

Dynamic Predicate Logic (1) Composing DRSs by Dynamic Sequencing

• LINEARIZED DRT SYNTAX WITH SEQUENCING

(1) John entered a bar. He found a chair.

Canonical DRS (K&R '93)
box format, concealed identity for names,
anaphora as identity condition

	x	y	$x' z$
	$John(x)$		
	$bar(y)$		
	$enter(x, y)$		
	$x' = x$		
	$chair(z)$		
	$find(x', z)$		

Linearized DRS (G&S '91 w/Muskens notation)
linear format, usu. with overt identity for names,
anaphora as choice of dref

$[x y z \mid x = john, bar y, enter xy, chair z, find xz]$

compositionally derived by *dynamic sequencing* (;)

$[x y \mid x = john, bar y, enter xy] ; [z \mid chair z, find xz]$

(2) John entered a bar. He didn't find a chair.

Canonical DRS

	x	y	x'
	$John(x)$		
	$bar(y)$		
	$enter(x, y)$		
	$x' = x$		
	z		
	$\neg chair(z)$		
	$find(x', z)$		

Linearized DRS

$[x y \mid x = john, bar y, enter xy, \neg[z \mid chair z, find xz]]$

compositionally derived by *dynamic sequencing* (;)

$[x y \mid x = john, bar y, enter xy] ; [\mid \neg[z \mid chair z, find xz]]$

(3) If a man owns a donkey, he beats it.

Canonical DRS

	x	y	$x' y'$
	$man(x)$		
	$donkey(y)$		
	$own(x, y)$		
	$x' = x$		
	$y' = y$		
	$beat(x', y')$		

Linearized DRS

$[\mid [x y \mid man x, donkey y, own xy] \Rightarrow [\mid beat x y]]$

DEFINITION 1 (DPL Syntax)

(notational variant of G&S '91)

- i. If R is an n -ary relation constant and t_1, \dots, t_n are terms, then $Rt_1\dots t_n$ is a condition.
If t_1 and t_2 are terms, then $t_1 = t_2$ is a condition.
- ii. If D and D' are DRSs then $\neg D, D \vee D'$ and $D \Rightarrow D'$ are conditions.
- iii. If ϕ_1, \dots, ϕ_m are conditions ($m \geq 1$) and x_1, \dots, x_n are variables ($n \geq 0$), then $[x_1 \dots x_n \mid \phi_1, \dots, \phi_m]$ is a DRS.
- iv. If D and D' are DRSs, then $D ; D'$ is a DRS.

• DPL SEMANTICS: TOTAL ASSIGNMENTS AND DYNAMIC SEQUENCING

NOTATION CONVENTIONS

(a la G&S '91)

- For any term t , model $M = \langle D_M, I_M \rangle$, assignment g :
 $\llbracket t \rrbracket^{M,g} = I_M(t)$ if t is an (individual) constant
 $= g(t)$ if t is a variable.
- For any assignments g and h , variables $\mathbf{x}_1 \dots \mathbf{x}_n$:
 $g[\mathbf{x}_1 \dots \mathbf{x}_n]h$ abbreviates $\forall v (v \neq \mathbf{x}_1 \wedge \dots \wedge v \neq \mathbf{x}_n \rightarrow h(v) = g(v))$
 $g[\]h$ abbreviates $\forall v (h(v) = g(v))$

We say that $\llbracket t \rrbracket^{M,g}$ is the *value* of t in M under g .

Also, given $g[\mathbf{x}_1 \dots \mathbf{x}_n]h$ we say that the *output (assignment)* h differs from the *input (assignment)* g at most in the values of $\mathbf{x}_1 \dots \mathbf{x}_n$.

DEFINITION 2 (DPL Semantics)

(G&S '91)

