Ordering semantics:
Stalnaker 1968, Lewis 1973

1. INTRODUCTION

• Ramsey test á la Stalnaker 1968

(***R***) "This is how to evaluate a conditional: First add the antecedent (hypothetically) to your stock of beliefs; second [MB: this is Stalnaker's amendment], make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true." (Stalnaker 1968)

• Some desiderata for formal analysis

(***w***) *Contingency*:
Conditionals are contingent statements—true or false, depending on the *world*:

(1) If Oswald hadn't killed Kennedy, someone else would have. (Adams 1975)

(***B***) *Modal base* (aka *accessibility relation*):
The truth value of a conditional depends on the background *stock of facts/beliefs*, e.g.

(2) i. (Given the law,) if Tom buys a car, he must buy insurance.
ii. But (given his finances,) if he buys a car, he won't have money for insurance.

Example from a Mayan myth (http://www.rci.rutgers.edu/~mbittner/ym.html)

(3) i. Once upon a time there was a little rabbit who was chased by a hunter.
ii. He was already tired of running, when he saw a cave and went inside.
iii. "Oh my!" he thought,
iv. "If the man comes here, he'll shoot me.
v. So what am I to do?
vi. I am going to hold up this cave.
vii. If he comes in here, I'll tell him I've been holding up this cave since I was born."
viii. And then he started holding up the cave.

(***A***) *A-strengthening*
The following argument form is invalid (*A-fallacy*):

	if A, then C	e.g.	If I strike this match, it will light.
⊨	*if AB*, *then C*	⊨	If I soak this match in water and then strike it, it will light.

(4) i. If I strike this match, it will light. (*coherent discourse*)
ii. But if I soak it in water and then strike it, it won't.

Example from *Pippi Longstocking* (by Astrid Lindgren):

(5) [While playing Thing-Finders with T. and A., P. finds an old can & puts it on her head.]
i. With the can on her head she wandered around the block like a little metal tower and never stopped until she stumbled over a low wire fence and fell flat on her stomach.
ii. "Now, see that!" said Pippi and took off the can.
iii. "If I hadn't had this thing on me, I'd have fallen flat on my face and hurt myself terribly."
iv. "Yes," said Annika,
v. "but if you hadn't had the can on your head, then you wouldn't have tripped on the wire fence in the first place."

(***T***) *Transitivity failure*:
The following argument form is also invalid (*T-fallacy*):

	if A, then B	e.g.	If Tom were honest, people would trust him.
	if B, then C		If people trusted Tom, he would cheat them.
$\models$	*if A, then C*	$\models$	If Tom were honest, he would cheat people.

(6) i. Tom comes from a respected family, so (*coherent discourse*)
ii. if he were honest, people would trust him.
iii. But he is a rascal, so
iv. if people trusted him, he would cheat them.
v. Of course, if he were honest, he wouldn't cheat anybody.

(~) *Equally close A-worlds*
Lewis 1973 counter-examples to Stalnaker 1968:

(7) a. if Bizet and Verdi were compatriots, they would (both) be French. **F**
b. If Bizet and Verdi were compatriots, they would be (both) French or (both) Italian. **T**

(8) i. Adam, Bill, and Charles all had equally good motive and opportunity to kill Lord David.
ii. Nobody else had any grudge against him.
iii. So, if Adam didn't kill him,
a. then Bill did. **F**
b. then either Bill or Charles did. **T**

(***L***) *No closest A-worlds?*
According to Lewis 1973, Stalnaker's *limit assumption*—that there is a closest *A*-world —may also be invalid. He suggests that antecedents with comparatives are a case in point.

(9) i. Al ran out onto the platform just in time to see the tail lights of the departing train.
ii. "Shucks," he exclaimed,
iii. "if I had left earlier, I would've caught this train!"

(10) A: I'll get this down for you. It must be a nuisance to be so short.
B: No, if I were taller, I wouldn't be comfortable sleeping on two airline seats.

• Proposed analyses
If A, then C is true in *w*, iff

		(*w*)	(*B*)	(*A*)	(*T*)	(~)	(*L*)
($\subseteq$)	*Strict Implication* every *A*-world is a *C*-world	–	–	–	–	–	–
(**K**)	**K**ripke 1963 every $A \cap B_w$-world is a *C*-world	+	+	–	–	+	$+_{MB}$
(**S**)	**S**talnaker 1968 the $\leq_w$-closest $A \cap B_w$-world is a *C*-world	+	+	+	+	–	–
(**L2**)	**L**ewis 1973, Analysis 2 every $\leq_w$-closest $A \cap B_w$-world is a *C*-world	+	+	+	+	+	$-_{DL}$ $+_{MB}$
(**L3**)	**L**ewis 1973, Analysis 3 every $A \cap B_w$-world that is $\leq_w$-close enough is a *C*-world.	+	+	+	+	+	$+_{DL}$ $-_{MB}$

2. **L2** AS THE BEST ANALYSIS

(***L***) *No closest A-worlds?*

(9) a. If I had left earlier, I would've caught this train.
    b. If I had left earlier, I would've turned into a crocodile.

