# Encapsulated quantification: Stone 1997, 1999

## 1. DATA & BASIC IDEAS

- Referential parallels

*Deictic reference*

(1) *She*$_{u1}$ left me. *nominal* (Partee 1973)
[uttered by a man sitting alone with his head in his hands]

(2) My neighbours *would*$_{\omega 1}$ kill me. *modal* (Stone 1997)
[looking at a high-end stereo in an electronics store]

*Anaphoric reference within a clause*

($3^1$) Pedro owns $a^{u2}$ [donkey]$_{u2}$ *nominal*

($4^1$) I *must*$_{\omega}^{\omega 1}$ [buy a car]$_{\omega 1}$ *modal* (Stone 1997)

*Anaphoric reference in discourse*

(3) Pedro$^{u1}$ owns $a^{u2}$ donkey. *He*$_{u1}$ beats *it*$_{u2}$. *nominal*

(4) I *must*$_{\omega}^{\omega 1}$ buy a car. But I *can't*$_{\omega,\, \omega 1}$. *modal* (Stone 1997)

*Anaphoric reference in conditionals*

(5) *If*$_{\omega}^{\omega 1}$ $\underline{a}^{u1}$ man commits a murder, ***weak rdg. only!***
$\underline{\text{he}}_{u1}$ *should*$_{\omega,\, \omega 1}^{\omega 2}$ go to prison.

*Nominal anaphora across modalities* (Stone 1999)

(6) *If*$_{\omega}^{\omega 1}$ there are bears in this area, $\underline{a}^{u2}$ bear *might*$_{\omega,\, \omega 1}^{\omega 2}$ come by.
But $\underline{\text{he}}_{u2}$ *wouldn't*$_{\omega,\omega 2}^{\omega 3}$ hurt you. I **have**$_{\omega}$ $\underline{a}^{u3}$ gun.
If$_{\omega,\omega 2}^{\omega 5}$ $\underline{\text{the}}_{u2}$ bear attacked you, I **would**$_{\omega,\, \omega 5}^{\omega 6}$ use $\underline{\text{it}}_{u3}$ to shoot $\underline{\text{him}}_{u2}$.

- Stone's (1997, 1999) theory

Adapting and extending Muskens (1995), Stone (1997, 1999) proposes:

(a) parallel analysis of nominal and modal reference in terms of drefs for *individual concepts* (type $s\mathbf{w}e$) and *modal concepts* (type $s\mathbf{w}(\mathbf{w}t)$).

(b) nouns and verbs relate their nominal arguments to a modal domain (*encapsulated quant.* $\forall w$)

(c) modals (*may*, *would*, etc) do not quantify over worlds; instead they relate modal drefs.

(d) representation of reality:
"Assume that the initial environment *i* assigns a value to ω corresponding to the information implicitly available to the participants in the conversation, which we can represent by a proposition or set of worlds *E*. Formally, then, for each world $w \in E$, $\omega(i, w) = E$, and for any other world $\omega(i, w)$ is empty." (Stone 1999:20)

NOTATION:

| | | | | |
|---|---|---|---|---|
| $a^*$ | for the speaker (type $e$) | $u_n$ | for | nominal drefs (type $s\mathbf{w}e$) |
| $p^*$ | for $E$ of Stone 1999:20 (type $\mathbf{w}t$) | $u$ | := | $\lambda i. (\lambda w.\ a^*)$ |
| $M$ | for modal base functions (type $\mathbf{w}(\mathbf{w}t)$) | $\omega_n$ | for | modal drefs (type $s\mathbf{w}(\mathbf{w}t)$) |
| $w' <_w w''$ | $:= (w' \leq_w w'' \wedge \neg w'' \leq_w w')$ | $\omega$ | := | $\lambda i. (\lambda w \in p^*.\ p^*)$ |
| $\mathbf{min}_{<w}\, p$ | $:= \{w' \in p \mid \neg\exists w'' \in p(w'' <_w w')\}$ | Dom $M$ | := | $\{w \mid Mw \neq \varnothing\}$ |
| $\lambda w \in p.\ f_w$ | $:= \{\langle w, f_w\rangle : w \in p\} \cup \{\langle w, \varnothing\rangle : w \notin p\}$ | Ran $M$ | := | $\{w' \mid \exists w(w' \in Mw)\}$ |

## 2. DEICTIC REFERENCE

(1) *She* left me.

($1^0$) Start-up a la Bittner 2007
$i_0$
$u$ $\lambda w.\ a^*$
ω $\lambda w \in p^*.\ p^*$

($1^1$) Accommodate presupposition of $\text{she}_{u1}$
$i_1$
$u$ $\lambda w.\ a^*$
ω $\lambda w \in p^*.\ p^*$
$u_1$ $\lambda w.\ a_1$

($1^2$) $\text{She}_{u1}$
$[female_\omega\{u_1\}]i_1i_1$
iff $\forall w \in \text{Ran}\ \omega i_1(female_w(u_1i_1w))$
iff $\forall w \in p^*(female_w(u_1i_1w))$
[In every $p^*$-world, $a_1$ is female.]

The past tense left further adds the presupposition that $a_1$ used to be with Bill, but this cannot be represented in this tenseless system.

($1^3$) $\text{left}_\omega$ me.
$[leave_\omega\{u_1, u\}]i_1i_1$
iff $\forall w \in p^*(leave_w(a_1, a^*))$
[In every $p^*$-world, $a_1$-female leaves the speaker $a^*$.]

(2) My neighbours *would* kill me.
[uttered while looking at high-end stereo, $c$, in electronics store]

($2^0$) Start-up
$i_0$
$u$ $\lambda w.\ a^*$
$u_1$ $\lambda w.\ c$
$\omega$ $\lambda w \in p^*.\ p^*$

($2^1$) Accommodate presup. of $\text{would}_{\omega,\ \omega 1}{}^{\omega 2}$ by pragmatic inference ($pfb$ = play for full blast)
$\mathbf{max}_\omega{}^{\omega 1}([buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}])i_0 i_1$
iff $i_0[\omega\text{: }\omega_1]i_1 \wedge \text{Dom } \omega_1 i_1 = p^* \wedge \forall w \in p^*(\omega_1 i_1 w = \{w' \mid buy_{w'}(a^*, c) \wedge pfb_{w'}(a^*, c)\})$
$i_1$
$u$ $\lambda w.\ a^*$
$u_1$ $\lambda w.\ c$
$\omega$ $\lambda w \in p^*.\ p^*$
$\omega_1$ $\lambda w \in p^*.\ \mathbf{B}$ s.t. • $\mathbf{B} := \{w' \mid buy_{w'}(a^*, c) \wedge pfb_{w'}(a^*, c)\}$

($2^2$) My neighbours
$[\omega_1\text{: } u_2 \mid neighbours.of_{\omega 1}\{u_2, u\}]i_1 i_2$
iff $i_1[\omega_1\text{: } u_2]i_2 \wedge \text{Dom } u_2 i_2 = \mathbf{B} \wedge \forall w \in \mathbf{B}(neighbours.of_w(u_2 i_2 w, u i_2 w))$
$i_2$
$u$ $(a^*)^\circ$
$u_1$ $\lambda w.\ c$
$u_2$ $\lambda w \in \mathbf{B}.\ b_w$ s.t. • $\forall w \in \mathbf{B}(neighbours.of_w(b_w, a^*)$
$\omega$ $\lambda w \in p^*.\ p^*$
$\omega_1$ $\lambda w \in p^*.\ \mathbf{B}$ s.t. • $\mathbf{B} := \{w' \mid buy_{w'}(a^*, c) \wedge pfb_{w'}(a^*, c)\}$

($2^3$) $\textit{would}_{\omega,\ \omega 1}{}^{\omega 2}$ kill me.
*Notation*: '<' for ranking by similarity to $w$, '$<_1$' for ranking $a^*$'s expectations in $w$
($[\mathbf{remote}_{<1}(\omega, \omega_1)]$;
$\mathbf{max}_\omega{}^{\omega 2}([\omega_2 \subseteq \omega_1, kill_{\omega 2}\{u_2, u\}]); [\mathbf{necc}_<(\omega_1, \omega_2)])i_2 i_3$
iff $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ p^*) \cap \mathbf{B} = \varnothing)$
$i_2[\omega\text{: }\omega_2]i_3 \wedge \text{Dom } \omega_2 i_3 = p^* \wedge \forall w \in p^*(\omega_2 i_3 w = \{w' \in \mathbf{B} \mid kill_{w'}(b_{w'}, a^*)\})$
$\wedge\ \forall w \in p^*((\mathbf{min}_{<,\ w}\ \mathbf{B}) \subseteq \mathbf{K})$
$i_2$
$u$ $(a^*)^\circ$
$u_1$ $\lambda w.\ c$
$u_2$ $\lambda w \in \mathbf{B}.\ b_w$ s.t. • $\forall w \in \mathbf{B}(neighbours.of_w(b_w, a^*)$
$\omega$ $\lambda w \in p^*.\ p^*$
$\omega_1$ $\lambda w \in p^*.\ \mathbf{B}$ s.t. • $\mathbf{B} := \{w' \mid buy_{w'}(a^*, c) \wedge pfb_{w'}(a^*, c)\}$
• $\forall w \in p^*((\mathbf{min}_{<1,w}\ p^*) \cap \mathbf{B} \neq \varnothing)$
$\omega_2$ $\lambda w \in p^*.\ \mathbf{K}$ s.t. • $\mathbf{K} := \{w' \in \mathbf{B} \mid kill_{w'}(b_{w'}, a^*)\}$
• $\forall w \in p^*((\mathbf{min}_{<w}\ \mathbf{B}) \subseteq \mathbf{K})$
[For every $p^*$-world $w$, the most $<_{1,\ w}$-likely $p^*$-worlds are not $\mathbf{B}$-worlds, and in every $w$-closest $\mathbf{B}$-world $w'$, where the speaker $a^*$ buys the stereo $c$ and plays it for full blast, he is killed by his $w'$-neighbours $b_{w'}$.]