- i. $\llbracket R t_1 \dots t_n \rrbracket^M = \{g \mid \langle \llbracket t_1 \rrbracket^{M,g}, \dots, \llbracket t_n \rrbracket^{M,g} \rangle \in I_M(R)\}$
 $\llbracket t_1 = t_2 \rrbracket^M = \{g \mid \llbracket t_1 \rrbracket^{M,g} = \llbracket t_2 \rrbracket^{M,g}\}$
- ii. $\llbracket \neg D \rrbracket^M = \{g \mid \neg \exists h (\langle g, h \rangle \in \llbracket D \rrbracket^M)\}$
 $\llbracket D \vee D \rrbracket^M = \{g \mid \exists h (\langle g, h \rangle \in \llbracket D \rrbracket^M \cup \llbracket D \rrbracket^M)\}$
 $\llbracket D \Rightarrow D \rrbracket^M = \{g \mid \forall h (\langle g, h \rangle \in \llbracket D \rrbracket^M \rightarrow \exists k (\langle h, k \rangle \in \llbracket D \rrbracket^M))\}$
- iii. $v[\mathbf{x}_1 \dots \mathbf{x}_n \mid \phi_1, \dots, \phi_m]b^M = \{\langle g, h \rangle \mid g[\mathbf{x}_1 \dots \mathbf{x}_n]h \wedge h \in \llbracket \phi_1 \rrbracket^M \wedge \dots \wedge h \in \llbracket \phi_m \rrbracket^M\}$
- iv. $vD ; D \text{ } b^M = \{\langle g, h \rangle \mid \exists k (\langle g, k \rangle \in \llbracket D \rrbracket^M \wedge \langle k, h \rangle \in \llbracket D \rrbracket^M)\}$

DEFINITION 3 (Truth)

(G&S '91)

- i. A condition ϕ is *true* in M under g iff $g \in \llbracket \phi \rrbracket^M$.
- ii. A DRS D is *true* in M under g iff $\exists h (\langle g, h \rangle \in \llbracket D \rrbracket^M)$.

REFERENCES

Groenendijk, J. and M. Stokhof (1991) 'Dynamic Predicate Logic'. *Linguistics and Philosophy* **14**:39–100.
 Muskens, R. (1995) 'Tense and the Logic of Change'. In U. Egli et al (eds.) *Lexical Knowledge in the Organization of Language*, pp. 147–183. John Benjamins, Amsterdam.

DPL (2)
Examples of Simple DRS's and Dynamic Sequencing

• **EXAMPLE 1: SIMPLE DRS**

Given any first order model $M = \langle D_M, I_M \rangle$ and assignment g , the DRS

$$[x \ y \ z \mid x = \text{john}, \text{bar } y, \text{enter } xy, \text{chair } z, \text{find } xz]$$

is true in M under g iff the equivalent statements (1–7) hold.

- | | | |
|----|--|---|
| 1. | $\exists h(\langle g, h \rangle \in \llbracket [x \ y \ z \mid x = \text{john}, \text{bar } y, \text{enter } xy, \text{chair } z, \text{find } xz] \rrbracket^M)$ | G&S.df 3.ii |
| 2. | $\exists h(g[x \ y \ z]h$
$\wedge h \in \llbracket x = \text{john} \rrbracket^M \wedge h \in \llbracket \text{bar } y \rrbracket^M \wedge h \in \llbracket \text{enter } xy \rrbracket^M$
$\wedge h \in \llbracket \text{chair } z \rrbracket^M \wedge h \in \llbracket \text{find } xz \rrbracket^M)$ | G&S.df. 2.iii |
| 3. | $\exists h(g[x \ y \ z]h$
$\wedge h \in \{kl \llbracket x \rrbracket^{M,k} = \llbracket \text{john} \rrbracket^{M,k}\}$
$\wedge h \in \{kl \llbracket y \rrbracket^{M,k} \in I_M(\text{bar})\}$
$\wedge h \in \{kl \langle \llbracket x \rrbracket^{M,k}, \llbracket y \rrbracket^{M,k} \rangle \in I_M(\text{enter})\}$
$\wedge h \in \{kl \llbracket z \rrbracket^{M,k} \in I_M(\text{chair})\}$
$\wedge h \in \{kl \langle \llbracket x \rrbracket^{M,h}, \llbracket z \rrbracket^{M,h} \rangle \in I_M(\text{find})\})$ | G&S.df. 2.i
G&S.df. 2.i
G&S;df. 2.i
G&S.df. 2.i
G&S.df. 2.i |
| 4. | $\exists h(g[x \ y \ z]h$
$\wedge \llbracket x \rrbracket^{M,h} = \llbracket \text{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\text{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\text{enter})$
$\wedge \forall z b^{M,h} \in I_M(\text{chair}) \wedge \langle \forall x b^{M,h}, \forall z b^{M,h} \rangle \in I_M(\text{find}))$ | abs. equ.
abs. equ. |
| 5. | $\exists h(\forall v(v \neq x \wedge v \neq y \wedge v \neq z \rightarrow g(v) = h(v))$
$\wedge h(x) = I_M(\text{john}) \wedge h(y) \in I_M(\text{bar}) \wedge \langle h(x), h(y) \rangle \in I_M(\text{enter})$
$\wedge h(z) \in I_M(\text{chair}) \wedge \langle h(x), h(z) \rangle \in I_M(\text{find}))$ | G&S.df. $g[\dots]h$
G&S.df. $\llbracket t \rrbracket^{M,g}$
G&S.df. $\llbracket t \rrbracket^{M,g}$ |
| 6. | $\exists a \ b \ c (a = I_M(\text{john}) \wedge b \in I_M(\text{bar}) \wedge \langle a, b \rangle \in I_M(\text{enter})$
$\wedge c \in I_M(\text{chair}) \wedge \langle a, c \rangle \in I_M(\text{find}))$ | eliminate h |
| 7. | $\exists b \ c (b \in I_M(\text{bar}) \wedge \langle I_M(\text{john}), b \rangle \in I_M(\text{enter})$
$\wedge c \in I_M(\text{chair}) \wedge \langle I_M(\text{john}), c \rangle \in I_M(\text{find}))$ | eliminate a |