• ***Problem*** for **L2** *if* comparatives *continuous*:
If *leave.earlier* means "leave earlier by any amount", then there are no closest worlds. Therefore, we predict (9a) as well as (9b) to be *trivially true*—a counterintuitive result.

• ***No problem*** for **L2** *if* comparatives *contextually coarse-grained*:
Contextual coarse-graining is independently needed to account e.g. for:

(9′) a. If I had left *earlier*, I would've caught this train.
    b. If homo sapiens had evolved *earlier*, it would've destroyed the world by now.
    c. If this neutrino had disintegrated *earlier*, our experiment would've failed.

What *earlier* means is "one instant earlier", where the size of the "instant" depends on the context — of the order of minutes in (9′a), millenia in (9′b), miliseconds in (9′c).

Without coarse-graining, comparatives are also a ***problem*** for Lewis's final analysis **L3**. Intuitively, **L3** says: Every *A*-world in B_w that is $\leq_w$-close enough is a *C*-world. More precisely, there is some *A*-world w' in B_w s.t. every *A*-world in B_w that is $\leq_w$-closer than w' is a *C*-world.

Now, consider the following context: I want to catch the 12:09 train. I estimate it'll take me 15 min to walk to the station so I leave at 11:54. Alas, I've miscalculated the time and arrive on the platform only to see the rear end of my train disappearing into the tunnel. Upset, I say:

(9a) Shucks, if I had left earlier I would've caught this train.

Intuitively, (9a) is *true* in this context. But **L3** wrongly predicts it to be *false*. For if *earlier* means "earlier by any amount" then, no matter how we pick the limit world (w'), the universal (every *A*-world…$\leq_w$-closer than w') will quantify over worlds where I leave *less than 1 second earlier*. But in those worlds, of course, I still miss the train. So Lewis's **L3-**analysis of (9a) makes counterintuitive predictions.

This problem generalizes to other conditionals with comparative antecedents (including Lewis's own *If I were taller*, … see (10B)). So, in general, **L3** is not the right way to go.

3. CONCLUSION

- I vote for **L2** as the best version of static ordering semantics for conditionals.
- But since **L2** is static, "contextual variables", ≤ and *B*, have to be assigned "suitable values" by magic. By the end of this seminar, I hope to replace such magic with incremental update for least some discourses like (3), (5), (6) and corresponding discourses in Yukatek, Kalaallisut, etc:

(***B***) *Modal base* (aka *accessibility relation*):
(3) i. Once upon a time there was a little rabbit who was chased by a hunter.
ii. He was already tired of running, when he saw a cave and went inside.
iii. "Oh my!" he thought,
iv. "If the man comes here, he'll shoot me.
v. So what am I to do?
vi. I am going to hold up this cave.
vii. If he comes in here, I'll tell him I've been holding up this cave since I was born."
viii. And then he started holding up the cave.

(***A***) *A-strengthening*
(5) i. With the can on her head [Pippi] wandered around the block like a little metal tower and never stopped until she stumbled over a low wire fence and fell flat on her stomach.
ii. "Now, see that!" said Pippi and took off the can.
iii. "If I hadn't had this thing on me, I'd have fallen flat on my face and hurt myself terribly."
iv. "Yes," said Annika,
v. "but if you hadn't had the can on your head, then you wouldn't have tripped on the wire fence in the first place."

(***T***) *Transitivity failure*:
(6) i. Tom comes from a respected family, so (*coherent discourse*)
ii. if he were honest, people would trust him.
iii. But he is a rascal, so
iv. if people trusted him, he would cheat them.
v. Of course, if he were honest, he wouldn't cheat anybody.

