

Kamp & Reyle 1993, Ch. 1 (1)
Introduction to Discourse Representation Theory

• BASIC PATTERNS OF DISCOURSE ANAPHORA

In Ch. 1 K&R analyze discourse anaphora to names and indefinite NPs in simple affirmative and negated sentences like (1)–(5) and failure of anaphora in (6).

- (1) Jones owns Ulysses. It fascinates him.
- (2) Jones owns a Porsche. It fascinates him.
- (3) A stockbroker who knows Bill likes him.
- (4) Jones owns a Porsche. He does not like it.
- (5) Jones doesn't own Ulysses. He likes it (however).
- (6) Jones doesn't own a Porsche. He likes it (however). (* anaphoric *it*)

• BASIC IDEA (Karttunen 1976)

Discourse anaphora is mediated by discourse referents (*drefs*). These are introduced by NPs (superscripted x, y , etc below) and may have their 'lifespan' cut off by higher operators such as negation. Within their lifespan, *dref*'s can be picked up by anaphoric pronouns (subscripted x, y), as in (1')–(5'). The problem with (6') is that the lifespan of a *dref* set up by an indefinite, a^y , under negation is limited to the scope of the negation operator — it_y comes too late.

- (1') Jones^x owns Ulysses^y. It_y fascinates him_x.
- (2') Jones^x owns a^y Porsche. It_y fascinates him_x.
- (3') A^x stockbroker who knows Bill^y likes him_y.
- (4') Jones^x owns a^y Porsche. He_x does not like it_y.
- (5') Jones^x doesn't own Ulysses^y. He_x likes it_y (however).
- (6') Jones^x doesn't [own a^y Porsche]. He_x likes it_y (however). ([...] = lifespan of *dref* set up by a^y)

• FORMALIZATION (K&R 1993, Ch. 1, elaborating Kamp 1981)

Explicate Karttunen's intuitive idea by defining a suitable formal language, *Discourse Representation Theory* (DRT), and a *translation algorithm* from a relevant fragment of English into DRT (K&R Construction Rules pp. 121–4).

DRT Syntax (= K&R Definition 1.4.1, p. 110)

- (i) A *Discourse Representation Structure (DRS)* K confined to a vocabulary V and *dref set* R is a pair, consisting of a possibly empty subset $U_K \subseteq R$ and a possibly empty set Con_K of DRS-conditions confined to V and R .
- (ii) A *DRS-condition confined to* V and R is an expression of one of the following forms:
 - (a) $x = y$, where $x, y \in R$
 - (b) $\pi(x)$, where $x \in R$ and π is an individual constant from V
 - (c) $\eta(x)$, where $x \in R$ and η is a unary predicate (corresponding to a common noun) from V
 - (d) $x\zeta$, where $x \in R$ and ζ is a unary predicate (corresponding to an intransitive verb) from V
 - (e) $x\xi y$, where $x, y \in R$ and ξ is a binary predicate from V
 - (f) $\neg K$, where K is a DRS confined to V and R .

Note:

Only *proper DRS's* — i.e. DRS's where no *dref's* are *free* in the sense of Definition 1.4.2 (p. 111) — are interpreted in DRT.

DRT Models

(= K&R Definition 1.2.1, p. 94)

A model M for a vocabulary V is a triple $\langle U_M, \text{Name}_M, \text{Pred}_M \rangle$ consisting of:

- (i) M 's universe U_M
- (ii) a function Name_M which assigns an element of U_M to each individual constant of V .
For each constant $c \in V$, $\text{Name}_M(c)$ is called the *bearer of c according to M* .
- (iii) a function Pred_M which assigns:
 - a subset of U_M to each 1-place predicate constant of V
 - a set of n -tuples of elements of U_M to each n -place predicate constant of V ($n \geq 2$)
 For each predicate constant $P \in V$, $\text{Pred}_M(P)$ is called the *extension of P in M* .

DRT Semantics

(= K&R Definitions 1.4.4–5, p. 112–4)

Let $K = \langle U_K, \text{Con}_K \rangle$ be a proper DRS confined to V and R , let $M = \langle U_M, \text{Name}_M, \text{Pred}_M \rangle$ be a DRT model for V , and let f be an *embedding from R into M* (i.e. f is a function such that $\text{Dom}(f) \subseteq R$ and $\text{Ran}(f) \subseteq U_M$).