3. ANAPHORIC REFERENCE

(3) Pedro owns a donkey. *He* beats *it*. *nominal anaphora*

($3^0$) Start-up

$i_0$

$u$ $\lambda w.\ a^*$

ω $\lambda w \in p^*.\ p^*$

($3^1$) Pedro

$[\omega\text{: } u_1 |\ u_1 = Pedro,\ male_\omega\{u_1\}]i_0i_1$

iff $i_0[\omega\text{: } u_1]i_1 \wedge \text{Dom } u_1i_1 = p^* \wedge \forall w \in p^*(u_1i_1w = pedro \wedge male_w\ u_1i_1w)$

$i_1$

$u$ $\lambda w.\ a^*$

$u_1$ $\lambda w \in p^*.\ pedro$ s.t. $\forall w \in p^*(male_w\ pedro)$

ω $\lambda w \in p^*.\ p^*$

($3^2$) owns

$[\omega\text{: } u_2 |\ own_\omega\{u_1, u_2\}]i_1i_2$

iff $i_1[\omega\text{: } u_2]i_2 \wedge \text{Dom } u_2i_2 = p^* \wedge \forall w \in p^*(own_w(pedro, u_2i_2w))$

$i_2$

$u$ $\lambda w.\ a^*$

$u_1$ $\lambda w \in p^*.\ pedro$ s.t. $\forall w \in p^*(male_w\ pedro)$

$u_2$ $\lambda w \in p^*.\ b_w$ s.t. $\forall w \in p^*(own_w(pedro, b_w))$

ω $\lambda w \in p^*.\ p^*$

($3^3$) a donkey

$[donkey_\omega\{u_2\}]i_2i_2$

iff $\forall w \in p^*(donkey_w(b_w))$

$i_2$

$u$ $\lambda w.\ a^*$

$u_1$ $\lambda w \in p^*.\ pedro$ s.t. $\forall w \in p^*(male_w\ pedro)$

$u_2$ $\lambda w \in p^*.\ b_w$ s.t. $\forall w \in p^*(own_w(pedro, b_w) \wedge donkey_w\ b_w)$

ω $\lambda w \in p^*.\ p^*$

($3^4$) He beats it

$[male_\omega\{u_1\},\ beat_\omega\{u_1, u_2\},\ u_2 \neq_\omega u_1]i_2i_2$

iff $\forall w \in p^*(male_w(pedro) \wedge beat_w(pedro, b_w) \wedge b_w \neq pedro)$

$i_2$

$u$ $\lambda w.\ a^*$

$u_1$ $\lambda w \in p^*.\ pedro$ s.t. $\forall w \in p^*(male_w\ pedro)$

$u_2$ $\lambda w \in p^*.\ b_w$ s.t. $\forall w \in p^*(own_w(pedro, b_w) \wedge donkey_w\ b_w \wedge beat_w(pedro, b_w) \wedge b_w \neq pedro)$

ω $\lambda w \in p^*.\ p^*$

(4) I must buy a car. But I *can't.* *modal anaphora*

($4^0$) Start-up
$i_0$
$u$ $\lambda w.\, a^*$
$\omega$ $\lambda w \in p^*.\, p^*$

($4^1$) [I _ buy a car]$_{\omega}^{\omega 1}$
$\mathbf{max}_{\omega}^{\omega 1}([\omega_1 \subseteq \omega]; [\omega{:}\ u_1|\ car_{\omega 1}\{u_1\}, buy_{\omega 1}\{u, u_1\}])i_0 i_1$
iff $\exists k(i_0[\omega{:}\ \omega_1]k \wedge k[\omega{:}\ u_1]i_1$
$\wedge$ Dom $\omega_1 i_1 = p^* \wedge \forall w \in p^*(\omega_1 i_1 w = \{w' \in p^*|\ \exists y(car_{w'}\, y \wedge buy_w(a^*, y)\}$
$\wedge$ Dom $u_1 i_1$ = Ran $\omega_1 i_1 \wedge \forall w' \in$ Ran $\omega_1 i_1(car_{w'}\, u_1 i_1 w' \wedge buy_w(a^*, u_1 i_1 w'))$
$i_1$
$u$ $\lambda w.\, a^*$
$u_1$ $\lambda w \in \mathbf{B}.\, c_w$ s.t. • $\forall w' \in \mathbf{B}(car_{w'}\, c_{w'} \wedge buy_w(a^*, c_{w'}))$
$\omega$ $\lambda w \in p^*.\, p^*$
$\omega_1$ $\lambda w \in p^*.\, \mathbf{B}$ s.t. • $\mathbf{B} = \{w' \in p^* |\ \exists y(car_{w'}\, y \wedge buy_w(a^*, y))\}$

($4^2$) must$_{\omega,\, \omega 1}$
*Notation*: '$<_2$' for ranking by $a^*$'s needs in $w$
$([\mathbf{vivid}_{<2}(\omega, \omega_1)]; [\mathbf{necc}_{<2}(\omega, \omega_1)])i_1 i_1$
iff $\forall w \in p^*((\mathbf{min}_{<2,\, w}\, p^*) \cap \mathbf{B} \neq \varnothing)$
$\forall w \in p^*((\mathbf{min}_{<2,\, w}\, p^*) \subseteq \mathbf{B})$
iff $\forall w \in p^*(\varnothing \subset (\mathbf{min}_{<2,\, w}\, p^*) \subseteq \mathbf{B})$
$i_1$
$u$ $\lambda w.\, a^*$
$u_1$ $\lambda w \in \mathbf{B}.\, c_w$ s.t. • $\forall w' \in \mathbf{B}(car_{w'}\, c_{w'} \wedge buy_w(a^*, c_{w'}))$
$\omega$ $\lambda w \in p^*.\, p^*$
$\omega_1$ $\lambda w \in p^*.\, \mathbf{B}$ s.t. • $\mathbf{B} = \{w' \in p^* |\ \exists y(car_{w'}\, y \wedge buy_w(a^*, y))\}$
• $\forall w \in p^*(\varnothing \subset (\mathbf{min}_{<2,\, w}\, p^*) \subseteq \mathbf{B})$

($4^3$) But I can't$_{\omega,\, \omega 1}$
*Notation*: '$<_1$' for ranking by $a^*$'s expectations in $w$
$([\mathbf{vivid}_{<2}(\omega, \omega_1)]; [\mathbf{not}_{<1}(\omega, \omega_1)])i_1 i_1$
iff $\forall w \in p^*((\mathbf{min}_{<2,\, w}\, p^*) \cap \omega_1 i_1 w \neq \varnothing)$
$\wedge\ \forall w \in p^*((\mathbf{min}_{<1,\, w}\, p^*) \cap \omega_1 i_1 w = \varnothing)$
$i_1$
$u$ $\lambda w.\, a^*$
$u_1$ $\lambda w \in \mathbf{B}.\, c_w$ s.t. • $\forall w' \in \mathbf{B}(car_{w'}\, c_{w'} \wedge buy_w(a^*, c_{w'}))$
$\omega$ $\lambda w \in p^*.\, p^*$
$\omega_1$ $\lambda w \in p^*.\, B$ s.t. • $\mathbf{B} = \{w' \in p^* |\ \exists y(car_{w'}\, y \wedge buy_w(a^*, y))\}$
• $\forall w \in p^*(\varnothing \subset (\mathbf{min}_{<2}\, p^*) \subseteq \mathbf{B})$
• $\forall w \in p^*((\mathbf{min}_{<1}\, p^*) \cap \mathbf{B} = \varnothing)$

## 4. DONKEY CONDITIONALS: WEAK READING ONLY

(5) If a man commits a murder, he should go to prison.

($5^0$) Start-up

| | $i_0$ |
|---|---|
| $u$ | $\lambda w.\ a^*$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ |

($5^1$) If$_\omega{}^{\omega 1}$ a$^{u1}$ man commits a murder

*Notation*: $mrd$ = commit a murder

$\mathbf{max}_\omega{}^{\omega 1}([\omega_1 \colon u_1 | \ man_{\omega 1}\{u_1\},\ mrd_{\omega 1}\{u_1\}])i_0 i_1$

iff $i_0[\omega \colon \omega_1]i_1 \wedge \text{Dom}\ \omega_1 i_1 = p^* \wedge \forall w \in p^*(\omega_1 i_1 w = \{w' \mid \exists x(man_{w'}\, x \wedge mrd_{w'}\, x)\}$
$\wedge\ \text{Dom}\ u_1 i_1 = \text{Ran}\ \omega_1 i_1 \wedge \forall w \in \text{Ran}\ \omega_1 i_1(man_{w'}\, u_1 i_1 w' \wedge mrd_{w'}\, u_1 i_1 w')$

| | $i_1$ | |
|---|---|---|
| $u$ | $\lambda w.\ a^*$ | |
| $u_1$ | $\lambda w \in \mathbf{M}.\ b_w$ | s.t. $\forall w' \in \mathbf{M}(man_{w'}\, b_{w'} \wedge mrd_{w'}\, b_{w'})$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{M}$ | s.t. $\mathbf{M} = \{w' \mid \exists x(man_{w'}\, x \wedge mrd_w x)\}$ |

($5^2$) he$_{u1}$ should$_{\omega,\ \omega 1,\ \omega 2}$ go to prison

*Notation*: $gtp$ = go to prison
'$<_3$' for ranking by conformity to the laws of $w$

$([\mathbf{remote}_{<3}(\omega, \omega_1)];$ — *presup. test on $i_1$*
$\mathbf{max}_\omega{}^{\omega 2}([\omega_2 \subseteq \omega_1,\ gtp_{\omega 2}\{u_1\}]);$ — *update to $i_2$*
$[\mathbf{necc}_{<3}(\omega_1, \omega_2)])i_1 i_2$ — *test $i_2$*

iff $\forall w \in p^*((\mathbf{min}_{<3,\ w}\, p^*) \cap \mathbf{M} = \varnothing)$
$\wedge\ i_1[\omega \colon \omega_2]i_2 \wedge \text{Dom}\ \omega_2 i_2 = p^* \wedge \forall w \in p^*(\omega_2 i_2 w = \{w' \in \mathbf{B} | \ gtp_{w'}\, b_{w'}\})$
$\wedge \forall w \in p^*((\mathbf{min}_{<3,\ w}\, \mathbf{M}) \subseteq \omega_2 i_2 w)$

| | $i_2$ | |
|---|---|---|
| $u$ | $\lambda w.\ a^*$ | |
| $u_1$ | $\lambda w \in \mathbf{M}.\ b_w$ | s.t. • $\forall w' \in \mathbf{M}(man_{w'}\, b_{w'} \wedge mrd_{w'}\, b_{w'})$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{M}$ | s.t. • $\mathbf{M} = \{w' \mid \exists x(man_{w'}\, x \wedge mrd_w x)\}$ <br> • $\forall w \in p^*((\mathbf{min}_{<3,\ w}\, p^*) \cap \mathbf{M} = \varnothing)$ <br> [UNREALISTIC: in the most law-abiding worlds, *among the speaker's doxastic alternatives* $p^*$, there is no murder] |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{P}$ | s.t. • $\mathbf{P} = \{w' \in \mathbf{M} \mid gtp_{w'}\, b_{w'}\}$ <br> • $\forall w \in p^*((\mathbf{min}_{<3,\ w}\, \mathbf{M}) \subseteq \mathbf{P}$ <br> [BAD: in $\mathbf{M}$-worlds $w'$, with one or more murderers, the laws of $w$ require that at least one murderer, $b_{w'}$, go to prison—**oops!!…**] |

## 5. HOW TO SHOOT A HYPOTHETICAL BEAR WITH A REAL GUN

(6a) If there are bears in this area, a bear might come by.

