

## (Dynamic) Hamblin semantics

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### 1 FROM ENGLISH QUESTIONS TO Ty2 (Hamblin 1973)

- TWO-SORTED TYPE THEORY (Ty2): Basic terms

$a \in Typ$	$Var_a$	$Con_a$	Name of objects
$s$	$i, j$		indices (worlds)
$st$	$p, q$		propositions (sets of worlds)
$(st)t$	$Q$		sets of propositions ( $p$ -sets)
$e$	$x, y, z$	$al, bill, \dots$	entities
$set$		$man, sick, \dots$	relations from worlds to entities
$seet$		$like, \dots$	relations from worlds to pairs of entities
$est$	$P$		iv-denotations
$eest$	$R$		tv-denotations

- ASSERTION

(1) Al is\_sick.

(1') Hamblin 1973: Sec. 1: indicative  $\rightsquigarrow$  *proposition*

cat	phrase	Ty2-translation	type
pn	Al	$al$	$e$
iv	is_sick	$\lambda x \lambda i. sick(i, x)$	$est$
s	(1)	$\lambda i. sick(i, al)$	$st$

(1'') Hamblin 1973: Sec. 8: indicative  $\rightsquigarrow$  *SG-set of propositions*

cat	phrase	Ty2-translation	type
pn	Al	$\lambda x. x = al$ (=: pn')	$et$
iv	is_sick	$\lambda P. P = \lambda x \lambda i. sick(i, x)$ (=: iv')	$(est)t$
s	(1)	$\lambda p. \exists x \exists P (pn'(x) \wedge iv'(P) \wedge p = P(x))$ = $\lambda p. p = \lambda i. sick(i, al)$	$(st)t$

(2) Al likes Sue.

(2'')

cat	phrase	Ty2-translation	type
pn	Sue	$\lambda x. x = sue$ (=: pn')	$et$
tv	likes	$\lambda R. R = \lambda y \lambda x \lambda i. like(i, x, y)$ (=: tv')	$(eest)t$
iv	likes_S.	$\lambda P. \exists y \exists R (pn'(y) \wedge tv'(R) \wedge P = R(y))$ = $\lambda P. P = \lambda x \lambda i. like(i, x, sue)$	$(est)t$
pn	Al	$\lambda x. x = al$	$et$
s	(2)	$\lambda p. p = \lambda i. like(i, al, sue)$	

• YN QUESTION: Hamblin 1973: Sec. 8: yn que  $\rightsquigarrow$  DU-set of propositions

(3) Is\_it\_the\_case\_that (=: YN) Al is\_sick?

(3'')

<u>cat</u>	<u>phrase</u>	<u>Ty2-translation</u>		<u>type</u>
pn	Al	$\lambda x. x = al$	(=: pn')	<i>et</i>
iv	is_sick	$\lambda P. P = \lambda x \lambda i. sick(i, x)$	(=: iv')	<i>(est)t</i>
s	(1)	$\lambda p. \exists x \exists P (pn'(x) \wedge iv'(P) \wedge p = P(x))$ $= \lambda p. p = \lambda i. sick(i, al)$		<i>(st)t</i>
s/s	YN	$\lambda Q \lambda p. \exists r (Q(r) \wedge (p = r \vee p = \lambda i. \neg r(i)))$		<i>((st)t)(st)t</i>
s	(3)	$\lambda p. p = \lambda i. sick(i, al)$ $\vee p = \lambda i. \neg sick(i, al)$		<i>(st)t</i>

• WH QUESTIONS: Hamblin 1973: Sec. 8: wh que  $\rightsquigarrow$  PL-set of propositions

(4) Who is sick?

(4'')

<u>cat</u>	<u>phrase</u>	<u>Ty2-translation</u>		<u>type</u>
pn	who	$\lambda x. person(i, x)$	(=: pn')	<i>et</i>
iv	is_sick	$\lambda P. P = \lambda x \lambda j. sick(j, x)$	(=: iv')	<i>(est)t</i>
s	(4)	$\lambda p. \exists x \exists P (pn'(x) \wedge iv'(P) \wedge p = P(x))$ $= \lambda p. \exists x (person(i, x) \wedge p = \lambda j. sick(j, x))$		<i>(st)t</i>

(5) Who does Al like?

(5'')

<u>cat</u>	<u>phrase</u>	<u>Ty2-translation</u>		<u>type</u>
pn	who	$\lambda x. person(i, x)$	(=: pn')	<i>et</i>
tv	likes	$\lambda R. R = \lambda y \lambda x \lambda j. like(j, x, y)$	(=: tv')	<i>(eest)t</i>
iv	likes_wh	$\lambda P. \exists y \exists R (pn'(y) \wedge tv'(R) \wedge P = R(y))$ $= \lambda P. \exists y (person(i, y) \wedge P = \lambda x \lambda j. like(j, x, y))$		<i>(est)t</i>
pn	Al	$\lambda x. x = al$		<i>et</i>
s	(5)	$\lambda p. \exists y (person(i, y) \wedge p = \lambda j. like(j, al, y))$		

(6) Who likes whom?

(6'')

<u>cat</u>	<u>phrase</u>	<u>Ty2-translation</u>		<u>type</u>
pn	whom	$\lambda x. person(i, x)$	(=: pn')	<i>et</i>
tv	likes	$\lambda R. R = \lambda y \lambda x \lambda j. like(j, x, y)$	(=: tv')	<i>(eest)t</i>
iv	likes wh	$\lambda P. \exists y (person(i, y) \wedge P = \lambda x \lambda j. like(j, x, y))$		<i>(est)t</i>
pn	who	$\lambda x. person(i, x)$		<i>et</i>
s	(6)	$\lambda p. \exists x \exists y (person(i, x) \wedge person(i, y)$ $\wedge p = \lambda j. like(j, x, y))$		<i>(st)t</i>