- (i) for any DRS condition $\gamma \in \text{Con}_K$, f *verifies* γ in M iff
 - (a) γ is of the form $\mathbf{x} = \mathbf{y}$ and $f(\mathbf{x}) = f(\mathbf{y})$.
 - (b) γ is of the form $\pi(\mathbf{x})$ and $f(\mathbf{x}) = \text{Name}_M(\pi)$.
 - (c) γ is of the form $\eta(\mathbf{x})$ and $f(\mathbf{x}) \in \text{Pred}_M(\eta)$.
 - (d) γ is of the form $\mathbf{x}\zeta$ and $f(\mathbf{x}) \in \text{Pred}_M(\zeta)$.
 - (e) γ is of the form $\mathbf{x}\xi\mathbf{y}$ and $\langle f(\mathbf{x}), f(\mathbf{y}) \rangle \in \text{Pred}_M(\xi)$
 - (f) γ is of the form $\neg K'$ and there is no embedding g from R into M such that
 - g extends f (i.e. $f \subseteq g$)
 - $\text{Dom}(g) = \text{Dom}(f) \cup U_K$
 - g verifies K' in M
- (ii) f *verifies* the DRS K in M iff f verifies each condition $\gamma \in \text{Con}_K$ in M .
- (iii) K is *true in M* iff there is an embedding f from R into M such that $\text{Dom}(f) = U_K$ and f verifies K in M .

Translation from English into DRT

(= K&R Construction Rules, pp. 86, 121–4)

Most of K&R's construction rules require little comment except for CR.PRO, the rule for anaphoric pronouns. Note that this rule requires the antecedent to be an *accessible dref* — a formalization of Karttunen's idea of limited lifespan. K&R define dref accessibility as a syntactic relation in terms of an auxiliary notion of *DRS subordination*:

DRS Subordination

(= K&R Definition 1.4.10, pp. 119–120)

Let K and K' be DRS's.

- (i) K is *immediately subordinate to K'* iff $\neg K \in \text{Con}_{K'}$
- (ii) K is *subordinate to K'* , $K < K'$, iff
 - either (a) K is immediately subordinate to K'
 - or (b) there is a DRS K'' such that K'' is subordinate to K' and K is immediately subordinate to K'' .
- (iii) K is *weakly subordinate to K'* , $K \leq K'$, iff $K = K'$ or $K < K'$.

Dref Accessibility

(= K&R Definition 1.4.11, pp. 120)

Let K be a DRS, \mathbf{x} a dref, and γ a DRS-condition.

We say that \mathbf{x} is *accessible from γ in K* iff, for some K' and K'' , (i) $K'' \leq K' \leq K$, (ii) $\gamma \in \text{Con}_{K'}$, and (iii) $\mathbf{x} \in U_{K'}$.

REFERENCES

- Kamp, H. (1981) 'A Theory of Truth and Semantic Representation'. In J. Groenendijk et al (eds.) *Formal Methods in the Study of Language*, pp. 277–322. Mathematical Centre, Amsterdam.
- Kamp, H. and U. Reyle (1993) *From Discourse to Logic*. Kluwer, Dordrecht. [= K&R]
- Karttunen, L. (1976) 'Discourse Referents'. In J. McCawley (ed.) *Syntax and Semantics* 7, pp. 363–85. Academic Press, New York.

Kamp & Reyle 1993, Ch. 1 (2)
Verifying DRSs for Simple Discourses

- DRT SEMANTICS OF (1)

(1) Jones owns Ulysses. It fascinates him.