($6^0$) Start-up

| | $i_0$ |
|---|---|
| $u$ | $\lambda w.\ a^*$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ |

($6^1$) If$_\omega{}^{\omega 1}$ there are bears in this area

$\mathbf{max}_\omega{}^{\omega 1}([\omega_1 \text{: } u_1 |\ bear_{\omega 1}\{u_1\}, near_{\omega 1}\{u_1, u\}])i_0 i_1$

iff $i_0[\omega\text{: } \omega_1]i_1$
$\wedge$ Dom $\omega_1 i_1 = p^* \wedge \forall w \in p^*(\omega_1 i_1 w = \{w' \mid \exists x(bear_{w'}\, x \wedge near_{w'}(x, a^*))\}$
$\wedge$ Dom $u_1 i_1 =$ Ran $\omega_1 i_1 \wedge \forall w' \in$ Ran $\omega_1 i_1(bear_{w'}\, u_1 i_1 w' \wedge near_{w'}(u_1 i_1 w', a^*))$

| | $i_1$ | |
|---|---|---|
| $u$ | $\lambda w.\ a^*$ | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. $\forall w' \in \mathbf{B}(bear_{w'}\, b_{w'} \wedge near_{w'}\, b_{w'})$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\, x \wedge near_{w'}(x, a^*))\}$ |

($6^2$) a$^{u2}$ bear might$_{\omega,\ \omega 1,\ \omega 2}$ come by
*Notation*: '$<_4$' for ranking by $a^*$'s hopes in $w$, '$<_1$' for ranking by $a^*$'s expectations in $w$

| | |
|---|---|
| $([\mathbf{remote}_{<4}(\omega, \omega_1)];$ | *presup. test on* $i_1$ |
| $\mathbf{max}_\omega{}^{\omega 2}([\omega_2 \subseteq \omega_1]; [\omega_2\text{: } u_2 \mid bear_{\omega 2}\{u_2\}, come_{\omega 2}\{u_2\}]);$ | *update to* $i_2$ |
| $[\mathbf{poss}_{<1}(\omega_1, \omega_2)])i_1 i_2$ | *test* $i_2$ |

iff $\forall w \in p^*((\mathbf{min}_{<4,\ w}\, p^*) \cap \mathbf{B} = \varnothing$
$\wedge\ i_1[\omega\text{: } \omega_2]i_2 \wedge$ Dom $\omega_2 i_2 = p^* \wedge \forall w \in p^*(\omega_2 i_2 w = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\, y \wedge come_{w'}\, y)\})$
$\wedge$ Dom $u_2 i_2 =$ Ran $\omega_2 i_2 \wedge \forall w \in$ Ran $\omega_2 i_2(bear_w\, u_2 i_2 w \wedge come_w\, u_2 i_2 w)$
$\wedge\ \forall w \in p^*((\mathbf{min}_{<1,\ w}\, \mathbf{B}) \cap \omega_2 i_2 w \neq \varnothing)$

| | $i_2$ | |
|---|---|---|
| $u$ | $\lambda w.\ a^*$ | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. • $\forall w' \in \mathbf{B}(bear_{w'}\, b_{w'} \wedge near_{w'}\, b_{w'})$ |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. • $\forall w' \in \mathbf{C}(bear_{w'}\, c_{w'} \wedge come_{w'}\, c_{w'})$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\, x \wedge near_{w'}(x, a^*))\}$ <br> • $\forall w \in p^*((\mathbf{min}_{<4,\ w}\, p^*) \cap \mathbf{B} = \varnothing$ <br> [in $w$, $a^*$ hopes there are no bears nearby] |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\, y \wedge come_{w'}\, y)\}$ <br> • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\, \mathbf{B}) \cap \mathbf{C} \neq \varnothing$ <br> [in $w$, $a^*$'s expected **B**-worlds include some **C**-worlds.] |

(6b) But he wouldn't hurt you.

($6^3$) he$_{u2}$ wouldn't$_{\omega,\ \omega 2,\ \omega 3}$ hurt you

*Notation*: '$<_4$' for ranking by $a^*$'s hopes in $w$,
'$<_1$' for ranking by $a^*$'s expectations in $w$

$([\mathbf{remote}_{<4}(\omega, \omega_2)];$ *presup. test on $i_2$*
$\mathbf{max}_{\omega}{}^{\omega 3}([\omega_3 \subseteq \omega_2]; [hurt_{\omega 3}\{u_3, \mathbf{adr}\{u\}\}])$ *update to $i_3$*
$[\mathbf{not}_{<1}(\omega_2, \omega_3)])i_2i_3$ *test $i_3$*

iff $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ p^*) \cap \mathbf{C} = \varnothing$
$\wedge\ i_2[\omega: \omega_3]i_3 \wedge \text{Dom}\ \omega_3 i_3 = p^* \wedge \forall w \in p^*(\omega_3 i_3 w = \{w' \in \mathbf{C} |\ hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\})$
$\wedge\ \forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \omega_3 i_3 w = \varnothing)$

| | $i_3$ | | |
|---|---|---|---|
| $u$ | $\lambda w.\ a^*$ | | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. | • $\forall w' \in \mathbf{B}(bear_{w'}\ b_{w'} \wedge near_{w'}\ b_{w'})$ |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. | • $\forall w' \in \mathbf{C}(bear_{w'}\ c_{w'} \wedge come_{w'}\ c_{w'})$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. | • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\ x \wedge near_{w'}(x, a^*))\}$<br>• $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ p^*) \cap \mathbf{B} = \varnothing$<br>[in $w$, $a^*$ hopes there are no bears nearby] |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. | • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\ y \wedge come_{w'}\ y)\}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{B}) \cap \mathbf{C} \neq \varnothing$<br>[in $w$, $a^*$'s expected **B**-worlds include some **C**-worlds.]<br>• $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ \mathbf{B}) \cap \mathbf{C} = \varnothing$<br>[in $w$, $a^*$'s hoped for **B**-worlds do not include any **C**-worlds.] |
| $\omega_3$ | $\lambda w \in p^*.\ \mathbf{H}$ | s.t. | • $\mathbf{H} = \{w' \in \mathbf{C} \mid hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \mathbf{H} = \varnothing$<br>[in $w$, $a^*$'s expected **C**-worlds do not include any **H**-worlds.] |

(6c) I (do) have a gun.

($6^4$) [I _ have a$^{u3}$ gun]$_\omega^{\omega 4}$

$\mathbf{max}_\omega^{\omega 4}([\omega_4{:}\ u_3 |\ gun_{\omega 4}\{u_3\},\ have_{\omega 4}\{u, u_3\}]))i_3 i_4$

iff $i_3[\omega{:}\ \omega_4]i_4 \wedge \text{Dom}\ \omega_4 i_4 = p^* \wedge \forall w \in p^*(\omega_4 i_4 w = \{w' \mid \exists y(gun_{w'}\, y \wedge have_{w'}(a^*, y))\})$

$\wedge\ \text{Dom}\ {}_3 i_4 = \text{Ran}\ \omega_4 i_4 \wedge \forall w' \in \text{Ran}\ \omega_4 i_4(gun_{w'}\, u_3 i_4 w' \wedge have_{w'}(a^*, u_3 i_4 w'))$

| | $i_4$ | | |
|---|---|---|---|
| $u$ | $\lambda w.\ a^*$ | | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. | • $\forall w' \in \mathbf{B}(bear_{w'}\, b_{w'} \wedge near_{w'}\, b_{w'})$ |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. | • $\forall w' \in \mathbf{C}(bear_{w'}\, c_{w'} \wedge come_{w'}\, c_{w'})$ |
| $u_3$ | $\lambda w \in \mathbf{G}.\ d_w$ | s.t. | • $\forall w' \in \mathbf{G}(gun_{w'}\, d_{w'} \wedge have_{w'}(a^*, d_{w'}))$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. | • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\, x \wedge near_{w'}(x, a^*))\}$<br>• $\forall w \in p^*((\mathbf{min}_{<4,\, w}\, p^*) \cap \mathbf{B} = \varnothing$ |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. | • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\, y \wedge come_{w'}\, y)\}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\, w}\, \mathbf{B}) \cap \mathbf{C} \neq \varnothing$<br>• $\forall w \in p^*((\mathbf{min}_{<4,\, w}\, \mathbf{B}) \cap \mathbf{C} = \varnothing$ |
| $\omega_3$ | $\lambda w \in p^*.\ \mathbf{H}$ | s.t. | • $\mathbf{H} = \{w' \in \mathbf{C} \mid hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\, w}\, \mathbf{C}) \cap \mathbf{H} = \varnothing$ |
| $\omega_4$ | $\lambda w \in p^*.\ \mathbf{G}$ | s.t. | • $\mathbf{G} = \{w' \mid \exists z(gun_{w'}\, z \wedge have_{w'}(a^*, z))\}$ |

($6^5$) do$_{\omega,\ \omega 4}$

$[\mathbf{real}(\omega, \omega_4)]i_4 i_4$

iff $\omega i_4 \subseteq \omega_4 i_4$

iff $p^* \subseteq \mathbf{G}$

($6^6$) Pragmatic strengthening

$[\mathbf{real}(\omega_2, \omega_4)]$

iff $\omega_2 i_4 \subseteq \omega_4 i_4$

iff $\mathbf{C} \subseteq \mathbf{G}$

| | $i_4$ | | | |
|---|---|---|---|---|
| $u$ | $\lambda w.\ a^*$ | | | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. | • $\forall w' \in \mathbf{B}(bear_{w'}\, b_{w'} \wedge near_{w'}\, b_{w'})$ | |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. | • $\forall w' \in \mathbf{C}(bear_{w'}\, c_{w'} \wedge come_{w'}\, c_{w'})$ | |
| $u_3$ | $\lambda w \in \mathbf{G}.\ d_w$ | s.t. | • $\forall w' \in \mathbf{G}(gun_{w'}\, d_{w'} \wedge have_{w'}(a^*, d_{w'}))$ | |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | s.t. | • $p^* \subseteq \mathbf{G}$ | ($6^5$) |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. | • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\, x \wedge near_{w'}(x, a^*))\}$<br>• $\forall w \in p^*((\mathbf{min}_{<4,\, w}\, p^*) \cap \mathbf{B} = \varnothing$ | |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. | • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\, y \wedge come_{w'}\, y)\} \subseteq \mathbf{G}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\, w}\, \mathbf{B}) \cap \mathbf{C} \neq \varnothing$<br>• $\forall w \in p^*((\mathbf{min}_{<4,\, w}\, \mathbf{B}) \cap \mathbf{C} = \varnothing$ | ($6^6$) |
| $\omega_3$ | $\lambda w \in p^*.\ \mathbf{H}$ | s.t. | • $\mathbf{H} = \{w' \in \mathbf{C} \mid hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$<br>• $\forall w \in p^*((\mathbf{min}_{<1,\, w}\, \mathbf{C}) \cap \mathbf{H} = \varnothing$ | |
| $\omega_4$ | $\lambda w \in p^*.\ \mathbf{G}$ | s.t. | • $\mathbf{G} = \{w' \mid \exists z(gun_{w'}\, z \wedge have_{w'}(a^*, z))\}$ | |