## 2 FROM ENGLISH QUESTIONS TO UC2

## • TWO-SORTED UPDATE WITH CENTERING (UC2): Basic terms

$a \in Typ$	$\uparrow Var_a$	$\downarrow Var_a$	$Con_a$	Name of objects
$s$		$i, j$		indices ( $\tau\perp$ -lists)
$st$		$I, J$		infotention states
$e$	$\mathbf{x}$	$y$	$al, bill, \dots$	entities
$\omega$	$\mathbf{w}$	$v$		worlds
$\omega t := \Omega$	$\mathbf{p}$	$q$		propositions (sets of worlds)
$\Omega t$	$\mathbf{Q}$	$Q$		sets of propositions
$\omega et$			$man, sick, \dots$	$\omega et$ -relations
$\omega eet$			$like, \dots$	$\omega eet$ -relations
$se$			$\mathbf{de}_n, de_n$	$e$ -projections
$s\omega$			$\mathbf{d}\omega_n, d\omega_n$	$\omega$ -projections
$s\Omega$			$\mathbf{d}\Omega_n, d\Omega_n$	$\Omega$ -projections

## • DEFAULT CENTERING

$$*p_0 := \{\langle\langle w, p_0 \rangle, \langle \rangle\rangle : w \in p_0\}$$

## • ASSERTION

(1) Al be.IND sick . (prosody)  
 UC2  $[x | (x = al)^\circ]$ ;  $P[\mathbf{d}\omega \in \mathbf{d}\omega \{\}]; [sick(\mathbf{d}\omega, \mathbf{de})]$ ;  $[\mathbf{p}]$ ;  $[\mathbf{d}\Omega = \mathbf{d}\omega \{\}]$

Ex1:

$$p_0 = \{w_0, w_1\} =:_{01} \quad \langle w_0, [al] \rangle \in \mathcal{B}[sick]$$

$$\langle w_1, [al] \rangle \notin \mathcal{B}[sick]$$

\*<sub>(01)</sub>

$$\langle\langle w_0, 01 \rangle, \langle \rangle\rangle$$

$$\langle\langle w_1, 01 \rangle, \langle \rangle\rangle$$

Al be.IND sick . (prosody)  
 $[x | (x = al)^\circ]$ ;  $P[\mathbf{d}\omega \in \mathbf{d}\omega \{\}]; [sick(\mathbf{d}\omega, \mathbf{de})]$ ;  $[\mathbf{p}]$ ;  $[\mathbf{d}\Omega = \mathbf{d}\omega \{\}]$

$$c_1 \quad c_1 \quad c_2 \quad c_3 \quad (c_0 := \{w_0\})$$

$$\langle\langle [al], w_0, 01 \rangle, \langle \rangle\rangle \quad \langle\langle [al], w_0, 01 \rangle, \langle \rangle\rangle \quad \langle\langle c_0, [al], w_0, 01 \rangle, \langle \rangle\rangle$$

$$\langle\langle [al], w_1, 01 \rangle, \langle \rangle\rangle$$

(2) Al like- -IND Sue  
 $[x | (x = al)^\circ]$ ;  $[y | like(\mathbf{d}\omega, \mathbf{de}, y)]$ ;  $P[\mathbf{d}\omega \in \mathbf{d}\omega \{\}]; [(de = sue)^\circ]$ ;  
 . (prosody)  
 $[\mathbf{p}]$ ;  $[\mathbf{d}\Omega = \mathbf{d}\omega \{\}]$



\*<sub>(0-3)</sub> $\langle\langle w_{0, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle w_{1, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle w_{2, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle w_{3, 0-3} \rangle, \langle \rangle\rangle$ 

who

 $[x | person \{\omega, x\}]$ ; $C_1$  $\langle\langle [aI], w_{0, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [aI], w_{1, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [aI], w_{2, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [aI], w_{3, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [sue], w_{0, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [sue], w_{1, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [sue], w_{2, 0-3} \rangle, \langle \rangle\rangle$  $\langle\langle [sue], w_{3, 0-3} \rangle, \langle \rangle\rangle$ 

be.IND

 $P[w | w \in d\omega \{\}]$ ; $C_2$  $\langle\langle [aI], w_{0, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [aI], w_{0, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [aI], w_{1, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [aI], w_{1, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [aI], w_{2, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [aI], w_{2, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [aI], w_{3, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [aI], w_{3, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [sue], w_{0, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [sue], w_{0, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [sue], w_{1, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [sue], w_{1, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [sue], w_{2, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [sue], w_{2, 0-3} \rangle, \langle w_3 \rangle\rangle$  $\langle\langle [sue], w_{3, 0-3} \rangle, \langle w_0 \rangle\rangle$ 

⋮

 $\langle\langle [sue], w_{3, 0-3} \rangle, \langle w_3 \rangle\rangle$ 

sick

 $[sick \langle d\omega, de \rangle]$ ; $C_3$  $\langle\langle [aI], w_{0, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [aI], w_{0, 0-3} \rangle, \langle w_1 \rangle\rangle$  $\langle\langle [aI], w_{1, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [aI], w_{1, 0-3} \rangle, \langle w_1 \rangle\rangle$  $\langle\langle [aI], w_{2, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [aI], w_{2, 0-3} \rangle, \langle w_1 \rangle\rangle$  $\langle\langle [aI], w_{3, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [aI], w_{3, 0-3} \rangle, \langle w_1 \rangle\rangle$  $\langle\langle [sue], w_{0, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [sue], w_{0, 0-3} \rangle, \langle w_2 \rangle\rangle$  $\langle\langle [sue], w_{1, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [sue], w_{1, 0-3} \rangle, \langle w_2 \rangle\rangle$  $\langle\langle [sue], w_{2, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [sue], w_{2, 0-3} \rangle, \langle w_2 \rangle\rangle$  $\langle\langle [sue], w_{3, 0-3} \rangle, \langle w_0 \rangle\rangle$  $\langle\langle [sue], w_{3, 0-3} \rangle, \langle w_2 \rangle\rangle$

? (prosody)

$[q]; [d\Omega = d\omega \{ |_{de} \}];$

$[\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{ | \} ]$

$C_4$

$\langle\langle [al], w_{0, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle [al], w_{0, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle [al], w_{1, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle [al], w_{1, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle [al], w_{2, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle [al], w_{2, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle [al], w_{3, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle [al], w_{3, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle [sue], w_{0, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle [sue], w_{0, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle [sue], w_{1, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle [sue], w_{1, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle [sue], w_{2, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle [sue], w_{2, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle [sue], w_{3, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle [sue], w_{3, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$C_5$