Cf. corresponding derivation in K&R '93 system:

Given any DRT model $M = \langle U_M, Name_M, Pred_M \rangle$, the DRS K

$x \ y \ z$
$\text{John}(x)$
$\text{bar}(y)$
$x \ \text{enter } y$
$\text{chair}(z)$
$x \ \text{find } z$

is true in M iff the equivalent statements (1–4) hold.

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|----|---|------------------------|
| 1. | $\exists h(\text{Dom}(h) = \{x, y, z\} \wedge h \text{ verifies } K \text{ in } M)$ | K&R.Sem.iii |
| 2. | $\exists h(\text{Dom}(h) = \{x, y, z\}$
$\wedge h \text{ verifies } \text{John}(x) \text{ in } M \wedge h \text{ verifies } \text{bar}(y) \text{ in } M \wedge h \text{ verifies } x \ \text{enter } y \text{ in } M$
$\wedge h \text{ verifies } \text{chair}(z) \text{ in } M \wedge h \text{ verifies } x \ \text{find } z \text{ in } M)$ | K&R.Sem.ii |
| 3. | $\exists h(\text{Dom}(h) = \{x, y, z\}$
$\wedge h(x) = Name_M(\text{John}) \wedge h(y) \in Pred_M(\text{bar}) \wedge \langle h(x), h(y) \rangle \in Pred_M(\text{enter})$
$\wedge h(z) \in Pred_M(\text{chair}) \wedge \langle h(x), h(z) \rangle \in Pred_M(\text{find}))$ | K&R.Sem.i
K&R.Sem.i |
| 4. | $\exists b \ c (b \in Pred_M(\text{bar}) \wedge \langle Name_M(\text{John}), b \rangle \in Pred_M(\text{enter}))$ | eliminate $h, h(x)$ |

- $\wedge c \in Pred_M(chair) \wedge \langle Name_M(John), c \rangle \in Pred_M(find)$
 EXAMPLE 2: EQUIVALENT DRS SEQUENCE

Given any first order model $M = \langle D_M, I_M \rangle$ and assignment g , the DRS

$$[x \ y \mid x = john, \ bar \ y, \ enter \ xy] ; [z \mid chair \ z, \ find \ xz]$$

is true in M under g iff the equivalent statements (1–11) hold.