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Lewis's premise semantics:
(Lewis 1973 a la 1981)

1. LEWIS'S **L2** IN TWO-SORTED TYPE LOGIC

- Basic idea of **L2** (Lewis 1973, Analysis 2):
  i. the context "somehow determines" two parameters:
     - a world-dependent *modal base* (aka set of *accessible worlds*) B_w, and
     - a partial order on worlds, $w' \leq_w w''$ (w' is at least as close to w as w'' is), where higher-ranked (closer) worlds are more similar to w (cf. more general orders in Kratzer 1981)
     - following Lewis 1981, we define an order $\leq_{O,\,w}$ induced by a set of propositions O_w:
       $w' \leq_{O,\,w} w''$ iff every proposition in O_w that is true in w'' is also true in w'
  ii. Truth condition wrt. modal base B_w and ordering relation $\leq_w$:
     If A, then $nec_{B,\leq}$ C is true in w, iff every $\leq_w$-closest A-world in B_w is a C-world
     If A, then $pos_{B,\leq}$ C is true in w, iff some $\leq_w$-closest A-world in B_w is a C-world
  iii. In English $nec_{B,\leq}$ is realized as *will*, *would*, …, and $pos_{B,\leq}$, as *may, might, …*

- Formalization in Ty_2 (= Ty_1 + type **w** for worlds):
  i. We model contexts as variable assignments.
     B_n abbreviates a variable of type $\mathbf{ww}t$. Hence, for all w, $B_n w$ is of type $\mathbf{w}t$ (set of worlds)
     O_n abbreviates a variable of type $\mathbf{w}((\mathbf{w}t)t)$. Hence, for all w, $O_n w$ is of type $(\mathbf{w}t)t$ (set of propositions). $O_n w$ induces an order $\leq_{n,\,w}$ as follows:
       $w' \leq_{n,\,w} w''$ iff $\{p|\, p \in O_{n,\,w} \,\&\, w'' \in p\} \subseteq \{p|\, p \in O_{n,\,w} \,\&\, w' \in p\}$
     $\leq_{n,\,w}$ is a *weak partial order* because it is:
       transitive: $w_0 \leq_{n,\,w} w_1 \wedge w_1 \leq_{n,\,w} w_2 \rightarrow w_0 \leq_{n,\,w} w_2$
       [viewed from w, if w_0 is at least as close as w_1 & w_1 at least as close as w_2, then w_0 is at least as close as w_2]
       reflexive: $w_1 \leq_{n,\,w} w_1$
       [viewed from w, w_1 is at least as close as w_1]
       anti-symmetric: $w_0 \leq_{n,\,w} w_1 \wedge w_1 \leq_{n,\,w} w_0 \rightarrow w_0 \sim_{n,\,w} w_1$
       [viewed from w, if w_0 is at least as close as w_1 & w_1 at least as close as w_0, then w_0 and w_1 are equally close]
     We write
       $w' <_{n,\,w} w''$ for $w' \leq_{n,\,w} w'' \wedge \neg\, w'' \leq_{n,\,w} w'$
     and say that $\leq_{n,\,w}$ is *centered* iff it further satisfies the following condition:
       centered: $\forall w, w_0$: $w \neq w_0 \rightarrow w <_{n,\,w} w_0$
       [w is closer (more similar) to w than any other world is]
  ii. Notation:
     $\alpha_a \in \beta_{at}$:= $\beta\alpha$
     $\neg\beta_{at}$:= $\lambda v_a \neg\beta v$
     $\alpha_{at} \cap \beta_{at}$:= $\lambda v_a(\alpha v \wedge \beta v)$
     $\alpha_{at} \subseteq \beta_{at}$:= $\forall v_a(v \in \alpha \rightarrow v \in \beta)$
     $\mathbf{min}_{\leq n,\,w}\, \beta_{\mathbf{w}t}$:= $\lambda w'(w' \in \beta \wedge \forall w''(w'' \in \beta \rightarrow w' \leq_{n,\,w} w''))$
  iii. From English to Ty_2:
     If A, then $nec_{n,\,m}$ C $\rightsquigarrow$ $\lambda w\, \forall w'(w' \in \mathbf{min}_{\leq n,\,w}(B_m w \cap A) \rightarrow w' \in C)$
     If A, then $pos_{n,\,m}$ C $\rightsquigarrow$ $\lambda w\, \exists w'(w' \in \mathbf{min}_{\leq n,\,w}(B_m w \cap A) \wedge w' \in C)$

2. **L2** ANALYSIS OF KEY EXAMPLES

(***w***) *Contingency*:
Conditionals are contingent statements—true or false, depending on the *world*:

(1) If$_m$ Oswald hadn't killed Kennedy, someone else would have. (Adams 1975)

• Ty_2-translation
$\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\, w}\ (B_m w \cap \neg p_0) \rightarrow w' \in p_1)$
where:
$p_0 \ := \ \lambda w\ kill_w(osw, knd)$
$p_1 \ := \ \lambda w\ \exists x(x \neq osw \wedge kill_w(x, knd))$