$\langle\langle \{01, 02\}, [al], w_{0, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{0, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{1, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{1, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{2, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{2, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{3, 0-3}, \langle 01, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [al], w_{3, 0-3}, \langle 01, w_1 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{0, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{0, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{1, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{1, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{2, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{2, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{3, 0-3}, \langle 02, w_0 \rangle \rangle\rangle$

$\langle\langle \{01, 02\}, [sue], w_{3, 0-3}, \langle 02, w_2 \rangle \rangle\rangle$

where  $_{01} := \{w_0, w_1\} = \{v \in p_0 | \langle v, [al] \rangle \in \mathfrak{B}[sick]\}$

$_{02} := \{w_0, w_2\} = \{v \in p_0 | \langle v, [sue] \rangle \in \mathfrak{B}[sick]\}$

(5) who(m) do.IND Al like  
 $[y | person \{ \omega, y \}]; {}^P[w | w \in \mathbf{d}\omega \{ | \}]; [x | x = al]; [like \langle d\omega, \mathbf{de}, de \rangle]$   
 ? (prosody)  
 $[q]; [d\Omega = d\omega \{ |_{de} \}]; [\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{ | \} ]$

(6) who like- -IND  
 $[x | person \{ \omega, x \}]; [w y | like \langle w, \mathbf{de}, y \rangle]; {}^P[d\omega \in \mathbf{d}\omega \{ | \}];$   
 who(m) ? (prosody)  
 $[person \{ \omega, de \}]; [q]; [d\Omega = d\omega \{ |_{de, de} \}]; [\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{ | \} ]$

## 3 FROM ENGLISH QUESTIONS TO UC (cf. Bittner 2008)

## • UPDATE WITH CENTERING (UC, 6-sorted): Basic terms

$a \in Typ$	$\top Var_a$	$\perp Var_a$	$Con_a$	Name of objects
$s$		$i, j$		indices ( $\top\perp$ -lists)
$st$		$I, J$		infotention states
$\alpha$	<b>a</b>	$a$	<i>john</i>	animate (entities)
$\beta$	<b>b</b>	$b$	<i>John</i>	inanimate (entities)
$\varepsilon$	<b>e</b>	$e$		events
$\sigma$	<b>s</b>	$s$		states (of entities)
$\tau$	<b>t</b>	$t$		times
$\omega$	<b>w, v</b>	$w, v$		worlds
$\omega\varepsilon$	<b>e</b>	$e$		event concepts
$\omega t =: \Omega$	<b>p, q</b>	$p, q$		propositions (sets of worlds)
$\Omega t$	<b>Q</b>	$Q$		sets of propositions
$\omega\alpha t$			<i>person, ...</i>	$\omega\alpha$ -relations
$\omega\sigma\alpha t$			<i>sick, ...</i>	$\omega\sigma\alpha$ -relations
$\omega\sigma\alpha\alpha t$			<i>like, ...</i>	$\omega\sigma\alpha\alpha$ -relations
$\varepsilon\alpha$			AGT	$\varepsilon$ -dependent animates
$\varepsilon\sigma$			CON	$\varepsilon$ -dependent states
$\sigma\varepsilon$			BEG, END	$\sigma$ -dependent events
$sa$			$\mathbf{da}_n, \mathbf{da}_n, \dots$	a-projections ( $a \in DTyp$ )

## • DEFAULT CENTERING

$$*\langle p_0, e_0 \rangle := \{ \langle \langle t, w, p_0, e_0 \rangle, \langle \underline{e} \rangle \rangle \mid w \in \mathfrak{P}_{p_0} \wedge t = \llbracket \vartheta \rrbracket(w, e_0) \wedge \underline{e} = \{ \langle v, e_0 \rangle \mid v \in \mathfrak{P}_{p_0} \} \}$$

$$\top_w \in \top_{p_0} \quad \bullet \quad \begin{array}{l} e_0 = \underline{e}_0(w): e_0\text{-agt speaks up} \\ \mid \\ \llbracket \vartheta \rrbracket(w, e_0): e_0\text{-instant} \end{array}$$

## • ASSERTION

$$(1) \quad \text{Al} \quad \text{be-} \quad \text{-IND.}$$

$$\text{UC} \quad [\mathbf{a} \mid (\mathbf{a} = \mathbf{al})^\circ]; [s \mid (\text{EXP } s = \text{CTR } s)^\circ]; \text{P}[\mathbf{d}\omega \in \mathbf{d}\omega \{ \}];$$

$$\text{.NPST}$$

$$\text{P}[(\vartheta(\mathbf{d}\omega, \mathbf{d}\varepsilon) \leq \mathbf{d}\tau)^\circ]; [\text{C}\langle \mathbf{d}\omega, \mathbf{d}\sigma, \mathbf{d}\tau \rangle, (\text{CTR } \mathbf{d}\sigma = \mathbf{d}\alpha)^\circ];$$

$$\text{sick} \quad \cdot \quad (\text{prosody})$$

$$[\text{sick}\langle \mathbf{d}\omega, \mathbf{d}\sigma: \text{EXP} \rangle]; [\mathbf{p}]; [\mathbf{d}\Omega = \mathbf{d}\omega \{ \} ]$$

$$\top_w \in \top_{p_1} \subseteq p_0 \quad \bullet \quad \begin{array}{l} e_0 = \underline{e}_0(w): e_0\text{-agt updates CG to } p_1 \subseteq p_0 \\ \mid \\ \llbracket \vartheta \rrbracket(w, e_0): e_0\text{-instant} \\ \text{-----} \\ s_1: \text{Al is sick} \end{array}$$

(2) Al like- -IND.  
 UC [a | (a = al)<sup>o</sup>]; [s a | like⟨dω, EXP, a⟩, (EXP s = CTR s)<sup>o</sup>]; <sup>P</sup>[(dω ∈ dΩ)<sup>o</sup>]  
 .NPST  
 ; <sup>P</sup>[(ϑ(dω, dε) ≤ dτ)<sup>o</sup>]; [C⟨dω, dσ, dτ⟩, (CTR dσ = dα)<sup>o</sup>];  
 Sue . (prosody)  
 [(dα = sue)<sup>o</sup>]; [p]; [dΩ = dω {}]

