INDIVIDUALS AND POSSIBILITIES (2):
Modal Anaphora as New Path to Familiar Results

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1. ORDERING SEMANTICS FOR CONDITIONALS = **if**-ANTECEDENT; **if**-CONSEQUENT; MODAL TEST

(1) If John were honest, [(1)] $\mathbf{if}(\omega, \omega_1, [\,|\, honest_{\omega 1}\{john\}])$

$$:= \lambda ij.\exists h(i[\omega{:}\ \omega_1]h \wedge [\,|\, honest_{\omega 1}\{john\}]hj) \\ \wedge \forall j'(\exists h(i[\omega{:}\ \omega_1]h \wedge [\,|\, honest_{\omega 1}\{john\}]hj') \rightarrow \\ \forall w(\exists w_0(w \in \omega i w_0) \rightarrow \\ \forall w_j w_{j'}(w_j \in \omega_1 jw \wedge w_{j'} \in \omega_1 j'w \wedge w_{j'} \leq_w w_j \rightarrow w_j \leq_w w_{j'})) \\ \wedge \forall w_{j'}(w_{j'} \in \omega_1 j'w \rightarrow \exists w_j(w_j \in \omega_1 jw \wedge w_j \leq_w w_{j'}))))$$

you *might* trust him. [(2)] $\mathbf{if}(\omega_1, \omega_2, [\,|\, trust_{\omega 2}\{you, john\}])$

$$:= \lambda jk.\exists h(j[\omega_1{:}\ \omega_2]h \wedge [\,|\, trust_{\omega 2}\{you, john\}]hk) \\ \wedge \forall k'(\exists h(j[\omega_1{:}\ \omega_2]h \wedge [\,|\, trust_{\omega 2}\{you, john\}]hk') \rightarrow \\ \forall w(\exists w_0(w \in \omega_1 j w_0) \rightarrow \\ \forall w_k w_{k'}(w_k \in \omega_2 kw \wedge w_{k'} \in \omega_2 k'w \wedge w_{k'} \leq_w w_k \rightarrow w_k \leq_w w_{k'})) \\ \wedge \forall w_{k'}(w_{k'} \in \omega_2 k'w \rightarrow \exists w_k(w_k \in \omega_2 kw \wedge w_k \leq_w w_{k'}))))$$

[(3)] $[\,|\, \mathbf{poss}_{\}, -}(\omega_1, \omega_2)]$

$$:= \lambda kl.k = l \wedge \\ \forall w(\exists w'(w' \in \omega_1 kw) \rightarrow \\ \exists w'(w' \in \omega_1 kw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 kw \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'') \\ \wedge w' \in \omega_2 kw'))$$

That is, the *antecedent update* [(1)] takes us from i to j by introducing a possibility ω_1, the *consequent update* [(2)] takes us on to k by introducing a possible elaboration ω_2 of ω_1, and finally we submit k to the **poss**-*test* of [(3)]. These developments are summarized in *Table* (1′) below (assuming that (1) is addressed to Bill):

Key to table (1′):

K	=	$\cap_a K_a$ (*our combined knowledge*), where K_a are the epistemic alternatives of $a \in \{me, you\}$
$\min_w \mathbf{H}$	=	the set of w-closest worlds from $\lambda w'.honest_{w'}(john)$
$\min_w \mathbf{T}$	=	the set of w-closest worlds from $\lambda w'.trust_{w'}(bill, john)$
$\lambda w \in X.S_w$	=	the function that maps w to w-accessible sphere S_w if $w \in X$, and to $\varnothing$ if $w \notin X$.
$\rightarrow$		same value as for the last environment

(1′)	i	j	k
you	$\lambda w.bill$	$\rightarrow$	$\rightarrow$
ω	$\lambda w \in K.K$	$\rightarrow$	$\rightarrow$
ω_1	?	$\lambda w \in K.\ \min_w \mathbf{H}$	$\rightarrow$
ω_2	?	?	$\lambda w' \in \cup_{w \in K} \min_w \mathbf{H}.\ \min_{w'} \mathbf{T}$

The test in [(3)] checks whether k satisfies the condition $\mathbf{poss}_{\}, -}(\omega_1, \omega_2)$, which amounts to this:

$$\forall w(\exists w'(w' \in \omega_1 kw) \rightarrow \\ \exists w'(w' \in \omega_1 kw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 kw \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'') \\ \wedge w' \in \omega_2 kw')) \qquad \text{from (1)}$$

$$= \forall w(w \in K \rightarrow \\ \exists w'(w' \in \min_w \mathbf{H} \wedge \}w' \wedge \forall w''(w'' \in \min_w \mathbf{H} \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'') \\ \wedge w' \in \min_{w'} \mathbf{T}) \qquad \text{subst. from (1′)}$$

That is, we get the *ordering semantics* of Lewis (1973): For every $w \in K$, some w-closest accessible **H**-world is also a **T**-world (not necessarily a *w-closest* **T**-world).