1. $\exists h(\langle g, h \rangle \in \llbracket [x \ y \mid x = john, \ bar \ y, \ enter \ xy] ; [z \mid chair \ z, \ find \ xz] \rrbracket^M)$ df 3.ii
2. $\exists h(\langle g, h \rangle \in \{ \langle g', h' \rangle \mid \exists k(\langle g', k \rangle \in \llbracket [x \ y \mid x = john, \ bar \ y, \ enter \ xy] \rrbracket^M \wedge \langle k, h' \rangle \in \llbracket [z \mid chair \ z, \ find \ xz] \rrbracket^M) \}$ df 2.iv
3. $\exists h \exists k(\langle g, k \rangle \in \llbracket [x \ y \mid x = john, \ bar \ y, \ enter \ xy] \rrbracket^M \wedge \langle k, h \rangle \in \llbracket [z \mid chair \ z, \ find \ xz] \rrbracket^M)$ abs. equ.
4. $\exists h \exists k(\langle g, k \rangle \in \{ \langle g', k' \rangle \mid g[x \ y]k' \wedge k' \in \llbracket [x = john] \rrbracket^M \wedge k' \in \llbracket [bar \ y] \rrbracket^M \wedge k' \in \llbracket [enter \ xy] \rrbracket^M \}$ df 2.iii
 $\wedge \langle k, h \rangle \in \{ \langle k', h' \rangle \mid k[z]h' \wedge h' \in \llbracket [chair \ z] \rrbracket^M \wedge h' \in \llbracket [find \ xz] \rrbracket^M \}$ df 2.iii
5. $\exists h \exists k(g[x \ y]k \wedge k \in \llbracket [x = john] \rrbracket^M \wedge k \in \llbracket [bar \ y] \rrbracket^M \wedge k \in \llbracket [enter \ xy] \rrbracket^M \wedge k[z]h \wedge h \in \llbracket [chair \ z] \rrbracket^M \wedge h \in \llbracket [find \ xz] \rrbracket^M)$ abs. equ.
abs. equ.
6. $\exists h \exists k(g[x \ y]k \wedge k[z]h \wedge k \in \llbracket [x = john] \rrbracket^M \wedge k \in \llbracket [bar \ y] \rrbracket^M \wedge k \in \llbracket [enter \ xy] \rrbracket^M \wedge h \in \llbracket [chair \ z] \rrbracket^M \wedge h \in \llbracket [find \ xz] \rrbracket^M)$ rearrange
7. $\exists h \exists k(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = k(v)) \wedge \forall v(v \neq z \rightarrow k(v) = h(v)) \wedge k \in \llbracket [x = john] \rrbracket^M \wedge k \in \llbracket [bar \ y] \rrbracket^M \wedge k \in \llbracket [enter \ xy] \rrbracket^M \wedge h \in \llbracket [chair \ z] \rrbracket^M \wedge h \in \llbracket [find \ xz] \rrbracket^M)$ df. $g[\dots]h$
8. $\exists h(\forall v(v \neq x \wedge v \neq y \wedge v \neq z \rightarrow g(v) = h(v)) \wedge h \in \llbracket [x = john] \rrbracket^M \wedge h \in \llbracket [bar \ y] \rrbracket^M \wedge h \in \llbracket [enter \ xy] \rrbracket^M \wedge h \in \llbracket [chair \ z] \rrbracket^M \wedge h \in \llbracket [find \ xz] \rrbracket^M)$ eliminate k
9. $\exists h(g[x \ y \ z]h \wedge \llbracket [x] \rrbracket^{M, h} = \llbracket [john] \rrbracket^{M, h} \wedge \llbracket [y] \rrbracket^{M, h} \in I_M(bar) \wedge \langle \llbracket [x] \rrbracket^{M, h}, \llbracket [y] \rrbracket^{M, h} \rangle \in I_M(enter) \wedge \llbracket [z] \rrbracket^{M, h} \in I_M(chair) \wedge \langle \llbracket [x] \rrbracket^{M, h}, \llbracket [z] \rrbracket^{M, h} \rangle \in I_M(find))$ df. $g[\dots]h$
df. 2.i, abs. equ.
df. 2.i, abs. equ.
10. $\exists h(\forall v(v \neq x \wedge v \neq y \wedge v \neq z \rightarrow g(v) = h(v)) \wedge h(x) = I_M(john) \wedge h(y) \in I_M(bar) \wedge \langle h(x), h(y) \rangle \in I_M(enter) \wedge h(z) \in I_M(chair) \wedge \langle h(x), h(z) \rangle \in I_M(find))$ df. $g[\dots]h$
df. $\llbracket [t] \rrbracket^{M, g}$
df. $\llbracket [t] \rrbracket^{M, g}$
11. $\exists b \ c (b \in I_M(bar) \wedge \langle I_M(john), b \rangle \in I_M(enter) \wedge c \in I_M(chair) \wedge \langle I_M(john), c \rangle \in I_M(find))$ eliminate $h, h(x)$

DPL (3)
Negation and Implication. Dynamic Sequencing Revisited.

• EXAMPLE 3: SIMPLE DRS WITH NEGATION

Given any first order model $M = \langle D_M, I_M \rangle$ and assignment g , the DRS

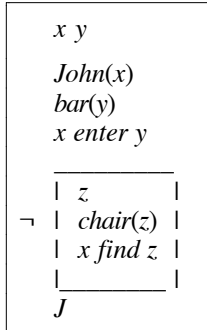
$$\langle x \ y \mid x = \textit{john}, \textit{bar} \ y, \textit{enter} \ xy, \neg[z \mid \textit{chair} \ z, \textit{find} \ xz] \rangle$$

is true in M under g iff the equivalent statements (1–9) hold.