## • YN QUESTION

(3) be- -IND.  
 UC [s | (EXP s = CTR s)<sup>o</sup>]; <sup>P</sup>[w | w ∈ dω {}];  
 .NPST  
<sup>P</sup>[(ϑ(dω, dε) ≤ dτ)<sup>o</sup>]; [C⟨dω, dσ, dτ⟩]; [a | (CTR dσ = a)<sup>o</sup>];  
 Al sick  
 [(dα = al)<sup>o</sup>]; [sick⟨dω, dσ: EXP⟩];  
 ? (prosody)  
 [q]; [dΩ = dω {}]; [Q]; [dΩt = dΩ {}]

<sup>T</sup>w ∈ <sup>T</sup>p<sub>0</sub> • e<sub>0</sub> = e<sub>0</sub>(w): e<sub>0</sub>-agt introduces <sup>T</sup>Q<sub>1</sub> = {p<sub>1</sub>}  
 (asks for which q ∈ <sup>T</sup>Q<sub>1</sub>, w ∈ q)  
 | ||ϑ|| (w, e<sub>0</sub>): e<sub>0</sub>-instant  
 ~~~~~  
 v ∈ p<sub>1</sub> \_\_\_\_\_ yes-answer to e<sub>0</sub>-question <sup>T</sup>Q<sub>1</sub>  
 s<sub>1</sub>: Al is sick

## • WH QUESTION

(4) who be- -IND.  
 UC [a | person {ω, a}]; [s | (EXP s = CTR s)<sup>o</sup>]; <sup>P</sup>[w | w ∈ dω {}];  
 .NPST  
<sup>P</sup>[(ϑ(dω, dε) ≤ dτ)<sup>o</sup>]; [C⟨dω, dσ, dτ⟩]; [(CTR dσ = dα)<sup>o</sup>];  
 sick ? (prosody)  
 [sick⟨dω, dσ: EXP⟩]; [q]; [dΩ = dω {|d<sub>α</sub>}]; [Q]; [dΩt = dΩ {}]

(6) who  
 UC [a | person {ω, a}];  
 like- -IND.  
 [s w a | like⟨w, CTR, a⟩ (EXP s = CTR s)<sup>o</sup>]; <sup>P</sup>[(dω ∈ dΩ)<sup>o</sup>];  
 .NPST  
<sup>P</sup>[(ϑ(dω, dε) ≤ dτ)<sup>o</sup>]; [C⟨dω, dσ, dτ⟩, (CTR dσ = dα)<sup>o</sup>];  
 who(m) ? (prosody)  
 [person {ω, dα}]; [q]; [dΩ = dω {|d<sub>α</sub>, d<sub>α</sub>}]; [Q]; [dΩt = dΩ {}]

## 4 FROM KALAALLISUT QUESTIONS TO UC (Bittner 2008)

## • ASSERTION

(1K) *Ole naparsima-pu-q.*

Ole sick-DEC.IV-3SG

Ole is sick.

Ole sick-

UC  $[a | (a = ole)^\circ]; [s | sick\langle d\omega, s: EXP \rangle, (EXP s = CTR s)^\circ];$ 

.DEC.IV-3SG

 $^P[d\omega\varepsilon =_\omega d\varepsilon]; [C\langle d\omega, d\sigma, d\tau \rangle]; [BEG d\sigma \leq_{\theta d\omega} d\omega\varepsilon, (CTR d\sigma = d\alpha)^\circ];$  $[p]; [d\Omega = d\omega \{\}]$  ${}^T w \in {}^T p_1 \subseteq p_0$ 

•

 $e_0 = \underline{e}_0(w): e_0\text{-agt updates CG to } p_1 \subseteq p_0$  $\llbracket \vartheta \rrbracket(w, e_0): e_0\text{-instant}$  $s_1: \text{Ole is sick}$ (2K) *Siut-nngu-pu-nga.*

ear-ache-DEC.IV-1SG

I have an earache.

ear- -ache

UC  $[b | ear\langle d\omega, b \rangle]; [s | feel.pain.in\langle d\omega, s: EXP: d\beta \rangle (EXP s = CTR s)^\circ];$ 

.DEC.IV-1SG

 $^P[d\omega\varepsilon =_\omega d\varepsilon]; [C\langle d\omega, d\sigma, d\tau \rangle]; [BEG d\sigma \leq_{\theta d\omega} d\omega\varepsilon, (CTR d\sigma = AGT d\varepsilon)^\circ]$  $; [p]; [d\Omega = d\omega \{\}]$ 

## • YN QUESTION

(3K) *Ole naparsima-pi-a?*

Ole sick-QUE-3SG

Is Ole sick?

Ole sick-

UC  $[a | (a = ole)^\circ]; [s w | sick\langle w, s: EXP \rangle, (EXP s = CTR s)^\circ];$ 

-QUE-3SG

 $^P[d\omega\varepsilon =_\omega d\varepsilon]; [C\langle d\omega, d\sigma, d\tau \rangle]; [BEG d\sigma \leq_{\theta d\omega} d\omega\varepsilon, (CTR d\sigma = d\alpha)^\circ];$  $[q]; [d\Omega = d\omega \{\}]; [Q]; [d\Omega t = d\Omega \{\}]; [ask\langle \omega, d\omega\varepsilon: AGT: d\Omega t \rangle]$  ${}^T w \in {}^T p_0$ 

•

 $e_0 = \underline{e}_0(w): e_0\text{-agt asks question } {}^T \{p_1\}$  $\llbracket \vartheta \rrbracket(w, e_0): e_0\text{-instant}$  $v \in p_1$ yes-answer to  $e_0\text{-question } {}^T \{p_1\}$  $s_1: \text{Ole is sick}$

## • WH QUESTIONS

(4K) *Kina naparsimava?*  
*kina naparsima-pi-a*  
 who.SG sick-QUE-3SG  
 Who is sick?

who.SG sick-  
 UC [a] *person* { $\omega$ , **a**}; [*s w* | *sick*⟨*w*, *s*: EXP⟩, (EXP *s* = CTR *s*)<sup>o</sup>];  
 -QUE-3SG  
<sup>P</sup>[*d* $\omega$  $\epsilon$  =  $\omega$  **d** $\epsilon$ ]; [**C**⟨*d* $\omega$ , *d* $\sigma$ , **d** $\tau$ ⟩]; [BEG *d* $\sigma$   $\leq_{\partial d\omega}$  *d* $\omega$  $\epsilon$ , (CTR *d* $\sigma$  = **d** $\alpha$ )<sup>o</sup>];  
 [*q*]; [*d* $\Omega$  = *d* $\omega$  {**d** $\alpha$ }]; [**Q**]; [**d** $\Omega$ *t* = *d* $\Omega$  {**|**}]; [*ask*⟨ $\omega$ , *d* $\omega$  $\epsilon$ : AGT: **d** $\Omega$ *t*⟩]

(5K) *Ole sunnguva?*  
*Ole su-nngu-pi-a*  
 Ole what-ache-QUE-3SG  
 Where is Ole hurting?

Ole  
 UC [a] (**a** = *ole*)<sup>o</sup>];  
 what- -ache  
 [*b*]; [*s w* | *feel.pain.in*⟨*w*, *s*: EXP: *d* $\beta$ ⟩, (EXP *s* = CTR *s*)<sup>o</sup>];  
 -QUE-3SG  
<sup>P</sup>[*d* $\omega$  $\epsilon$  =  $\omega$  **d** $\epsilon$ ]; [**C**⟨*d* $\omega$ , *d* $\sigma$ , **d** $\tau$ ⟩]; [BEG *d* $\sigma$   $\leq_{\partial d\omega}$  *d* $\omega$  $\epsilon$ , (CTR *d* $\sigma$  = **d** $\alpha$ )<sup>o</sup>];  
 [*q*]; [*d* $\Omega$  = *d* $\omega$  {**d** $\beta$ }]; [**Q**]; [**d** $\Omega$ *t* = *d* $\Omega$  {**|**}]; [*ask*⟨ $\omega$ , *d* $\omega$  $\epsilon$ : AGT: **d** $\Omega$ *t*⟩]