- DRS K_1 for (1):

(derived in class)

x	y	u	v
Jones(x)			
Ulysses(y)			
x owns y			
u = y			
v = x			
u fascinates v			

- DRT *model* M_1 for the vocabulary of (1):

(K&R Def. 1.2.1)

$$M_1 = \langle U_{M_1}, \text{Name}_{M_1}, \text{Pred}_{M_1} \rangle,$$

where

$$U_{M_1} = \{a, b\}$$

$$\text{Name}_{M_1}(\text{Jones}) = a$$

$$\text{Name}_{M_1}(\text{Ulysses}) = b$$

$$\text{Pred}_{M_1}(\text{owns}) = \{\langle a, b \rangle\}$$

$$\text{Pred}_{M_1}(\text{fascinates}) = \{\langle b, a \rangle\}$$

- The following *embedding* f_1 from $\{x, y, u, v\}$ into M_1 *verifies* K_1 in M_1 :

(K&R Def. 1.4.4)

$$f_1(x) = f_1(v) = a$$

$$f_1(y) = f_1(u) = b$$

Specifically, f_1 verifies every condition of K_1 in M_1 as follows:

Jones(x) verified by clause (b): $f_1(x) = a = \text{Name}_{M_1}(\text{Jones})$

Ulysses(y) verified by clause (b): $f_1(y) = b = \text{Name}_{M_1}(\text{Ulysses})$

x owns y verified by clause (e): $\langle f_1(x), f_1(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_1}(\text{owns}) = \{\langle a, b \rangle\}$

u = y verified by clause (a): $f_1(u) = b = f_1(y)$

v = x verified by clause (a): $f_1(v) = a = f_1(x)$

u fascinates v verified by clause (e): $\langle f_1(u), f_1(v) \rangle = \langle b, a \rangle \in \text{Pred}_{M_1}(\text{fascinates}) = \{\langle b, a \rangle\}$

- Therefore, K_1 is *true* in M_1 .

(K&R Def. 1.4.5)

- DRT SEMANTICS OF (2)
- (2) Jones owns a Porsche. It fascinates him.

- DRS K_2 for (2):

(DRT construction rules)

x	y	u	v
Jones(x)			
Porsche(y)			
x owns y			
$u = y$			
$v = x$			
u fascinates v			

- DRT *model* M_2 for the vocabulary of (2):

(K&R Def. 1.2.1)

$$M_2 = \langle U_{M_2}, \text{Name}_{M_2}, \text{Pred}_{M_2} \rangle,$$

where

$$U_{M_2} = \{a, b\}$$

$$\text{Name}_{M_2}(\text{Jones}) = a$$

$$\text{Pred}_{M_2}(\text{Porsche}) = \{b\} \quad \text{Pred}_{M_2}(\text{owns}) = \{\langle a, b \rangle\} \quad \text{Pred}_{M_2}(\text{fascinates}) = \{\langle b, a \rangle\}$$

- The following *embedding* f_2 from $\{x, y, u, v\}$ into M_2 *verifies* K_2 in M_2 :

(K&R Def. 1.4.4)

$$f_2(x) = f_2(v) = a \quad f_2(y) = f_2(u) = b$$

Specifically, f_2 verifies every condition of K_2 in M_2 as follows:

Jones(x) verified by clause (b): $f_2(x) = a = \text{Name}_{M_2}(\text{Jones})$

Porsche(y) verified by clause (c): $f_2(y) = b \in \text{Pred}_{M_2}(\text{Porsche}) = \{b\}$

x owns y verified by clause (e): $\langle f_2(x), f_2(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_2}(\text{owns}) = \{\langle a, b \rangle\}$

$u = y$ verified by clause (a): $f_2(u) = b = f_2(y)$

$v = x$ verified by clause (a): $f_2(v) = a = f_2(x)$

u fascinates v verified by clause (e): $\langle f_2(u), f_2(v) \rangle = \langle b, a \rangle \in \text{Pred}_{M_2}(\text{fascinates}) = \{\langle b, a \rangle\}$

- Therefore, K_2 is *true* in M_2 .

(K&R Def. 1.4.5)

- DRT SEMANTICS OF (3)

(3) A stockbroker who knows Bill likes him.