1. $\exists h(\langle g, h \rangle \in \llbracket [x \ y \mid x = \textit{john}, \textit{bar} \ y, \textit{enter} \ xy, \neg[z \mid \textit{chair} \ z, \textit{find} \ xz]]^M \rrbracket)$ G&S.df 3.ii
2. $\exists h(g[x \ y]h$
 $\wedge h \in \llbracket x = \textit{john} \rrbracket^M \wedge h \in \llbracket \textit{bar} \ y \rrbracket^M \wedge h \in \llbracket \textit{enter} \ xy \rrbracket^M$
 $\wedge h \in \llbracket \neg[z \mid \textit{chair} \ z, \textit{find} \ xz] \rrbracket^M)$ G&S.df. 2.iii,
abs. equ.
3. $\exists h(g[x \ y]h$
 $\wedge h \in \{k \mid \llbracket x \rrbracket^{M,k} = \llbracket \textit{john} \rrbracket^{M,k}\}$ G&S.df. 2.i
 $\wedge h \in \{k \mid \llbracket y \rrbracket^{M,k} \in I_M(\textit{bar})\}$ G&S.df. 2.i
 $\wedge h \in \{k \mid \langle \llbracket x \rrbracket^{M,k}, \llbracket y \rrbracket^{M,k} \rangle \in I_M(\textit{enter})\}$ G&S;df. 2.i
 $\wedge h \in \{l \mid \neg \exists k(\langle l, k \rangle \in \llbracket [z \mid \textit{chair} \ z, \textit{find} \ xz] \rrbracket^M)\}$ G&S.df. 2.ii
4. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \textit{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\textit{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\textit{enter})$ abs. equ.
 $\wedge \neg \exists k(\langle h, k \rangle \in \llbracket [z \mid \textit{chair} \ z, \textit{find} \ xz] \rrbracket^M)$ abs. equ.
5. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \textit{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\textit{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\textit{enter})$
 $\wedge \neg \exists k(h[z]k \wedge k \in \{l \mid \llbracket z \rrbracket^{M,l} \in I_M(\textit{chair})\} \wedge k \in \{l \mid \langle \llbracket x \rrbracket^{M,l}, \llbracket z \rrbracket^{M,l} \rangle \in I_M(\textit{find})\}))$ G&S.df. 2.iii
6. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \textit{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\textit{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\textit{enter})$
 $\wedge \neg \exists k(h[z]k \wedge \llbracket z \rrbracket^{M,k} \in I_M(\textit{chair}) \wedge \langle \llbracket x \rrbracket^{M,k}, \llbracket z \rrbracket^{M,k} \rangle \in I_M(\textit{find}))$ abs. equ.
7. $\exists h(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = h(v))$ G&S.df. $g[\dots]h$
 $\wedge h(x) = I_M(\textit{john}) \wedge h(y) \in I_M(\textit{bar}) \wedge \langle h(x), h(y) \rangle \in I_M(\textit{enter})$ G&S.df. $\llbracket t \rrbracket^{M,g}$
 $\wedge \neg \exists k(\forall v(v \neq z \rightarrow h(v) = k(v))$ G&S.df. $g[\dots]h$
 $\wedge k(z) \in I_M(\textit{chair}) \wedge \langle k(x), k(z) \rangle \in I_M(\textit{find}))$ G&S.df. $\llbracket t \rrbracket^{M,g}$
8. $\exists a(b(a = I_M(\textit{john}) \wedge b \in I_M(\textit{bar}) \wedge \langle a, b \rangle \in I_M(\textit{enter})$ eliminate h
 $\wedge \neg \exists c(c \in I_M(\textit{chair}) \wedge \langle a, c \rangle \in I_M(\textit{find})))$ eliminate k
9. $\exists b(b \in I_M(\textit{bar}) \wedge \langle I_M(\textit{john}), b \rangle \in I_M(\textit{enter})$ eliminate a
 $\wedge \neg \exists c(c \in I_M(\textit{chair}) \wedge \langle I_M(\textit{john}), c \rangle \in I_M(\textit{find}))$

Cf. corresponding derivation in K&R '93 system:

Given any DRT model $M = \langle U_M, Name_M, Pred_M \rangle$, the DRS K



is true in M iff the equivalent statements (1–6) hold.