• Modal base:
For all worlds $w \in W$,
$Bw \ = \ W$

• Similarity order on W:
Let:
q_0 = Oswald plots to kill Kennedy
q_1 = more than one gunman plots to kill Kennedy

Consider worlds $w_0, \ldots w_7$ such that:

	p_0	q_0	q_1
w_0	1	1	1
w_1	1	1	0
w_2	1	0	1
w_3	1	0	0
w_4	0	1	1
w_5	0	1	0
w_6	0	0	1
w_7	0	0	0

For each world, let $O_w = \{p \in \{p_0, q_0, q_1, \neg p_0, \neg q_0, \neg q_1\}: w \in p\}$ (more precisely, let g be an assignment s.t. $g(O_w)$ is a characteristic function of this set.) Then:

$\leq_{w0}$ induced by $O_{w0} = \{p_0, q_0, q_1\}$:

	w_7	
w_5	w_3	w_6
w_1	w_2	w_4
	w_0	

$\leq_{w1}$ induced by $O_{w1} = \{p_0, q_0, \neg q_1\}$

	w_7	
w_4	w_2	w_5
w_0	w_3	w_5
	w_1	

• Predicted truth values:
(1) ***false*** in w_1 wrt. $\leq_{w1}$ and $Bw_1 = W$: (Oswald acts alone)
because $\mathbf{min}_{\leq,\, w1}\ (Bw_1 \cap \neg p_0) = \{w_5\} \not\subseteq p_1$

If $w_4 \in p_1$, then (1) ***true*** in w_0 wrt. $\leq_{w0}$ and $Bw_0 = W$: (conspiracy)
because $\mathbf{min}_{\leq,\, w0}\ (Bw_0 \cap \neg p_0) = \{w_4\} \subseteq p_1$

(***B***) *Modal base* (aka *accessibility relation*):
The truth value of a conditional depends on the background *stock of facts/beliefs*, e.g.

(2) i. (Given the law,) if Tom buys a car, he must$_{1,,n}$ buy car insurance.
ii. But (given his finances,) if he buys a car, he will$_{0,\,n}$ not have money for insurance.

• Ty_2-translation
$\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq 0,\, w}\,(B_1 w \cap p_0) \rightarrow w' \in p_1)$
$\wedge\ \neg\exists w'(w' \in \mathbf{min}_{\leq 0,\, w}\,(B_2 w \cap p_0) \rightarrow w' \in p_1)$

where:
$p_0 \quad := \quad \lambda w\ \exists x(car_w\, x \wedge buy_w(tom, x))$
$p_1 \quad := \quad \lambda w\ \exists y(car.ins_w\, y \wedge buy_w(tom, y))$

• Two modal bases:
Let:
q_0 = Tom lives in NJ & has less than \$5000
q_1 = in NJ a car plus car insurance cost at least \$5000
q_2 = all drivers in NJ have car insurance

$B_0 w_1 = q_2$ (ideal worlds, by the laws of w_1)
$B_1 w_1 = ((q_0 \cap q_1) - q_2)$ (worlds like w_1 wrt. key facts)

• Similarity order on $B_n w_1$:
Consider worlds $w_0, \ldots w_7$ such that:

	p_0	q_0	q_1	q_2
w_0	1	1	1	1
w_1	1	1	1	0
w_2	1	1	0	1
w_3	1	1	0	0
w_4	1	0	1	1
w_5	1	0	1	0
w_6	1	0	0	1
w_7	1	0	0	0

	p_0	q_0	q_1	q_2
w_8	0	1	1	1
w_9	0	1	1	0
…	…	…	…	…

For each world, let $O_w = \{p \in \{p_0, q_0, q_1, q_2, \neg p_0, \neg q_0, \neg q_1, \neg q_2\}: w \in p\}$. Then:

$\leq_{w1}$ induced by $O_{w1} = \{p_0, q_0, q_1, \neg q_2\}$:

$w_0 \quad w_3 \quad w_5 \quad w_9$

w_1

• Predicted truth values:
(1) ***true*** in w_1 wrt. order $\leq_{w1}$, *law-base* $B_0 w_1$ and *fact-base* $B_1 w_1$:
because (i) and (ii) hold:

i. $\mathbf{min}_{\leq,\, w1}\,(B_0 w_1 \cap p_0)$
$= \mathbf{min}_{\leq,\, w1}\,(q_2 \cap p_0)$
$= \mathbf{min}_{\leq,\, w1}\,\{w_0, w_2, w_4, w_6, w_8\}$
$= \{w_0\}$
$\subseteq p_1$

ii. $\mathbf{min}_{\leq,\, w1}\,(B_1 w_1 \cap p_0)$
$= \mathbf{min}_{\leq,\, w1}\,((q_0 \cap q_1) - q_2) \cap p_0$
$= \mathbf{min}_{\leq,\, w1}\,\{w_1\}$
$= \{w_1\}$
$\subseteq (W - p_1)$

(***A***) *A-strengthening*
The following argument form is invalid (*A-fallacy*):

(4) If I strike this match, it will light.
$\models$ If I soak this match in water and then strike it, it will light.