2. DONKEY ANAPHORA BETWEEN **if**-ANTECEDENT AND **if**-CONSEQUENT

(2) If *a bear* comes in,

(1) $\mathbf{if}(\omega, \omega_1, [\omega_1{:}\ u_1 \mid bear_{\omega 1}\{u_1\}, come\text{-}in_{\omega 1}\{u_1\}])$

$$:= \lambda ij.\exists h(i[\omega{:}\ \omega_1]h \wedge [\omega_1{:}\ u_1 \mid bear_{\omega 1}\{u_1\}, come\text{-}in_{\omega 1}\{u_1\}]hj)$$
$$\quad \wedge \forall j'(\exists h(i[\omega{:}\ \omega_1]h \wedge [\omega_1{:}\ u_1 \mid bear_{\omega 1}\{u_1\}, come\text{-}in_{\omega 1}\{u_1\}]hj') \rightarrow$$
$$\quad\quad \forall w(\exists w_0(w \in \omega i w_0) \rightarrow$$
$$\quad\quad\quad \forall w_j w_{j'}(w_j \in \omega_1 jw \wedge w_{j'} \in \omega_1 j'w \wedge w_{j'} \leq_w w_j \rightarrow w_j \leq_w w_{j'}))$$
$$\quad\quad\quad \wedge \forall w_{j'}(w_{j'} \in \omega_1 j'w \rightarrow \exists w_j(w_j \in \omega_1 jw \wedge w_j \leq_w w_{j'}))))$$

I will not shoot *it*.

(2) $\mathbf{if}(\omega_1, \omega_2, [\ \mid shoot_{\omega 2}\{me, u_1\}])$

$$:= \lambda jk.\exists h(j[\omega_1{:}\ \omega_2]h \wedge [\ \mid shoot_{\omega 2}\{me, u_1\}]hk)$$
$$\quad \wedge \forall k'(\exists h(j[\omega_1{:}\ \omega_2]h \wedge [\ \mid shoot_{\omega 2}\{me, u_1\}]hk') \rightarrow$$
$$\quad\quad \forall w(\exists w_0(w \in \omega_1 j w_0) \rightarrow$$
$$\quad\quad\quad \forall w_k w_{k'}(w_k \in \omega_2 kw \wedge w_{k'} \in \omega_2 k'w \wedge w_{k'} \leq_w w_k \rightarrow w_k \leq_w w_{k'}))$$
$$\quad\quad\quad \wedge \forall w_{k'}(w_{k'} \in \omega_2 k'w \rightarrow \exists w_k(w_k \in \omega_2 kw \wedge w_k \leq_w w_{k'}))))$$

(3) $[\ \mid \mathbf{not}_{\}, -}(\omega_1, \omega_2)]$

$$:= \lambda kl.k = l \wedge$$
$$\quad \forall w(\exists w'(w' \in \omega_1 kw) \rightarrow$$
$$\quad\quad \forall w'(w' \in \omega_1 kw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 kw \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'')$$
$$\quad\quad\quad \rightarrow w' \notin \omega_2 kw'))$$

With donkey anaphora in the picture, it's harder to say in plain English what the possibility concepts ω_1 and ω_2 pick out. The local environment now plays a crucial role in identifying different potential antecedents. The following key is a rough guide to what's going on.

Key to table (2′):

$\min_{j,w}$ **B** = the *w*-closest worlds with an entering bear, with the relevant bear concept identified by *j* (as $u_1 j$)
$\min_{j,w}$ **S** = the *w*-closest worlds w' such that John shoots ujw' in w'

(2′)	*i*	*j*	*k*
me	$\lambda w.john$	$\rightarrow$	$\rightarrow$
ω	$\lambda w \in K.K$	$\rightarrow$	$\rightarrow$
ω_1	?	$\lambda w \in K.\ \min_{j,\,w} \mathbf{B}$	$\rightarrow$
u_1	?	$\lambda w' \in \cup_w \min_{j,w} \mathbf{B}.u_1 jw'$, s.t. $(bear_{\omega 1}\{u_1\}, come\text{-}in_{\omega 1}\{u_1\})j$	$\rightarrow$
ω_2	?	?	$\lambda w' \in \cup_w \min_{j,\,w} \mathbf{B}.\ \min_{w'} \mathbf{S}$

The test in (3) checks whether *k* satisfies the condition $\mathbf{not}_{\}, -}(\omega_1, \omega_2)$, which amounts to this:

$$\forall w(\exists w'(w' \in \omega_1 kw) \rightarrow$$
$$\quad \neg\exists w'(w' \in \omega_1 kw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 kw \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'')$$
$$\quad\quad \wedge w' \in \omega_2 kw')) \qquad \text{from (2)}$$

$$= \forall w(w \in K \rightarrow$$
$$\quad \neg\exists w'(w' \in \min_{k,\,w} \mathbf{B} \wedge \}w' \wedge \forall w''(w'' \in \min_{k,\,w} \mathbf{B} \wedge \}w'' \wedge w'' -_w w' \rightarrow w' -_w w'')$$
$$\quad\quad \wedge w' \in \min_{k,\,w'} \mathbf{S}) \qquad \text{subst. from (2′)}$$

i.e., roughly, for every $w \in K$, no *w*-closest accessible **B**-world is an **S**-world (cf. Kamp 1981)

