_r $e_□$) ≠ □*i*[*man ri* $e_□$]



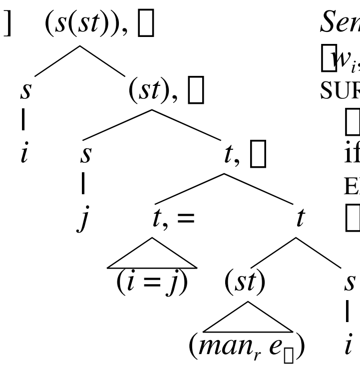
not a term: *type mismatch*!

	$(□_r\ □)$	:=	$□i[□\ ri\ □i]$	**if this is a term**
•	$(man_r\ e_i)$	:=	$□i[man\ ri\ e_i i]$	



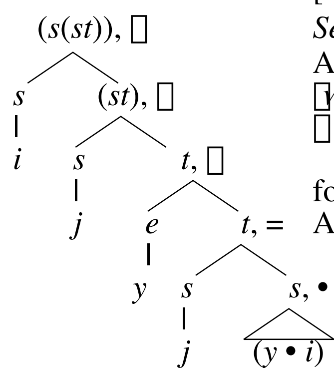
Sem preview:
function from states to truth values s.t.:
$□w_i, \top_i, □_i□□$ 1,
if 1st entity on $\top_i$ is a w_i-man,
otherwise, □ 0

T.	$(□_r\ □\ □□)$	:=	$□i[□\ ri\ □\ □□]$	**if this is a term**	[*relational condition*]
		:=	$□i[□\ ri\ □\ □□i]$	"	
		:=	$□i[□\ ri\ □i\ □□]$	"	
		:=	$□i[□\ ri\ □i\ □□i]$	"	
•	**H3**				
□.	$(□\ □\ □□)$	:=	$□i[(□ = □□i)]$	**if this is a term**	[*identity condition*]
			$□i[(□i = □□i)]$	"	
•	**H3**				
[l.	$[\ l\ □]$	:=	$□i[□j[((j = i)\ □\ □i)]]$	**if this is a term**	[*test box*]
•	$[l\ man_r\ e_□]$	:=	$[l\ (man_r\ e_□)]$		N1.()
		:=	$□i[□j[((j = i)\ □\ (man_r\ e_□)i]]$		



Sem preview:
$□w_i, \top_i, □_i□$
SURVIVES:
□ $□w_i, \top_i, □_i□□$ 1,
if it passes □-test
ELIMINATED:
□ any state □ 0,
otherwise

[u.	$[u l\ ]$	:=	$□i[□j[□u(j = u \bullet i)]]$	**if this is a term**	[*recentering box*]
•	$[y l\ ]$	:=	$□i[□j[□y(j = y \bullet i)]]$		



Sem preview:
ADD AN ENTITY to $□_i$:
$□w_i, \top_i, □_i□$
□ $□w_i, \top_i, d \cdot □_i□$
□ 1,
for any entity *d*.
Any other transition rejected, i.e., □ 0

	$[u l\ □]$	:=	$□i[□j[□u((j = u \bullet i)\ □\ □i)]]$	"	
	$[u_1\ u_2 l\ □]$	:=	$□i[□j[□u_1□u_2((j = u_1 \bullet u_2 \bullet i)\ □\ □i)]]$	"	
;.	$(□_1\ ;\ □_2)$	:=	$□i[□j[□k(□_1 ik\ □\ □_2 kj)]]$	"	[***s**equencing*]
;;.	$□_1\ ;\ □_2\ ;\ □_3$	:=	$((□_1\ ;\ □_2)\ ;\ □_3)$	"	[default assoc. to L]
•	**H3**				

• EXAMPLE 1 (*English*). Sugar-coated translation. i = □w_i, □□, □□□

Eng1. Once there was a man. i_1 = □w_i, □□, □□□□
LC: [*y*|]; [| man_r $e_□$]; • □ is a man in w_i

Eng2. He $i_{1□}$= □w_i, □□□, □□□□
LC: [| $male_r$ $e_□$]; [**x**| **x** □ $e_□$]; ⋮

had an enemy. i_2 = □w_i, □□□, □@, □□□
[*y*|]; [| $enemy_r$ $e_⊤$ $e_□$] ⋮

• @ is an enemy of □ in w_i

• EXAMPLE 2 (*Kalaallisut*). Sugar-coated translation. i_0 = □w_i, □□, □□□

Kal1. once man-exist-IND.IV-3SG i_1 = □w_i, □□, □□□□
LC: [*y*| man_r *y*]; • □ is a man in w_i

Kal2. man that $i_{1□}$= □w_i, □□□, □□□□
LC: [| man_r $e_□$]; [**x**| **x** □ $e_□$]; ⋮

enemy-exist-IND.IV-**3SG**. i_2 = □w_i, □□□, □@, □□□
[*y*| $enemy_r$ $e_□$ *y*] ⋮

• @ is an enemy of □ in w_i

• EXAMPLE 3: Desugaring LC translations:

LC: [*y*|]; [| man_r $e_□$] 1st line in EX1

:= [*y*|]; [| (man_r $e_□$)] N1.()

:= □*i*[□*j*[□*y*(*j* = *y* • *i*)]]; N1.[*u* (1st line)
□*i*[□*j*[((*j* = *i*) □ (man_r $e_□$)*i*)]] N1.**[|**

:= □*i*[□*j*[□*y*(*j* = *y* • *i*)]];
□*i*[□*j*[((*j* = *i*) □ □*i*[*man ri* $e_□$*i*]*i*)]] N1.I (2nd line)

:= □*i*[□*j*[□*k*(□*i*[□*j*[□*y*(*j* = *y* • *i*)]]*ik* N1.;
□ □*i*[□*j*[((*j* = *i*) □ □*i*[*man ri* $e_□$*i*]*i*)]]*kj*)]]

which will reduce to:
≡ □*i*[□*j*[□*y*((*j* = *y* • *i*) □ *man ri y*)]] by LC sem. (**Lecture 7**)

This will update an input IA-state (*i*) to an output (*j*) obtained by adding, to the bottom stack (*j* = *y* • *i*), an entity that in the input reality is a man (*man ri y*), e.g.:

input *i*	output *j*
□w_i, □□, □□□	□w_i, □□, □□□□
	• □ is a man in w_i

Homework 3

• EXAMPLE 4: Desugaring LC translations:

LC: $[\ |\ male_r\ e_{\square}];\ [\mathbf{x}|\ \mathbf{x}\ \square\ e_{\square}]$ 2nd line in EX1

:= ______________________________ ______________

:= ______________________________ ______________

______________________________ ______________

:= ______________________________ ______________

______________________________ ______________

:= ______________________________ ______________

____

which will reduce to:

$\equiv\ \square i[\square j[(male\ ri\ e_{\square}i\ \square\ \square\mathbf{x}((j = \mathbf{x} \bullet i)\ \square\ (\mathbf{x} = e_{\square}i)))]]$ by LC sem. (**Lecture 7**)

This will update an input IA-state (i), s.t. the 1st entity on the bottom stack ($e_{\square}i$) is male in the input reality, to an output (j) where that entity has been added to the top stack:

input i	output j
$\square w_i$, □□, □□□	$\square w_i$, □□□, □□□
• □ is a man in w_i	• □ is a man in w_i

• EXAMPLE 5: Desugaring LC translations:

LC: $[y|\ ];\ [\ |\ enemy_r\ e_{\top}\ e_{\square}]$ 3rd line in EX1

:= ______________________________ ______________

:= ______________________________ ______________

______________________________ ______________

:= ______________________________ ______________

______________________________ ______________

:= ______________________________ ______________

____

which will reduce to:

$\equiv\ \square i[\square j[\square y((j = y \bullet i)\ \square\ enemy\ ri\ e_{\top}i\ y)]]$ by LC sem. (**Lecture 7**)

This will update an input IA-state (i) to an output (j) obtained by adding, to the bottom stack, an entity that in the input reality is an enemy of the 1st entity on the top stack, e.g.:

input i	output j
$\square w_i$, □□□, □□□	$\square w_i$, □□□, □@, □□□
	• @ is an enemy of □ in w_i

Solution to Homework 3

• EXAMPLE 4: Desugaring LC translations:

LC:	[ǀ $male_r\ e_{\square}$]; [**x**ǀ **x** □ $e_{\square}$]	2nd line in EX1
:=	[ǀ ($male_r\ e_{\square}$)]; [**x**ǀ (**x** □ $e_{\square}$)]	N1.()
:=	□*i*[□*j*[((*j* = *i*) □ ($male_r\ e_{\square}$)*i*]];	N1.[ǀ
	□*i*[□*j*[□**x**((*j* = **x** • *i*) □ (**x** □ $e_{\square}$)*i*)]]	N1.[*u*ǀ (2nd line)
:=	□*i*[□*j*[((*j* = *i*) □ □*i*[*male ri* $e_{\square}$*i*]*i*)]];	N1.I (2nd line)
	□*i*[□*j*[□**x**((*j* = **x** • *i*) □ □*i*[(**x** = $e_{\square}$*i*)]*i*)]]	N1.□ (1st line)
:=	□*i*[□*j*[□*k*(□*i*[□*j*[((*j* = *i*) □ □*i*[*male ri* $e_{\square}$*i*]*i*)]]*ik*	N1.;
	□ □*i*[□*j*[□**x**((*j* = **x** • *i*) □ □*i*[(**x** = $e_{\square}$*i*)]*i*)]]*kj*)]]	

which will reduce to:

≡ □*i*[□*j*[*male ri* $e_{\square}$*i* □ □**x**(*j* = **x** • *i* □ **x** = $e_{\square}$*i*)]] — by LC sem. (**Lecture 7**)

This will update an input IA-state (*i*), s.t. the 1st entity on the bottom stack ($e_{\square}$*i*) is male in the input reality, to an output where that entity has been added to the top stack:

input *i*	output *j*
□w_i, □□, □□□	□w_i, □□□, □□□
• □ is a man in w_i	• □ is a man in w_i

• EXAMPLE 5: Desugaring LC translations:

LC:	[*y*ǀ]; [ǀ $enemy_r\ e_{\top}\ e_{\square}$]	3rd line in EX1
:=	[*y*ǀ]; [ǀ ($enemy_r\ e_{\top}\ e_{\square}$)]	N1.()
:=	□*i*[□*j*[□*y*(*j* = *y* • *i*)]];	N1.[*u* (1st line)
	□*i*[□*j*[((*j* = *i*) □ ($enemy_r\ e_{\top}\ e_{\square}$)*i*)]]	N1.**[ǀ**
:=	□*i*[□*j*[□*y*(*j* = *y* • *i*)]];	
	□*i*[□*j*[((*j* = *i*) □ □*i*[*enemy ri* $e_{\top}$*i* $e_{\square}$*i*]*i*)]]	N1.T (4th line)
:=	□*i*[□*j*[□*k*(□*i*[□*j*[□*y*(*j* = *y* • *i*)]]*ik*	N1.;
	□ □*i*[□*j*[((*j* = *i*) □ □*i*[*enemy ri* $e_{\top}$*i* $e_{\square}$*i*]*i*)]]*kj*)]]	

which will reduce to:

≡ □*i*[□*j*[□*y*(*j* = *y* • *i* □ *enemy ri* $e_{\top}$*i y*)]] — by LC sem. (**Lecture 7**)

This will update an input IA-state (*i*) to an output (*j*) obtained by adding, to the bottom stack, an entity that in the input reality is an enemy of the 1st entity on the top stack, e.g.:

input *i*	output *j*
□w_i, □□□, □□□	□w_i, □□□, □@, □□□
	• @ is an enemy of □ in w_i

Lecture 7
SEMANTICS OF LOGIC OF CENTERING (LC)

I. Mathematical preliminaries

- n-tuples (or n-sequences)
  $\langle\rangle$ — 0-tuple (or *empty tuple*)
  $\langle\square\rangle$ — 1-tuple
  $\langle\square, \%\rangle \neq \langle\%, \square\rangle$ — 2-ple (or *pair*), **order matters**
  $\langle\square, \%, \square\rangle \neq \langle\square, \%\rangle$ — 3-tuple (or *triple*), **repeats matter**
  ⋮

- $A^n := \{\langle a_1, \ldots, a_n\rangle: a_1, \ldots, a_n \in A\}$ — A to the n'th (or *n-fold product* of A)
  e.g., for $A = \{1, 2\}$
  $A^0 = \{\langle\rangle\}$
  $A^1 = \{\langle 1\rangle, \langle 2\rangle\}$
  $A^2 = \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 2\rangle\}$
  $A^3 = \{\langle 1, 1, 1\rangle, \langle 1, 1, 2\rangle, \langle 1, 2, 1\rangle, \langle 1, 2, 2\rangle, \ldots, \langle 2, 2, 2\rangle\}$
  ⋮

- $^1[\langle a_1, \ldots a_n\rangle] := a_1$ — 1st *coordinate* of $\langle a_1, \ldots, a_n\rangle$
  ⋮
  $^n[\langle a_1, \ldots, a_n\rangle] := a_n$ — nth *coordinate* of $\langle a_1, \ldots, a_n\rangle$

- sequence-building operation:
  $b \cdot \langle\rangle = \langle b\rangle$
  $b \cdot \langle a_1, \ldots, a_n\rangle = \langle b, a_1, \ldots, a_n\rangle$

- A *function* is a set of ordered pairs f such that, for all x, y, z, if $\langle x, y\rangle \in f$ and $\langle x, z\rangle \in f$, then $y = z$. — e.g. square.of-function on $\{1, 2, \ldots\}$: $(\cdot)^2 = \{\langle 1, 1^2\rangle, \langle 2, 2^2\rangle, \ldots\}$

- Dom f := $\{x$: for some y, $\langle x, y\rangle \in f\}$ — *domain* of function f
  Ran f := $\{y$: for some x, $\langle x, y\rangle \in f\}$ — *range* of function f
  $f(x) = y$ iff $\langle x, y\rangle \in f$ — f of x is y (or f*-value* at x is y)

- Sample abstracts:
  $\{x \in \{1, 2, 3\}: x^2 \leq 5\} = \{1, 2\}$ — *set abstract*
  $\{x^2: x \in \{1, 2, 3\}\} = \{1^2, 2^2, 3^2\}$ — *image abstract*
  $\langle x^2: x \in \{1, 2, 3\}\rangle = \{\langle 1, 1^2\rangle, \langle 2, 2^2\rangle, \langle 3, 3^2\rangle\}$ — *function abstract*

  In general:
- $z \in \{x \in A: \varphi\}$ iff $z \in A$ & $[z/x]\varphi$ — $1 \in \{x \in \{1, 2, 3\}$ & $x^2 \leq 5\}$ because $1 \in \{1, 2, 3\}$ & $1^2 \leq 5$

- $z \in \{f(x): x \in A\}$ iff, for some $x \in A$, $z = f(x)$ — $4 \in \{x^2: x \in \{1, 2, 3\}\}$ because $2 \in \{1, 2, 3\}$ & $4 = 2^2$

- $\langle f(x): x \in A\rangle := \{\langle x, f(x)\rangle: x \in A\}$ — $\langle x^2: x \in \{1, 2, 3\}\rangle := \{\langle 1, 1^2\rangle, \langle 2, 2^2\rangle, \langle 3, 3^2\rangle\}$

II. Logic of Centering (LC): Model theory and semantics

D2.0 (LC frames) An LC *frame* is a set $D = \{D_a: a \in \mathbf{Typ}\}$ of a-*domains*, D_a, such that:

b. $D_t = \{1, 0\}$ [1 for *true*, 0 for *false*]
D_e is a non-empty set disjoint from D_t. [domain of *entities*]
D_ω is a non-empty set disjoint from D_t and D_e. [domain of *worlds*]
$D_s = \{\langle w, \top, \bot\rangle: w \in D_\omega$ [domain of IA-*states*]
& for some $n \geq 0$, $\top \in D_e^n$
& for some $n' \geq 0$, $\bot \in D_e^{n'}\}$

f. $D_{(ab)} = \{f: \text{Dom } f = D_a \text{ \& Ran } f \subseteq D_b\}$ [domain of (ab)-*functions*]

D2.1 (LC models and assignments).