ANALYSIS: Exercise.

(***T***) *Transitivity failure*:
The following argument form is also invalid (*T-fallacy*):

(6) If Tom were honest, people would trust him.
If people trusted Tom, he would cheat them.
$\models$ If Tom were honest, he would cheat people.

ANALYSIS: Exercise.

(**~**) *Equally close A-worlds*

(7) a. if Bizet and Verdi were compatriots, they would (both) be French. **F**
b. If Bizet and Verdi were compatriots, they would be (both) French or (both) Italian. **T**

ANALYSIS: Exercise.

APPENDIX: SOLUTIONS TO EXERCISES

(***A***) *A-strengthening*:
A-fallacy:

(4) <u>If I strike this match, it will light.</u>
$\models$ If I soak this match in water and then strike it, it will light.

ANALYSIS:

• Ty_2-translation

Let $g(x_0)$ = the speaker, $g(x_1)$ = this match

$p_0 \;:=\; \lambda w\ strike_w(x_0, x_1)$
$p_1 \;:=\; \lambda w\ wet_w(x_1)$
$q \;:=\; \lambda w\ light_w(x_1)$

(4′) $\underline{\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\,w}\ (Bw \cap p_0) \to w' \in q)}$ premise
$\models$ $\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\,w}\ (Bw \cap p_0 \cap p_1) \to w' \in q)$ conclusion

• Modal base: for all w,
$Bw = r_0 \cap r_1 \cap r_2$
where:
$r_0 \;:=\; \lambda w\ match_w\, x_1$
$r_1 \;:=\; \lambda w\ \forall x \forall y(match_w(y) \wedge \neg wet_w(y) \wedge strike_w(x, y) \to light_w(y))$
$r_2 \;:=\; \lambda w\ \forall x \forall y(match_w(y) \wedge wet_w(y) \wedge strike_w(x, y) \to \neg light_w(y))$

• Similarity order in Bw:
Let:

	r_0	r_1	r_2	p_0	p_1
w_0	1	1	1	1	0
w_1	1	1	1	1	1
w_2	1	1	1	0	0
w_3	1	1	1	0	1

$\leq_{w0}$ induced by $O_{w0} = \{p_0, \neg p_1\}$:

w_3
w_1 w_2
w_0

$\leq_{w2}$ induced by $O_{w2} = \{\neg p_0, \neg p_1\}$:

w_1
w_0 w_3
w_2

• Fallacy of (4′): True premise but false conclusion in any world where the match is dry.

In w_0:
$\mathbf{min}_{\leq w0}\ (Bw_0 \cap p_0) = \mathbf{min}_{\leq w0}(r_0 \cap r_1 \cap r_2 \cap p_0) = \{w_0\} \subseteq q$ prem. ***true***
but $\mathbf{min}_{\leq w0}\ (Bw_0 \cap p_0 \cap p_1) = \mathbf{min}_{\leq w0}(r_0 \cap r_1 \cap r_2 \cap p_0 \cap p_1) = \{w_1\} \nsubseteq q$ concl. ***false***

Similarly in w_2:
$\mathbf{min}_{\leq w2}\ (Bw_2 \cap p_0) = \mathbf{min}_{\leq w2}(r_0 \cap r_1 \cap r_2 \cap p_0) = \{w_0\} \subseteq q$ prem. ***true***
but $\mathbf{min}_{\leq w2}\ (Bw_2 \cap p_0 \cap p_1) = \mathbf{min}_{\leq w2}(r_0 \cap r_1 \cap r_2 \cap p_0 \cap p_1) = \{w_1\} \nsubseteq q$ concl. ***false***

(***T***) *Transitivity failure*:
T-fallacy:

(6) If Tom were honest, people would trust him.
If people trusted Tom, he would cheat them.
$\models$ If Tom were honest, he would cheat people.