- DRS K_3 for (3):

(DRT construction rules)

x	y	u
stockbroker(x)		
Bill(y)		
x knows y		
$u = y$		
x likes u		

- DRT model M_3 for the vocabulary of (3):

(K&R Def. 1.2.1)

$$M_3 = \langle U_{M_3}, \text{Name}_{M_3}, \text{Pred}_{M_3} \rangle,$$

where

$$U_{M_3} = \{a, b\}$$

$$\text{Name}_{M_3}(\text{Bill}) = b$$

$$\text{Pred}_{M_3}(\text{stockbroker}) = \{a\} \quad \text{Pred}_{M_3}(\text{knows}) = \{\langle a, b \rangle\} \quad \text{Pred}_{M_3}(\text{likes}) = \{\langle a, b \rangle\}$$

- The following *embedding* f_3 from $\{x, y, u\}$ into M_3 *verifies* K_3 in M_3 :

(K&R Def. 1.4.4)

$$f_3(x) = a \quad f_3(y) = f_3(u) = b$$

Specifically, f_3 verifies every condition of K_3 in M_3 as follows:

$$\text{stockbroker}(x) \quad \text{verified by clause (c): } f_3(x) = a \in \text{Pred}_{M_3}(\text{stockbroker}) = \{a\}$$

$$\text{Bill}(y) \quad \text{verified by clause (b): } f_3(y) = b = \text{Name}_{M_3}(\text{Bill})$$

$$x \text{ knows } y \quad \text{verified by clause (e): } \langle f_3(x), f_3(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_3}(\text{knows}) = \{\langle a, b \rangle\}$$

$$u = y \quad \text{verified by clause (a): } f_3(u) = b = f_3(y)$$

$$x \text{ likes } u \quad \text{verified by clause (e): } \langle f_3(x), f_3(u) \rangle = \langle a, b \rangle \in \text{Pred}_{M_3}(\text{likes}) = \{\langle a, b \rangle\}$$

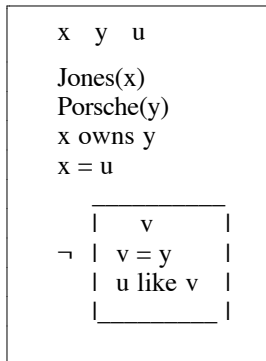
- Therefore, K_3 is *true* in M_3 .

(K&R Def. 1.4.5)

- DRT SEMANTICS OF (4)
- (4) Jones owns a Porsche. He does not like it.

- DRS K_4 for (4):

(DRT construction rules)



- DRT model M_4 for the vocabulary of (4):

(K&R Def. 1.2.1)

$$M_4 = \langle U_{M_4}, \text{Name}_{M_4}, \text{Pred}_{M_4} \rangle,$$

where

$$U_{M_4} = \{a, b\}$$

$$\text{Name}_{M_4}(\text{Jones}) = a$$

$$\text{Pred}_{M_4}(\text{Porsche}) = \{b\} \quad \text{Pred}_{M_4}(\text{owns}) = \{\langle a, b \rangle\} \quad \text{Pred}_{M_4}(\text{like}) = \{\langle a, a \rangle\}$$

- The following embedding f_4 from $\{x, y, u\}$ into M_4 verifies K_4 in M_4 :

(K&R Def. 1.4.4)

$$f_4(x) = f_4(u) = a \quad f_4(y) = b$$

Specifically, f_4 verifies every condition of K_4 in M_4 as follows:

Jones(x)	verified by clause (b):	$f_4(x) = a = \text{Name}_{M_4}(\text{Jones})$
Porsche(y)	verified by clause (c):	$f_4(y) = b \in \text{Pred}_{M_4}(\text{Porsche}) = \{b\}$
x owns y	verified by clause (e):	$\langle f_4(x), f_4(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_4}(\text{owns}) = \{\langle a, b \rangle\}$
$x = u$	verified by clause (a):	$f_4(x) = a = f_4(u)$

<table border="1" style="border-collapse: collapse; width: 80%; margin: auto;"> <tr> <td style="padding: 2px;">v</td> </tr> <tr> <td style="padding: 2px;">\neg $v = y$ </td> </tr> <tr> <td style="padding: 2px;"> u like v </td> </tr> </table>	v	\neg $v = y$	u like v	verified by clause (f):	there is no embedding g into M_4 such that: <ul style="list-style-type: none"> • $f_4 \subseteq g$, i.e., $g(x) = g(u) = a$ and $g(y) = b$ • $\text{Dom}(g) = \{x, u, y\} \cup \{v\} = \{x, u, y, v\}$ • $g(v) = g(y)$, i.e., $g(v) = b$ $\langle g(u), g(v) \rangle = \langle a, b \rangle \in \text{Pred}_{M_4}(\text{like}) = \{\langle a, a \rangle\}$ (No such g because the last condition cannot be met.)
v					
\neg $v = y$					
u like v					

- Therefore, K_4 is true in M_4 .