M. An LC *model* is a triple $M = \langle D^M, [\![\cdot]\!]^M, \#^M\rangle$ such that:
- $\bullet_1$ $D^M = \{D_a^M: a \in \mathbf{Typ}\}$ is an LC frame
- $\bullet_2$ $[\![\cdot]\!]^M$ is a function, from LC constants, such that: [*M-interpretation*]
  $[\![\alpha]\!]^M \in D_a^M$ if $\alpha \in \mathbf{Con}_a$
- $\bullet_3$ For any $\langle w, \top, \bot\rangle \in D_s^M$:
  $[\![e_\top]\!]^M(\langle w, \top, \bot\rangle) = {}^1[\top]$ if $\top \neq \langle\rangle$
  $= \#^M$ otherwise
  $[\![e_\bot]\!]^M(\langle w, \top, \bot\rangle) = {}^1[\bot]$ if $\bot \neq \langle\rangle$
  $= \#^M$ otherwise
  $[\![r]\!]^M(\langle w, \top, \bot\rangle) = w$
- $\bullet_4$ $\{\#^M\} = \{d \in D_e^M: \text{for all } w \in D_\omega^M, [\![E]\!]^M(w)(d) = 0\}$ [*non-existent entity*]

g. An *M-assignment* is a function g, from LC variables, such that:
$g(u) \in D_a^M$ if $u \in \mathbf{Var}_a$

N2. (*u-to-d* alternatives). Let g be an M-assignment g, $u \in \mathbf{Var}_a$ and $d \in D_a^M$. We write $g[u/d]$ for the M-assignment such that: (i) $g[u/d](u) = d$, and (ii) $g[u/d](v) = g(v)$ for any variable $v \neq u$.

D2.2 (LC semantics) For any LC-model $M = \langle D^M, [\![\cdot]\!]^M, \#^M\rangle$ and M-assignment g:

r.	$[\![\alpha]\!]^{M,g}$	$\in$	D_a^M	if $\alpha \in \mathbf{Term}_a$
b.	$[\![\alpha]\!]^{M,g}$	$=$	$[\![\alpha]\!]^M$	if $\alpha \in \mathbf{Con}_a$
		$=$	$g(\alpha)$	if $\alpha \in \mathbf{Var}_a$
¬.	$[\![\neg\varphi]\!]^{M,g}$	$=$	1	iff $[\![\varphi]\!]^{M,g} = 0$
∧.	$[\![(\varphi \wedge \psi)]\!]^{M,g}$	$=$	1	iff $[\![\varphi]\!]^{M,g} = 1$ & $[\![\psi]\!]^{M,g} = 1$
=.	$[\![(\alpha = \beta)]\!]^{M,g}$	$=$	1	iff $[\![\alpha]\!]^{M,g} = [\![\beta]\!]^{M,g}$
a.	$[\![\alpha\beta]\!]^{M,g}$	$=$	$[\![\alpha]\!]^{M,g}([\![\beta]\!]^{M,g})$	
λ.	$[\![\lambda u[\alpha]]\!]^{M,g}$	$=$	$\{\langle d, [\![\alpha]\!]^{M,g[u/d]}\rangle: d \in D_a^M\}$	if $u \in \mathbf{Var}_a$
∃.	$[\![\exists u\varphi]\!]^{M,g}$	$=$	1	iff $\{d \in D_a^M: [\![\varphi]\!]^{M,g[u/d]} = 1\} \neq \{\}$
•.	$[\![u \bullet \alpha]\!]^{M,g}$	$=$	$\langle {}^1[[\![\alpha]\!]^{M,g}], g(u) \cdot {}^2[[\![\alpha]\!]^{M,g}], {}^3[[\![\alpha]\!]^{M,g}]\rangle$	if $u = \mathbf{x}$
		$=$	$\langle {}^1[[\![\alpha]\!]^{M,g}], {}^2[[\![\alpha]\!]^{M,g}], g(u) \cdot {}^3[[\![\alpha]\!]^{M,g}]\rangle$	if $u \in (\mathbf{Var}_e - \{\mathbf{x}\})$

D3 (Truth).

φ. A t-term φ (*formula*) is *true* in M under g, written $\models_{M,g} \varphi$, iff $[\![\varphi]\!]^{M,g} = 1$.
φ. A (st)-term φ (*condition*) is *true* in a IA-state $\mathbf{s}$ in M under g, written $\models_{M,g,\mathbf{s}} \varphi$, iff $[\![\varphi]\!]^{M,g}(\mathbf{s}) = 1$.
φ. A $(s(st))$-term φ (*update*) is *true* in a IA-state $\mathbf{s}$ in M under g, written $\models_{M,g,\mathbf{s}} \varphi$,
iff, for some $\mathbf{s}'$, $[\![\varphi]\!]^{M,g}(\mathbf{s})(\mathbf{s}') = 1$.

A1. For any IA-state **s**, write

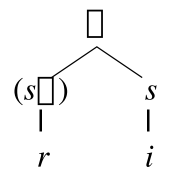
w_s	for	$^1[\mathbf{s}]$	*world* of **s**
$\top_s$	for	$^2[\mathbf{s}]$	*top stack* of **s**
$□_s$	for	$^3[\mathbf{s}]$	*bottom stack* of **s**

III. From LC syntax to semantics

EXAMPLE 0 (Sugar coating). Sample spell-out of *Sem preview*.

□. $□_□$:= □□ **if this is a term** [*argument subscript*]

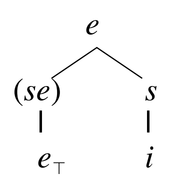
- r_i := ri



Sem preview: *i-reality*, w_i, of state □w_i, $\top_i$, $□_i$□ assigned to i (written □(i))

$[\![ri]\!]^{M,□}$ = $[\![r]\!]^{M,□}([\![i]\!]^{M,□})$ D2.2.***a***
= $[\![r]\!]^{M}(□(i))$ D2.2.***b***, ***b***
= $^1[□(i)]$ D2.1.•$_3$
= $w_{□(i)}$ A1

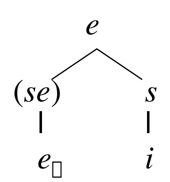
- $e_{\top i}$:= $e_\top i$



Sem preview: 1*st entity* on $\top_i$ if $\top_i \neq$ □□, *nonexistent entity* (i.e. error), otherwise

$[\![e_\top i]\!]^{M,□}$ = $[\![e_\top]\!]^{M,□}([\![i]\!]^{M,□})$ D2.2.***a***
= $[\![e_\top]\!]^{M}(□(i))$ D2.2.***b***, ***b***
= $^1[\top_{□(i)}]$ if $\top_{□(i)} \neq$ □□ D2.1.•$_3$, A1
$\#^M$ otherwise

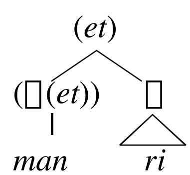
- $e_{□i}$:= $e_□ i$



Sem preview: 1*st entity* on $□_i$ if $□_i \neq$ □□, *nonexistent entity* (i.e. error) otherwise

$[\![e_□ i]\!]^{M,□}$ = $[\![e_□]\!]^{M,□}([\![i]\!]^{M,□})$ D2.2.***a***
= $[\![e_□]\!]^{M}(□(i))$ D2.2.***b***, ***b***
= $^1[□_{□(i)}]$ if $□_{□(i)} \neq$ □□ D2.1.•$_3$, A1
$\#^M$ otherwise

- man_{ri} := *man ri*



Sem preview: function from entities d to truth values s.t.:
d □ 1 if d is a w_i-man
□ 0 otherwise

$[\![man\ ri]\!]^{M,□}$ = $[\![man]\!]^{M,□}([\![ri]\!]^{M,□})$ D2.2.***a***
= $[\![man]\!]^{M}(w_{□(i)})$ D2.2.***b***, $[\![ri]\!]^{M,□}$ above

FACT 1: $[\![\exists y((j = y \bullet i) \wedge man\ ri\ y)]\!]^{M,g} = 1$
iff $\{d \in D_e^M: g(j) = \langle w_{g(i)}, \top_{g(i)}, d \cdot \bot_{g(i)}\rangle \ \&\ [\![man]\!]^M(w_{g(i)})(d) = 1\} \neq \{\}$

PROOF: **H4**

EXAMPLE 2 (*Kalaallisut*). From LC translation to semantics.

input: $\mathbf{s}_0 = \langle w_0, \langle\rangle, \langle\rangle\rangle$
such that $w_0 \in D_\omega^M$

Kal1. Once man-exist-IND.IV-3SG
LC. $[y|\ man_r\ y]$ sugar-coated
:= $\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]$ sugar-free

output: $\mathbf{s}_1$
such that $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]]\!]^{M,g}(\mathbf{s}_0)(\mathbf{s}_1) = 1$

i.e., for some M-entity $d \in D_e^M$, (i) $\mathbf{s}_1 = \langle w_0, \langle\rangle, \langle d\rangle\rangle$ F2 below
& (ii) $[\![man]\!]^M(w_0)(d) = 1$

FACT 2: $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]]\!]^{M,g}(\mathbf{s}_0)(\mathbf{s}_1) = 1$
iff $\{d \in D_e^M: \mathbf{s}_1 = \langle w_{s0}, \top_{s0}, d \cdot \bot_{s0}\rangle \ \&\ [\![man]\!]^M(w_{s0})(d) = 1\} \neq \{\}$

PROOF: Define: $g' := g[i/\mathbf{s}_0][j/\mathbf{s}_1]$. Then (1) iff (11):

1. $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]]\!]^{M,g}(\mathbf{s}_0)(\mathbf{s}_1) = 1$
2. $\{[\![\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]\!]^{M,g[i/d]}: d \in D_s^M\}(\mathbf{s}_0)(\mathbf{s}_1) = 1$ D2.3.λ
3. $[\![\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]\!]^{M,g[i/s0]}(\mathbf{s}_1) = 1$ df. {–:–}
4. $\{[\![\exists y((j = y \bullet i) \wedge man\ ri\ y)]\!]^{M,g[i/s0][j/d]}: d \in D_s^M\}(\mathbf{s}_1) = 1$ D2.3.λ
5. $[\![\exists y((j = y \bullet i) \wedge man\ ri\ y)]\!]^{M,g[i/s0][j/s1]} = 1$ df. {–:–}
6. $[\![\exists y((j = y \bullet i) \wedge man\ ri\ y)]\!]^{M,g'} = 1$ df. g'
7. $\{d \in D_e^M: g'(j) = \langle w_{g'(i)}, \top_{g'(i)}, d \cdot \bot_{g'(i)}\rangle$ F1
   $\&\ [\![man]\!]^M(w_{g'(i)})(d) = 1\} \neq \{\}$
8. $\{d \in D_e^M: g'(j) = \langle {}^1[g'(i)], {}^2[g'(i)], d \cdot {}^3[g'(i)]\rangle$ A1
   $\&\ [\![man]\!]^M({}^1[g'(i)])(d) = 1\} \neq \{\}$ A
9. $\{d \in D_e^M: g[i/\mathbf{s}_0][j/\mathbf{s}_1](j) = \langle {}^1[g[i/\mathbf{s}_0][j/\mathbf{s}_1](i)],$ df. g'
   ${}^2[g[i/\mathbf{s}_0][j/\mathbf{s}_1](i)],$
   $d \cdot {}^3[g[i/\mathbf{s}_0][j/\mathbf{s}_1](i)]\rangle$
   $\&\ [\![man]\!]^M({}^1[g[i/\mathbf{s}_0][j/\mathbf{s}_1](i))(d) = 1\} \neq \{\}$
10. $\{d \in D_e^M: \mathbf{s}_1 = \langle {}^1[\mathbf{s}_0], {}^2[\mathbf{s}_0], d \cdot {}^3[\mathbf{s}_0]\rangle \ \&\ [\![man]\!]^M({}^1[\mathbf{s}_0])(d) = 1\} \neq \{\}$ N2.i, ii
11. $\{d \in D_e^M: \mathbf{s}_1 = \langle w_{s0}, \top_{s0}, d \cdot \bot_{s0}\rangle \ \&\ [\![man]\!]^M(w_{s0})(d) = 1\} \neq \{\}$ A1 □

That is, given an input state $\mathbf{s}_0$, this update accepts an output state $\mathbf{s}_1$ just in case:

- $\mathbf{s}_1 = \langle w_{s0}, \ldots\rangle$
  i.e., same world in the output $\mathbf{s}_1$ as in the input $\mathbf{s}_0$
- $\mathbf{s}_1 = \langle \ldots, \top_{s0}, \ldots\rangle$
  i.e., same top stack in the output $\mathbf{s}_1$ as in the input $\mathbf{s}_0$
- $\{d \in D_e^M: \mathbf{s}_1 = \langle \ldots, \ldots, d \cdot \bot_{s0}\rangle \ \&\ [\![man]\!]^M(w_{s0})(d) = 1\} \neq \{\}$
  i.e., there is ($\neq \{\}$) an M-entity ($d \in D_e^M$) which is a man in this world ($[\![man]\!]^M(w_{s0})(d) = 1$)
  and is the new 1st coordinate on the bottom stack in the output ($d \cdot \bot_{s0}$)

Homework 4

FACT 1: $[\![\square y((j = y \bullet i) \square\ man\ ri\ y)]\!]^{M,\square} = 1$

iff $\{d \square D_e^M: \square(j) = \square w_{\square(i)}, \top_{\square(i)}, d \cdot \square_{\square(i)} \square \& [\![man]\!]^M(w_{\square(i)})(d) = 1\} \neq \{\}$

PROOF: (1) iff (15)

1.	$[\![\square y((j = y \bullet i) \square\ man\ ri\ y)]\!]^{M,\square} = 1$	
2.	______	D2.2.□
3.	______	D2.2.□
4.	______	D2.2.=
5.	______	D2.2.***b***
6.	______	N2.ii
7.	______	D2.2.•
	______	D2.2.***a***
8.	______	N2.i
	______	D2.2.***a***
9.	______	D2.2.***b***, ***b***, ***b***
	______	D2.2.***b***, ***b***
10.	______	N2.ii, ii, ii
	______	N2.i
11.	______	
	______	D2.2.***a***
12.	______	
	______	D2.2.***b***, ***b***
13.	______	
	______	N2.ii
14.	______	
	______	$D2.1.\bullet_3$
15.	______	A1
	______	A1 □

Solution to Homework 4

FACT 1: $[\![\square y((j = y \bullet i) \square\ man\ ri\ y)]\!]^{M, \square} = 1$

iff $\{d \square D_e^M: \square(j) = \square w_{\square(i)}, \top_{\square(i)}, d \cdot \square_{\square(i)}\square \& [\![man]\!]^M(w_{\square(i)})(d) = 1\} \neq \{\}$ abbrev.