(6[7]) If$_{\omega,\ \omega2}{}^{\omega5}$ the$_{u2}$ bear attacked you
$(\mathbf{max}_{\omega}{}^{\omega5}([\omega_5 \subseteq \omega_2]; [bear_{\omega5}\{u_2\}, attack_{\omega5}\{u_2, \mathbf{adr}\ u\}])); [\mathbf{remote}_{<1}(\omega_2, \omega_5)])i_4 i_5$
iff $i_4[\omega: \omega_5]i_5 \wedge \text{Dom}\ \omega_5 i_5 = p^* \wedge \forall w \in p^*(\omega_5 i_5 w = \{w' \in \mathbf{C} | \ attack_{w'}(c_{w'}, \mathbf{adr}\ a^*)\})$
$\wedge\ \forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \omega_5 i_4 w = \varnothing)$

| | $i_5$ | | |
|---|---|---|---|
| $u$ | $\lambda w.\ a^*$ | | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. | • $\forall w' \in \mathbf{B}(bear_{w'}\ b_{w'} \wedge near_{w'}\ b_{w'})$ |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. | • $\forall w' \in \mathbf{C}(bear_{w'}\ c_{w'} \wedge come_{w'}\ c_{w'})$ |
| $u_3$ | $\lambda w \in \mathbf{G}.\ d_w$ | s.t. | • $\forall w' \in \mathbf{G}(gun_{w'}\ d_{w'} \wedge have_{w'}(a^*, d_{w'}))$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | s.t. | • $p^* \subseteq \mathbf{G}$ |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. | • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\ x \wedge near_{w'}(x, a^*))\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ p^*) \cap \mathbf{B} = \varnothing$ |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. | • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\ y \wedge come_{w'}\ y)\} \subseteq \mathbf{G}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{B}) \cap \mathbf{C} \neq \varnothing$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ \mathbf{B}) \cap \mathbf{C} = \varnothing$ |
| $\omega_3$ | $\lambda w \in p^*.\ \mathbf{H}$ | s.t. | • $\mathbf{H} = \{w' \in \mathbf{C} \mid hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \mathbf{H} = \varnothing$ |
| $\omega_4$ | $\lambda w \in p^*.\ \mathbf{G}$ | s.t. | • $\mathbf{G} = \{w' \mid \exists z(gun_{w'}\ z \wedge have_{w'}(a^*, z))\}$ |
| $\omega_5$ | $\lambda w \in p^*.\ \mathbf{A}$ | s.t. | • $\mathbf{A} = \{w' \in \mathbf{C} \mid attack_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \mathbf{A} = \varnothing$ |

(6[8]) I would$_{\omega,\ \omega5}{}^{\omega6}$ use it$_{u3}$ to shoot him$_{u2}$ *uts* = use to shoot
$([\mathbf{remote}_{<1}(\omega, \omega_5)]; \mathbf{max}_{\omega}{}^{\omega6}([\omega_6 \subseteq \omega_5]; [uts_{\omega6}\{u, u_3, u_4\}]); [\mathbf{necc}_{<1}(\omega_5, \omega_6)])i_5 i_6$
iff $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ p^*) \cap \mathbf{A} = \varnothing \wedge i_5[\omega: \omega_6]i_6 \wedge \text{Dom}\ \omega_6 i_6 = p^*$
$\wedge\ \forall w \in p^*(\omega_6 i_6 w = \{w' \in \mathbf{A} | \ uts_{w'}(a^*, d_{w'}, c_{w'})\}) \wedge \forall w \in p^*(\mathbf{min}_{<1,\ w}\ \mathbf{A}) \subseteq \omega_6 i_6 w)$

| | $i_6$ | | |
|---|---|---|---|
| $u$ | $\lambda w.\ a^*$ | | |
| $u_1$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. | • $\forall w' \in \mathbf{B}(bear_{w'}\ b_{w'} \wedge near_{w'}\ b_{w'})$ |
| $u_2$ | $\lambda w \in \mathbf{C}.\ c_w$ | s.t. | • $\forall w' \in \mathbf{C}(bear_{w'}\ c_{w'} \wedge come_{w'}\ c_{w'})$ |
| $u_3$ | $\lambda w \in \mathbf{G}.\ d_w$ | s.t. | • $\forall w' \in \mathbf{G}(gun_{w'}\ d_{w'} \wedge have_{w'}(a^*, d_{w'}))$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | s.t. | • $p^* \subseteq \mathbf{G}$ |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. | • $\mathbf{B} = \{w' \mid \exists x(bear_{w'}\ x \wedge near_{w'}(x, a^*))\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ p^*) \cap \mathbf{B} = \varnothing$ |
| $\omega_2$ | $\lambda w \in p^*.\ \mathbf{C}$ | s.t. | • $\mathbf{C} = \{w' \in \mathbf{B} \mid \exists y(bear_{w'}\ y \wedge come_{w'}\ y)\} \subseteq \mathbf{G}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{B}) \cap \mathbf{C} \neq \varnothing$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<4,\ w}\ \mathbf{B}) \cap \mathbf{C} = \varnothing$ |
| $\omega_3$ | $\lambda w \in p^*.\ \mathbf{H}$ | s.t. | • $\mathbf{H} = \{w' \in \mathbf{C} \mid hurt_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \mathbf{H} = \varnothing$ |
| $\omega_4$ | $\lambda w \in p^*.\ \mathbf{G}$ | s.t. | • $\mathbf{G} = \{w' \mid \exists z(gun_{w'}\ z \wedge have_{w'}(a^*, z))\}$ |
| $\omega_5$ | $\lambda w \in p^*.\ \mathbf{A}$ | s.t. | • $\mathbf{A} = \{w' \in \mathbf{C} \mid attack_{w'}(c_{w'}, \mathbf{adr}\ a^*)\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{C}) \cap \mathbf{A} = \varnothing$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ p^*) \cap \mathbf{A} = \varnothing$ |
| $\omega_6$ | $\lambda w \in p^*.\ \mathbf{S}$ | s.t. | • $\mathbf{S} = \{w' \in \mathbf{A} \mid uts_{w'}(a^*, d_{w'}, c_{w'})\}$ |
| | | | • $\forall w \in p^*((\mathbf{min}_{<1,\ w}\ \mathbf{A}) \subseteq \mathbf{S}$ |

## APPENDIX 1: THREE VERSIONS OF KEY DEFINITIONS

Version 1: Stone 1997 (our notation)

- *Discourse referents* as concepts restricted to modal domain $\omega$

D1. $i[\omega\text{: } u]j \;:=\; \forall v(\mathbf{mk}(v) \wedge v \neq u \rightarrow vi = vj)$ *nominal dref*
$\quad\wedge\; \forall w(w \in \text{Ran } \omega j \rightarrow ujw\ \ \mathbf{in}\ w)$ p. 29

$^1$D2. $i[\omega\text{: } \omega']j \;:=\; \forall v(\mathbf{mk}(v) \wedge v \neq \omega' \rightarrow vi = vj)$ *modal dref*
$\quad\wedge\; \text{Dom } \omega'j \subseteq \text{Ran } \omega j$ p. 29

- *Lexical relations* (relativized to modal domain $\omega$)

D3. $R_\omega\{\delta_1, \ldots, \delta_n\} \;:=\; \lambda i.\, \forall w(w \in \text{Ran } \omega i \rightarrow R_w(\delta_1 iw, \ldots, \delta_n iw))$ p. 28

- *Modal relations* (parametrized by modal base $M$ and weak optimality ranking $\leq$)

$^1$D4. "$\omega'$ elaborates on $\omega$" (cf. Brasoveanu's 2007 "$\sqsubseteq$")
$\omega \rightsquigarrow \omega' \;:=\; \lambda i.\forall w \in \text{Dom } \omega i(\omega' iw \subseteq \omega iw)$ cf. p. 23, 30

$^1$D5. $\mathbf{necc}_{M,\leq}(\omega, \omega') \;:=\; \lambda i.\forall w \in \text{Dom } \omega i((\mathbf{min}_\leq(\omega iw \cap M)) \subseteq \omega' iw)$ p. 31
(For all $w \in \text{Dom } \omega i$, every $M$-accessible $\leq_w$-ideal world in $\omega iw$ is also in $\omega' iw$.)

$^1$D6. $\mathbf{poss}_{M,\leq}(\omega, \omega') \;:=\; \lambda i.\forall w \in \text{Dom } \omega i((\mathbf{min}_\leq(\omega iw \cap M)) \cap \omega' iw \neq \emptyset)$ p. 31
(For every $w$, some $M$-accessible $\leq_w$-ideal world in $\omega iw$ is also in $\omega' iw$.)

- *Modal presuppositions*

> "Modals are classified not only by the relationship they impose between $\omega_r$ [MB: reference modality] and $\omega_e$ [MB: event modality], but also by the relationship they presuppose between $\omega_r$ and $\omega_s$ [MB: speech modality]. These are given by further conditions: **real**($\omega_s$, $\omega_r$, *), **vivid**($\omega_s$, $\omega_r$, *) and **remote**($\omega_s$, $\omega_r$, *), describe the presuppositions of factual, vivid and remote modals, respectively." p. 25

$^1$D8. $\mathbf{real}_{M,\leq}(\omega, \omega') \;:=\; \mathbf{necc}_{M,\leq}(\omega, \omega')$ cf. p. 31
$\mathbf{vivid}_{M,\leq}(\omega, \omega') \;:=\; \mathbf{poss}_{M,\leq}(\omega, \omega')$ cf. p. 31
$\mathbf{remote}_{M,\leq}(\omega, \omega') \;:=\; \lambda i.\neg\mathbf{poss}_{M,\leq}(\omega, \omega')i$ (idea 1) cf. p. 31
$\quad := \lambda i.\ i = i$ (idea 2)

- *Conditional update* ($\angle$ for weak similarity ranking) and "*default inference*" (*implicature*?)

$^1$D9. $\mathbf{if}_{\angle}(\omega, \omega', D) \;:=$ p. 30
$\lambda \overline{ij}.\exists k(i[\omega\text{: } \omega']k \wedge Dkj)$
$\quad \wedge\; \forall h(\exists k(i[\omega\text{: } \omega']k \wedge Dkh)$
$\quad\quad \rightarrow \forall w(w \in \text{Ran } \omega i$
$\quad\quad\quad \rightarrow \forall w'w''(w' \in \omega' jw \wedge w'' \in \omega' hw \wedge w'' \angle_w w' \rightarrow w' \angle_w w'')$
$\quad\quad\quad\quad \wedge\; \forall w''(w'' \in \omega' hw \rightarrow \exists w'(w' \in \omega' jw \wedge w' \angle_w w''))))$

Version 2: Stone and Hardt 1999 (our notation)

• *Discourse referents* as concepts restricted to modal domain ω

D1. $i[\omega\!: u]j$ := $\forall v(\mathbf{mk}(v) \wedge v \neq u \rightarrow vi = vj)$ $\wedge\ \forall w(w \in \text{Ran}\ \omega j \rightarrow ujw\ \ \mathbf{in}\ w)$ — *nominal dref* in (6): 1st clause

[1]D2. $i[\omega\!: \omega']j$ := $\forall v(\mathbf{mk}(v) \wedge v \neq \omega' \rightarrow vi = vj)$ $\wedge\ \text{Dom}\ \omega' j \subseteq \text{Ran}\ \omega j$ — *modal dref* in (6): 2nd clause

• *Lexical relations* (relativized to modal domain ω)

D3. $R_\omega\{\delta_1, \ldots, \delta_n\}$ := $\lambda i.\ \forall w(w \in \text{Ran}\ \omega i \rightarrow R_w(\delta_1 iw, \ldots, \delta_n iw))$ — in (7)

• *Modal relations* (no parameters)

[2]D6. $\mathbf{poss}(\omega, \omega')$ := $\lambda i.\exists w(w \in \text{Ran}\ \omega i \wedge w \in \omega' iw)$ — in (10)
(Some world $w$ in Ran $\omega i$ is in $\omega' iw$.)

[2]D7. $\mathbf{not}(\omega, \omega')$ := $\lambda i.\neg\exists w(w \in \text{Ran}\ \omega i \wedge w \in \omega' iw)$ — in (10)
(No world $w$ in Ran $\omega i$ is in $\omega' iw$.)