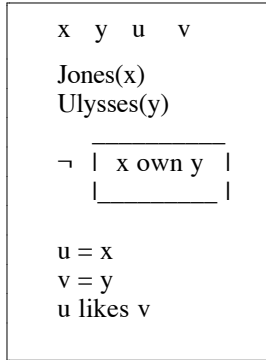
(K&R Def. 1.4.5)

NOTE: More generally, K_4 will be true in any DRT model in which there is no Porsche that (the bearer of the name) Jones owns and likes.

- DRT SEMANTICS OF (5)
- (5) Jones doesn't own Ulysses. He likes it (however).

• DRS K_5 for (5):

(DRT construction rules)



• DRT model M_5 for the vocabulary of (5):

(K&R Def. 1.2.1)

$$M_5 = \langle U_{M_5}, \text{Name}_{M_5}, \text{Pred}_{M_5} \rangle,$$

where

$$U_{M_5} = \{a, b, c\}$$

$$\text{Name}_{M_5}(\text{Jones}) = a \qquad \text{Name}_{M_5}(\text{Ulysses}) = b$$

$$\text{Pred}_{M_5}(\text{owns}) = \{\langle c, b \rangle\} \qquad \text{Pred}_{M_5}(\text{likes}) = \{\langle a, b \rangle\}$$

• The following embedding f_5 from $\{x, y, u, v\}$ into M_5 verifies K_5 in M_5 :

(K&R Def. 1.4.4)

$$f_5(x) = f_5(u) = a \qquad f_5(y) = f_5(v) = b$$

Specifically, f_5 verifies every condition of K_5 in M_5 as follows:

$$\text{Jones}(x) \qquad \text{verified by clause (b):} \qquad f_5(x) = a = \text{Name}_{M_5}(\text{Jones})$$

$$\text{Ulysses}(y) \qquad \text{verified by clause (b):} \qquad f_5(y) = b = \text{Name}_{M_5}(\text{Ulysses})$$

$$\neg \text{span style="border: 1px solid black; padding: 2px 10px;"> x own $y$$$
 verified by clause (f): there is no embedding g into M_5 such that:

- $f_5 \subseteq g$, i.e., $g(x) = g(u) = a$ and $g(y) = g(v) = b$
- $\text{Dom}(g) = \{x, u, y, v\} \cup \{\}$ = $\{x, u, y, v\}$
- $\langle g(x), g(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_5}(\text{own}) = \{\langle c, b \rangle\}$

 (No such g because the last condition cannot be met.)

$$u = x \qquad \text{verified by clause (a):} \qquad f_5(u) = a = f_5(x)$$

$$v = y \qquad \text{verified by clause (a):} \qquad f_5(v) = b = f_5(y)$$

$$u \text{ likes } v \qquad \text{verified by clause (e):} \qquad \langle f_5(u), f_5(v) \rangle = \langle a, b \rangle \in \text{Pred}_{M_5}(\text{likes}) = \{\langle a, b \rangle\}$$

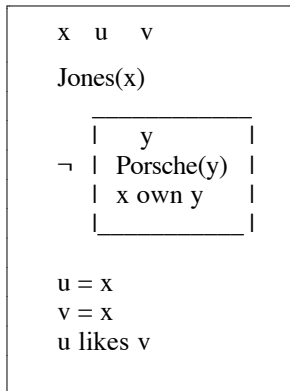
• Therefore, K_5 is true in M_5 .

(K&R Def. 1.4.5)

- DRT SEMANTICS OF (6)
- (6) Jones doesn't own a Porsche. He likes it (however).

- DRS K_6 for (6):

(DRT construction rules)



NOTE: y is **not** accessible to v , x is the only accessible antecedent. [Therefore, this discourse is wrongly assigned the same DRS as 'Jones doesn't own a Porsche. He likes himself.']