PROOF: (1) iff (14)

1. $[\![\square y((j = y \bullet i) \square\ man\ ri\ y)]\!]^{M, \square} = 1$
2. $\{d \square D_e^M: [\![((j = y \bullet i) \square\ man\ ri\ y)]\!]^{M, \square[y/d]} = 1\} \neq \{\}$ D2.2.$\square$
3. $\{d \square D_e^M: [\![(j = y \bullet i)]\!]^{M, \square[y/d]} = 1\ \&\ [\![man\ ri\ y]\!]^{M, \square[y/d]} = 1\} \neq \{\}$ D2.2.$\square$
4. $\{d \square D_e^M: [\![j]\!]^{M, \square[y/d]} = [\![y \bullet i]\!]^{M, \square[y/d]}\ \&\ [\![man\ ri\ y]\!]^{M, \square[y/d]} = 1\} \neq \{\}$ D2.2.=
5. $\{d \square D_e^M: \square[y/d](j) = [\![y \bullet i]\!]^{M, \square[y/d]}\ \&\ [\![man\ ri\ y]\!]^{M, \square[y/d]} = 1\} \neq \{\}$ D2.2.***b***
6. $\{d \square D_e^M: \square(j) = [\![y \bullet i]\!]^{M, \square[y/d]}\ \&\ [\![man\ ri\ y]\!]^{M, \square[y/d]} = 1\} \neq \{\}$ N2.ii
7. $\{d \square D_e^M: \square(j) = \square^1[[\![i]\!]^{M, \square[y/d]}], {}^2[[\![i]\!]^{M, \square[y/d]}], \square[y/d](y) \cdot {}^3[[\![i]\!]^{M, \square[y/d]}]\square$ D2.2.•
   $\&\ [\![man\ ri]\!]^{M, \square[y/d]}([\![y]\!]^{M, \square[y/d]}) = 1\} \neq \{\}$ D2.2.***a***
8. $\{d \square D_e^M: \square(j) = \square^1[[\![i]\!]^{M, \square[y/d]}], {}^2[[\![i]\!]^{M, \square[y/d]}], d \cdot {}^3[[\![i]\!]^{M, \square[y/d]}]\square$ N2.i
   $\&\ [\![man]\!]^{M, \square[y/d]}([\![ri]\!]^{M, \square[y/d]})([\![y]\!]^{M, \square[y/d]}) = 1\} \neq \{\}$ D2.2.***a***
9. $\{d \square D_e^M: \square(j) = \square^1[\square[y/d](i)], {}^2[\square[y/d](i)], d \cdot {}^3[\square[y/d](i)]\square$ D2.2.***b***, ***b***, ***b***
   $\&\ [\![man]\!]^{M}([\![ri]\!]^{M, \square[y/d]})(\square[y/d](y)) = 1\} \neq \{\}$ D2.2.***b***, ***b***
10. $\{d \square D_e^M: \square(j) = \square^1[\square(i)], {}^2[\square(i)], d \cdot {}^3[\square(i)]\square$ N2.ii, ii, ii
   $\&\ [\![man]\!]^{M}([\![ri]\!]^{M, \square[y/d]})(d) = 1\} \neq \{\}$ N2.i
11. $\{d \square D_e^M: \square(j) = \square^1[\square(i)], {}^2[\square(i)], d \cdot {}^3[\square(i)]\square$
   $\&\ [\![man]\!]^{M}([\![r]\!]^{M, \square[y/d]}([\![i]\!]^{M, \square[y/d]}))(d) = 1\} \neq \{\}$ D2.2.***a***
12. $\{d \square D_e^M: \square(j) = \square^1[\square(i)], {}^2[\square(i)], d \cdot {}^3[\square(i)]\square$
   $\&\ [\![man]\!]^{M}([\![r]\!]^{M}(\square[y/d](i)))(d) = 1\} \neq \{\}$ D2.2.***b***, ***b***
13. $\{d \square D_e^M: \square(j) = \square^1[\square(i)], {}^2[\square(i)], d \cdot {}^3[\square(i)]\square$
   $\&\ [\![man]\!]^{M}([\![r]\!]^{M}(\square(i)))(d) = 1\} \neq \{\}$ N2.ii
14. $\{d \square D_e^M: \square(j) = \square^1[\square(i)], {}^2[\square(i)], d \cdot {}^3[\square(i)]\square$
   $\&\ [\![man]\!]^{M}({}^1[\square(i)])(d) = 1\} \neq \{\}$ D2.1.$\bullet_3$
15. $\{d \square D_e^M: \square(j) = \square w_{\square(i)}, \top_{\square(i)}, d \cdot \square_{\square(i)}\square$ A1
   $\&\ [\![man]\!]^{M}(w_{\square(i)})(d) = 1\} \neq \{\}$ A1 □

EXAMPLE 3 (*Kalaallisut*). From LC translation to semantics ctd.

input: $\mathbf{s}_0 = \langle w_0, \langle\rangle, \langle\rangle\rangle$
such that $w_0 \in D_\omega{}^M$

Kal1. Once man-exist-IND.IV-3SG ‘Once there was a man.’
LC. $[y|\ man_r\ y]$ *sugar-coated*
$\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]$ *sugar-free*
(UPDATE: add a w_i-man d_0 to $\bot_i$, i.e., d_0 becomes ${}^1\bot_i$) *Sem preview*

*output*1: $\mathbf{s}_1 = \langle w_0, \langle\rangle, d_0 \cdot \langle\rangle\rangle$
$= \langle w_0, \langle\rangle, \langle d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$ FACT 1, 2
(in w_0, d_0 is a man)

Kal2. Man that … ‘He …
LC: $[|\ man_r\ e_\bot]$; $[\mathbf{x}|\ \mathbf{x} \in e_\bot]$ *sugar-coated*
$\lambda i[\lambda j[((j = i) \wedge man\ ri\ e_\bot i)]]$; $\lambda i[\lambda j[\exists \mathbf{x}((j = \mathbf{x} \cdot i) \wedge (\mathbf{x} = e_\bot i))]]$ *sugar-free*
(TEST: ${}^1\bot_i$ is a w_i-man) (UPDATE: add ${}^1\bot_i$ to $\top_i$) *Sem preview*

*output*2: $\mathbf{s}_2 = \langle w_0, d_0 \cdot \langle\rangle, \langle d_0\rangle\rangle$
$= \langle w_0, \langle d_0\rangle, \langle d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$ FACT 3, 4 below

… enemy-exist-IND.IV-**3SG** …had an enemy.’
…; $[y|\ enemy_r\ e_\top\ y]$ *sugar-coated*
…; $\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge enemy\ ri\ e_\top i\ y)]]$ *sugar-free*
… (UPDATE: add a w_i-enemy d_1 of d_0 to $\bot_i$, i.e. d_1 becomes ${}^1\bot_i$) *Sem preview*

*output*3: $\mathbf{s}_3 = \langle w_0, \langle d_0\rangle, d_1 \cdot \langle d_0\rangle\rangle$
$= \langle w_0, \langle d_0\rangle, \langle d_1, d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$
• $[\![enemy]\!]^M(w_0)(d_0)(d_1) = 1$ FACT 5 below
(in w_0, d_1 is an enemy of d_0)

FACT 1: $[\![\exists y((j = y \bullet i) \wedge man\ ri\ y)]\!]^{M, g} = 1$
iff $\{d \in D_e{}^M: g(j) = \langle w_{g(i)}, \top_{g(i)}, d \cdot \bot_{g(i)}\rangle$
$\&\ [\![man]\!]^M(w_{g(i)})(d) = 1\} \neq \{\}$

FACT 2: $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge man\ ri\ y)]]]\!]^{M, g}(\mathbf{s})(\mathbf{s}') = 1$ Above: $\mathbf{s} = \mathbf{s}_0$, $\mathbf{s}' = \mathbf{s}_1$
iff $\{d \in D_e{}^M: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle$ $w_\mathbf{s} = w_0$, $\top_\mathbf{s} = \bot_\mathbf{s} = \langle\rangle$
$\&\ [\![man]\!]^M(w_\mathbf{s})(d) = 1\} \neq \{\}$

FACT 3: $[\![\lambda i[\lambda j[((j = i \wedge man\ ri\ e_\bot i)]\!]^{M, g}(\mathbf{s})(\mathbf{s}') = 1$ Above: $\mathbf{s} = \mathbf{s}_1$, $\mathbf{s}' = \mathbf{s}_1$,
iff $\mathbf{s}' = \mathbf{s}\ \&\ [\![man]\!]^M(w_\mathbf{s})({}^1\bot_\mathbf{s}) = 1$ $w_\mathbf{s} = w_0$, ${}^1\bot_\mathbf{s} = d_0$

FACT 4: $[\![\lambda i[\lambda j[\exists \mathbf{x}((j = \mathbf{x} \bullet i) \wedge (\mathbf{x} = e_\bot i)]]]\!]^{M, g}(\mathbf{s})(\mathbf{s}') = 1$ Above: $\mathbf{s} = \mathbf{s}_1$, $\mathbf{s}' = \mathbf{s}_2$
iff $\{d \in D_e{}^M: \mathbf{s}' = \langle w_\mathbf{s}, d \cdot \top_\mathbf{s}, \bot_\mathbf{s}\rangle$ $w_\mathbf{s} = w_0$, $\top_\mathbf{s} = \langle d_0\rangle$,
$\&\ d = {}^1\bot_\mathbf{s}\} \neq \{\}$ $\bot_\mathbf{s} = \langle d_0\rangle$, ${}^1\bot_\mathbf{s} = d_0$
iff $\mathbf{s}_2 = \langle w_{\mathbf{s}1}, {}^1\bot_{\mathbf{s}1} \cdot \top_{\mathbf{s}1}, \bot_{\mathbf{s}1}\rangle$ df. {–:–}

FACT 5: $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge enemy\ ri\ e_\top i\ y)]]]\!]^{M, g}(\mathbf{s})(\mathbf{s}') = 1$ Above: $\mathbf{s} = \mathbf{s}_2$, $\mathbf{s}' = \mathbf{s}_3$
iff $\{d \in D_e{}^M: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle$ $w_\mathbf{s} = w_0$, $\top_\mathbf{s} = \langle d_0\rangle$,
$\&\ [\![enemy]\!]^M(w_\mathbf{s})({}^1\top_\mathbf{s})(d) = 1\} \neq \{\}$ $\bot_\mathbf{s} = \langle d_1, d_0\rangle$, ${}^1\bot_\mathbf{s} = d_1$

EXAMPLE 4 (*English*). From LC translation to semantics ctd.

input: $\mathbf{s}_0 = \langle w_0, \langle\rangle, \langle\rangle\rangle$
such that $w_0 \in D_\omega{}^M$

Eng1. Once there was a man.
LC. $[y|\ ]$; $[|\ man_r\ e_\bot]$ *sugar-coated*
$\lambda i[\lambda j[\exists y(j = y \bullet i)]]$; $\lambda i[\lambda j[i = j \wedge man\ ri\ e_\bot i]]$ *sugar-free*
(UPD: add arb d_0 to $\bot_i$) ; (TEST: ${}^1\bot_i$ is a w_i-man) *Sem preview*

*output*1: $\mathbf{s}_1 = \langle w_0, \langle\rangle, d_0 \cdot \langle\rangle\rangle$
$= \langle w_0, \langle\rangle, \langle d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$ FACT 2′, 3
(in w_0, d_0 is a man)

Kal2. He …
LC: $[|\ male_r\ e_\bot]$; $[\mathbf{x}|\ \mathbf{x} \in e_\bot]$ *sugar-coated*
$\lambda i[\lambda j[((j = i) \wedge male\ ri\ e_\bot i)]]$; $\lambda i[\lambda j[\exists\mathbf{x}((j = \mathbf{x} \cdot i) \wedge (\mathbf{x} = e_\bot i))]]$ *sugar-free*
(TEST: ${}^1\bot_i$ is a w_i-male) (UPD: add ${}^1\bot_i$ to $\top_i$) *Sem preview*

*output*2: $\mathbf{s}_2 = \langle w_0, d_0 \cdot \langle\rangle, \langle d_0\rangle\rangle$
$= \langle w_0, \langle d_0\rangle, \langle d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$ FACT 3[*male*], 4

… had an enemy.
…; $[y|\ ]$; $[|\ enemy_r\ e_\top\ e_\bot]$ *sugar-coated*
…; $\lambda i[\lambda j[\exists y(j = y \bullet i)]]$; $\lambda i[\lambda j[i = j \wedge enemy\ ri\ e_\top i\ e_\bot i]]$ *sugar-free*
… (UPD: add arb d_1 to $\bot_i$); (TEST: ${}^1\bot_i$ is a w_i-enemy of ${}^1\top_i$) *Sem preview*

*output*3: $\mathbf{s}_3 = \langle w_0, \langle d_0\rangle, d_1 \cdot \langle d_0\rangle\rangle$
$= \langle w_0, \langle d_0\rangle, \langle d_1, d_0\rangle\rangle$ s.t. • $[\![man]\!]^M(w_0)(d_0) = 1$
• $[\![enemy]\!]^M(w_0)(d_0)(d_1) = 1$ FACT 2′, 5′
(in w_0, d_1 is an enemy of d_0)

FACT 2′: $[\![\lambda i[\lambda j[\exists y(j = y \bullet i)]]]\!]^{M,\,g}(\mathbf{s})(\mathbf{s}') = 1$
iff $\{d \in D_e{}^M: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle\} \neq \{\}$

FACT 5′: $[\![\lambda i[\lambda j[j = i \wedge enemy\ ri\ e_\top i\ e_\bot i]]]\!]^{M,\,g}(\mathbf{s})(\mathbf{s}') = 1$
iff $[\![enemy]\!]^M(w_\mathbf{s})({}^1\top_\mathbf{s})({}^1\bot_\mathbf{s}) = 1$

• EXAMPLE 5. (Simplifying LC translations). LC semantics validates two useful equivalences:

λ-conversion: $\lambda u[\ldots u\ldots](\alpha) \equiv \ldots\alpha\ldots$ u, α of same type, no accidental binding
Alphabetic variance: $\lambda u[\ldots u\ldots] \equiv \lambda u'[\ldots u'\ldots]$ u, u' of same type, no $\lambda u'$ or $\exists u'$ in […]

Note that **every** occurrence of u in $[\ldots u\ldots]$ must be replaced by α (or by the new variable u').
e.g.,

$\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge \lambda i[(enemy\ ri\ y)]i)]]ik$
$\equiv$ $\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge \lambda k[(enemy\ rk\ y)]i)]]ik$ *alph var*. (optional)
$\equiv$ $\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge (enemy\ ri\ y))]]ik$ *λ-cnv*
$\equiv$ $\lambda j[\exists y((j = y \bullet i) \wedge (enemy\ ri\ y))]k$ *λ-cnv*
$\equiv$ $\exists y((k = y \bullet i) \wedge (enemy\ ri\ y))$ *λ-cnv*

IV. *Logic of Centering* (LC): Repeated for ease of reference

D1.0 (LC types) The set of LC-*types* is the smallest set **Typ** such that:

b.	$t, e, \omega, s \in$ **Typ**	(***t**ruth values*, ***e**ntities*, ***w**orlds*, IA-*states*)
f.	$(ab) \in$ **Typ** if $a, b \in$ **Typ**	(functional types)