(6K) *Kina sunnguva?*  
*kina su-nngu-pi-a*  
 who.SG what-ache-QUE-3SG  
 Who is hurting where?

who.SG  
 UC [a] *person* { $\omega$ , **a**};  
 what- -ache  
 [*b*]; [*s w* | *feel.pain.in*⟨*w*, *s*: EXP: *d* $\beta$ ⟩, (EXP *s* = CTR *s*)<sup>o</sup>];  
 -QUE-3SG  
<sup>P</sup>[*d* $\omega$  $\epsilon$  =  $\omega$  **d** $\epsilon$ ]; [**C**⟨*d* $\omega$ , *d* $\sigma$ , **d** $\tau$ ⟩]; [BEG *d* $\sigma$   $\leq_{\partial d\omega}$  *d* $\omega$  $\epsilon$ , (CTR *d* $\sigma$  = **d** $\alpha$ )<sup>o</sup>];  
 [*q*]; [*d* $\Omega$  = *d* $\omega$  {**d** $\alpha$ , **d** $\beta$ }]; [**Q**]; [**d** $\Omega$ *t* = *d* $\Omega$  {**|**}]; [*ask*⟨ $\omega$ , *d* $\omega$  $\epsilon$ : AGT: **d** $\Omega$ *t*⟩]



## Questions: Partitions, answer sets, or both?

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### 1 QUESTIONS AS PARTITIONS (GS 1982, 1984)

- INFERENCES FROM EMBEDDED QUESTIONS

- (I) Ed knows whether Al is sick. (factive)  $iv/s + YN-s$   
Al is sick.  
 Ed knows that Al is sick.
- (II) Ed knows whether Al is sick.  
Al is not sick.  
 Ed knows that Al is not sick.
- (III) Ed told Bill whether Al is sick. (non-factive)  $iv/s + YN-s$   
Al is sick.  
 Ed told Bill that Al is sick.
- (IV) Ed told Bill whether Al is sick.  
Al is not sick.  
 Ed told Bill that Al is not sick.
- (V) Ed knows who is sick.  $iv/s + WH-s$   
(Only) Al is sick.  
 Ed knows that (only) Al is sick.
- (VII) Ed knows who likes whom.  
Bill likes Al (and that's the full answer).  
 Ed knows that Bill likes Al (and that's the full answer).

- BASIC IDEA: To know  $p$  is to know any entailment of  $p$  (ditto for other attitude and speech reports.) In any world, an (embedded) question denotes the *exhaustive answer*, which entails all of the related *that*-complements.

- TWO-SORTED TYPE THEORY (Ty2): Basic terms

| $a \in Typ$ | $Var_a$   | $Con_a$               | <u>Name of objects</u>                     |
|-------------|-----------|-----------------------|--------------------------------------------|
| $s$         | $i, j, k$ |                       | indices (worlds)                           |
| $st$        | $p, q$    |                       | propositions (sets of worlds)              |
| $(st)t$     | $Q$       |                       | sets of propositions ( $p$ -sets)          |
| $e$         | $x, y, z$ | $al, bill, ed, \dots$ | entities                                   |
| $set$       |           | $man, sick, \dots$    | relations from worlds to entities          |
| $seet$      |           | $like, \dots$         | relations from worlds to pairs of entities |

• (EMBEDDED) QUESTIONS AS PARTITIONS (minus inessential complexities)<sup>1</sup>

| cat      | phrase             | Ty2-translation                                            | type      |
|----------|--------------------|------------------------------------------------------------|-----------|
| <u>s</u> | that AI is sick    | $\lambda i[sick(i, al)]$                                   | <i>st</i> |
| <u>s</u> | whether AI is sick | $\lambda j[sick(j, al) = sick(i, al)]$                     | <i>st</i> |
| <u>s</u> | who is sick        | $\lambda j[\lambda x. sick(j, x) = \lambda x. sick(i, x)]$ | <i>st</i> |

Terminology

A *partition* of a set  $A$  is a family  $B$  of non-empty pairwise disjoint subsets of  $A$  such that  $\cup B = A$ .

Ex1:

|                                                               |                                                                                              |                                                                                               |
|---------------------------------------------------------------|----------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
| $D_e = \{\llbracket a \rrbracket, \llbracket ed \rrbracket\}$ | $\langle w_0, \llbracket a \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket$    | $\langle w_0, \llbracket ed \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket$    |
| $D_s = \{w_0, w_1, w_2, w_3\}$                                | $\langle w_1, \llbracket a \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket$    | $\langle w_1, \llbracket ed \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket$ |
|                                                               | $\langle w_2, \llbracket a \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket$ | $\langle w_2, \llbracket ed \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket$    |