• *Conditional update* ($\underline{\angle}$ for weak similarity ranking)

[1]D9. $\mathbf{if}_{\underline{\angle}}(\omega, \omega', D)$ := — (9)

$$\begin{aligned}&\lambda ij.\exists k(i[\omega\!: \omega']k \wedge Dkj)\\ &\quad \wedge \forall h(\exists k(i[\omega\!: \omega']k \wedge Dkh)\\ &\qquad \rightarrow \forall w(w \in \text{Ran}\ \omega i\\ &\qquad\quad \rightarrow \forall w'w''(w' \in \omega' jw \wedge w'' \in \omega' hw \wedge w'' \underline{\angle}_w w' \rightarrow w' \underline{\angle}_w w'')\\ &\qquad\qquad \wedge \forall w''(w'' \in \omega' hw \rightarrow \exists w'(w' \in \omega' jw \wedge w' \underline{\angle}_w w''))))\end{aligned}$$

Version 3: Stone 1999 (our notation)

• *Discourse referents* as concepts restricted to modal domain ω

D1. $i[\omega\text{: } u]j$ := $\forall v(\mathbf{mk}(v) \wedge v \neq u \rightarrow vi = vj)$ $\wedge\ \forall w(w \in \text{Ran } \omega j \rightarrow ujw\ \ \mathbf{in}\ w)$ *nominal dref* (28)

[3]D2. $i[\omega\text{: } \omega']j$ := $\forall v(\mathbf{mk}(v) \wedge v \neq \omega' \rightarrow vi = vj)$ $\wedge\ \text{Dom } \omega'j \subseteq \text{Dom } \omega i$ *modal dref* (27)

• *Lexical relations* (relativized to modal domain ω)

D3. $R_\omega\{\delta_1, \ldots, \delta_n\}$ := $\lambda i.\ \forall w(w \in \text{Ran } \omega i \rightarrow R_w(\delta_1 iw, \ldots, \delta_n iw))$ (26): 1st line

• *Modal relations* (parametrized by modal base *relation M* and weak optimality ranking ≤)

[3]D5. $\mathbf{necc}_{M, \leq}(\omega, \omega')$ :=

$$\begin{aligned}&\lambda i.\forall w(w \in \text{Dom } \omega i\\ &\quad \rightarrow \forall w'(w' \in \omega iw \wedge w' \in Mw\\ &\qquad \wedge\ \exists w''(w'' \in \omega iw \vee w'' \in \omega' iw) \wedge w'' \in Mw \wedge w'' \leq_w w'\\ &\qquad\quad \wedge\ \forall w'''(w''' \in \omega iw \wedge w''' \in Mw \wedge w''' \leq_w w' \rightarrow w''' \in \omega' iw))))\end{aligned}$$

(For all $w \in \text{Dom } \omega i$, for every $w' \in (Mw \cap \omega iw)$ there is some approximation $w''$ to the $\leq_w$-ideal s.t. every $(Mw \cap \omega iw)$-world $w'''$ as good as or better than $w''$ is in $\omega' iw$.
This is a "close enough" theory, which as we've seen gets into trouble with comparatives.)

[3]D6. $\mathbf{poss}_{M, \leq}(\omega, \omega')$ :=

$$\begin{aligned}&\lambda i.\forall w(w \in \text{Dom } \omega i\\ &\quad \rightarrow \exists w'(w' \in \omega iw \wedge w' \in Mw\\ &\qquad \wedge\ \forall w''(w'' \in \omega iw \vee w'' \in \omega' iw) \wedge w'' \in Mw \wedge w'' \leq_w w'\\ &\qquad\quad \rightarrow \exists w'''(w''' \in \omega iw \wedge w''' \in Mw \wedge w''' \leq_w w' \rightarrow w''' \in \omega' iw))))\end{aligned}$$

(Dual of "close enough" necessity. Hard to paraphrase and, in my view, not worth the trouble.)

• *Conditional update*

[3]D9. $\mathbf{if}_{\underline{\angle}}(\omega, \omega', D)$ :=

$$\begin{aligned}&\lambda ij.\exists k(i[\omega\text{: }\omega']k \wedge Dkj)\\ &\quad \wedge\ \forall h(\exists k(i[\omega\text{: }\omega']k \wedge Dkh)\\ &\qquad \rightarrow \forall ww'(w' \in \omega iw\\ &\qquad\quad \rightarrow \forall w_j w_h(w_j \in \omega' jw \wedge w_h \in \omega' hw \wedge w_h \underline{\angle}_{w'} w_j \rightarrow w_j \underline{\angle}_{w'} w_h)\\ &\qquad\quad\ \ \wedge\ \forall w_h(w_h \in \omega' hw \rightarrow \exists w_j(w_j \in \omega' jw \wedge w_j \underline{\angle}_{w'} w_h))))\end{aligned}$$

## APPENDIX 2: DYNAMIC $Ty_2$ WITH ENCAPSULATED QUANTIFICATION ($EDy_2$)

D01. Basic terms

| Variables | Constants | Type | Name of objects |
|---|---|---|---|
| $i, j, k, h$ | | $s$ | atomic (info) states (*aka* indices) |
| $x, y, z$ | $a^*$, $john$, … | $e$ | entities |
| $w$ | $w^*$, … | $\mathbf{w}$ | worlds |
| | $b, c$, … | $\mathbf{w}e$ | entity-concepts |
| $p$ | $p^*$, $\mathbf{A}$, $\mathbf{B}$, … | $\mathbf{w}t$ | propositions |
| | $M$, … | $\mathbf{ww}t$ | accessibility relations |
| | $\leq_n$ | $\mathbf{www}t$ | ordering relations |
| | $u_n$ | $s(\mathbf{w}e)$ | $e$-stores ($u_0$ for the speaker) |
| | $\omega_n$ | $s(\mathbf{ww}t)$ | $\mathbf{ww}t$-stores ($\omega_0$ for the reality) |

D02. Auxiliary definitions

- $i[\omega: u]j$ for (for every dref $\delta$ other than $u$, $\delta i = \delta j$) & $\text{Dom } uj = \text{Ran } \omega i$
  $i[\omega: \omega']j$ for (for every dref $\delta$ other than $\omega$, $\delta i = \delta j$) & $\text{Dom } \omega' j = \text{Dom } \omega i$
- $\mathbf{min}_{<, w}(p)$ := $\{w' \mid w' \in p \wedge \neg\exists w''(w'' \in p \wedge w'' <_w w')\}$
- $\lambda w \in p.\, f_w$ := $\{\langle w, f_w\rangle: w \in p\} \cup \{\langle w, \varnothing\rangle: w \notin p\}$
- $M \subseteq M'$ := $\forall ww'(w' \in Mw \rightarrow w' \in Mw)$
- $\text{Dom } M$ := $\{w \mid Mw \neq \varnothing\}$
  $\text{Ran } M$ := $\{w' \mid \exists w(w' \in Mw)\}$

D03. Defined dref's

- $John$ := $\lambda i.\, (\lambda w.\, john)$
- $u_0$ := $\lambda i.\, (\lambda w.\, a^*)$
- $\omega_0$ := $\lambda i.\, (\lambda w \in p^*.\, p^*)$

D1. Conditions (type $st$)

- $R_\omega\{\delta_1, \ldots, \delta_n\}$ := $\lambda i.\, \forall w \in \text{Ran } \omega i(R_w(\delta_1 iw, \ldots, \delta_n iw))$
- $\delta = \delta'$ := $\lambda i.\, \delta i = \delta' i$
- $\omega \subseteq \omega'$ := $\lambda i.\, \omega i \subseteq \omega' i$ — cf. S97: $\leadsto$
- $\mathbf{necc}_{<_n}(\omega, \omega')$ := $\lambda i. \forall w \in \text{Dom } \omega i((\mathbf{min}_{<_n, w}\omega iw) \subseteq \omega' iw)$ — S97:31
  $\mathbf{poss}_{<_n}(\omega, \omega')$ := $\lambda i. \forall w \in \text{Dom } \omega i((\mathbf{min}_{<_n, w}\omega iw) \cap \omega' iw \neq \varnothing)$ — S97:31
  $\mathbf{not}_{<_n}(\omega, \omega')$ := $\lambda i. \forall w \in \text{Dom } \omega i((\mathbf{min}_{<_n, w}\omega iw) \cap \omega' iw = \varnothing)$ — S97:31
- $\mathbf{real}(\omega, \omega')$ := $(\omega \subseteq \omega')$ — cf. S97:25
  $\mathbf{vivid}_{<_n}(\omega, \omega')$ := $\mathbf{poss}_{<_n}(\omega, \omega')$ — cf. S97:25
  $\mathbf{remote}_{<_n}(\omega, \omega')$ := $\mathbf{not}_{<_n}(\omega, \omega')$ — cf. S97:25
- $(C, C')$ := $\lambda i.\, Ci \wedge C'i$

D2. Updates (type $s(st)$)

- $[C]$ := $\lambda ij.\, i = j \wedge Cj$
  $[\omega: \delta \mid C]$ := $\lambda ij.\, i[\omega: \delta]j \wedge Cj)$
- $(D_1; D_2)$ := $\lambda ij.\, \exists k(D_1 ij \wedge D_2 kj)$
- $\mathbf{max}_{\omega}^{\omega'}(D)$ := $\lambda ij.(\exists k(i[\omega: \omega']k \wedge Dkj) \wedge \forall h(\exists k([\omega: \omega']ik \wedge Dkh) \rightarrow \omega' h \subseteq \omega' j)$

D3. Truth (relative to default state)

- $\models D$ := $\exists ij\, Dij$

## APPENDIX 3: SAMPLE DERIVATIONS

(2) My neighbours *would* kill me.
[uttered while looking at high-end stereo, $c$, in electronics store]

($2^0$) Start-up
$i_0$
$u$ $\lambda w.\ a^*$
$u_1$ $\lambda w.\ c$
$\omega$ $\lambda w \in p^*.\ p^*$

($2^1$) Accommodate presup. of $\text{would}_{\omega,\ \omega 1}{}^{\omega 2}$ by pragmatic inference ($pfb$ = play for full blast)
$\mathbf{max}_{\omega}{}^{\omega 1}([buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}])i_0 i_1$

(1) iff (9)

1. $\mathbf{max}_{\omega}{}^{\omega 1}([buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}])i_0 i_1$

2. $\exists k(i_0[\omega\text{: }\omega_1]k \wedge [buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}]ki_1)$ D2.**max**
$\wedge\ \forall h(\exists k(i_0[\omega\text{: }\omega_1]k \wedge [buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}]kh) \rightarrow \omega_1 h \subseteq \omega_1 i_1)$

3. $\exists k(i_0[\omega\text{: }\omega_1]k \wedge k = i_1 \wedge \forall w \in \text{Ran}\ \omega_1 i_1(buy_w(ui_1w, u_1i_1w) \wedge pfb_w(ui_1w, u_1i_1w))$ D2.$C$, D1
$\wedge\ \forall h(\exists k(i_0[\omega\text{: }\omega_1]k \wedge [buy_{\omega 1}\{u, u_1\}, pfb_{\omega 1}\{u, u_1\}]kh) \rightarrow \omega_1 h \subseteq \omega_1 i_1)$

4. $i_0[\omega\text{: }\omega_1]i_1 \wedge \forall w \in \text{Ran}\ \omega_1 i_1(buy_w(ui_1w, u_1i_1w) \wedge pfb_w(ui_1w, u_1i_1w))$ simplify
$\wedge\ \forall h([\omega\text{: }\omega_1]i_0i_1 \wedge \forall w \in \text{Ran}\ \omega_1 h(buy_w(uhw, u_1hw) \wedge pfb_w(uhw, u_1hw))$ D2, 1, smp
$\rightarrow \omega_1 h \subseteq \omega_1 i_1)$