ANALYSIS:

• Ty_2-translation

$p_0 \;=\; \lambda w\ honest_w(tom)$

$p_1 \;=\; \lambda w\ \forall x(deal.with_w(x, tom) \rightarrow trust_w(x, tom))$

$p_2 \;=\; \lambda w \forall x(trust_w(x, tom) \rightarrow cheat_w(tom, x))$

(6′)

$\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\, w}\ (Bw \cap p_0) \rightarrow w' \in p_1)$ premise 1

$\underline{\lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\, w}\ (Bw \cap p_1) \rightarrow w' \in p_2)}$ premise 2

$\models\ \lambda w\ \forall w'(w' \in \mathbf{min}_{\leq,\, w}\ (Bw \cap p_0) \rightarrow w' \in p_2)$ conclusion

• Modal base:
For all w,

$Bw \;=\; r_0 \cap r_1 \cap r_2$

$r_0 \;=\; \lambda w\ \forall x \forall y(deal.with_w(x, y) \wedge honest_w(y) \rightarrow trust_w(x, y))$

$r_1 \;=\; \lambda w\ \forall x \forall y(trust_w(x, y) \wedge honest_w(y) \rightarrow \neg cheat_w(x, y))$

$r_2 \;=\; \lambda w\ \forall x \forall y(trust_w(x, y) \wedge \neg honest_w(y) \rightarrow cheat_w(y, x))$

• Similarity order on Bw:
Let:

	r_0	r_1	r_2	p_0	p_1
w_0	1	1	1	0	0
w_1	1	1	1	0	1
~~w_2~~	~~1~~	~~1~~	~~1~~	~~1~~	~~0~~
w_3	1	1	1	1	1

$\leq_{w0}$ induced by $O_{w0} = \{\neg p_0, \neg p_1\}$:

w_3

w_1

w_0

• Fallacy (6′):
True premises but false conclusion in w_0, because

$\mathbf{min}_{\leq w0}\ (Bw_0 \cap p_0) = \mathbf{min}_{\leq w0}(r_0 \cap r_1 \cap r_2 \cap p_0) = \{w_3\} \subseteq p_1$ prem. 1 ***true***

$\mathbf{min}_{\leq w0}\ (Bw_0 \cap p_1) = \mathbf{min}_{\leq w0}(r_0 \cap r_1 \cap r_2 \cap p_1) = \{w_1\} \subseteq p_2$ prem. 2 ***true***

but $\mathbf{min}_{\leq w0}\ (Bw_0 \cap p_0 \cap p_1) = \mathbf{min}_{\leq w0}(r_0 \cap r_1 \cap r_2 \cap p_0) = \{w_3\} \not\subseteq p_2$ concl. ***false***

(~) *Equally close A-worlds*

(7) a. if Bizet and Verdi were compatriots, they would (both) be French. **F**

b. If Bizet and Verdi were compatriots, they would be (both) French or (both) Italian. **T**

ANALYSIS:

• Ty_2-translation

$p_0 \;=\; \lambda w\, \exists P((P = fr \vee P = it \vee \dots) \wedge P_w(b) \wedge P_w(v))$

$q_0 \;=\; \lambda w\, fr_w(b)$

$q_1 \;=\; \lambda w\, fr_w(v)$

$q_2 \;=\; \lambda w\, it_w(b)$

$q_3 \;=\; \lambda w\, it_w(v)$

(7′) a. $\lambda w\, \forall w'(w' \in \mathbf{min}_{\leq,\, w}\, (Bw \cap p_0) \rightarrow w' \in q_0 \cap q_1)$

b. $\lambda w\, \forall w'(w' \in \mathbf{min}_{\leq,\, w}\, (Bw \cap p_0) \rightarrow w' \in (q_0 \cap q_1) \vee w' \in (q_2 \cap q_3))$

• Modal base: for all w,

$Bw \;= r_0$

$r_0 \;=\; \lambda w\, \forall P \forall Q \forall x((P = fr \vee P = it \vee \dots) \wedge (Q = fr \vee Q = it \vee \dots) \wedge P \neq Q \wedge P_w x \rightarrow \neg Q_w x)$

(no dual nationality)

• Similarity order on Bw:

	r_0	q_0	q_1	q_2	q_3
w_0	1	1	0	0	1
w_1	1	1	0	0	0
w_2	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~
w_3	~~1~~	~~1~~	~~0~~	~~1~~	~~1~~
w_4	~~1~~	~~1~~	~~1~~	~~0~~	~~1~~
w_5	1	1	1	0	0
w_6	~~1~~	~~1~~	~~1~~	~~1~~	~~0~~
w_7	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~
w_8	1	0	0	0	1
w_9	1	0	0	0	0
w_{10}	1	0	0	1	0
w_{11}	1	0	0	1	1
w_{12}	~~1~~	~~0~~	~~1~~	~~0~~	~~1~~
w_{13}	1	0	1	0	0
w_{14}	1	0	1	1	0
w_{15}	~~1~~	~~0~~	~~1~~	~~1~~	~~1~~