- DRT model M_6 for the vocabulary of (6):

(K&R Def. 1.2.1)

$$M_6 = \langle U_{M_6}, \text{Name}_{M_6}, \text{Pred}_{M_6} \rangle,$$

where

$$U_{M_6} = \{a, b, c\}$$

$$\text{Name}_{M_6}(\text{Jones}) = a$$

$$\text{Pred}_{M_6}(\text{Porsche}) = \{b\} \quad \text{Pred}_{M_6}(\text{own}) = \{\langle c, b \rangle\} \quad \text{Pred}_{M_6}(\text{like}) = \{\langle a, a \rangle\}$$

- The following embedding f_6 from $\{x, u, v\}$ into M_6 verifies K_6 in M_6 :

(K&R Def. 1.4.4)

$$f_6(x) = f_6(u) = f_6(v) = a$$

Specifically, f_6 verifies every condition of K_6 in M_6 as follows:

Jones(x)	verified by clause (b):	$f_6(x) = a = \text{Name}_{M_6}(\text{Jones})$
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\neg	<table border="1" style="border-collapse: collapse; margin: auto;"> <tr> <td style="padding: 2px 5px;">y</td> </tr> <tr> <td style="padding: 2px 5px;">Porsche(y)</td> </tr> <tr> <td style="padding: 2px 5px;">x own y</td> </tr> </table>	y	Porsche(y)	x own y			
y							
Porsche(y)							
x own y							

$u = x$	verified by clause (a):	$f_6(u) = a = f_6(x)$
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$v = x$	verified by clause (a):	$f_6(v) = a = f_6(x)$
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u likes v	verified by clause (e):	$\langle f_6(u), f_6(v) \rangle = \langle a, a \rangle \in \text{Pred}_{M_6}(\text{like}) = \{\langle a, a \rangle\}$
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- Therefore, K_6 is true in M_6 .

(K&R Def. 1.4.5)

Kamp & Reyle 1993, Ch. 1 (ctd)

• DRT SEMANTICS OF (1)

(1) Jones owns Ulysses. It fascinates him.

• DRS K_1 for (1):

(derived in class)

<p>x y u v</p> <p>Jones(x)</p> <p>Ulysses(y)</p> <p>x owns y</p> <p>u = y</p> <p>v = x</p> <p>u fascinates v</p>
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• DRT *model* M_1 for the vocabulary of (1):

(K&R Def. 1.2.1)

$$M_1 = \langle U_{M_1}, \text{Name}_{M_1}, \text{Pred}_{M_1} \rangle,$$

where

$$U_{M_1} = \{a, b\}$$

$$\text{Name}_{M_1}(\text{Jones}) = a$$

$$\text{Name}_{M_1}(\text{Ulysses}) = b$$

$$\text{Pred}_{M_1}(\text{owns}) = \{\langle a, b \rangle\}$$

$$\text{Pred}_{M_1}(\text{fascinates}) = \{\langle b, a \rangle\}$$

• The following *embedding* f_1 from $\{x, y, u, v\}$ into M_1 *verifies* K_1 in M_1 :

(K&R Def. 1.4.4)

$$f_1(x) = f_1(v) = a$$

$$f_1(y) = f_1(u) = b$$

Specifically, f_1 verifies every condition of K_1 in M_1 as follows:

Jones(x) verified by clause (b): $f_1(x) = a = \text{Name}_{M_1}(\text{Jones})$

Ulysses(y) verified by clause (b): $f_1(y) = b = \text{Name}_{M_1}(\text{Ulysses})$

x owns y verified by clause (e): $\langle f_1(x), f_1(y) \rangle = \langle a, b \rangle \in \text{Pred}_{M_1}(\text{owns}) = \{\langle a, b \rangle\}$

u = y verified by clause (a): $f_1(u) = b = f_1(y)$

v = x verified by clause (a): $f_1(v) = a = f_1(x)$

u fascinates v verified by clause (e): $\langle f_1(u), f_1(v) \rangle = \langle b, a \rangle \in \text{Pred}_{M_1}(\text{fascinates}) = \{\langle b, a \rangle\}$

• Therefore, K_1 is *true* in M_1 .

(K&R Def. 1.4.5)