D1.1 (Basic LC terms)

c.	$\mathbf{Con}_{(se)}$	=	$\{e_{\top}, e_{\bot}\}$	(*se*)-constants	(*demonstratives*)
	$\mathbf{Con}_{(s\omega)}$	=	$\{r\}$	($s\omega$)-constant	(*current reality*)
	$\mathbf{Con}_{(\omega(et))}$	=	$\{E, hunt, \ldots, man, \ldots\}$	($\omega(et)$)-constants	(*property pred's*)
	$\mathbf{Con}_{(\omega(e(et)))}$	=	$\{see, kill, \ldots, enemy, \ldots\}$	($\omega(e(et))$)-constants	(*relational pred's*)
v.	$\mathbf{Var}_e$	=	$\{\mathbf{x}, y, y', z, z'\}$	*e*-variables	(var's over *entities*)
	$\mathbf{Var}_{\omega}$	=	$\{w, w', w''\}$	ω-variables	(var's over *worlds*)
	$\mathbf{Var}_s$	=	$\{i, i', j, j', k, k', h, h'\}$	*s*-variables	(var's over IA-*states*)

D1.2 (LC syntax) For any $a \in$ **Typ** the set of *a-terms* is the smallest set $\mathbf{Term}_a$ such that:

b.	$\alpha \in \mathbf{Term}_a$	if $\alpha \in \mathbf{Con}_a$ or $\alpha \in \mathbf{Var}_a$	[***b**asic term*]
¬.	$\neg\varphi \in \mathbf{Term}_t$	if $\varphi \in \mathbf{Term}_t$	[***n**egation*]
∧.	$(\varphi \wedge \psi) \in \mathbf{Term}_t$	if $\varphi, \psi \in \mathbf{Term}_t$	[***c**onjunction*]
=.	$(\alpha = \beta) \in \mathbf{Term}_t$	if $\alpha, \beta \in \mathbf{Term}_a$	[***i**dentity*]
a.	$\alpha\beta \in \mathbf{Term}_b$	if $\alpha \in \mathbf{Term}_{(ab)}$ and $\beta \in \mathbf{Term}_a$	[***a**pplication*]
λ.	$\lambda u[\alpha] \in \mathbf{Term}_{(ab)}$	if $u \in \mathbf{Var}_a$ and $\alpha \in \mathbf{Term}_b$	[***λ**-abstraction*]
∃.	$\exists u\varphi \in \mathbf{Term}_t$	if $u \in \mathbf{Var}_a$ and $\varphi \in \mathbf{Term}_t$	[***∃**-quantification*]
•.	$u \bullet \alpha \in \mathbf{Term}_s$	if $u \in \mathbf{Var}_e$ and $\alpha \in \mathbf{Term}_s$	[***s**tacking*]

N1 (Sugar coating). The following abbreviations may be used for LC terms:

(). ( ) may be omitted if the result is unambiguous, e.g.,

	$\varphi = \psi$	:=	$(\varphi = \psi)$		
$_\beta$.	α_β	:=	$\alpha\beta$	if this is a term	[*argument subscript*]
I.	$(\delta_r\, \alpha)$	:=	$\lambda i[\delta\ ri\ \alpha]$	"	[*property condition*]
		:=	$\lambda i[\delta\ ri\ \alpha i]$	"	
T.	$(\delta_r\, \alpha\, \beta)$	:=	$\lambda i[\delta\ ri\ \alpha\ \beta]$	"	[*relational condition*]
		:=	$\lambda i[\delta\ ri\ \alpha\ \beta i]$	"	
		:=	$\lambda i[\delta\ ri\ \alpha i\ \beta]$	"	
		:=	$\lambda i[\delta\ ri\ \alpha i\ \beta i]$	"	
≐.	$(\alpha \doteq \beta)$	:=	$\lambda i[(\alpha = \beta i)]$	"	[*identity condition*]
			$\lambda i[(\alpha i = \beta i)]$	"	
[|.	$[\,\mid \varphi]$	:=	$\lambda i[\lambda j[((j = i) \wedge \varphi i)]]$	"	[*test box*]
[*u*.	$[u\mid\,]$	:=	$\lambda i[\lambda j[\exists u(j = u \bullet i)]]$	"	[*recentering box*]
	$[u\mid \varphi]$	:=	$\lambda i[\lambda j[\exists u((j = u \bullet i) \wedge \varphi i)]]$	"	
	$[u_1\, u_2\mid \varphi]$	:=	$\lambda i[\lambda j[\exists u_1 \exists u_2((j = u_1 \bullet u_2 \bullet i) \wedge \varphi i)]]$	"	
;.	$(\varphi_1\, ;\, \varphi_2)$	:=	$\lambda i[\lambda j[\exists k(\varphi_1 ik \wedge \varphi_2 kj)]]$	"	[***s**equencing*]
;;.	$\varphi_1\, ;\, \varphi_2\, ;\, \varphi_3$	:=	$((\varphi_1\, ;\, \varphi_2)\, ;\, \varphi_3)$	"	[default assoc. to L]

D2.0 (LC frames) An LC *frame* is a set $D = \{D_\Box: \Box\Box$ **Typ**$\}$ of □-*domains*, $D_\Box$, such that:

b. $D_t = \{1, 0\}$ [1 for *true*, 0 for *false*]
D_e is a non-empty set disjoint from D_t. [domain of *entities*]
$D_\Box$ is a non-empty set disjoint from D_t and D_e. [domain of *worlds*]
$D_s = \{\Box w, \top, \Box\Box: w \Box D_\Box$ [domain of IA-*states*]
& for some $n \geq 0$, $\top \Box D_e^{\,n}$
& for some $n\Box \geq 0$, $\Box \Box D_e^{\,n\Box}\}$

f. $D_{(\Box\Box)} = \{f$: Dom $f = D_\Box$ & Ran $f \Box D_\Box\}$ [domain of (□□)-*functions*]

D2.1 (LC models and assignments).

M. An LC *model* is a triple $M = \Box D^M, [\![\cdot]\!]^M, \#^M\Box$ such that:
- $\bullet_1$ $D^M = \{D_\Box^{\,M}: \Box\Box$ **Typ**$\}$ is an LC frame
- $\bullet_2$ $[\![\cdot]\!]^M$ is a function, from LC constants, such that: [*M-interpretation*]
 $[\![\Box]\!]^M \Box D_\Box^{\,M}$ if $\Box \Box$ **Con**$_\Box$
- $\bullet_3$ For any $\Box w, \top, \Box\Box\Box D_s^{\,M}$:

$[\![e_\top]\!]^M(\Box w, \top, \Box\Box)$	=	$^1[\top]$	if $\top \neq \Box\Box$
	=	$\#^M$	otherwise
$[\![e_\Box]\!]^M(\Box w, \top, \Box\Box)$	=	$^1[\Box]$	if $\Box \neq \Box\Box$
	=	$\#^M$	otherwise
$[\![r]\!]^M(\Box w, \top, \Box\Box)$	=	w	

- $\bullet_4$ $\{\#^M\} = \{d \Box D_e^{\,M}$: for all $w \Box D_\Box^{\,M}$, $[\![E]\!]^M(w)(d) = 0\}$ [*non-existent entity*]

□. An *M-assignment* is a function □, from LC variables, such that:
$\Box(u) \Box D_\Box^{\,M}$ if $u \Box$ **Var**$_\Box$

N2. (*u-to-d* alternatives). Let □ be an *M*-assignment □, $u \Box$ **Var**$_\Box$ and $d \Box D_\Box^{\,M}$. We write $\Box[u/d]$ for the *M*-assignment such that: (i) $\Box[u/d](u) = d$, and (ii) $\Box[u/d](v) = \Box(v)$ for any variable $v \neq u$.

D2.2 (LC semantics) For any LC-model $M = \Box D^M, [\![\cdot]\!]^M, \#^M\Box$ and *M*-assignment □:

r.	$[\![\Box]\!]^{M,\Box}$	□	$D_\Box^{\,M}$	if □ □ **Term**$_\Box$
b.	$[\![\Box]\!]^{M,\Box}$	=	$[\![\Box]\!]^M$	if □ □ **Con**$_\Box$
		=	$\Box(\Box)$	if □ □ **Var**$_\Box$
¬.	$[\![\neg\Box]\!]^{M,\Box}$	=	1	iff $[\![\Box]\!]^{M,\Box} = 0$
□.	$[\![(\Box \Box \Box)]\!]^{M,\Box}$	=	1	iff $[\![\Box]\!]^{M,\Box} = 1$ & $[\![\Box]\!]^{M,\Box} = 1$
=.	$[\![(\Box = \Box)]\!]^{M,\Box}$	=	1	iff $[\![\Box]\!]^{M,\Box} = [\![\Box]\!]^{M,\Box}$
a.	$[\![\Box\Box]\!]^{M,\Box}$	=	$[\![\Box]\!]^{M,\Box}([\![\Box]\!]^{M,\Box})$	
□.	$[\![\Box u[\Box]]\!]^{M,\Box}$	=	$\Box[\![\Box]\!]^{M,\Box[u/d]}: d \Box D_\Box^{\,M}\Box$	if $u \Box$ **Var**$_\Box$
□.	$[\![\Box u\Box]\!]^{M,\Box}$	=	1	iff $\{d \Box D_\Box^{\,M}: [\![\Box]\!]^{M,\Box[u/d]} = 1\} \neq \{\}$
•.	$[\![u \bullet \Box]\!]^{M,\Box}$	=	$\Box^1[[\![\Box]\!]^{M,\Box}], \Box(u) \cdot {}^2[[\![\Box]\!]^{M,\Box}], {}^3[[\![\Box]\!]^{M,\Box}]\Box$	if u = **x**
		=	$\Box^1[[\![\Box]\!]^{M,\Box}], {}^2[[\![\Box]\!]^{M,\Box}], \Box(u) \cdot {}^3[[\![\Box]\!]^{M,\Box}]\Box$	if $u \Box$ (**Var**$_e$ – {**x**})

D3 (Truth).

□. A *t*-term □ (*formula*) is *true* in *M* under □, written $\models_{M,\Box} \Box$, iff $[\![\Box]\!]^{M,\Box} = 1$.

□. A (*st*)-term □ (*condition*) is *true* in a IA-state **s** in *M* under □, written $\models_{M,\Box,\mathbf{s}} \Box$, iff $[\![\Box]\!]^{M,\Box}(\mathbf{s}) = 1$.

□. A (*s*(*st*))-term □ (*update*) is *true* in a IA-state **s** in *M* under □, written $\models_{M,\Box,\mathbf{s}} \Box$,
iff, for some $\mathbf{s}\Box$, $[\![\Box]\!]^{M,\Box}(\mathbf{s})(\mathbf{s}\Box) = 1$.

EXAMPLE 6 (*English*). From a model and a set of input states to the output set.

Consider a model $M1$ such that:

$D_{□}^{M1} = \{w_0, w_1, w_2, w_3\}$

$D_e^{M1} = \{□, \$\}$

$[\![man]\!]^{M1}(w)(d) = 1$, if □$w, d$□□ {□$w_0$, □□ □$w_0$, \$□ □w_1, □□ □w_2, \$□}
= 0, otherwise

$[\![woman]\!]^{M1}(w)(d) = 1$, if □$w, d$□□ {□$w_1$, \$□ □w_2, □□ □w_3, □□ □w_3, \$□}
= 0, otherwise

$[\![enemy]\!]^{M1}(w)(d)(d□) = 1$, if □$w, d, d$□□□ {□$w_0$, □, \$□ □w_0, \$, □□ □w_1, □, \$□ □w_1, \$, □□}
= 0, otherwise

$[\![kill]\!]^{M1}(w)(d)(d□)$ = 1, if □w, d, d□□□ {□w_0, \$, □□ □w_1, □, \$□ □w_3, \$, □□}
= 0, otherwise

At the outset, suppose all worlds are live options & nothing is attended to, i.e.:

Input set: {□w_0, □□ □□□ □w_1, □□ □□□ □w_2, □□ □□□ □w_3, □□ □□□}

Eng1. Once there was a man.

LC. [\[y| man_r y]\]^{M1}(**s**)(**s**□) = 1 *sugar-coated*

iff [\[□i[□j[□y(i = y • j □ man ri y)]]]\]^{M1}(**s**)(**s**□) = 1 *sugar-free*

iff {d □ D_e^{M1}: **s**□= □$w_\mathbf{s}$, $\top_\mathbf{s}$, $d \cdot$ □$_\mathbf{s}$□& [\[man]\]$^{M1}(w_\mathbf{s})(d) = 1$ *truth condition*

*Output*1: {□w_0, □□ □□□□ □w_0, □□ □\$□□ □w_1, □□ □□□□ □w_2, □□ □\$□□}

Eng2. He …

LC: [\[| man_r $e_□$]; [**x**| **x** □ $e_□$]\]^{M1}(**s**)(**s**□) = 1 *sugar-coated*

iff [\[□i[□j[(j = i) □ man ri $e_□$i]];
□i[□j[□**x**((j = **x** · i) □ (**x** = $e_□$i))]]\]^{M1}(**s**)(**s**□) = 1 *sugar-free*

iff [\[man]\]$^{M1}(w_\mathbf{s})(^1□_\mathbf{s}) = 1$
& {d □ D_e^{M1}: **s**□= □$w_\mathbf{s}$, $d \cdot \top_\mathbf{s}$, □$_\mathbf{s}$□& $d = {}^1□_\mathbf{s}$} ≠ {} *truth condition*

*Output*2: {□w_0, □□□ □□□□ □w_0, □\$□ □\$□□ □w_1, □□□ □□□□ □w_2, □\$□ □\$□□}

… had an enemy.

iff [\[y| $enemy_r$ $e_\top$ y]\]^{M1}(**s**)(**s**□) = 1 *sugar coated*

iff [\[□i[□j[□y(j = y • i) □ enemy ri $e_\top$i y)]]]\]^{M1}(**s**)(**s**□) = 1 *sugar-free*

iff {d □ D_e^{M1}: **s**□= □$w_\mathbf{s}$, $\top_\mathbf{s}$, $d \cdot$ □$_\mathbf{s}$□& [\[enemy]\]$^{M1}(w_\mathbf{s})(^1\top_\mathbf{s})(d) = 1$} ≠ {} *truth condition*

*Output*3: {□w_0, □□□ □\$, □□□ □w_0, □\$□ □□, \$□□ □w_1, □□□ □\$, □□□}

Eng3. The guy killed him.

LC: [\[| man_r $e_□$, $kill_r$ $e_\top$ $e_□$]; [**x**| **x** □ $e_□$]\]^{M1}(**s**)(**s**□) = 1 *sugar-coated*

iff [\[□i[□j[(j = i) □ man ri $e_□$i □ kill ri $e_\top$i $e_□$i]];
□i[□j[□**x**((j = **x** · i) □ (**x** = $e_□$i)]]]\]^{M1}(**s**)(**s**□) = 1 *sugar-free*

iff [\[man]\]$^{M1}(w_\mathbf{s})(^1□_\mathbf{s}) = 1$
& [\[kill]\]$^{M1}(w_\mathbf{s})(^1\top_\mathbf{s})(^1□_\mathbf{s}) = 1$
& {d □ D_e^{M1}: **s**□= □$w_\mathbf{s}$, $d \cdot \top_\mathbf{s}$, d □$_\mathbf{s}$□& $d = {}^1□_\mathbf{s}$} ≠ {} *truth condition*

*Output*4: {□w_0, □□, \$□ □□, \$□□}

Homework 5

EXAMPLE 7 (*English*). From a model and a set of input states to the output set.