5. $i_0[\omega\text{: }\omega_1]i_1 \wedge \text{Dom}\ \omega_1 i_1 = \text{Dom}\ \omega i_0 \wedge \forall w \in \text{Ran}\ \omega_1 i_1(buy_w(a^*, c) \wedge pfb_w(a^*, c))$ D02.[ω: ω′]
$\wedge\ \forall h(i_0[\omega\text{: }\omega_1]h \wedge \text{Dom}\ \omega_1 h = \text{Dom}\ \omega i_0 \wedge \forall w \in \text{Ran}\ \omega_1 h(buy_w(a^*, c) \wedge pfb_w(a^*, c))$
$\rightarrow \omega_1 h \subseteq \omega_1 i_1)$

6. $i_0[\omega\text{: }\omega_1]i_1 \wedge \text{Dom}\ \omega_1 i_1 = p^* \wedge \forall w \in \text{Ran}\ \omega_1 i_1(buy_w(a^*, c) \wedge pfb_w(a^*, c))$ D02, df. $i_0$
$\wedge\ \forall h(i_0[\omega\text{: }\omega_1]h \wedge \text{Dom}\ \omega_1 h = p^* \wedge \forall w \in \text{Ran}\ \omega_1 h(buy_w(a^*, c) \wedge pfb_w(a^*, c))$
$\rightarrow \forall ww'(w' \in \omega_1 hw \rightarrow w' \in \omega_1 i_1 w)$ D03.⊆

7. $i_0[\omega\text{: }\omega_1]i_1 \wedge \text{Dom}\ \omega_1 i_1 = p^* \wedge \forall w \in \text{Ran}\ \omega_1 i_1(buy_w(a^*, c) \wedge pfb_w(a^*, c))$
$\wedge\ \forall h(i_0[\omega\text{: }\omega_1]h \wedge \text{Dom}\ \omega_1 h = p^* \wedge \forall w \in \text{Ran}\ \omega_1 h(buy_w(a^*, c) \wedge pfb_w(a^*, c))$
$\rightarrow \forall w \in p^*(\omega_1 hw \subseteq \omega_1 i_1 w)$ simplify

8. $i_0[\omega\text{: }\omega_1]i_1 \wedge \text{Dom}\ \omega_1 i_1 = p^*$
$\wedge\ \forall w \in p^*(\omega_1 i_1 w = \{w' \mid buy_w(a^*, c) \wedge pfb_w(a^*, c)\})$ simplify

$i_1$
$u$ $\lambda w.\ a^*$
$u_1$ $\lambda w.\ c$
$\omega$ $\lambda w \in p^*.\ p^*$
$\omega_1$ $\lambda w \in p^*.\ \mathbf{B}$ s.t. • $\mathbf{B} := \{w' \mid buy_w(a^*, c) \wedge pfb_w(a^*, c)\}$

($2^2$) My neighbours
$[\omega_1: u_2 | \textit{neighbours.of}_{\omega 1}\{u_2, u\}]i_1 i_2$

(1) iff (4)

| | | |
|---|---|---|
| 1. | $[\omega_1: u_2 | \textit{neighbours.of}_{\omega 1}\{u_2, u\}]i_1 i_2$ | |
| 2. | $i_1[\omega_1: u_2]i_2 \wedge (\textit{neighbours.of}_{\omega 1}\{u_2, u\})i_2$ | D2 |
| 3. | $i_1[\omega_1: u_2]i_2 \wedge \text{Dom } u_2 i_2 = \text{Ran } \omega_1 i_1$ <br> $\wedge\ \forall w \in \text{Ran } \omega_1 i_2(\textit{neighbours.of}_w(u_2 i_2 w, u i_2 w))$ | D02.[ω: *u*] <br> D1.*R* |
| 4. | $i_1[\omega_1: u_2]i_2 \wedge \text{Dom } u_2 i_2 = \mathbf{B} \wedge \forall w \in \mathbf{B}(\textit{neighbours.of}_w(u_2 i_2 w, a^*))$ | D02, df. $i_1$ |

| | $i_2$ | |
|---|---|---|
| $u$ | $\lambda w.\ a^*$ | |
| $u_1$ | $\lambda w.\ c$ | |
| $u_2$ | $\lambda w \in \mathbf{B}.\ b_w$ | s.t. • $\forall w \in \mathbf{B}(\textit{neighbours.of}_w(b_w, a^*)$ |
| $\omega$ | $\lambda w \in p^*.\ p^*$ | |
| $\omega_1$ | $\lambda w \in p^*.\ \mathbf{B}$ | s.t. • $\mathbf{B} := \{w' | \textit{buy}_{w'}(a^*, c) \wedge \textit{pfb}_{w'}(a^*, c)\}$ |

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# Strong readings in $UTy_3$

0. OUTLINE OF ANALYSIS

(**!**) Start-up update — a la Bittner, 2007

$[u|\ u = Al]; [r|\ speaking_r\langle u\rangle]; [p]; [p \sim r];$ — $u := u_0, r := r_0, p := p_0$

(**∀**) Strong readings as maximizing anaphora

(1) 0. presupposition accommodation

$[u_1|\ u_1 = country.of_r\langle u\rangle]; [p_1|\ law.of_r\langle p_1, u_1\rangle];$

i. [According to$^{S1, p2}$ the law]

$[S_1]; [S_1 \sim_r p_1]; [p_2|\ p_2 = \cup S_1];$

Output if the GB-law in $w_1$ is $\pi_0$, the laws in $w_2$ are $\pi_0$, $\pi_1$, and $\pi_2$, and in $w_3$, $\pi_0$ and $\pi_1$, where $\pi_0 := \{w_0\}$, $\pi_1 := \{w_0, w_1\}$ and $\pi_2 := \{w_0, w_1, w_2\}$.

| $J_1$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | |
|---|---|---|---|---|---|---|---|---|
| $i_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0\}$ | $\pi_0$ | $J_{1, r = w1}$ |
| $i_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $J_{1, r = w2}$ |
| $i_3$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | |
| $i_4$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | |
| $i_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $J_{1, r = w3}$ |
| $i_6$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | |

ii. if$_{p2}{}^{r2}$ [**a**$^{u2, A2}$ man commits a murder]$^{p3}$

$[r_2|\ r_2 \in p_2]; [u_2|\ man_{r2}\langle u_2\rangle, mrd_{r2}\langle u_2\rangle];$ $\mathbf{[A_2]; [A_2 \sim_{r2} u_2];}$ $[p_3]; [p_3 \sim_r r_2]$

Output if no man commits murder in $w_0$, men $a$ and $b$ commit murders in $w_1$, men $a$ and $c$ in $w_2$ (and men $b$ and $c$ in $w_3$):

| $J_2$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i_{21}$ | $al$ | $w_2$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $J_{2, r = w2}$ |
| $i_{22}$ | $al$ | $w_2$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | |
| $i_{31}$ | $al$ | $w_2$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | |
| $i_{32}$ | $al$ | $w_2$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | |
| $i_{41}$ | $al$ | $w_2$ | … | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | |
| $i_{42}$ | $al$ | $w_2$ | … | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | |
| $i_{23}$ | $al$ | $w_2$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{24}$ | $al$ | $w_2$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{33}$ | $al$ | $w_2$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{34}$ | $al$ | $w_2$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{43}$ | $al$ | $w_2$ | … | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{44}$ | $al$ | $w_2$ | … | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | |
| $i_{51}$ | $al$ | $w_3$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $J_{2, r = w3}$ |
| $i_{52}$ | $al$ | $w_3$ | … | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | |
| $i_{61}$ | $al$ | $w_3$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | |
| $i_{62}$ | $al$ | $w_3$ | … | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | |

iii. [**he**$_{u2}$ should$_{S1,\,p2,\,p3}{}^{p4}$ go to prison].
[**min**$_{S1}\langle p_2\rangle \varnothing p_3\rangle$]; [$p_4$| $p_4$ = **min**$_{S1}\langle p_3\rangle$]; [$gtp_{r2}\langle u_2\rangle$]; **[$A_2$ ~$_{r2}$ $u_2$];** [$p_4$ ~$_r$ $r_2$]; [$p$]; [$p \sim r$]

Output if $a$ and $b$ go to prison (gtp) in $w_1$; only $a$ gtp in $w_2$; (and nobody gtp in $w_3$).

| $J_3$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i_{21}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{22}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{31}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{32}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{41}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{42}$ | $al$ | $w_2$ | $\{w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{51}$ | $al$ | $w_3$ | $\{w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_{52}$ | $al$ | $w_3$ | $\{w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_{61}$ | $al$ | $w_3$ | $\{w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_{62}$ | $al$ | $w_3$ | $\{w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |

## 1. START-UP UPDATE

*Default info state* (dummy)

| | $u$ | … | $r$ | … | $p$ | … | $S_1$ | … | $A_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $i_@$ | $@_e$ | | $@_{\mathbf{w}}$ | | {} | | {} | | {} |

($^{-2}$) **[$u$| $u = Al$]**
$= \lambda I\lambda j.\ \exists i \in I(i[u]j \wedge uj = al)$

| $J_{-2}$ | $u$ | … |
|---|---|---|
| $i_{-2}$ | $al$ | |

($^{-1}$) **[$r$| $speaking_r\langle u\rangle$]**
$= \lambda I\lambda j.\ \exists i \in I(i[r]j \wedge speaking_{rj}(uj))$

Output if Al is speaking in $w_0$, $w_1$, $w_2$ and $w_4$:

| $J_{-1}$ | $u$ | $r$ | … |
|---|---|---|---|
| $i_{-11}$ | $al$ | $w_1$ | |
| $i_{-12}$ | $al$ | $w_2$ | |
| $i_{-13}$ | $al$ | $w_3$ | |

($^0$) **[$p$]; [$p \sim r$]**
$= \lambda I\lambda j.\ \exists i \in I(i[p]j \wedge @_{\mathbf{w}} \notin pj \wedge pj = rI)$

| $J_0$ | $u$ | $r$ | $p$ | … |
|---|---|---|---|---|
| $i_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | |
| $i_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | |
| $i_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | |

## 2. STRONG READING AS MAXIMIZING ANAPHORA

**($1^0$)** **presupposition accommodation** (*the law* presupposes a salient region with laws)
$[u_1|\ live.in_r\langle u, u_1\rangle]; [p_1|\ law.of_r\langle p_1, u_1\rangle]$

Output $K_{11} := [u_1|\ live.in_r\langle u, u_1\rangle]J_0$
if the speaker (Al) lives in Great Britain (GB):

| $K_{11}$ | $u$ | $r$ | $p$ | $u_1$ |
|---|---|---|---|---|
| $k_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ |
| $k_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ |
| $k_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ |

Output $K_{12} := [p_1|\ law.of_r\langle p_1, u_1\rangle]K_{11}$
$= \lambda j.\ \exists i \in K_{11}(i[p_1]j \wedge law.of_{rj}(p_1j, u_1j))$ D2, D1.*R*, D02
if the laws of GB are the propositions $\pi_0$ in $w_1$, $\pi_0$, $\pi_1$, and $\pi_2$ in $w_2$, and $\pi_0$ and $\pi_1$ in $w_3$:

| $K_{12}$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | |
|---|---|---|---|---|---|---|
| $k_{10}$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $K_{12,\ r = w1}$ |
| $k_{20}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $K_{12,\ r = w2}$ |
| $k_{21}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | |
| $k_{22}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | |
| $k_{30}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $K_{12,\ r = w3}$ |
| $k_{31}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | |

**($1^1$)** **According to**$^{S1, p2}$
$[S_1]; [S_1 \sim_r p_1]; [p_2|\ p_2 = \cup S_1]$

Output $I_{11} := ([S_1]; [S_1 \sim_r p_1])K_{12}$
$= \lambda j.\ \exists i \in K_{12}(i[S_1]j \wedge S_1j = p_1K_{12,\ r = rj})$ D2, D3.$\sim_r$

| $I_{11}$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ |
|---|---|---|---|---|---|---|
| $i_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0\}$ |
| $i_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ |
| $i_3$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ |
| $i_4$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ |
| $i_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ |
| $i_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ |

Output $I_{12} := [p_2|\ p_2 = \cup S_1]I_{11}$
$= \lambda j.\ \exists i \in I_{11}(i[p_2]j \wedge p_2j = \cup(S_1j))$ D2, D05.$\cup S$
if $\pi_0 := \{w_0\}$, $\pi_1 := \{w_0, w_1\}$ and $\pi_2 := \{w_0, w_1, w_2\}$:

| $I_{12}$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ |
|---|---|---|---|---|---|---|---|
| $j_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0\}$ | $\pi_0$ |
| $j_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_3$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_4$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ |
| $j_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ |

**($1^2$)** **the** **law**

$[S_1 \sim_r p_1]$; $[law.of_r\langle p_1, u_1\rangle]$

Output $I_{13} := ([S_1 \sim_r p_1]; [law.of_r\langle p_1, u_1\rangle])I_{12}$
$= I_{12}$ D2, D3.$\sim_r$, D1.$R$

| $I_{13}$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ |
|---|---|---|---|---|---|---|---|
| $j_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0\}$ | $\pi_0$ |
| $j_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_3$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_4$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ |
| $j_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ |
| $j_5$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ |

**(2[1])** $\mathbf{if}_{p2}{}^{r2}$ **[**
$[r_2 | r_2 \in p_2]$

Output $H_1 := [r_2 | r_2 \in p_2]I_{13}$
$= \lambda j.\ \exists i \in I_{13}(i[r_2]j \wedge r_2 j \in p_2 j)$ D2, D1.$\in$

given $\pi_0 := \{w_0\}$, $\pi_1 := \{w_0, w_1\}$ and $\pi_2 := \{w_0, w_1, w_2\}$:

| $H_1$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ |
|---|---|---|---|---|---|---|---|---|
| $h_1$ | $al$ | $w_1$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0\}$ | $\pi_0$ | $w_0$ |
| $h_2$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ |
| $h_3$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ |
| $h_4$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ |
| $h_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ |
| $h_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ |
| $h_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ |
| $h_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ |
| $h_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ |
| $h_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ |
| $h_{11}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_0$ |
| $h_{12}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ |
| $h_{13}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_0$ |
| $h_{14}$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ |

**($2^2$) [a$^{u2}$ man**

$[u_2 | \, man_{r2}\langle u_2\rangle]$

Output $H_2 := [u_2 | \, man_{r2}\langle u_2\rangle]H_1$

$= \lambda j.\, \exists i \in H_1(i[u_2]j \wedge man_{r2j}\, u_2 j)$ D2, D1.*R*

if the men are *a* in $w_0$, *a* and *b* and $w_1$, and *a*, *b*, and *c* in $w_2$ (and $w_3$):

| $H_2$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $i_1$ | *al* | $w_1$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0\}$ | $\pi_0$ | $w_0$ | *a* |
| $i_2$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ | *a* |
| $i_3$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{3'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_4$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{4'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *b* |
| $i_{4''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_5$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ | *a* |
| $i_6$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{6'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_7$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{7'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *b* |
| $i_{7''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_8$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_0$ | *a* |
| $i_9$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{9'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_{10}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{10'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *b* |
| $i_{10''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_{11}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_0$ | *a* |
| $i_{12}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* |
| $i_{12'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* |
| $i_{13}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_0$ | *a* |
| $i_{14}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* |
| $i_{14'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* |

**(2[3]) commits a murder]$^{p3}$**
$[mrd_{r2}\langle u_2\rangle]$

Output $H_3 := [mrd_{r2}\langle u_2\rangle]H_2$
$= \lambda j.\ j \in H_2 \wedge mrd_{r2j}\ u_2 j$ D2, D1.*R*

if no men commits murder in $w_0$, *a* and *b* commit murders and $w_1$, and *a* and *c* in $w_2$ (and *b* and *c* in $w_3$)

| $H_3$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ |
|---|---|---|---|---|---|---|---|---|---|
| $i_3$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{3'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_4$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{4''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_6$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{6'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_7$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{7''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_9$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* |
| $i_{9'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* |
| $i_{10}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* |
| $i_{10''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* |
| $i_{12}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* |
| $i_{12'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* |
| $i_{14}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* |
| $i_{14'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* |

i.e., rearranging the rows to highlight the $r_2$-substates:

| $H_3$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| $i_{12}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* | $H_{3,\, r2 = w1}$ |
| $i_{12'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* | |
| $i_{14}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* | |
| $i_{14'}$ | *al* | $w_3$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* | |
| $i_3$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | |
| $i_{3'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | |
| $i_6$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | |
| $i_{6'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | |
| $i_9$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | |
| $i_{9'}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | |
| $i_4$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* | $H_{3,\, r2 = w2}$ |
| $i_{4''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* | |
| $i_7$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* | |
| $i_{7''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* | |
| $i_{10}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *a* | |
| $i_{10''}$ | *al* | $w_2$ | $\{w_1, w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | *c* | |

**($2^4$)** $]_{r,\,u3}{}^{A3}$ scope of $\mathbf{a}^{u3}$

$[A_2]; [A_2 \sim_{r2} u_2]$

Output $H_4 := ([A_2]; [A_2 \sim_{r2} u_2])H_3$

$= \lambda j.\ \exists i \in H_3(i[A_2]j \wedge A_2 j = u_2 H_{3,\, r2 = r2j})$ D2, D3.$\sim_r$

| $H_4$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $j_1$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $H_{4,\, w = w3}$ |
| $j_2$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | |
| $j_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | |
| ~~$j_4$~~ | ~~$al$~~ | ~~$w_3$~~ | ~~$\{w_1, w_2, w_3\}$~~ | ~~$gb$~~ | ~~$\pi_1$~~ | ~~$\{\pi_0, \pi_1\}$~~ | ~~$\pi_1$~~ | ~~$w_1$~~ | ~~$b$~~ | ~~$\{a, b\}$~~ | |
| $j_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $H_{4,\, w = w2}$ |
| $j_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | |
| $j_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | |
| $j_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | |
| $j_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | |
| $j_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | |
| $j_{11}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | |
| $j_{12}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | |
| $j_{13}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | |
| $j_{14}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | |
| $j_{15}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | |
| $j_{16}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | |

**($2^5$)** $]_{r,\,r2}{}^{p3}$ scope of $\mathbf{if}_{p1}{}^{p2}$

$[p_3]; [p_3 \sim_r r_2]$

Output $H_5 := ([p_3]; [p_3 \sim_r r_2])H_4$

$= \lambda j.\ \exists i \in H_4(i[p_3]j \wedge p_3 j = r_2 H_{4,\, r = rj})$ D2, D3.$\sim_r$

| $H_5$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ |
| $k_2$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ |
| $k_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ |
| $k_4$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ |
| $k_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ |
| $k_{11}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ |
| $k_{12}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ |
| $k_{13}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ |
| $k_{14}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ |
| $k_{15}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ |
| $k_{16}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ |

**($3^1$)** $\mathbf{he}_{u2,\, A2}$ **[** — maximizing **he**
$[male_{r2}\langle u_2\rangle]$; $[A_2 \sim_{r2} u_2]$;

Output $J_1 :=$ $([male_{r2}\langle u_2\rangle]; [A_2 \sim_{r2} u_2])H_5$
$= \lambda j.\, j \in H_5(male_{r2j}\, u_2 j \wedge A_2 j = u_2 H_{5,\, r2\, =\, r2j})$ — D2, D1.$R$, D02, D3.$\sim_r$
$= H_5$

**($3^2$)** $[\mathbf{should}_{S1,\, p2,\, p3}{}^{p4}$
$[\mathbf{min}_{S1}\langle p_2\rangle \varnothing\, p_3\rangle]$; $[p_4 |\, p_4 = \mathbf{min}_{S1}\langle p_3\rangle]$

Output $J_{21} :=$ $[\mathbf{min}_{S1}\langle p_2\rangle \varnothing\, p_3\rangle]J_1$ — remoteness-presup.
$= \lambda j.\, j \in J_1 \wedge (\mathbf{min}_{S1j}\, p_2 j) \cap p_3 j = \varnothing$ — D2, D1.$\varnothing$
$= J_1$

for, given $\pi_0 := \{w_0\}$, $\pi_1 := \{w_0, w_1\}$, and $\pi_2 := \{w_0, w_1, w_2\}$, we have:

- $(\mathbf{min}_{\{\pi 0,\, \pi 1\}}\, \pi_1) \cap \{w_1\}$
  $= \{w_0\} \cap \{w_1\}$ — D03
  $= \varnothing$
- $(\mathbf{min}_{\{\pi 0,\, \pi 1,\, \pi 2\}}\, \pi_2) \cap \{w_1, w_2\}$
  $= \{w_0\} \cap \{w_1, w_2\}$ — D03
  $= \varnothing$

The $p_3$-antecedent modality is remote from the $S_1$-ideal of the $p_2$-modal base, i.e. at each index, the $p_3$-modality is disjoint from the $S_1$-ideal of $p_2$—$\{w_0\}$, where nobody commits any murder.