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|----|--|--|
| 1. | $\exists h(\text{Dom}(h) = \{x, y\} \wedge h \text{ verifies } K \text{ in } M)$ | K&R.Sem.iii |
| 2. | $\exists h(\text{Dom}(h) = \{x, y\} \wedge h \text{ verifies } John(x) \text{ in } M \wedge h \text{ verifies } bar(y) \text{ in } M \wedge h \text{ verifies } x \text{ enter } y \text{ in } M \wedge h \text{ verifies } \neg J \text{ in } M)$ | K&R.Sem.ii |
| 3. | $\exists h(\text{Dom}(h) = \{x, y\} \wedge h \text{ verifies } John(x) \text{ in } M \wedge h \text{ verifies } bar(y) \text{ in } M \wedge h \text{ verifies } x \text{ enter } y \text{ in } M \wedge \neg k(h \subseteq k \wedge \text{Dom}(k) = \text{Dom}(h) \cup \{z\} \wedge k \text{ verifies } J \text{ in } M))$ | K&R.Sem.i(f) |
| 4. | $\exists h(\text{Dom}(h) = \{x, y\} \wedge h \text{ verifies } John(x) \text{ in } M \wedge h \text{ verifies } bar(y) \text{ in } M \wedge h \text{ verifies } x \text{ enter } y \text{ in } M \wedge \neg k(h \subseteq k \wedge \text{Dom}(k) = \text{Dom}(h) \cup \{z\} \wedge k \text{ verifies } chair(z) \text{ in } M \wedge k \text{ verifies } x \text{ find } z \text{ in } M))$ | K&R.Sem.ii |
| 5. | $\exists h(\text{Dom}(h) = \{x, y\} \wedge h(x) = Name_M(John) \wedge h(y) \in Pred_M(bar) \wedge \langle h(x), h(y) \rangle \in Pred_M(enter) \wedge \neg k(h \subseteq k \wedge \text{Dom}(k) = \text{Dom}(h) \cup \{z\} \wedge k(z) \in Pred_M(chair) \wedge \langle k(x), k(z) \rangle \in Pred_M(find))$ | K&R.Sem.i
K&R.Sem.i |
| 6. | $\exists b(b \in Pred_M(bar) \wedge \langle Name_M(John), b \rangle \in Pred_M(enter) \wedge \neg \exists c(c \in Pred_M(chair) \wedge \langle Name_M(John), c \rangle \in Pred_M(find))$ | eliminate $h, h(x)$
eliminate $k, k(x)$ |

• EXAMPLE 4: EQUIVALENT DRS SEQUENCE

Given any first order model $M = \langle D_M, I_M \rangle$ and assignment g , the DRS

$$[x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy] ; [\mid \neg[z \mid \text{chair } z, \text{find } xz]]$$

is true in M under g iff the equivalent statements (1–14) hold.