Consider again *M*1 such that:

$D_{□}^{M1} = \{w_0, w_1, w_2, w_3\}$
$D_e^{M1} = \{□, \$\}$
⟦*man*⟧$^{M1}(w)(d)$ = 1, if □w, d□□ {□w_0, □□ □w_0, \$□ □w_1, □□ □w_2, \$□}
= 0, otherwise
⟦*woman*⟧$^{M1}(w)(d)$ = 1, if □w, d□□ {□w_1, \$□ □w_2, □□ □w_3, □□ □w_3, \$□}
= 0, otherwise
⟦*enemy*⟧$^{M1}(w)(d)(d$□$)$ = 1, if □w, d, d□□□ {□w_0, □, \$□ □w_0, \$, □□ □w_1, □, \$□ □w_1, \$, □□}
= 0, otherwise
⟦*kill*⟧$^{M1}(w)(d)(d$□$)$ = 1, if □w, d, d□□□ {□w_0, \$, □□ □w_1, □, \$□ □w_3, \$, □□}
= 0, otherwise

Again at the outset, suppose all worlds are live options & nothing is attended to, i.e.:

Input set: {□w_0, □□ □□□ □w_1, □□ □□□ □w_2, □□ □□□ □w_3, □□ □□□}

Eng1. Once there was a woman.

LC. ⟦[*y*| *woman*$_r$ *y*]⟧^{M1}(**s**)(**s**□) = 1 *sugar-coated*
iff ⟦□*i*[□*j*[□*y*(*i* = *y* • *j* □ *woman ri y*)]]⟧^{M1}(**s**)(**s**□) = 1 *sugar-free*
iff {*d* □ D_e^{M1}: **s**□= □$w_{\mathbf{s}}$, $\top_{\mathbf{s}}$, *d* · □$_{\mathbf{s}}$□& ⟦*woman*⟧^{M1}($w_{\mathbf{s}}$)(*d*) = 1 *truth condition*

*Output*1: ____

Eng2. She …

LC: ⟦[| *woman*$_r$ *e*$_□$]; [**x**| **x** □ *e*$_□$]⟧^{M1}(**s**)(**s**□) = 1 *sugar-coated*
iff ⟦□*i*[□*j*[(*j* = *i*) □ *woman ri e*$_□$*i*]];
□*i*[□*j*[□**x**((*j* = **x** · *i*) □ (**x** = *e*$_□$*i*))]]⟧^{M1}(**s**)(**s**□) = 1 *sugar-free*
iff ⟦*woman*⟧^{M1}($w_{\mathbf{s}}$)(^{1}□$_{\mathbf{s}}$) = 1
& {*d* □ D_e^{M1}: **s**□= □$w_{\mathbf{s}}$, *d* ·$\top_{\mathbf{s}}$, □$_{\mathbf{s}}$□& *d* = ^{1}□$_{\mathbf{s}}$} ≠ {} *truth condition*

*Output*2: ____

… killed…
iff ⟦[*y*| *kill*$_r$ *y e*$_\top$]⟧^{M1}(**s**)(**s**□) = 1 *sugar coated*
iff ⟦□*i*[□*j*[□*y*(*j* = *y* • *i*) □ *kill ri y e*$_\top$*i*)]]⟧^{M1}(**s**)(**s**□) = 1 *sugar-free*
iff {*d* □ D_e^{M1}: **s**□= □$w_{\mathbf{s}}$, $\top_{\mathbf{s}}$, *d* · □$_{\mathbf{s}}$□& ⟦*kill*⟧^{M1}($w_{\mathbf{s}}$)(*d*)($^1\top_{\mathbf{s}}$) = 1} ≠ {} *truth condition*

*Output*3: ____

… a man.
⟦[| *man*$_r$ *e*$_□$]⟧^{M1}(**s**)(**s**□) = 1 *sugar-coated*
iff ⟦□*i*[□*j*[(*j* = *i*) □ *man ri e*$_□$*i*]⟧^{M1}(**s**)(**s**□) = 1 *sugar-free*
iff **s**□= **s** & ⟦*man*⟧^{M1}($w_{\mathbf{s}}$)(^{1}□$_{\mathbf{s}}$) = 1

*Output*4: ____

Solution to Homework 5

EXAMPLE 7 (*English*). From a model and a set of input states to the output set.

Consider again *M*1 such that:
$D_{\square}^{M1} = \{w_0, w_1, w_2, w_3\}$
$D_e^{M1} = \{\square, \$\}$
$[\![man]\!]^{M1}(w)(d) = 1$, if $\langle w, d\rangle \in \{\langle w_0, \square\rangle, \langle w_0, \$\rangle, \langle w_1, \square\rangle, \langle w_2, \$\rangle\}$
= 0, otherwise
$[\![woman]\!]^{M1}(w)(d) = 1$, if $\langle w, d\rangle \in \{\langle w_1, \$\rangle, \langle w_2, \square\rangle, \langle w_3, \square\rangle, \langle w_3, \$\rangle\}$
= 0, otherwise
$[\![enemy]\!]^{M1}(w)(d)(d') = 1$, if $\langle w, d, d'\rangle \in \{\langle w_0, \square, \$\rangle, \langle w_0, \$, \square\rangle, \langle w_1, \square, \$\rangle, \langle w_1, \$, \square\rangle\}$
= 0, otherwise
$[\![kill]\!]^{M1}(w)(d)(d')$ = 1, if $\langle w, d, d'\rangle \in \{\langle w_0, \$, \square\rangle, \langle w_1, \square, \$\rangle, \langle w_3, \$, \square\rangle\}$
= 0, otherwise

Again at the outset, suppose all worlds are live options & nothing is attended to, i.e.:

Input set: $\{\langle w_0, \langle\rangle, \langle\rangle\rangle, \langle w_1, \langle\rangle, \langle\rangle\rangle, \langle w_2, \langle\rangle, \langle\rangle\rangle, \langle w_3, \langle\rangle, \langle\rangle\rangle\}$

Eng1. Once there was a woman.
LC. $[\![[y|\ woman_r\ y]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*
iff $[\![\lambda i[\lambda j[\exists y(i = y \bullet j \wedge woman\ ri\ y)]]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*
iff $\{d \in D_e^{M1}: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle\ \&\ [\![woman]\!]^{M1}(w_\mathbf{s})(d) = 1$ *truth condition*

*Output*1: $\{\langle w_1, \langle\rangle, \langle\$\rangle\rangle, \langle w_2, \langle\rangle, \langle\square\rangle\rangle, \langle w_3, \langle\rangle, \langle\$\rangle\rangle, \langle w_3, \langle\rangle, \langle\square\rangle\rangle\}$

Eng2. She …
LC: $[\![[|\ woman_r\ e_\bot];\ [\mathbf{x}|\ \mathbf{x} \in e_\bot]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*
iff $[\![\lambda i[\lambda j[(j = i) \wedge woman\ ri\ e_\bot i]];$
$\lambda i[\lambda j[\exists \mathbf{x}((j = \mathbf{x} \cdot i) \wedge (\mathbf{x} = e_\bot i))]]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*
iff $[\![woman]\!]^{M1}(w_\mathbf{s})(^1\bot_\mathbf{s}) = 1$
$\&\ \{d \in D_e^{M1}: \mathbf{s}' = \langle w_\mathbf{s}, d \cdot \top_\mathbf{s}, \bot_\mathbf{s}\rangle\ \&\ d = {}^1\bot_\mathbf{s}\} \neq \{\}$ *truth condition*

*Output*2: $\{\langle w_1, \langle\$\rangle, \langle\$\rangle\rangle, \langle w_2, \langle\square\rangle, \langle\square\rangle\rangle, \langle w_3, \langle\$\rangle, \langle\$\rangle\rangle, \langle w_3, \langle\square\rangle, \langle\square\rangle\rangle\}$

… killed…
iff $[\![[y|\ kill_r\ y\ e_\top]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar coated*
iff $[\![\lambda i[\lambda j[\exists y(j = y \bullet i) \wedge kill\ ri\ y\ e_\top i)]]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*
iff $\{d \in D_e^{M1}: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle\ \&\ [\![kill]\!]^{M1}(w_\mathbf{s})(d)(^1\top_\mathbf{s}) = 1\} \neq \{\}$ *truth condition*

*Output*3: $\{\langle w_1, \langle\$\rangle, \langle\square, \$\rangle\rangle, \langle w_3, \langle\square\rangle, \langle\$, \square\rangle\rangle\}$

… a man.
$[\![[|\ man_r\ e_\bot]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*
iff $[\![\lambda i[\lambda j[(j = i) \wedge man\ ri\ e_\bot i]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*
iff $\mathbf{s}' = \mathbf{s}\ \&\ [\![man]\!]^{M1}(w_\mathbf{s})(^1\bot_\mathbf{s}) = 1$

*Output*4: $\{\langle w_1, \langle\$\rangle, \langle\square, \$\rangle\rangle\}$

EXAMPLE 8 (*English*).

Problem: Using LC we cannot capture the following CROSS-LINGUISTIC GENERALIZATION:

- *Agent-patient verbs* (e.g. 'kill') *relate agent* ${}^1\top$ (subject) *to patient* ${}^1\bot$ (object).

We can only maintain this for *She killed a man* in **H5,** but not e.g. for *The guy killed him* in **Ex 6**.

Solution: LC$_n$, with unlimited stack access. LC$_n$ is basically LC except that $\mathbf{Con}_{(se)} = \{e_\top, e_\bot\}$ becomes $\mathbf{Con}_{(se)} = \{1_\top, 2_\top, \ldots, 1_\bot, 2_\bot, \ldots\}$ & the related semantic rule D2.1.$\bullet_3$ is revised to match.

LC$_n$ analysis of *The guy killed him* from EXAMPLE 6**:**
Same model *M*1, same input set, same discourse. For **Eng1–2**, analysis **as in** EX 6 **except that** $e_\bot$ and $e_\top$ are **renamed** $1_\bot$ and $1_\top$, respectively:

Input set: $\{\langle w_0, \langle\rangle, \langle\rangle\rangle, \langle w_1, \langle\rangle, \langle\rangle\rangle, \langle w_2, \langle\rangle, \langle\rangle\rangle, \langle w_3, \langle\rangle, \langle\rangle\rangle\}$

⋮

*Output*3: $\{\langle w_0, \langle\rangle, \langle\$, \langle\rangle\rangle\rangle, \langle w_0, \langle\$\rangle, \langle\rangle, \$\rangle\rangle, \langle w_1, \langle\rangle, \langle\$, \langle\rangle\rangle\rangle\}$

Eng3. The guy … **cf. H5:** she

LC$_n$: $[\![[\,|\, man_r\ 1_\bot]; [\mathbf{x}|\ \mathbf{x} \in 1_\bot]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*

iff $[\![\lambda i[\lambda j[(j = i) \wedge man\ ri\ 1_\bot i]]; \lambda i[\lambda j[\exists \mathbf{x}(j = \mathbf{x} \bullet i \wedge \mathbf{x} = 1_\bot i)]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*

iff $[\![man]\!]^{M1}(w_\mathbf{s})({}^1\bot_\mathbf{s}) = 1$ & $\{d \in D_e^{M1}: \mathbf{s}' = \langle w_\mathbf{s}, d \cdot \top_\mathbf{s}, \bot_\mathbf{s}\rangle\ \&\ d = {}^1\bot_\mathbf{s}\} \neq \{\}$ *truth condition*

*Output*4a: $\{\langle w_0, \langle\$, \langle\rangle\rangle, \langle\$, \langle\rangle\rangle\rangle, \langle w_0, \langle\rangle, \$\rangle, \langle\rangle, \$\rangle\}$

… killed… **cf. H5:** killed

$[\![[y|\ kill_r\ y\ 1_\top]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*

iff $[\![\lambda i[\lambda j[\exists y((j = y \bullet i) \wedge kill\ ri\ y\ 1_\top i)]]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*

iff $\{d \in D_e^{M1}: \mathbf{s}' = \langle w_\mathbf{s}, \top_\mathbf{s}, d \cdot \bot_\mathbf{s}\rangle\ \&\ [\![kill]\!]^{M1}(w_\mathbf{s})(d)({}^1\top_\mathbf{s}) = 1\} \neq \{\}$ *truth condition*

*Output*4b: $\{\langle w_0, \langle\rangle, \$\rangle, \langle\$, \langle\rangle, \$\rangle\rangle\}$

…him. **cf. H5:** a man

$[\![[\,|\, man_r\ 1_\bot]; [\,|\ 1_\bot = 2_\top]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-coated*

iff $[\![\lambda i[\lambda j[(j = i) \wedge man\ ri\ 1_\bot i]]; \lambda i[\lambda j[(j = i \wedge 1_\bot i = 2_\top i)]]\!]^{M1}(\mathbf{s})(\mathbf{s}') = 1$ *sugar-free*

iff $[\![man]\!]^{M1}(w_\mathbf{s})({}^1\bot_\mathbf{s}) = 1$ & ${}^1\bot_\mathbf{s} = {}^2\top_\mathbf{s}$ *truth condition*

*Output*4c: $\{\langle w_0, \langle\rangle, \$\rangle, \langle\$, \langle\rangle, \$\rangle\rangle\}$ **cf**. EXAMPLE 6

V. *Logic of Centering* with unlimited stack access (LC_n):
Like LC **except for (i) richer** $\mathbf{Con}_{(se)}$ **and D2.1.•$_3$, (ii) simpler** $\mathbf{Var}_e$ **and D2.2.• Same A1.**

D1.0 (LC_n types) The set of LC_n-*types* is the smallest set **Typ** such that:

b.	$t, e, \omega, s \in$ **Typ**	(***t**ruth values*, ***e**ntities*, ***w**orlds*, IA-***s**tates*)
f.	$(ab) \in$ **Typ** if $a, b \in$ **Typ**	(functional types)

D1.1 (Basic LC_n terms)

c.	$\mathbf{Con}_{(se)}$	=	$\{1_\top, 2_\top, \ldots, 1_\bot, 2_\bot, \ldots\}$	(*se*)-constants	(*demonstratives*)
	$\mathbf{Con}_{(s\omega)}$	=	$\{r\}$	($s\omega$)-constant	(*current reality*)
	$\mathbf{Con}_{(\omega(et))}$	=	$\{E, hunt, \ldots, man, \ldots\}$	($\omega(et)$)-constants	(*property pred's*)
	$\mathbf{Con}_{(\omega(e(et)))}$	=	$\{see, kill, \ldots, enemy, \ldots\}$	($\omega(e(et))$)-constants	(*relational pred's*)
v.	$\mathbf{Var}_e$	=	$\{\mathbf{x}, y\}$	*e*-variables	(var's over *entities*)
	$\mathbf{Var}_\omega$	=	$\{w, w', w''\}$	ω-variables	(var's over *worlds*)
	$\mathbf{Var}_s$	=	$\{i, i', j, j', k, k', h, h'\}$	*s*-variables	(var's over IA-*states*)

D1.2 (LC_n syntax) For any $a \in$ **Typ** the set of a-*terms* is the smallest set $\mathbf{Term}_a$ such that:

b.	$A \in \mathbf{Term}_a$	if $A \in \mathbf{Con}_a$ or $A \in \mathbf{Var}_a$	[***b**asic term*]
¬.	$\neg\varphi \in \mathbf{Term}_t$	if $\varphi \in \mathbf{Term}_t$	[***n**egation*]
∧.	$(\varphi \wedge \psi) \in \mathbf{Term}_t$	if $\varphi, \psi \in \mathbf{Term}_t$	[***c**onjunction*]
=.	$(A = B) \in \mathbf{Term}_t$	if $A, B \in \mathbf{Term}_a$	[***i**dentity*]
a.	$BA \in \mathbf{Term}_b$	if $B \in \mathbf{Term}_{(ab)}$ and $A \in \mathbf{Term}_a$	[***a**pplication*]
λ.	$\lambda u[B] \in \mathbf{Term}_{(ab)}$	if $u \in \mathbf{Var}_a$ and $B \in \mathbf{Term}_b$	[***λ**-abstraction*]
∃.	$\exists u\varphi \in \mathbf{Term}_t$	if $u \in \mathbf{Var}_a$ and $\varphi \in \mathbf{Term}_t$	[***∃**-quantification*]
•.	$u \bullet A \in \mathbf{Term}_s$	if $u \in \mathbf{Var}_e$ and $A \in \mathbf{Term}_s$	[*stacking*]