Output $J_{22} :=$ $[p_4 |\, p_4 = \mathbf{min}_{S1}\langle p_3\rangle]J_{21}$ — necessity-update
$= \lambda j.\, \exists i \in J_{21}(i[p_4]j \wedge p_4 j = \mathbf{min}_{S1j}\, p_3 j)$ — D2, D1.$\varnothing$

given $\pi_0 := \{w_0\}$, $\pi_1 := \{w_0, w_1\}$, and $\pi_2 := \{w_0, w_1, w_2\}$:

| $J_{22}$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i_1$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_2$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_4$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $i_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{11}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{12}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{13}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{14}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{15}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $i_{16}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $c$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |

**($3^3$) go to prison**

$[gtp_{r2}\langle u_2\rangle]$

Output $J_3$ := $[gtp_{r2}\langle u_2\rangle]J_{22}$ necessity-update

= $\lambda j.\ j \in J_{22} \wedge gtp_{r2}\ u_2 j$ D2, D1.*R*

if all murderers go to prison (gtp) in $w_1$, only *a* does in $w_2$ (and no murderer does in $w_3$):

| $J_3$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | $p_4$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i_1$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | $J_{3,\, r2\, =\, w1}$ |
| $i_2$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_4$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_{11}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | $J_{3,\, r2\, =\, w2}$ |
| $i_{13}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_{15}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_2$ | $a$ | $\{a, c\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |

**($3^4$) $]_{A2}$** scope of maximizing **he[**

$[A_2 \sim_{r2} u_2]$

Output $J_4$ := $[A_2 \sim_{r2} u_2]J_3$

= $\lambda j.\ j \in J_3 \wedge A_2 j = u_2 J_{3,\, r2\, =\, r2j}$ D2, D3.$\sim_r$

| $J_4$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | $p_4$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i_1$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | $J_{4,\, r\, =\, w3}$ |
| $i_2$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_3$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_4$ | $al$ | $w_3$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ | |
| $i_5$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | $J_{4,\, r\, =\, w2}$ |
| $i_6$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_7$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_8$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_9$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $a$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |
| $i_{10}$ | $al$ | $w_2$ | $\{w_1, w_2, w_3\}$ | $gb$ | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | $b$ | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ | |

**(3[5])** $]_{p4}$ scope of [**should**

$[p_4 \sim_r r_2]$

Output $J_5$ := $[p_4 \sim_r r_2]J_4$

= $\lambda j.\ j \in J_4 \wedge p_4 j = r_2 J_{4,\, r = rj}$ D2, D3.$\sim_r$

= $J_4$

**(3[6])** $]^p$ common ground update

$[p]; [p \sim r]$

Output $J_6$ := $([p]; [p \sim r])J_5$

= $\lambda j.\ \exists i \in J_5(i[p]j \wedge pj = rJ_5$ D2, D3.~

| $J_6$ | $u$ | $r$ | $p$ | $u_1$ | $p_1$ | $S_1$ | $p_2$ | $r_2$ | $u_2$ | $A_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $j_1$ | *al* | $w_3$ | $\{w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $j_2$ | *al* | $w_3$ | $\{w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $j_3$ | *al* | $w_3$ | $\{w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *a* | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $j_4$ | *al* | $w_3$ | $\{w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1\}$ | $\pi_1$ | $w_1$ | *b* | $\{a, b\}$ | $\{w_1\}$ | $\{w_1\}$ |
| $j_5$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $j_6$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_0$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $j_7$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $j_8$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_1$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $j_9$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *a* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |
| $j_{10}$ | *al* | $w_2$ | $\{w_2, w_3\}$ | *gb* | $\pi_2$ | $\{\pi_0, \pi_1, \pi_2\}$ | $\pi_2$ | $w_1$ | *b* | $\{a, b\}$ | $\{w_1, w_2\}$ | $\{w_1\}$ |

## APPENDIX: UPDATE SEMANTICS + $Ty_3$ ($UTy_3$)

D01. Basic terms of $UTy_3$

| Variables | Constants | Type | Name of objects |
|---|---|---|---|
| $i, j, k, h$ | $i_@$ | $s$ | atomic (info) states (*aka* indices) |
| $x, y, z$ | $@_e$, *john*, … | $e$ | entities |
| $w$ | $@_\mathbf{w}$, $w_0$, … | $\mathbf{w}$ | worlds |
| $I, J, K, H$ | $\{i_@\}$ | $st$ | (plural) (info) states |
| $\pi$ | $\{w_0\}$, … | $\mathbf{w}t$ | propositions |
| $\Sigma$ | $\{\{w_0\}\}$, … | $(\mathbf{w}t)t$ | sets of propositions |
| | **fut** | $\mathbf{w}(\mathbf{w}t)$ | (possible) futures |
| | **adr** | $\mathbf{w}(ee)$ | |
| | $u_n$ | $se$ | $e$-stores ($u_0$ for the speaker) |
| | $A_n$ | $s(et)$ | $et$-stores |
| | $r_n$ | $s\mathbf{w}$ | $\mathbf{w}$-stores ($r_0$ for the reality) |
| | $p_n$ | $s(\mathbf{w}t)$ | $\mathbf{w}t$-stores ($p_0$ for the common ground) |
| | $S_n$ | $s((\mathbf{w}t)t)$ | $(\mathbf{w}t)t$-stores |

D02. Dummies

- $@_\mathbf{w} \neq w_n \wedge @_\mathbf{w} \notin \alpha_{\mathbf{w}(\mathbf{w}t)}\, w_n$
  $\wedge\ @_e \neq c_e \wedge @_e \neq \beta_{ee}\, c_e \ \wedge @_e \neq \gamma_{\mathbf{w}(ee)} w_n c_e$
- $R_w(x_1, \ldots x_n) \rightarrow w \neq @_\mathbf{w} \wedge @_e \notin \{x_1, \ldots x_n\}$ for any basic relation $R_{e\ldots e\mathbf{w}}$
  $R_w(x, \pi) \rightarrow w \neq @_\mathbf{w} \wedge x \neq @_e \wedge \pi \neq \{\} \wedge @_\mathbf{w} \notin \pi$ for any basic relation $R_{(\mathbf{w}t)e\mathbf{w}}$
- $\delta i_@ = @_a \wedge \Delta i_@ = \{\}$ for $\delta_{sa}$, $\Delta_{s(at)}$, $a \in \{e, \mathbf{w}, \mathbf{w}t\}$

D03. Order & minimal elements

- $w \leq_\Sigma w' \ := \ \{\pi / \pi \in \Sigma \wedge w' \in \pi\} \subseteq \{\pi / \pi \in \Sigma \wedge w \in \pi\}$
- $w <_\Sigma w' \ := \ w \leq_\Sigma w' \wedge \neg w' \leq_\Sigma w$
- $\mathbf{min}_\Sigma(\pi) \ := \ \{w | \, w \in \pi \wedge \neg\exists w'(w' \in \pi \wedge w' <_\Sigma w)\}$

D04. Defined *sa*-dref's ($a \in \{e, \mathbf{w}\}$)

- $John \ := \ \lambda i_s.\, john_e$
- $\alpha_r\langle\delta\rangle \ := \ \lambda i_s.\, \alpha_{ri}\, \delta i$ for $\alpha_{\mathbf{w}ba}$, $\delta_{sb}$
- $\beta\langle\delta\rangle \ := \ \lambda i_s.\, \beta\, \delta i$ for $\beta_{ba}$, $\delta_{sb}$

D05. Defined $s(\mathbf{w}t)$-dref's

- $\mathbf{min}_S\langle p\rangle \ := \ \lambda i_s.\, \mathbf{min}_{Si}(pi)$
- $\mathbf{dox}_r\langle u\rangle \ := \ \lambda i_s.\, \cap\{\pi | \, believe_{ri}(ui, \pi)\} \ := \ \lambda i_s.\, \{w | \, \forall\pi(believe_{ri}(ui, \pi) \rightarrow w \in \pi)\}$
- $(p - p') \ := \ \lambda i_s.\, (pi - p'i) \ := \ \lambda i_s.\, \{w | \, w \in pi \wedge w \notin p'i\}$
- $\cup S \ := \ \lambda i_s.\, \cup(Si) \ := \ \lambda i_s.\, \{w | \, \exists\pi(\pi \in Si \wedge w \in \pi)\}$

D06. Global values, substates

- $\delta I \ := \ \{\delta i: i_s \in I_{st}\}$ for $\delta_{sa}$, $a \in \{e, \mathbf{w}, et, \mathbf{w}t, (\mathbf{w}t)t\}$
- $I_{\delta = d} \ := \ \{i_s \in I | \, \delta i = d\}$ for $\delta_{sa}$, $d_a$, $a \in \{e, \mathbf{w}, et, \mathbf{w}t, (\mathbf{w}t)t\}$

D1. Conditions (type *st*)

- $\text{PL}\langle A\rangle \;:=\; \lambda i_s.\ |Ai| > 1$
  $\text{SG}\langle A\rangle \;:=\; \lambda i_s.\ |Ai| = 1$
- $\delta = \delta' \;:=\; \lambda i_s.\ \delta i = \delta' i$ for $\delta_{sa}, \delta'_{sa}, a \in \{e, \mathbf{w}, et, \mathbf{w}t, (\mathbf{w}t)t\}$
  $\delta \in \Delta \;:=\; \lambda i_s.\ \delta i \in \Delta i$ for $\delta_{sa}, \Delta_{s(st)}, a \in \{e, \mathbf{w}, \mathbf{w}t\}$
- $\text{SM}\langle \Delta, \Delta'\rangle \;:=\; \lambda i_s.\ \varnothing \subset \Delta' i \subseteq \Delta i$
- $\Delta \varnothing \Delta' \;:=\; \lambda i_s.\ \Delta' i \cap \Delta i = \varnothing$
  $\Delta \bigcirc \Delta' \;:=\; \lambda i_s.\ \Delta' i \cap \Delta i \neq \varnothing$
- $R_r\langle \delta_1, \ldots \delta_n\rangle \;:=\; \lambda i_s.\ R_{ri}(\delta_1 i, \ldots \delta_n i)$ for $R_{an\ldots a1\,\mathbf{w}}, \delta_{1,\,sa1}, \ldots \delta_{n,\,san}$
  $R_r\langle u, r'\rangle \;:=\; \lambda i_s.\ \exists\pi(R_{ri}(ui, \pi) \wedge r'i \in \pi)$ for $R_{(\mathbf{w}t)e\mathbf{w}}$

D2. Local updates (type (*st*)(*st*))

- $[\delta] \;:=\; \lambda I_{st}\lambda j_s.\ \exists i_s \in I(i[\delta]j \wedge @_a \neq \delta j)$ for $\delta_{sa}, a \in \{e, \mathbf{w}\}$
- $[\Delta] \;:=\; \lambda I_{st}\lambda j_s.\ \exists i_s \in I(i[\Delta]j \wedge @_a \notin \Delta j)$ for $\Delta_{sa}, a \in \{et, \mathbf{w}t, (\mathbf{w}t)t\}$
- $[C_1, \ldots C_n] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge C_1 j \wedge \ldots C_n j$
- $(D_1; D_2) \;:=\; \lambda I_{st}.\ D_2(D_1 I)$
  $(D_1 \vee D_2) \;:=\; \lambda I_{st}.(D_1 I \cup D_2 I)$
- $[\delta|\ C_1, \ldots C_n] \;:=\; [\delta]; [C_1, \ldots C_n]$ for $\delta_{sa}, a \in \{e, \mathbf{w}, et, \mathbf{w}t, (\mathbf{w}t)t\}$

D3. Non-local updates (type (*st*)(*st*))

- $[\Delta \sim \delta] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge \Delta j = \delta I$ for $\Delta_{s(at)}, \delta_{sa}, a \in \{e, \mathbf{w}, \mathbf{w}t\}$
  $[\Delta \sim_r \delta] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge \Delta j = \delta I_{r = rj}$ for $\Delta_{s(at)}, \delta_{sa}, a \in \{e, \mathbf{w}, \mathbf{w}t\}$
  $[S \sim_r r'] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge Sj = \{r'I_{r = rj}\}$
- $[\text{SG}\{\delta\}] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge |\delta I| = 1)$
  $[\text{PL}_r\{\delta\}] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge |\delta I_{r = rj}| > 1)$
  $[\text{SG}_r\{\delta\}] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge |\delta I_{r = rj}| = 1)$
  $[_u(\text{SG}_r\{\delta\})] \;:=\; \lambda I_{st}\lambda j_s.\ j \in I \wedge @ \notin uI_{r = rj} \wedge \forall x_e \in uI_{r = rj}(|\delta I_{r = rj,\ u = x}| = 1)$

D4. Truth (relative to default state)

- $\models D \;:=\; \exists j_s(j \in D\{i_@\})$