|                                                               | $\langle w_3, \llbracket a \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket$ | $\langle w_3, \llbracket ed \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket$ |

- $\llbracket \lambda i. sick(i, al) \rrbracket^g = \times\{w_0, w_1\}$
- $\llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^{g[i/w_0]}$   
 $= \times\{v \in D_s \mid \llbracket sick \rrbracket(v, \llbracket a \rrbracket) = \llbracket sick \rrbracket(w_0, \llbracket a \rrbracket)\}$   
 $= \llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^{g[i/w_1]} = \times\{w_0, w_1\}$
- $\llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^{g[i/w_2]}$   
 $= \llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^{g[i/w_3]} = \times\{w_2, w_3\}$
- $\llbracket \lambda j[\lambda x. sick(j, x) = \lambda x. sick(i, al)] \rrbracket^{g[i/w_0]}$   
 $= \times\{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\} = \{d \mid \langle w_0, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\}\} = \times\{w_0\}$
- $\llbracket \lambda j[\lambda x. sick(j, x) = \lambda x. sick(i, al)] \rrbracket^{g[i/w_1]}$   
 $= \times\{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\} = \{d \mid \langle w_1, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\}\} = \times\{w_1\}$
- $\llbracket \lambda j[\lambda x. sick(j, x) = \lambda x. sick(i, al)] \rrbracket^{g[i/w_2]}$   
 $= \times\{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\} = \{d \mid \langle w_2, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\}\} = \times\{w_2\}$
- $\llbracket \lambda j[\lambda x. sick(j, x) = \lambda x. sick(i, al)] \rrbracket^{g[i/w_3]}$   
 $= \times\{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\} = \{d \mid \langle w_3, d \rangle \in \mathfrak{B}\llbracket sick \rrbracket\}\} = \times\{w_3\}$

<sup>1</sup> Following Montague 1973, GS '82 assume cat-to-type correspondence, so all phrases of a given category translate into the highest type instantiated for that category, e.g. 'know' is first translated into *know'* of type *s(sst)(se)t* (*sst* needed for 'wonder', *se* for 'is rising') and then simplified to *know'\** of type *s(st)et* by a meaning postulate. Based on cross-linguistic evidence, I give up cat-to-type correspondence and build meaning postulates into the basic meaning assignment—e.g. my *know* corresponds to GS *know'\**.

## 2 FROM PARTITIONS TO EXHAUSTIVITY INFERENCES

## • YN-COMPLEMENTS

(I) Ed knows whether Al is sick.

Al is sick.  
Ed knows that Al is sick.

Ty2  $\frac{\text{know}(i, ed, \lambda j[\text{sick}(j, al) = \text{sick}(i, al)])}{\text{sick}(i, al)}$   
 $\text{know}(i, ed, \lambda j[\text{sick}(j, al)])$

PROOF. If (1) &amp; (2), then (7).

1.  $\llbracket \text{know}(i, ed, \lambda j[\text{sick}(j, al) = \text{sick}(i, al)]) \rrbracket^g = 1$  premise 1
2.  $\llbracket \text{sick}(i, al) \rrbracket^g = 1$  premise 2
3.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^g) = 1$  (1), Ty2
4.  $\llbracket \text{sick} \rrbracket(g(i), \llbracket al \rrbracket) = 1$  (2), Ty2
5.  $\llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^g$   
 $= \lambda \{v \in D_s \mid \llbracket \text{sick} \rrbracket(v, \llbracket al \rrbracket) = 1\}$  (4), Ty2  
 $= \llbracket \lambda j[\text{sick}(j, al)] \rrbracket^g$  Ty2
6.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\text{sick}(j, al)] \rrbracket^g) = 1$  (3), (5)
7.  $\llbracket \text{know}(i, ed, \lambda j[\text{sick}(j, al)]) \rrbracket^g = 1$  Ty2

(II) Ed knows whether Al is sick.

Al is not sick.  
Ed knows that Al is not sick.

Ty2  $\frac{\text{know}(i, ed, \lambda j[\text{sick}(j, al) = \text{sick}(i, al)])}{\neg \text{sick}(i, al)}$   
 $\text{know}(i, ed, \lambda j[\neg \text{sick}(j, al)])$

PROOF. If (1) &amp; (2), then (7).

1.  $\llbracket \text{know}(i, ed, \lambda j[\text{sick}(j, al) = \text{sick}(i, al)]) \rrbracket^g = 1$  premise 1
2.  $\llbracket \neg \text{sick}(i, al) \rrbracket^g = 1$  premise 2
3.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^g) = 1$  (1), Ty2
4.  $\llbracket \text{sick} \rrbracket(g(i), \llbracket al \rrbracket) = 0$  (2), Ty2
5.  $\llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^g$   
 $= \lambda \{v \in D_s \mid \llbracket \text{sick} \rrbracket(v, \llbracket al \rrbracket) = 0\}$  (4), Ty2  
 $= \llbracket \lambda j[\neg \text{sick}(j, al)] \rrbracket^g$  Ty2
6.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\neg \text{sick}(j, al)] \rrbracket^g) = 1$  (3), (5)