N1 (Sugar coating). The following abbreviations may be used for LC_n terms (same as for LC):

().	( ) may be omitted if the result is unambiguous, e.g.,				
	$A = B$	:=	$(A = B)$		
$_a$.	B_A	:=	BA	if this is a term	[*argument subscript*]
I.	$(P_r A)$	:=	$\lambda i[P\ ri\ A]$	"	[*property condition*]
		:=	$\lambda i[P\ ri\ Ai]$	"	
T.	$(R_r A B)$	:=	$\lambda i[R\ ri\ A\ B]$	"	[*relational condition*]
		:=	$\lambda i[R\ ri\ A\ Bi]$	"	
		:=	$\lambda i[R\ ri\ Ai\ B]$	"	
		:=	$\lambda i[R\ ri\ Ai\ Bi]$	"	
≡.	$(A \equiv B)$	:=	$\lambda i[(A = Bi)]$	"	[*identity condition*]
			$\lambda i[(Ai = Bi)]$	"	
¬.	$\neg K$	:=	$\lambda i[\neg Ki]$	"	[*negation condition*]
[|.	$[\ \vert K]$	:=	$\lambda i[\lambda j[((j = i) \wedge Ki)]]$	"	[*test box*]
[*u*.	$[u\vert\ ]$	:=	$\lambda i[\lambda j[\exists u(j = u \bullet i)]]$	"	[*recentering box*]
	$[u\vert K]$	:=	$\lambda i[\lambda j[\exists u((j = u \bullet i) \wedge Ki)]]$	"	
	$[u_1\ u_2\vert K]$	:=	$\lambda i[\lambda j[\exists u_1\exists u_2((j = u_1 \bullet u_2 \bullet i) \wedge Ki)]]$	"	
;.	$(K_1\ ;\ K_2)$	:=	$\lambda i[\lambda j[\exists k(K_1ik \wedge K_2kj)]]$	"	[***s**equencing*]
;;.	$K_1\ ;\ K_2\ ;\ K_3$	:=	$((K_1\ ;\ K_2)\ ;\ K_3)$	"	[default assoc. to L]

D2.0 (LC$_n$ frames) An LC$_n$ *frame* is a set $D = \{D_\tau: \tau \in \mathbf{Typ}\}$ of τ-*domains*, D_τ, such that:

b. $D_t = \{1, 0\}$ [1 for *true*, 0 for *false*]
D_e is a non-empty set disjoint from D_t. [domain of *entities*]
D_ω is a non-empty set disjoint from D_t and D_e. [domain of *worlds*]
$D_s = \{\langle w, \top, \bot\rangle: w \in D_\omega$ [domain of IA-*states*]
& for some $n \geq 0$, $\top \in D_e^n$
& for some $n' \geq 0$, $\bot \in D_e^{n'}\}$

f. $D_{(\sigma\tau)} = \{f: \text{Dom } f = D_\sigma \text{ \& Ran } f \subseteq D_\tau\}$ [domain of $(\sigma\tau)$-*functions*]

D2.1 (LC$_N$ models and assignments).

M. An LC$_n$ *model* is a triple $M = \langle D^M, [\![\cdot]\!]^M, \#^M\rangle$ such that:

- •$_1$ $D^M = \{D_\tau^M: \tau \in \mathbf{Typ}\}$ is an LC$_n$ frame
- •$_2$ $[\![\cdot]\!]^M$ is a function, from LC$_n$ constants, such that: [*M-interpretation*]
  $[\![\alpha]\!]^M \in D_\tau^M$ if $\alpha \in \mathbf{Con}_\tau$
- •$_3$ For any $n \in \{1, 2, \ldots\}$ and $\langle w, \top, \bot\rangle \in D_s^M$:
  $[\![n_\top]\!]^M(\langle w, \top, \bot\rangle) = {}^n[\top]$ if $\top$ has at least n coordinates
  $= \#^M$ otherwise
  $[\![n_\bot]\!]^M(\langle w, \top, \bot\rangle) = {}^n[\bot]$ if $\bot$ has at least n coordinates
  $= \#^M$ otherwise
  $[\![r]\!]^M(\langle w, \top, \bot\rangle) = w$
- •$_4$ $\{\#^M\} = \{d \in D_e^M: \text{for all } w \in D_\omega^M, [\![E]\!]^M(w)(d) = 0\}$ [*non-existent entity*]

g. An *M-assignment* is a function g, from LC$_n$ variables, such that:
$g(u) \in D_\tau^M$ if $u \in \mathbf{Var}_\tau$

N2. (*u-to-d* alternatives). Let g be an M-assignment g, $u \in \mathbf{Var}_\tau$ and $d \in D_\tau^M$. We write $g[u/d]$ for the M-assignment such that: (i) $g[u/d](u) = d$, and (ii) $g[u/d](v) = g(v)$ for any variable $v \neq u$.

D2.2 (LC$_n$ semantics) For any LC$_n$-model $M = \langle D^M, [\![\cdot]\!]^M, \#^M\rangle$ and M-assignment g:

r.	$[\![\alpha]\!]^{M,g}$	$\in D_\tau^M$	if $\alpha \in \mathbf{Term}_\tau$
b.	$[\![\alpha]\!]^{M,g}$	$= [\![\alpha]\!]^M$	if $\alpha \in \mathbf{Con}_\tau$
		$= g(\alpha)$	if $\alpha \in \mathbf{Var}_\tau$
¬.	$[\![\neg\varphi]\!]^{M,g}$	$= 1$	iff $[\![\varphi]\!]^{M,g} = 0$
∧.	$[\![(\varphi \wedge \psi)]\!]^{M,g}$	$= 1$	iff $[\![\varphi]\!]^{M,g} = 1$ & $[\![\psi]\!]^{M,g} = 1$
=.	$[\![(\alpha = \beta)]\!]^{M,g}$	$= 1$	iff $[\![\alpha]\!]^{M,g} = [\![\beta]\!]^{M,g}$
a.	$[\![\alpha\beta]\!]^{M,g}$	$= [\![\alpha]\!]^{M,g}([\![\beta]\!]^{M,g})$	
λ.	$[\![\lambda u[\alpha]]\!]^{M,g}$	$= \{\langle d, [\![\alpha]\!]^{M,g[u/d]}\rangle: d \in D_\sigma^M\}$	if $u \in \mathbf{Var}_\sigma$
∃.	$[\![\exists u\varphi]\!]^{M,g}$	$= 1$	iff $\{d \in D_\sigma^M: [\![\varphi]\!]^{M,g[u/d]} = 1\} \neq \{\}$
•.	$[\![u \bullet \alpha]\!]^{M,g}$	$= \langle {}^1[[\![\alpha]\!]^{M,g}], g(u) \cdot {}^2[[\![\alpha]\!]^{M,g}], {}^3[[\![\alpha]\!]^{M,g}]\rangle$	if $u = \mathbf{x}$
		$= \langle {}^1[[\![\alpha]\!]^{M,g}], {}^2[[\![\alpha]\!]^{M,g}], g(u) \cdot {}^3[[\![\alpha]\!]^{M,g}]\rangle$	if $u = y$

D3 (Truth).

t. A t-term φ (*formula*) is *true* in M under g, written $\models_{M,g} \varphi$, iff $[\![\varphi]\!]^{M,g} = 1$.

st. A (st)-term φ (*condition*) is *true* in a IA-state $\mathbf{s}$ in M under g, written $\models_{M,g,\mathbf{s}} \varphi$, iff $[\![\varphi]\!]^{M,g}(\mathbf{s}) = 1$.

s(st). A $(s(st))$-term φ (*update*) is *true* in a IA-state $\mathbf{s}$ in M under g, written $\models_{M,g,\mathbf{s}} \varphi$, iff, for some $\mathbf{s}'$, $[\![\varphi]\!]^{M,g}(\mathbf{s})(\mathbf{s}') = 1$.

Quiz 2

In Kalaallisut *Kal*1 may be followed by *Kal*2 or *Kal*2′. Throughout this quiz we will consider the LC_n translations and model *M*0 below:

*Kal*1: Arnaq allamik arna-sivuq.
LC_n: $[\mathbf{x}|\ woman_r\ \mathbf{x}]; [y|\ \neg(y \square 1_\top)]; [|\ woman_r\ 1_\bot]; [|\ meet_r\ 1_\bot\ 1_\top]$

*Kal*2: Tiituriartuqquvaa.
LC_n: $[|\ invite.to.tea_r\ 1_\bot\ 1_\top]$

*Kal*2′: Naapi-taa-ta tiituriartuqquvaa.
LC_n: $[|\ meet_r\ 1_\bot\ 1_\top]; [y|\ y \square 1_\top]; [\mathbf{x}|\ \mathbf{x} \square 2_\bot]; [|\ invite.to.tea_r\ 1_\bot\ 1_\top]$

LC_n model M0:
$D_\omega^{M0} = \{w_0, w_1, w_2, w_3\}$
$D_e^{M0} = \{\square, \$, \%\}$

$[\![woman]\!]^{M0}(w)(d)$ = 1, if $\langle w, d\rangle \in \{\langle w_0, \square\rangle, \langle w_1, \square\rangle, \langle w_1, \$\rangle, \langle w_2, \square\rangle, \langle w_2, \$\rangle, \langle w_2, \%\rangle\}$
= 0, otherwise

$[\![meet]\!]^{M0}(w)(d)(d')$ = 1, if $\langle w, d, d'\rangle \in \{\langle w_0, \$, \square\rangle, \langle w_0, \square, \$\rangle, \langle w_1, \$, \square\rangle, \langle w_1, \square, \$\rangle,$
$\langle w_2, \$, \square\rangle, \langle w_2, \square, \$\rangle, \langle w_2, \%, \square\rangle, \langle w_2, \square, \%\rangle\}$
= 0, otherwise

$[\![invite.to.tea]\!]^{M0}(w)(d)(d')$ = 1, if $\langle w, d, d'\rangle \in \{\langle w_0, \$, \square\rangle, \langle w_1, \$, \square\rangle, \langle w_2, \square, \%\rangle, \langle w_3, \%, \$\rangle\}$
= 0, otherwise

Usual ABBREVIATIONS for any *tuple* $\langle a_1, \ldots, a_n, \ldots\rangle$ and any IA-*state* **s**:

$^n[\langle a_1, \ldots, a_n, \ldots\rangle]$	:=	a_n	*n*th coordinate
$^n\langle a_1, \ldots, a_n, \ldots\rangle$	:=	$^n[\langle a_1, \ldots, a_n, \ldots\rangle]$	[] may be omitted
$w_\mathbf{s}$	:=	$^1[\mathbf{s}]$	A1: world of **s**
$\top_\mathbf{s}$	:=	$^2[\mathbf{s}]$	A1: top stack of **s**
$\bot_\mathbf{s}$	:=	$^3[\mathbf{s}]$	A1: bottom stack of **s**

1. **(**5 pts**)**. Complete the following fact of LC_n and its proof:

FACT : $[\![\square\mathbf{x}((j = \mathbf{x} \bullet i) \ \square\ (\mathbf{x} = 2_{\top} i))]\!]^{M,\square} = 1$

iff ______________________________

____

PROOF: (1) iff (10):

1. $[\![\square\mathbf{x}((j = \mathbf{x} \bullet i) \ \square\ (\mathbf{x} = 2_{\top} i)))]\!]^{M,\square} = 1$
2. ______________________________ D2.2.□
3. ______________________________ D2.2.□
4. ______________________________ D2.2.=, =
5. ______________________________ D2.2.***b***, ***b***
6. ______________________________ N2, N2
7. ______________________________ D2.2.•, ***a***
8. ______________________________ D2.2.***b***, ***b***, ***b***, ***b***, ***b***
9. ______________________________ N2, N2, N2, N2, N2

   ______________________________ A1: $\mathbf{s} = \square w_{\mathbf{s}}, \top_{\mathbf{s}}, \square_{\mathbf{s}}\square$
10. ______________________________ A1, D2.1.$\bullet_3$

    ______________________________ □

2 (5 pts**).** As usual at the outset, suppose all worlds are live options & nothing is attended to:

Input set: {⟨w_0, ⟨⟩, ⟨⟩⟩, ⟨w_1, ⟨⟩, ⟨⟩⟩, ⟨w_2, ⟨⟩, ⟨⟩⟩, ⟨w_3, ⟨⟩, ⟨⟩⟩}

Use the sugar-coating conventions for LC$_n$ (**N1 on p. 23**) to de-sugar these LC$_n$-translations:

Kal1. Arnaq …

LC$_n$: ⟦[**x**| (*woman$_r$* **x**)]⟧^{M0}(**s**)(**s**′) = 1 — *sugar-coated*

iff ⟦____⟧^{M0}(**s**)(**s**′) = 1 — N1.[*u*

iff ⟦____⟧^{M0}(**s**)(**s**′) = 1 — N1.I

iff {d ∈ D_e^{M0}: **s**′ = ⟨$w_{\mathbf{s}}$, $d \cdot \top_{\mathbf{s}}$, $\bot_{\mathbf{s}}$⟩ & ⟦*woman*⟧^{M0}($w_{\mathbf{s}}$)(d) = 1} ≠ {} — *truth condition*

*Output*1: {⟨w_0, ⟨⟩, ⟨⟩⟩, ⟨w_1, ⟨⟩, ⟨⟩⟩, ⟨w_1, ⟨\$⟩, ⟨⟩⟩,
⟨w_2, ⟨⟩, ⟨⟩⟩, ⟨w_2, ⟨\$⟩, ⟨⟩⟩, ⟨w_2, ⟨%⟩, ⟨⟩⟩}

… allamik…

iff ⟦[*y*| ¬(*y* = $1_\top$)]⟧^{M0}(**s**)(**s**′) = 1 — *sugar coated*

iff ⟦____⟧^{M0}(**s**)(**s**′) = 1 — N1.[*u*

iff ⟦____⟧^{M0}(**s**)(**s**′) = 1 — N1.¬

iff ⟦____⟧^{M0}(**s**)(**s**′) = 1 — N1. =

iff {d ∈ D_e^{M0}: **s**′ = ⟨$w_{\mathbf{s}}$, $\top_{\mathbf{s}}$, $d \cdot \bot_{\mathbf{s}}$⟩ & $d \neq {}^1\top_{\mathbf{s}}$} ≠ {} — *truth condition*

*Output*2: {⟨w_0, ⟨⟩, ⟨\$⟩⟩, ⟨w_0, ⟨⟩, ⟨%⟩⟩,
⟨w_1, ⟨⟩, ⟨\$⟩⟩, ⟨w_1, ⟨⟩, ⟨%⟩⟩, ⟨w_1, ⟨\$⟩, ⟨⟩⟩, ⟨w_1, ⟨\$⟩, ⟨%⟩⟩,
⟨w_2, ⟨⟩, ⟨\$⟩⟩, ⟨w_2, ⟨⟩, ⟨%⟩⟩, ⟨w_2, ⟨\$⟩, ⟨⟩⟩, ⟨w_2, ⟨\$⟩, ⟨%⟩⟩, ⟨w_2, ⟨%⟩, ⟨⟩⟩, ⟨w_2, ⟨%⟩, ⟨\$⟩⟩}

… arna-sivuq.