$\leq_{w0}$ induced, within $Bw = r_0$, by $\mathbf{O}_{w0} = \{q_0, \neg q_1, \neg q_2, q_3\}$:

… … … …

w_5 w_9 w_{11}

w_1 w_8

w_0

• In w_0:

$\mathbf{min}_{\leq w0}\, (Bw_0 \cap p_0) = \mathbf{min}_{\leq w0}(r_0 \cap p_0) = \{w_5, w_{11}\} \not\subseteq (q_0 \cap q_1)$ (7′a) ***false***

$\mathbf{min}_{\leq w0}\, (Bw_0 \cap p_0) = \mathbf{min}_{\leq w0}(r_0 \cap p_0) = \{w_5, w_{11}\} \subseteq ((q_0 \cap q_1) \cup (q_2 \cap q_3))$ (7′b) ***true***

Kratzer's premise semantics:
(quotes from Kratzer 1981)

- Kripke 1963 a la Kratzer 1981

Proposition
Let W be the set of all possible worlds. A proposition is a subset of W

Truth of a Proposition
A proposition p is true in a world $w \in W$ if, and only if, $w \in p$. Otherwise, p is false in w.

Logical Consequence
A proposition p follows from a set of propositions A if, and only if, p is true in all worlds of W where all propositions of A are true.

Consistency
A set of propositions A is consistent if, and only if, there is a world in W where all propositions of A are true.

Logical Compatibility
A set of propositions A is compatible with a set of propositions A if, and only if, $A \cup \{p\}$ is a consistent set of propositions.

Conversational Backgrounds
We know already that a conversational background is the kind of entity which might be referred to by the utterance of a phrase like *what is known*…What is known is different from one possible world to another. And what is known in a possible world is a set of propositions. In our semantics, a conversational background will therefore be constructure as a function which assigns sets of propositions to possible worlds.

Simple Necessity
A proposition is a simple necessity in a world w with respect to the conversational background f if, and only if, it follows from $f(w)$.

Simple Possibility
A proposition is a simple possibility in a world w with respect to the conversational background f if, and only if, it is compatible with $f(w)$.

- Graded modality

It is probable that…	Human Necessity
There is a good possibility that …	Human Possibility
There is a slight possibility that…	Slight Possibility
It is more likely that … than that…	Comparative Possibility

Quite generally, a set of propositions A can induce an ordering $\leq_A$ on the set of all possible worlds in the following way [MB: Lewis 1981]:

The Ordering $\leq_A$
For all worlds w and $z \in W$:
$w \leq_A z$ if and only if $\{p: p \in A \text{ and } z \in p\} \subseteq \{p: p \in A \text{ and } w \in p\}$

It is probable that… Human Necessity

Human Necessity
A proposition p is a human necessity in a world w with respect to a modal base f and an ordering source g if, and only if, the following condition is fulfilled:
For all $u \in \cap f(w)$ there is $v \in \cap f(w)$ such that
(i) $v \leq_{g(w)} u$
(ii) for all $z \in \cap f(w)$: If $z \leq_{g(w)} v$, then $z \in p$.

There is a good possibility that … Human Possibility

Human Possibility
A proposition is a human possibility in a world w with respect to a modal base f and an ordering source g if, and only if, its negation (that is its complement) is not a human necessity in w with respect to f and g.

There is a slight possibility that… Slight Possibility

Slight Possibility
A proposition p is a slight possibility in a world w with respect to a modal base f and an ordering source g if, and only if,
(i) p is compatible with $f(w)$
(ii) the negation of p is a human necessity in w with respect to f and g.

It is more likely that … than that… Comparative Possibility

Comparative Possibility
A proposition p is more possible that a proposition q in a world w in view of a modal base f and an ordering source g if, and only if, the following conditions are satisfied:
(i) For all $u \in \cap f(w)$:
If $u \in q$, then there is a world $v \in \cap f(w)$ such that $v \leq_{g(w)} u$ and $v \in p$.
(ii) There is a world $u \in \cap f(w)$ such that:
$u \in p$ and there is no world $v \in \cap f(w)$ such that $v \in q$ and $v \leq_{g(w)} u$.