APPENDIX: KEY DEFINITIONS

- *Discourse referents* as concepts restricted by reference possibility ω_r

$$i[\omega_r\!: u]j \quad := \forall v(\mathbf{mk}(v) \wedge v \neq u \rightarrow vi = vj) \wedge \forall w'(\exists w(w' \in \omega_r jw) \rightarrow ujw' \ \mathbf{in}\ w')$$

individual referent (Stone 1997)

$$i[\omega_r\!: \omega']j \quad := \forall v(\mathbf{mk}(v) \wedge v \neq \omega' \rightarrow vi = vj) \wedge \forall w'(\neg\exists w(w' \in \omega_r jw) \rightarrow \neg\exists w''(w'' \in \omega' jw'))$$

possibility referent (Stone 1997)

$$i[\omega_r\!: \delta_0, \delta_1, \ldots, \delta_n]j \quad := \exists k(i[\omega_r\!: \delta_0]k \wedge k[\omega_r\!: \delta_1, \ldots, \delta_n]j)$$

Stone 1999

- *Conditional update* (Stone 1997)

$$\begin{aligned}\mathbf{if}(\omega_1, \omega_2, D) := \lambda ij.\exists k(i[\omega_1\!: \omega_2]k \wedge Dkj) \wedge \\ \forall h(\exists k(i[\omega_1\!: \omega_2]k \wedge Dkh) \rightarrow \\ \forall w(\exists w_0(w \in \omega_1 iw_0) \rightarrow \\ \forall w_j w_h(w_j \in \omega_2 jw \wedge w_h \in \omega_2 hw \wedge w_h \leq_w w_j \rightarrow w_j \leq_w w_h)) \\ \wedge \forall w_h(w_h \in \omega_2 hw \rightarrow \exists w_j(w_j \in \omega_2 jw \wedge w_j \leq_w w_h))))\end{aligned}$$

- *Modal relations* parametrized by modal base } and ordering source –

$$\begin{aligned}\mathbf{necc}_{\}, –}(\omega_1, \omega_2) := \lambda i.\forall w(\exists w'(w' \in \omega_1 iw) \rightarrow \\ \forall w'(w' \in \omega_1 iw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 iw \wedge \}w'' \wedge w'' –_w w' \rightarrow w' –_w w'') \\ \rightarrow w' \in \omega_2 iw'))\end{aligned}$$

(For all $w \in \text{Dom}\ \omega_1 i$, every }-accessible –-best world w' in $\omega_1 iw$ is in $\omega_2 iw'$.)

$$\begin{aligned}\mathbf{poss}_{\}, –}(\omega_1, \omega_2) := \lambda i.\forall w(\exists w'(w' \in \omega_1 iw) \rightarrow \\ \exists w'(w' \in \omega_1 iw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 iw \wedge \}w'' \wedge w'' –_w w' \rightarrow w' –_w w'') \\ \wedge w' \in \omega_2 iw'))\end{aligned}$$

(For all $w \in \text{Dom}\ \omega_1 i$, some }-accessible –-best world w' in $\omega_1 iw$ is in $\omega_2 iw'$.)

$$\begin{aligned}\mathbf{not}_{\}, –}(\omega_1, \omega_2) := \lambda i.\forall w(\exists w'(w' \in \omega_1 iw) \rightarrow \\ \neg\exists w'(w' \in \omega_1 iw \wedge \}w' \wedge \forall w''(w'' \in \omega_1 iw \wedge \}w'' \wedge w'' –_w w' \rightarrow w' –_w w'') \\ \wedge w' \in \omega_2 iw'))\end{aligned}$$

(For all $w \in \text{Dom}\ \omega_1 i$, no }-accessible –-best world w' in $\omega_1 iw$ is in $\omega_2 iw'$.)

These definitions of modal relations do not match exactly any of the versions given in Stone (1997), Stone (1999) or Stone & Hardt (1997). Each of Stone's versions seems to me right in some respects and wrong in others, and this is my best attempt to put together the pieces that seem right in a way that makes sense — for the examples in this handout, the predictions in the tables are based on these put-together definitions.

Three points in particular are crucial for these predictions:

(1) w must be restricted to the domain of $\omega_1 i$ (1st line of each definition, as in Stone 1999); outside of this domain, $\omega_1 iw = \varnothing$ so the test for $\mathbf{poss}_{\}, –}(\omega_1, \omega_2)$ could never be passed.

(2) w' must satisfy the consequent (represented by ω_2), but it need not be a *w-closest* world that does so — formally, $w' \in \omega_2 iw'$ (3rd line, as in Stone & Hardt 1997), not $w' \in \omega_2 iw$ (contra Stone 1997, 1999).

(3) in the definition of possibility, we need universal quantification over w (as in Stone 1997, 1999), not existential (contra Stone & Hardt 1997), lest this test be too weak.