⟦[| *woman$_r$* $1_\bot$]; [| *meet$_r$* $1_\bot$ $1_\top$]⟧^{M0}(**s**)(**s**′) = 1 — *sugar-coated*

iff ⟦λ*i*[λ*j*[((*j* = *i*) ∧ *woman ri* $1_\bot i$)]];
λ*i*[λ*j*[((*j* = *i*) ∧ *meet ri* $1_\bot i$ $1_\top i$)]]⟧^{M0}(**s**)(**s**′) = 1 — *sugar-free*

iff **s**′ = **s** & ⟦*woman*⟧^{M0}($w_{\mathbf{s}}$)(${}^1\bot_{\mathbf{s}}$) = 1 & ⟦*meet*⟧^{M0}($w_{\mathbf{s}}$)(${}^1\bot_{\mathbf{s}}$)(${}^1\top_{\mathbf{s}}$) = 1 — *truth condition*

*Output*3: {⟨w_1, ⟨⟩, ⟨\$⟩⟩, ⟨w_1, ⟨\$⟩, ⟨⟩⟩,
⟨w_2, ⟨⟩, ⟨\$⟩⟩, ⟨w_2, ⟨\$⟩, ⟨⟩⟩, ⟨w_2, ⟨⟩, ⟨%⟩⟩, ⟨w_2, ⟨%⟩, ⟨⟩⟩}

3 (2 pts). If *Kal*1 is followed by *Kal*2:

*Output*3: $\{\square w_1, \square\square\square, \$\square\square, \square w_1, \$\square, \square\square\square\square,$ from **part 2**
$\square w_2, \square\square\square, \$\square\square, \square w_2, \$\square, \square\square\square\square, \square w_2, \square\square\square, \%\square\square, \square w_2, \%\square, \square\square\square\square\}$

Kal2: Tiituriartuqquvaa.

LC$_n$: $[\![[\mathrm{l}\ \mathit{invite.to.tea}_r\ 1_\square\ 1_\top]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-coated*

iff $[\![\square i[\square j[((j = i)\ \square\ \mathit{invite.to.tea}\ ri\ 1_\square i\ 1_\top i)]]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-free*

iff $\mathbf{s}\square = \mathbf{s}$ & $[\![\mathit{invite.to.tea}]\!]^{M0}(w_\mathbf{s})(^1\square_\mathbf{s})(^1\top_\mathbf{s}) = 1$ *truth condition*

*Output*4: ____

____

4 (8 pts). If *Kal*1 is followed by *Kal*2$\square$

*Output*3: $\{\square w_1, \square\square\square, \$\square\square, \square w_1, \$\square, \square\square\square\square,$ from **part 2**
$\square w_2, \square\square\square, \$\square\square, \square w_2, \$\square, \square\square\square\square, \square w_2, \square\square\square, \%\square\square, \square w_2, \%\square, \square\square\square\square\}$

Kal2$\square$ Naapi-…

LC$_n$: $[\![[\mathrm{l}\ \mathit{meet}_r\ 1_\square\ 1_\top]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-coated*

iff $[\![\square i[\square j[((j = i)\ \square\ \mathit{meet}\ ri\ 1_\square i\ 1_\top i)]]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-free*

iff $\mathbf{s}\square = \mathbf{s}$ & $[\![\mathit{meet}]\!]^{M0}(w_\mathbf{s})(^1\square_\mathbf{s})(^1\top_\mathbf{s}) = 1$ *truth condition*

*Output*4$\square$ ____

____

Kal2$\square$ …-taa- …

LC$_n$: $[\![[y]\ y\ \square\ 1_\top]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-coated*

iff $[\![\square i[\square j[\square y((j = y \cdot i)\ \square\ (y = 1_\top i))]]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-free*

iff $\{d\ \square\ D_e^{M0}: \mathbf{s}\square = \square w_\mathbf{s}, \top_\mathbf{s}, d \cdot \square_\mathbf{s}\square\ \&\ d = {}^1\top_\mathbf{s}\} \neq \{\}$ *truth condition*

*Output*5$\square$ ____

____

… -ta…

iff $[\![[\mathbf{x}\mathrm{l}\ \ \mathbf{x}\ \square\ 2_\square]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar coated*

iff $[\![\square i[\square j[\square \mathbf{x}((j = \mathbf{x} \bullet i)\ \square\ (\mathbf{x} = 2_\square i))]]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-free*

iff $\{d\ \square\ D_e^{M0}: \mathbf{s}\square = \square w_\mathbf{s}, d \cdot \top_\mathbf{s}, \square_\mathbf{s}\square\ \&\ d = {}^2\square_\mathbf{s}\} \neq \{\}$ *truth condition*

*Output*6$\square$ ____

____

… tiituriartuqquvaa.

$[\![[\mathrm{l}\ \mathit{invite.to.tea}_r\ 1_\square\ 1_\top]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-coated*

iff $[\![\square i[\square j[((j = i)\ \square\ \mathit{invite.to.tea}\ ri\ 1_\square i\ 1_\top i)]]]\!]^{M0}(\mathbf{s})(\mathbf{s}\square) = 1$ *sugar-free*

iff $\mathbf{s}\square = \mathbf{s}$ & $[\![\mathit{invite.to.tea}]\!]^{M0}(w_\mathbf{s})(^1\square_\mathbf{s})(^1\top_\mathbf{s}) = 1$ *truth condition*

*Output*7$\square$ ____

Solution to Quiz 2

In Kalaallisut *Kal*1 may be followed by *Kal*2 or *Kal*2′. Throughout this quiz we will consider the LC_n translations and model *M*0 below:

*Kal*1:	Arnaq	alla-mik	arna-si-vu-q.
	woman	other-MOD	woman-meet-IND.IV-**3SG**
LC_n:	$[\mathbf{x}\vert\ woman_r\ \mathbf{x}];\ [y\vert\ \neg(y \in 1_\top)];\ [\vert\ woman_r\ 1_\bot];\ [\vert\ meet_r\ 1_\bot\ 1_\top]$		
*Kal*2:	Tii-tur-iartu-qqu-va-a.		
	tea-drink-go.to-ask-IND.TV-**3SG**.3SG		
LC_n:	$[\vert\ invite.to.tea_r\ 1_\bot\ 1_\top]$		
*Kal*2′:	Naapi-ta-a-ta		tii-tur-iartu-qqu-va-a.
	meet-TV\RN-3SG-ERG		tea-drink-go.to-ask-IND.TV-**3SG.**3SG
LC_n:	$[\vert\ meet_r\ 1_\bot\ 1_\top];\ [y\vert\ y \in 1_\top];\ [\mathbf{x}\vert\ \mathbf{x} \in 2_\bot];\ [\vert\ invite.to.tea_r\ 1_\bot\ 1_\top]$		

LC_n model M0:

$D_\omega^{M0} = \{w_0, w_1, w_2, w_3\}$

$D_e^{M0} = \{\#, \$, \%\}$

$[\![woman]\!]^{M0}(w)(d)$ = 1, if $\langle w, d\rangle \in \{\langle w_0, \#\rangle, \langle w_1, \#\rangle, \langle w_1, \$\rangle, \langle w_2, \#\rangle, \langle w_2, \$\rangle, \langle w_2, \%\rangle\}$
= 0, otherwise

$[\![meet]\!]^{M0}(w)(d)(d')$ = 1, if $\langle w, d, d'\rangle \in \{\langle w_0, \$, \#\rangle, \langle w_0, \#, \$\rangle, \langle w_1, \$, \#\rangle, \langle w_1, \#, \$\rangle, \langle w_2, \$, \#\rangle, \langle w_2, \#, \$\rangle, \langle w_2, \%, \#\rangle, \langle w_2, \#, \%\rangle\}$
= 0, otherwise

$[\![invite.to.tea]\!]^{M0}(w)(d)(d')$ = 1, if $\langle w, d, d'\rangle \in \{\langle w_0, \$, \#\rangle, \langle w_1, \$, \#\rangle, \langle w_2, \#, \%\rangle, \langle w_3, \%, \$\rangle\}$
= 0, otherwise

Usual ABBREVIATIONS for any *tuple* $\langle a_1, \ldots, a_n, \ldots\rangle$ and any IA-*state* **s**:

$^n[\langle a_1, \ldots, a_n, \ldots\rangle]$	:=	a_n	*n*th coordinate
$^n\langle a_1, \ldots, a_n, \ldots\rangle$	:=	$^n[\langle a_1, \ldots, a_n, \ldots\rangle]$	[] may be omitted
$w_{\mathbf{s}}$	:=	$^1[\mathbf{s}]$	A1: world of **s**
$\top_{\mathbf{s}}$	:=	$^2[\mathbf{s}]$	A1: top stack of **s**
$\bot_{\mathbf{s}}$	:=	$^3[\mathbf{s}]$	A1: bottom stack of **s**

1. (5 pts). Complete the following fact of LC_n and its proof:

FACT : $[\![\exists \mathbf{x}((j = \mathbf{x} \bullet i) \wedge (\mathbf{x} = 2_{\top} i))]\!]^{M, \square} = 1$

iff $\{d \in D_e^M: \square(j) = \langle w_{\square(i)}, d \cdot \top_{\square(i)}, \square_{\square(i)}\rangle \;\&\; d = {}^2[\top_{\square(i)}]\} \neq \{\}$ **line 10** below

PROOF: (1) iff (10):

1.	$[\![\exists \mathbf{x}((j = \mathbf{x} \bullet i) \wedge (\mathbf{x} = 2_{\top} i)))]\!]^{M, \square} = 1$	
2.	$\{d \in D_e^M: [\![((j = \mathbf{x} \bullet i) \wedge (\mathbf{x} = 2_{\top} i))]\!]^{M, \square[\mathbf{x}/d]} = 1\} \neq \{\}$	D2.2.$\exists$
3.	$\{d \in D_e^M: [\![(j = \mathbf{x} \bullet i)]\!]^{M, \square[\mathbf{x}/d]} = 1 \;\&\; [\![(\mathbf{x} = 2_{\top} i)]\!]^{M, \square[\mathbf{x}/d]} = 1\} \neq \{\}$	D2.2.$\wedge$
4.	$\{d \in D_e^M: [\![j]\!]^{M, \square[\mathbf{x}/d]} = [\![\mathbf{x} \bullet i]\!]^{M, \square[\mathbf{x}/d]} \;\&\; [\![\mathbf{x}]\!]^{M, \square[\mathbf{x}/d]} = [\![2_{\top} i]\!]^{M, \square[\mathbf{x}/d]}\} \neq \{\}$	D2.2.=, =
5.	$\{d \in D_e^M: \square[\mathbf{x}/d](j) = [\![\mathbf{x} \bullet i]\!]^{M, \square[\mathbf{x}/d]} \;\&\; \square[\mathbf{x}/d](\mathbf{x}) = [\![2_{\top} i]\!]^{M, \square[\mathbf{x}/d]}\} \neq \{\}$	D2.2.***b***, ***b***
6.	$\{d \in D_e^M: \square(j) = [\![\mathbf{x} \bullet i]\!]^{M, \square[\mathbf{x}/d]} \;\&\; d = [\![2_{\top} i]\!]^{M, \square[\mathbf{x}/d]}\} \neq \{\}$	N2, N2
7.	$\{d \in D_e^M: \square(j) = \langle {}^1[[\![i]\!]^{M, \square[\mathbf{x}/d]}], \square[\mathbf{x}/d](\mathbf{x}) \cdot {}^2[[\![i]\!]^{M, \square[\mathbf{x}/d]}], {}^3[[\![i]\!]^{M, \square[\mathbf{x}/d]}]\rangle$	D2.2.$\bullet$
	$\& \; d = [\![2_{\top}]\!]^{M, \square[\mathbf{x}/d]}([\![i]\!]^{M, \square[\mathbf{x}/d]})\} \neq \{\}$	D2.2.***a***
8.	$\{d \in D_e^M: \square(j) = \langle {}^1[\square[\mathbf{x}/d](i)], \square[\mathbf{x}/d](\mathbf{x}) \cdot {}^2[\square[\mathbf{x}/d](i)], {}^3[\square[\mathbf{x}/d](i)]\rangle$	D2.2.***b***, ***b***, ***b***
	$\& \; d = [\![2_{\top}]\!]^{M}(\square[\mathbf{x}/d](i))\} \neq \{\}$	D2.2.***b***, ***b***
9.	$\{d \in D_e^M: \square(j) = \langle {}^1[\square(i)], d \cdot {}^2[\square(i)], {}^3[\square(i)]\rangle$	N2, N2, N2, N2
	$\& \; d = [\![2_{\top}]\!]^{M}(\langle w_{\square(i)}, \top_{\square(i)}, \square_{\square(i)}\rangle)\} \neq \{\}$	N2, A1: $\mathbf{s} = \langle w_{\mathbf{s}}, \top_{\mathbf{s}}, \square_{\mathbf{s}}\rangle$
10.	$\{d \in D_e^M: \square(j) = \langle w_{\square(i)}, d \cdot \top_{\square(i)}, \square_{\square(i)}\rangle$	A1
	$\& \; d = {}^2[\top_{\square(i)}]\} \neq \{\}$	D2.1.$\bullet_3$ □

2 (5 pts). As usual at the outset, suppose all worlds are live options & nothing is attended to:

Input set: {□w_0, □□, □□□. □w_1, □□, □□□. □w_2, □□, □□□. □w_3, □□, □□□}

Kal1. Arnaq …
woman…

LC$_n$: ⟦[**x**| ($woman_r$ **x**)]⟧^{M0}(**s**)(**s**□) = 1 — *sugar-coated*
iff ⟦□i[□j[□**x**((j = **x** • i) □ ($woman_r$ **x**)i)]]⟧^{M0}(**s**)(**s**□) = 1 — N1.[u
iff ⟦□i[□j[□**x**((j = **x** • i) □ □i[woman ri **x**]i)]]⟧^{M0}(**s**)(**s**□) = 1 — N1.I
iff ⟦□i[□j[□**x**((j = **x** · i) □ woman ri **x**)]]⟧^{M0}(**s**)(**s**□) = 1 — *□-conversion*
iff {d □ D_e^{M0}: **s**□= □$w_\mathbf{s}$, d · ⊤$_\mathbf{s}$, □$_\mathbf{s}$□& ⟦woman⟧^{M0}($w_\mathbf{s}$)(d) = 1} ≠ {} — *truth condition*

*Output*1: {□w_0, □□□, □□□. □w_1, □□□, □□□. □w_1, □$□, □□□.
□w_2, □□□, □□□. □w_2, □$□, □□□. □$w_2$, □%□, □□□}

…alla-mik…
…other-MOD
iff ⟦[y| ¬(y □ 1$_⊤$)]⟧^{M0}(**s**)(**s**□) = 1 — *sugar coated*
iff ⟦□i[□j[□y((j = y • i) □ ¬(y □ 1$_⊤$)i)]]⟧^{M0}(**s**)(**s**□) = 1 — N1.[u
iff ⟦□i[□j[□y((j = y • i) □ □i[¬(y □ 1$_⊤$)i]i)]]⟧^{M0}(**s**)(**s**□) = 1 — N1.¬
iff ⟦□i[□j[□y((j = y • i) □ □i[¬□i[(y = 1$_⊤$i)]i]i)]]⟧^{M0}(**s**)(**s**□) = 1 — N1. □
iff ⟦□i[□j[□y((j = y • i) □ □i[¬(y = 1$_⊤$i)]i)]]⟧^{M0}(**s**)(**s**□) = 1 — *□-conversion*
iff ⟦□i[□j[□y((j = y • i) □ ¬(y = 1$_⊤$i))]]⟧^{M0}(**s**)(**s**□) = 1 — *□-conversion*
iff {d □ D_e^{M0}: **s**□= □$w_\mathbf{s}$, ⊤$_\mathbf{s}$, d · □$_\mathbf{s}$□& d ≠ ^{1}⊤$_\mathbf{s}$} ≠ {} — *truth condition*