7.  $\llbracket \text{know}(i, ed, \lambda j[\neg \text{sick}(j, al)]) \rrbracket^g = 1$  Ty2

**(III)** Ed told Bill whether Al is sick.

Al is sick.

Ed told Bill that Al is sick.

Ty2  $tell(i, ed, bill, \lambda j[sick(j, al) = sick(i, al)])$   
 $sick(i, al)$   
 $tell(i, ed, bill, \lambda j[sick(j, al)])$

PROOF (as for **(I)**) If (1) & (2), then (7).

1.  $\llbracket tell(i, ed, bill, \lambda j[sick(j, al) = sick(i, al)]) \rrbracket^g = 1$  premise 1
2.  $\llbracket sick(i, al) \rrbracket^g = 1$  premise 2
3.  $\llbracket tel \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket bill \rrbracket, \llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^g) = 1$  (1), Ty2
4.  $\llbracket sick \rrbracket(g(i), \llbracket al \rrbracket) = 1$  (2), Ty2
5.  $\llbracket \lambda j[sick(j, al) = sick(i, al)] \rrbracket^g$   
 $= \lambda \{v \in D_s \mid \llbracket sick \rrbracket(v, \llbracket al \rrbracket) = 1\}$  (4), Ty2  
 $= \llbracket \lambda j[sick(j, al)] \rrbracket^g$  Ty2
6.  $\llbracket tel \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket bill \rrbracket, \llbracket \lambda j[sick(j, al)] \rrbracket^g) = 1$  (3), (5)
7.  $\llbracket tell(i, ed, bill, \lambda j[sick(j, al)]) \rrbracket^g = 1$  Ty2

**(IV)** Ed told Bill whether Al is sick.

Al is not sick.

Ed told Bill that Al is not sick.

Ty2  $tell(i, ed, bill, \lambda j[sick(j, al) = sick(i, al)])$   
 $\neg sick(i, al)$   
 $tell(i, ed, bill, \lambda j[\neg sick(j, al)])$

PROOF (as for **(II)**) Exercise.

REMARK.

In the proofs of **(I)**–**(IV)** the (non-)factivity of the main verb is irrelevant. If desired, factivity can be made explicit by means of meaning postulates e.g.:

$MP_{fct.} know(i, x, p) \leftrightarrow (p(i) \wedge believe(i, x, p))$

## • WH-COMPLEMENTS

- (V) Ed knows who is sick. (strong inf.)  
Only Al is sick.  
 Ed knows that only Al is sick.

$$\text{Ty2 } \frac{\text{know}(i, ed, \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)])}{\forall x(\text{sick}(i, x) \leftrightarrow x = al)} \\ \text{know}(i, ed, \lambda j[\forall x(\text{sick}(j, x) \leftrightarrow x = al)])$$

PROOF. If (1) & (2), then (7):

1.  $\llbracket \text{know}(i, ed, \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)]) \rrbracket^g = 1$  prem. 1
2.  $\llbracket \forall x(\text{sick}(i, x) \leftrightarrow x = al) \rrbracket^g = 1$  prem. 2
3.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)] \rrbracket^g) = 1$  (1), Ty2
4.  $\{d \mid \langle g(i), d \rangle \in \mathfrak{D}\llbracket \text{sick} \rrbracket\} = \{\llbracket al \rrbracket\}$  (2), Ty2
5.  $\llbracket \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)] \rrbracket^g$   
 $= \lambda \{w \mid \{d \mid \langle w, d \rangle \in \mathfrak{D}\llbracket \text{sick} \rrbracket\} = \{\llbracket al \rrbracket\}\}$  (4), Ty2  
 $= \llbracket \lambda j[\forall x(\text{sick}(j, x) \leftrightarrow x = al)] \rrbracket^g$  Ty2
6.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\forall x(\text{sick}(j, x) \leftrightarrow x = al)] \rrbracket^g) = 1$  (3), (5)
7.  $\llbracket \text{know}(i, ed, \lambda j[\forall x(\text{sick}(j, x) \leftrightarrow x = al)]) \rrbracket^g = 1$  Ty2

- (VII) Ed knows who likes whom. (strong inf.)  
Bill likes Al and that's the full answer.  
 Ed knows that [Bill likes Al and that's the full answer].

$$\text{Ty2 } \frac{\text{know}(i, ed, \lambda j[\lambda xy.\text{like}(j, x, y) = \lambda xy.\text{like}(i, x, y)])}{\forall xy(\text{like}(i, x, y) \leftrightarrow x = bill \wedge y = al)} \\ \text{know}(i, ed, \lambda j[\forall xy(\text{like}(i, x, y) \leftrightarrow x = bill \wedge y = al)])$$

PROOF. If (1) & (2), then (7):

1.  $\llbracket \text{know}(i, ed, \lambda j[\lambda xy.\text{like}(j, x, y) = \lambda xy.\text{like}(i, x, y)]) \rrbracket^g = 1$  prem. 1
2.  $\llbracket \forall xy(\text{like}(i, x, y) \leftrightarrow x = bill \wedge y = al) \rrbracket^g = 1$  prem. 2
3.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket,$   
 $\llbracket \lambda j[\lambda xy.\text{like}(j, x, y) = \lambda xy.\text{like}(i, x, y)] \rrbracket^g) = 1$  (1), Ty2
4.  $\{\langle d, d' \rangle \mid \langle g(i), d, d' \rangle \in \mathfrak{D}\llbracket \text{like} \rrbracket\} = \{\langle \llbracket bill \rrbracket, \llbracket al \rrbracket \rangle\}$  (2), Ty2
5.  $\llbracket \lambda j[\lambda xy.\text{like}(j, x, y) = \lambda xy.\text{like}(i, x, y)] \rrbracket^g$   
 $= \lambda \{w \mid \{\langle d, d' \rangle \mid \langle w, d, d' \rangle \in \mathfrak{D}\llbracket \text{like} \rrbracket\} = \{\langle \llbracket bill \rrbracket, \llbracket al \rrbracket \rangle\}\}$  (4), Ty2  