Conditional modality
Consider an utterance of a sentence of the following form:

(if α), (then modal…)
(i) The first part of the utterance requires one, and only one, modal base and one, and only one, ordering source to be correct.
(ii) If f is the modal base and g the ordering source for the first part of the utterance, then f^+ is the modal base and g the ordering source for the second part of the utterance. f^+ is that function from possible worlds to sets of proposition, such that for any world w, $f^+(w) = f(w) \cup \{p\}$.

Modal Presuppositions: Stalnaker 1975, Stone 1997

• Stalnaker 1975

(1) i. Sir Charles was murdered.
ii. Either the butler or the gardener did it.
iii. So, if the butler didn't do it, the gardener {*did*, *must*'ve}. *indicative* cnd.

(2) i. Sir Charles was murdered with an icepick.
ii. If the butler had done it, he *would*n't have used an icepick. *subjunctive* cnd.
iii. So the murderer *must*'ve been someone else. *indicative* modal

(3) (after Anderson 1951)
i. Sir Charles was murdered.
ii. If the butler had done it, we *would*'ve found just the clues which we in fact found. *subjunctive* cnd.
iii. So the murderer *must*'ve been the butler. *indicative* modal

Common ground (Stalnaker's "context set")
"A speaker inevitably takes certain information for granted which he speaks as the *common ground* [MB emphasis] of the participants in the conversation. …The presumed common ground in the sense intended—the *presuppositions* of the speaker—need not be the beliefs which are really common to the speaker and his audience; in fact, they need not be beliefs at all. The presuppositions will include whatever the speaker finds it convenient to take for granted, or to pretend to take for granted to facilitate his communication."

Indicative presupposition (default pragmatics)
"when a speaker says 'If *A*', then everything he is presupposing to hold in the actual situation is presupposed to hold in the hypothetical situation in which *A* is true.

[e.g.] Suppose it is an open question whether the butler did it or not, but it is established and accepted that whoever did it, he or she did it with an ice-pick. Then it may be taken as accepted and established that if the butler did it, he did it with an ice-pick.

Subjunctive presupposition (default defeated)
"[the above] principle is only a *defeasible* presumption [MB emphasis]…For some special purposes the speaker… may want to suspend temporarily some of the presuppositions made in that context. …I take it that the subjunctive mood in English and some other languages is a conventional device for indicating that presuppositions are being suspended, which means in the case of subjunctive *conditional* statements, that the selection function…may reach outside of the [common ground]"

• Stone 1997

(4) a. Pedro owns a donkey. *He* beats it. (nominal anaphor)
b. When John saw Mary, she cross*ed* the street. (temporal anaphor)
c. If the railroads merged, the line *would* face bankruptcy. (modal anaphor)

(5) a. #Pedro owns a donkey. *She* beats it. (nominal anaphor)
b. #When John saw Mary, she cross*es* the street. (temporal anaphor)
c. #If the railroads merged, the line *will* face bankruptcy. (modal anaphor)

p. 7 *Presuppositional account* of #(5a, b, c)
"In each case [in (2)], the anaphoric element—the pronoun *she*, the present tense in *crosses*, the 'vivid' modality in *will*—is incompatible with its intended referent. This is because the anaphoric forms impose presuppositions that their referents must satisfy."

Pronoun presuppositions
"PRONOUNS must agree with their antecedents in number, person, and in some cases gender; *she* can refer to an entity only if the entity is a single female other than the speaker or the addressee of the utterance"

p. 8 *Tense presuppositions*
"the PRESENT tense may refer only to the moment of speech, while the PAST tense may refer only to times that precede the moment of speech."

Modal presuppositions
"REAL modals may refer only to the scenario given by the actual perspective of the speaker; real modals include ordinary present tense indicative verbs like *is* and the epistemic modal *must*. In contrast, modals I shall call VIVID may refer to possible elaborations of the speaker information; *will* and *can* are representative of vivid modals. REMOTE modals [Isard's, 1974, term], like *would* or *should*, may even refer to scenarios that are not regarded as real possibilities given the state of the world at the moment of speech.

• Cross-linguistic evidence (see *Kalaallisut texts* at http://www.rci.rutgers.edu/~mbittner)

	REAL modality	VIVID modality	REMOTE modality
English	*(if…) is, must*	*(if…) will, can*	*(if…) would, might*
Kalaallisut [Bittner 2005]	(…HYP) IND/FCT	(…HYP) *prosp.stative*-IND/FCT	(…***galuar-***HYP) …*prosp.stative*-IND/FCT or (…-HYP) …*prosp.stative*-**galuar-**IND/FCT

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