*Output*2: {□w_0, □□□, □$□. □$w_0$, □□□, □%□.
□w_1, □□□, □$□. □$w_1$, □□□, □%□. □$w_1$, □$□, □□□. □w_1, □$□, □%□.
□w_2, □□□, □$□. □$w_2$, □□□, □%□. □$w_2$, □$□, □□□. □w_2, □$□, □%□. □$w_2$, □%□, □□□. □$w_2$, □%□, □$□}

…arna-si-vu-q.
…woman-meet-IND.IV-**3SG**
⟦[| $woman_r$ 1$_□$]; [| $meet_r$ 1$_□$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1 — *sugar-coated*
iff ⟦□i[□j[((j = i) □ woman ri 1$_□$i)]];
□i[□j[((j = i) □ meet ri 1$_□$i 1$_⊤$i)]]⟧^{M0}(**s**)(**s**□) = 1 — *sugar-free*
iff **s**□= **s** & ⟦woman⟧^{M0}($w_\mathbf{s}$)(^{1}□$_\mathbf{s}$) = 1 & ⟦meet⟧^{M0}($w_\mathbf{s}$)(^{1}□$_\mathbf{s}$)(^{1}⊤$_\mathbf{s}$) = 1

*Output*3: {□w_1, □□□, □$□. □$w_1$, □$□, □□□.
□w_2, □□□, □$□. □$w_2$, □$□, □□□. □w_2, □□□, □%□. □w_2, □%□, □□□}

3 (2 pts). If *Kal*1 is followed by *Kal*2:

*Output*3: {□w_1, □□□, $□□, □$w_1$, □$□, □□□□, □w_2, □□□, $□□, □$w_2$, □$□, □□□□, □w_2, □□□, □%□□, □w_2, □%□, □□□□}

Kal2: Tii-tur-iartu-qqu-va-a.
tea-drink-go.to-ask-IND.TV-**3SG**.3SG

LC$_n$:	⟦[ǀ *invite.to.tea*$_r$ 1$_□$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1	*sugar-coated*
iff	⟦□*i*[□*j*[((*j* = *i*) □ *invite.to.tea ri* 1$_□$*i* 1$_⊤$*i*)]]⟧^{M0}(**s**)(**s**□) = 1	*sugar-free*
iff	**s**□= **s** & ⟦*invite.to.tea*⟧^{M0}($w_{\mathbf{s}}$)(^{1}□$_{\mathbf{s}}$)(^{1}⊤$_{\mathbf{s}}$) = 1	*truth condition*

*Output*4: {□w_1, □□□, $□□, □$w_2$, □%□, □□□□}

4 (8 pts). If *Kal*1 is followed by *Kal*2□:

*Output*3: {□w_1, □□□, $□□, □$w_1$, □$□, □□□□, □w_2, □□□, $□□, □$w_2$, □$□, □□□□, □w_2, □□□, □%□□, □w_2, □%□, □□□□}

Kal2□: Naapi-…
meet-

LC$_n$:	⟦[ǀ *meet*$_r$ 1$_□$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1	*sugar-coated*
	iff ⟦□*i*[□*j*[((*j* = *i*) □ *meet ri* 1$_□$*i* 1$_⊤$*i*)]]⟧^{M0}(**s**)(**s**□) = 1	*sugar-free*
	iff **s**□= **s** & ⟦*meet*⟧^{M0}($w_{\mathbf{s}}$)(^{1}□$_{\mathbf{s}}$)(^{1}⊤$_{\mathbf{s}}$) = 1	*truth condition*

*Output*4□: {□w_1, □□□, $□□, □$w_1$, □$□, □□□□,
□w_2, □□□, $□□, □$w_2$, □$□, □□□□, □w_2, □□□, □%□□, □w_2, □%□, □□□□}

Kal2□: …-ta-a- …
…TV\RN-3SG-…

LC$_n$:	⟦[*y*] *y* □ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1	*sugar-coated*
	iff ⟦□*i*[□*j*[□*y*((*j* = *y* · *i*) □ (*y* = 1$_⊤$*i*))]]⟧^{M0}(**s**)(**s**□) = 1	*sugar-free*
	iff {*d* □ D_e^{M0}: **s**□= □$w_{\mathbf{s}}$, ⊤$_{\mathbf{s}}$, *d* · □$_{\mathbf{s}}$□& *d* = ^{1}⊤$_{\mathbf{s}}$} ≠ {}	*truth condition*

*Output*5□: {□w_1, □□□, □□, $□□, □$w_1$, □$□, □$, □□□,
□w_2, □□□, □□, $□□, □$w_2$, □$□, □$, □□□, □$w_2$, □□□, □□, %□□, □$w_2$, □%□, □%, □□□}

… -ta…
…-ERG…

	iff ⟦[**x**ǀ **x** □ 2$_□$]⟧^{M0}(**s**)(**s**□) = 1	*sugar coated*
	iff ⟦□*i*[□*j*[□**x**((*j* = **x** • *i*) □ (**x** = 2$_□$*i*))]]⟧^{M0}(**s**)(**s**□) = 1	*sugar-free*
	iff {*d* □ D_e^{M0}: **s**□= □$w_{\mathbf{s}}$, *d* · ⊤$_{\mathbf{s}}$, □$_{\mathbf{s}}$□& *d* = ^{2}□$_{\mathbf{s}}$} ≠ {}	*truth condition*

*Output*6□: {□w_1, □$, □□, □□, $□□, □$w_1$, □□, $□, □$, □□□,
□w_2, □$, □□, □□, $□□, □$w_2$, □□, $□, □$, □□□, □w_2, □%, □□, □□, %□□, □w_2, □□, %□, □%, □□□}

…tii-tur-iartu-qqu-va-a.
…tea-drink-go.to-ask-IND.TV-**3SG**.3SG

	⟦[ǀ *invite.to.tea*$_r$ 1$_□$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1	*sugar-coated*
	iff ⟦□*i*[□*j*[((*j* = *i*) □ *invite.to.tea ri* 1$_□$*i* 1$_⊤$*i*)]]⟧^{M0}(**s**)(**s**□) = 1	*sugar-free*
	iff **s**□= **s** & ⟦*invite.to.tea*⟧^{M0}($w_{\mathbf{s}}$)(^{1}□$_{\mathbf{s}}$)(^{1}⊤$_{\mathbf{s}}$) = 1	*truth condition*

*Output*7□: {□w_1, □□, $□, □$, □□□, □w_2, □%, □□, □□, %□□}

Lecture 8
ENGLISH VERSION OF **Quiz 2**

*Eng*1 might be followed by *Eng*2 or *Eng*2□ Consider the LC_n translations and model *M*0 below:

Eng1: A woman met another woman.
LC_n: [**x**|]; [| *woman*$_r$ $1_\top$]; [*y*| *meet*$_r$ *y* $1_\top$]; [| ¬($1_□$ □ $1_\top$)]; [| *woman*$_r$ $1_□$];

Eng2: She invited her to tea.
LC_n: [| *woman*$_r$ $1_\top$]; [| *invite*$_r$ $1_□$ $1_\top$]; [| *woman*$_r$ $1_□$]

Eng2□ The lady invited her to tea.
LC_n: [**x**| **x** □ $1_□$]; [| *woman*$_r$ $1_\top$]; [*y*| *invite*$_r$ *y* $1_\top$]; [| *woman*$_r$ $1_□$]; [| $1_□$ □ $2_\top$]

LC_n model M0:
$D_□^{M0} = \{w_0, w_1, w_2, w_3\}$
D_e^{M0} = {□, \$, %}
⟦*woman*⟧^{M0}(*w*)(*d*) = 1, if □*w*, *d*□□ {□w_0, □□ □w_1, □□ □w_1, \$□ □w_2, □□ □w_2, \$□ □w_2, %□}
= 0, otherwise
⟦*meet*⟧^{M0}(*w*)(*d*)(*d*□ = 1, if □*w*, *d*, *d*□□ {□w_0, \$, □□ □w_0, □, \$□ □w_1, \$, □□ □w_1, □, \$□
□w_2, \$, □□ □w_2, □, \$□ □w_2, %, □□ □w_2, □, %□}
= 0, otherwise
⟦*invite*⟧^{M0}(*w*)(*d*)(*d*□ = 1, if □*w*, *d*, *d*□□ {□w_0, \$, □□ □w_1, \$, □□ □w_2, □, %□ □w_3, %, \$□}
= 0, otherwise

Input set: {□w_0, □□ □□□ □w_1, □□ □□□ □w_2, □□ □□□ □w_3, □□ □□□}

*Eng*1. A woman …
LC_n: ⟦[**x**|]; [| *woman*$_r$ $1_\top$]⟧^{M0}(**s**)(**s**□ = 1 *sugar-coated*
iff ⟦□*i*[□*j*[□**x**(*j* = **x** · *i*)]]; □*i*[□*j*[*j* = *i* □ *woman ri* $1_\top$*i*)]]⟧^{M0}(**s**)(**s**□ = 1 *sugar-free*
iff {*d* □ D_e^{M0}: **s**□= □$w_\mathbf{s}$, *d* · $\top_\mathbf{s}$, □$_\mathbf{s}$□& ⟦*woman*⟧^{M0}($w_\mathbf{s}$)(^{1}□$_\mathbf{s}$) = 1} ≠ {} *truth condition*

*Output*1: {□w_0, □□□ □□□ □w_1, □□□ □□□ □w_1, □\$□ □□□
□w_2, □□□ □□□ □w_2, □\$□ □□□ □w_2, □%□ □□□}

…met
iff ⟦[*y*| *meet*$_r$ *y* $1_\top$]⟧^{M0}(**s**)(**s**□ = 1 *sugar coated*
iff ⟦□*i*[□*j*[□*y*((*j* = *y* • *i*) □ *meet ri y* $1_\top$*i*)]]⟧^{M0}(**s**)(**s**□ = 1 *sugar-free*
iff {*d* □ D_e^{M0}: **s**□= □$w_\mathbf{s}$, $\top_\mathbf{s}$, *d* · □$_\mathbf{s}$□& ⟦*meet*⟧^{M0}(*d*)($^1\top_\mathbf{s}$) = 1} ≠ {} *truth condition*

*Output*2: {□w_0, □□□ □\$□□ □w_1, □□□ □\$□□ □w_1, □\$□ □□□□
□w_2, □□□ □\$□□ □w_2, □□□ □%□□ □w_2, □\$□ □□□□ □w_2, □%□ □□□□}

…another woman.
⟦[| ¬($1_□$ □ $1_\top$)]; [| *woman*$_r$ $1_□$]⟧^{M0}(**s**)(**s**□ = 1 *sugar-coated*
iff ⟦□*i*[□*j*[*j* = *i* □ $1_□$*i* ≠ $1_\top$*i*]]; □*i*[□*j*[*j* = *i* □ *woman ri* $1_□$*i*]]⟧^{M0}(**s**)(**s**□ = 1 *sugar-free*
iff **s**□= **s** & ^{1}□$_\mathbf{s}$ ≠ $^1\top_\mathbf{s}$ & ⟦*woman*⟧^{M0}($w_\mathbf{s}$)(^{1}□$_\mathbf{s}$) = 1 *truth condition*

*Output*3: {□w_1, □□□ □\$□□ □w_1, □\$□ □□□□ **as for** ***Kal*1**
□w_2, □□□ □\$□□ □w_2, □\$□ □□□□ □w_2, □□□ □%□□ □w_2, □%□ □□□□}

If *Eng*1 **is followed by** *Eng*2:

*Output*3: {□w_1, □□□, $□□, □$w_1$, □$□, □□□□, □w_2, □□□, $□□, □$w_2$, □$□, □□□□, □w_2, □□□, □%□□, □w_2, □%□, □□□□}

*Eng*2: She invited…

LC$_n$: ⟦[I *woman*$_r$ 1$_⊤$]; [I *invite*$_r$ 1$_□$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[*j* = *i* □ *woman ri* 1$_⊤$*i*]]; □*i*[□*j*[*j* = *i* □ *invite ri* 1$_□$*i* 1$_⊤$*i*]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff **s**□= **s** & ⟦*woman*⟧^{M0}(w_s)($^1⊤_s$) = 1 & ⟦*invite*⟧^{M0}(w_s)($^1□_s$)($^1⊤_s$) = 1 *truth condition*

*Output*4: {□w_1, □□□, $□□, □$w_2$, □%□, □□□□}

…her

⟦[I *woman*$_r$ 1$_□$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[*j* = *i* □ *woman ri* 1$_□$*i*]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff **s**□= **s** & ⟦*woman*⟧^{M0}(w_s)($^1□_s$) = 1 *truth condition*

*Output*5: {□w_1, □□□, $□□, □$w_2$, □%□, □□□□} **as for** *Kal*2

If *Eng*1 **is followed by** *Eng*2□:

*Output*3: {□w_1, □□□, $□□, □$w_1$, □$□, □□□□, □w_2, □□□, $□□, □$w_2$, □$□, □□□□, □w_2, □□□, □%□□, □w_2, □%□, □□□□}

*Eng*2□: The…

LC$_n$: ⟦[**x**I **x** □ 1$_□$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[□**x**((*j* = **x** • *i*) □ (**x** = 1$_□$*i*))]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff {*d* □ D_e^{M0}: **s**□= □w_s, *d* · $⊤_s$, □$_s$□& *d* = $^1□_s$} ≠ {} *truth condition*

*Output*4□: {□w_1, □$, □□, $□□, □$w_1$, □□, $□, □□□□,
□w_2, □$, □□, $□□, □$w_2$, □□, $□, □□□□, □$w_2$, □%, □□, □%□□, □$w_2$, □□, %□, □□□□}

…lady …

⟦[I *woman*$_r$ 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[*j* = *i* □ *woman ri* 1$_⊤$*i*]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff **s**□= **s** & ⟦*woman*⟧^{M0}(w_s)($^1⊤_s$) = 1 *truth condition*

*Output*5□: {□w_1, □$, □□, $□□, □$w_1$, □□, $□, □□□□,
□w_2, □$, □□, $□□, □$w_2$, □□, $□, □□□□, □$w_2$, □%, □□, □%□□, □$w_2$, □□, %□, □□□□}

…invited…

⟦[*y*I *invite*$_r$ *y* 1$_⊤$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[□*y*(*j* = *y* • *i* □ *invite ri y* 1$_⊤$*i*)]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff {*d* □ D_e^{M0}: **s**□= □w_s, $⊤_s$, *d* · □$_s$□& ⟦*invite*⟧^{M0}(w_s)(*d*)($^1⊤_s$) = 1} ≠ {} *truth condition*

*Output*6□: {□w_1, □□, $□, □$, □□□, □w_2, □%, □□, □□, %□□}

…her

⟦[I *woman*$_r$ 1$_□$]; [I 1$_□$ □ 2$_⊤$]⟧^{M0}(**s**)(**s**□) = 1 *sugar-coated*

iff ⟦□*i*[□*j*[*j* = *i* □ *woman ri* 1$_□$*i*]]; □*i*[□*j*[*j* = *i* □ 1$_□$*i* = 2$_⊤$*i*]]⟧^{M0}(**s**)(**s**□) = 1 *sugar-free*

iff **s**□= **s** & ⟦*woman*⟧^{M0}(w_s)($^1□_s$) = 1 & $^1□_s$ = $^2⊤_s$ *truth condition*

*Output*7□: {□w_1, □□, $□, □$, □□□, □w_2, □%, □□, □□, %□□} **as for** *Kal*2□