1. $\exists h(\langle g, h \rangle \in \llbracket [x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy] ; [\mid \neg[z \mid \text{chair } z, \text{find } xz]] \rrbracket^M)$ df 3.ii
2. $\exists h(\langle g, h \rangle \in \{ \langle g', h' \rangle \mid \exists k(\langle g', k \rangle \in \llbracket [x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy] \rrbracket^M \wedge \langle k, h' \rangle \in \llbracket [\mid \neg[z \mid \text{chair } z, \text{find } xz]] \rrbracket^M) \}$ df 2.iv
3. $\exists h \exists k(\langle g, k \rangle \in \llbracket [x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy] \rrbracket^M \wedge \langle k, h \rangle \in \llbracket [\mid \neg[z \mid \text{chair } z, \text{find } xz]] \rrbracket^M)$ abs. equ.
4. $\exists h \exists k(\langle g, k \rangle \in \{ \langle g', k' \rangle \mid g[x \ y]k' \wedge k' \in \llbracket x = \text{john} \rrbracket^M \wedge k' \in \llbracket \text{bar } y \rrbracket^M \wedge k' \in \llbracket \text{enter } xy \rrbracket^M \} \wedge \langle k, h \rangle \in \{ \langle k', h' \rangle \mid k' \llbracket \]h' \wedge h' \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M \})$ df 2.iii
df 2.iii
5. $\exists h \exists k(g[x \ y]k \wedge k \in \llbracket x = \text{john} \rrbracket^M \wedge k \in \llbracket \text{bar } y \rrbracket^M \wedge k \in \llbracket \text{enter } xy \rrbracket^M \wedge k \llbracket \]h \wedge h \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ abs. equ.
abs. equ.
6. $\exists h \exists k(g[x \ y]k \wedge k \llbracket \]h \wedge k \in \llbracket x = \text{john} \rrbracket^M \wedge k \in \llbracket \text{bar } y \rrbracket^M \wedge k \in \llbracket \text{enter } xy \rrbracket^M \wedge h \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ rearrange
7. $\exists h \exists k(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = k(v)) \wedge \forall v(k(v) = h(v)) \wedge k \in \llbracket x = \text{john} \rrbracket^M \wedge k \in \llbracket \text{bar } y \rrbracket^M \wedge k \in \llbracket \text{enter } xy \rrbracket^M \wedge h \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ df $g[\dots]h$
8. $\exists h(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = h(v)) \wedge h \in \llbracket x = \text{john} \rrbracket^M \wedge h \in \llbracket \text{bar } y \rrbracket^M \wedge h \in \llbracket \text{enter } xy \rrbracket^M \wedge h \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ eliminate k
9. $\exists h(g[x \ y]h \wedge h \in \llbracket x = \text{john} \rrbracket^M \wedge h \in \llbracket \text{bar } y \rrbracket^M \wedge h \in \llbracket \text{enter } xy \rrbracket^M \wedge h \in \{ \llbracket \]h \neg k \llbracket [z]k \wedge k \in \llbracket \text{chair } z \rrbracket^M \wedge k \in \llbracket \text{find } xz \rrbracket^M \})$ df. $g[\dots]h$
df. 2.ii
10. $\exists h(g[x \ y]h \wedge h \in \llbracket x = \text{john} \rrbracket^M \wedge h \in \llbracket \text{bar } y \rrbracket^M \wedge h \in \llbracket \text{enter } xy \rrbracket^M \wedge \neg k \llbracket [z]k \wedge k \in \llbracket \text{chair } z \rrbracket^M \wedge k \in \llbracket \text{find } xz \rrbracket^M)$ abs. equ.
11. $\exists h(g[x \ y]h \wedge \llbracket [x] \rrbracket^{M, h} = \llbracket \text{john} \rrbracket^{M, h} \wedge \llbracket [y] \rrbracket^{M, h} \in I_M(\text{bar}) \wedge \langle \llbracket [x] \rrbracket^{M, h}, \llbracket [y] \rrbracket^{M, h} \rangle \in I_M(\text{enter}) \wedge \neg k \llbracket [z]k \wedge \llbracket [z] \rrbracket^{M, k} \in I_M(\text{chair}) \wedge \langle \llbracket [x] \rrbracket^{M, k}, \llbracket [z] \rrbracket^{M, k} \rangle \in I_M(\text{find}))$ df. 2.i, abs. equ
df. 2.i, abs. equ
12. $\exists h(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = h(v)) \wedge h(x) = I_M(\text{john}) \wedge h(y) \in I_M(\text{bar}) \wedge \langle h(x), h(y) \rangle \in I_M(\text{enter}) \wedge \neg k(\forall v(v \neq z \rightarrow h(v) = k(v)) \wedge k(z) \in I_M(\text{chair}) \wedge \langle k(x), k(z) \rangle \in I_M(\text{find}))$ df. $g[\dots]h$
df. $\llbracket [] \rrbracket^{M, g}$
df. $h[\dots]k$
df. $\llbracket [] \rrbracket^{M, g}$
13. $\exists a \ b(a = I_M(\text{john}) \wedge b \in I_M(\text{bar}) \wedge \langle a, b \rangle \in I_M(\text{enter}) \wedge \neg c (c \in I_M(\text{chair}) \wedge \langle a, c \rangle \in I_M(\text{find})))$ eliminate h
eliminate k
14. $\exists b(b \in I_M(\text{bar}) \wedge \langle I_M(\text{john}), b \rangle \in I_M(\text{enter}) \wedge \neg \exists c(c \in I_M(\text{chair}) \wedge \langle I_M(\text{john}), c \rangle \in I_M(\text{find})))$ eliminate a

• EXAMPLE 5: SIMPLE DRS WITH IMPLICATION

Given any first order model $M = \langle D_M, I_M \rangle$ and assignment g , the DRS

$$[x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy, \neg[z \mid \text{chair } z, \text{find } xz]]$$

is true in M under g iff the equivalent statements (1–9) hold.

1. $\exists h(\langle g, h \rangle \in \llbracket [x \ y \mid x = \text{john}, \text{bar } y, \text{enter } xy, \neg[z \mid \text{chair } z, \text{find } xz]] \rrbracket^M)$ G&S.df 3.ii
2. $\exists h(g[x \ y]h$
 $\wedge h \in \llbracket x = \text{john} \rrbracket^M \wedge h \in \llbracket \text{bar } y \rrbracket^M \wedge h \in \llbracket \text{enter } xy \rrbracket^M$
 $\wedge h \in \llbracket \neg[z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ G&S.df. 2.iii
abs. equ.
3. $\exists h(g[x \ y]h$
 $\wedge h \in \{kl \mid \llbracket x \rrbracket^{M,k} = \llbracket \text{john} \rrbracket^{M,k}\}$ G&S.df. 2.i
 $\wedge h \in \{kl \mid \llbracket y \rrbracket^{M,k} \in I_M(\text{bar})\}$ G&S.df. 2.i
 $\wedge h \in \{kl \mid \langle \llbracket x \rrbracket^{M,k}, \llbracket y \rrbracket^{M,k} \rangle \in I_M(\text{enter})\}$ G&S;df. 2.i
 $\wedge h \in \{ll \mid \neg \exists k(\langle l, k \rangle \in \llbracket [z \mid \text{chair } z, \text{find } xz] \rrbracket^M)\}$ G&S.df. 2.ii
4. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \text{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\text{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\text{enter})$ abs. equ.
 $\wedge \neg \exists k(\langle h, k \rangle \in \llbracket [z \mid \text{chair } z, \text{find } xz] \rrbracket^M)$ abs. equ.
5. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \text{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\text{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\text{enter})$
 $\wedge \neg \exists k(h[z]k \wedge k \in \{ll \mid \llbracket z \rrbracket^{M,l} \in I_M(\text{chair})\} \wedge k \in \{ll \mid \langle \llbracket x \rrbracket^{M,l}, \llbracket z \rrbracket^{M,l} \rangle \in I_M(\text{find})\}))$ G&S.df. 2.iii
6. $\exists h(g[x \ y]h$
 $\wedge \llbracket x \rrbracket^{M,h} = \llbracket \text{john} \rrbracket^{M,h} \wedge \llbracket y \rrbracket^{M,h} \in I_M(\text{bar}) \wedge \langle \llbracket x \rrbracket^{M,h}, \llbracket y \rrbracket^{M,h} \rangle \in I_M(\text{enter})$
 $\wedge \neg \exists k(h[z]k \wedge \llbracket z \rrbracket^{M,k} \in I_M(\text{chair}) \wedge \langle \llbracket x \rrbracket^{M,k}, \llbracket z \rrbracket^{M,k} \rangle \in I_M(\text{find}))$ abs. equ.
7. $\exists h(\forall v(v \neq x \wedge v \neq y \rightarrow g(v) = h(v))$ G&S.df. $g[\dots]h$
 $\wedge h(x) = I_M(\text{john}) \wedge h(y) \in I_M(\text{bar}) \wedge \langle h(x), h(y) \rangle \in I_M(\text{enter})$ G&S.df. $\llbracket t \rrbracket^{M,g}$
 $\wedge \neg \exists k(\forall v(v \neq z \rightarrow h(v) = k(v))$ G&S.df. $g[\dots]h$
 $\wedge k(z) \in I_M(\text{chair}) \wedge \langle k(x), k(z) \rangle \in I_M(\text{find}))$ G&S.df. $\llbracket t \rrbracket^{M,g}$
8. $\exists a(b = I_M(\text{john}) \wedge b \in I_M(\text{bar}) \wedge \langle a, b \rangle \in I_M(\text{enter})$ eliminate h
 $\wedge \neg \exists c(c \in I_M(\text{chair}) \wedge \langle a, c \rangle \in I_M(\text{find}))$ eliminate k
9. $\exists b(b \in I_M(\text{bar}) \wedge \langle I_M(\text{john}), b \rangle \in I_M(\text{enter})$ eliminate a
 $\wedge \neg \exists c(c \in I_M(\text{chair}) \wedge \langle I_M(\text{john}), c \rangle \in I_M(\text{find}))$