 $= \llbracket \lambda j[\forall xy(\text{like}(j, x, y) \leftrightarrow x = bill \wedge y = al)] \rrbracket^g$  Ty2
6.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket,$   
 $\llbracket \lambda j[\forall xy(\text{like}(j, x, y) \leftrightarrow x = bill \wedge y = al)] \rrbracket^g) = 1$  (3), (5)
7.  $\llbracket \text{know}(i, ed, \lambda j[\forall xy(\text{like}(j, x, y) \leftrightarrow x = bill \wedge y = al)]) \rrbracket^g = 1$  Ty2

( $\mathbf{V}_{\subseteq}$ ) Ed knows who is sick.

Al is sick.

Ed knows that Al is sick.

(*weak inf.*)

Ty2  $\frac{\text{know}(i, ed, \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)])}{\text{sick}(i, al)}$   
 $\text{know}(i, ed, \lambda j[\text{sick}(j, al)])$

$\text{MP}_{\subseteq}^2 \text{ know}(i, x, p) \wedge p \subseteq q \rightarrow \text{know}(i, x, q)$

PROOF of ( $\mathbf{V}_{\subseteq}$ ): Assuming  $\text{MP}_{\subseteq}$ , if (1) & (2), then (8):

1.  $\llbracket \text{know}(i, ed, \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)]) \rrbracket^g = 1$  prem. 1
2.  $\llbracket \text{sick}(i, al) \rrbracket^g = 1$  prem. 2
3.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)] \rrbracket^g) = 1$  (1), Ty2
4.  $\llbracket \text{sick} \rrbracket(g(i), \llbracket al \rrbracket) = 1$  (2), Ty2
5.  $\llbracket al \rrbracket \in \{d \mid \langle g(i), d \rangle \in \mathfrak{b}\llbracket \text{sick} \rrbracket\}$   $\{-|- \}$
6.  $\llbracket \lambda j[\lambda x.\text{sick}(j, x) = \lambda x.\text{sick}(i, x)] \rrbracket^g$   
 $= \lambda \{w \mid \{d \mid \langle w, d \rangle \in \mathfrak{b}\llbracket \text{sick} \rrbracket\} = \{d \mid \langle g(i), d \rangle \in \mathfrak{b}\llbracket \text{sick} \rrbracket\}\}$  Ty2  
 $\subseteq \lambda \{w \mid \llbracket al \rrbracket \in \{d \mid \langle w, d \rangle \in \mathfrak{b}\llbracket \text{sick} \rrbracket\}\}$  (5),  $\{-|- \}$   
 $= \lambda \{w \mid \langle w, \llbracket al \rrbracket \rangle \in \mathfrak{b}\llbracket \text{sick} \rrbracket\}$   $\{-|- \}$   
 $= \llbracket \lambda j[\text{sick}(j, al)] \rrbracket^g$  Ty2
7.  $\llbracket \text{know} \rrbracket(g(i), \llbracket ed \rrbracket, \llbracket \lambda j[\text{sick}(j, al)] \rrbracket^g) = 1$  (3), (5),  $\text{MP}_{\subseteq}$
8.  $\llbracket \text{know}(i, ed, \lambda j[\text{sick}(j, al)]) \rrbracket^g = 1$  Ty2

---

$^2 p \subseteq q := \forall j(p(j) \rightarrow q(j))$

### 3 FROM ANSWER SETS TO PARTITIONS

#### • FROM RANKING SETS TO ORDERING RELATIONS (Lewis 1981)

Given a set  $Q$  of propositions:

i<sub><</sub>. world  $v$  is  $Q$ -better than world  $w$ ,

$$v <_Q w \quad \text{iff} \quad \{q \mid q \in Q \wedge w \in q\} \subset \{q \mid q \in Q \wedge v \in q\}$$

ii<sub><</sub>. the set of  $Q$ -best  $p$ -worlds,

$$\text{MIN}_Q(p) \quad := \quad \{w \mid w \in p \wedge \neg \exists v (v \in p \wedge v <_Q w)\}$$

Terminology: For any relation  $R$  on a non-empty set  $A$ ,

- $R$  is a (*weak*) *partial order* (on  $A$ ) iff  $R$  is transitive ( $xRy \wedge yRz \rightarrow xRz$ ), reflexive ( $xRx$ , for all  $x \in A$ ) & antisymmetric ( $xRy \wedge yRx \rightarrow x = y$ )
- $R$  is a *strict (partial) order* (on  $A$ ) iff  $R$  is transitive & irreflexive ( $\neg xRx$ , for all  $x \in A$ )

#### • FROM ANSWER SETS TO EQUIVALENCE RELATIONS

Given a set  $Q$  of propositions:

i<sub>=</sub>. world  $v$  is  $Q$ -equivalent to world  $w$ ,

$$v \equiv_Q w \quad \text{iff} \quad \{q \mid q \in Q \wedge w \in q\} = \{q \mid q \in Q \wedge v \in q\}$$

ii<sub>=</sub>. the  $Q$ -equivalence class of  $v$ ,

$$\text{EQU}_Q(v) \quad := \quad \{w \mid v \equiv_Q w\}$$

Terminology:

- $R \subseteq A \times A$  is an *equivalence relation* on  $A$  iff  $\text{Dom } R = A$  and  $R$  is transitive, reflexive & symmetric ( $xRy \rightarrow yRx$ )
- $[x]_R := \{y \in A \mid xRy\}$  is the ( $R$ -) *equivalence class* of  $x$ , for all  $x \in A$ .
- $A/R := \{[x]_R \mid x \in A\}$  is called  *$A$  modulo  $R$* .

FACT: If  $R$  is an equivalence relation on  $A$ , then  $A/R$  is a partition of  $A$ .

PROOF (outline):

(a) For all  $x \in A$ ,  $xRx$  (by reflexivity). Hence  $x \in [x]_R$ , and so  $[x]_R \neq \emptyset$ .

(b)  $\cup(A/R) = \cup\{\{y \in A \mid xRy\} \mid x \in A\} = A$ , given (a) &  $\text{Dom } R = A$ .

(c)  $xRy \rightarrow [x]_R = [y]_R$ .

- ( $\subseteq$ ). Suppose  $z \in [x]_R$ . Then  $xRz$  (df.  $[x]_R$ ) and so  $zRx$  (symm.). Hence  $zRy$  (by tr., given  $xRy$ ), and so  $yRz$  (symm.). Therefore,  $z \in [y]_R$ .

- ( $\supseteq$ ). Analogous reasoning.

(d)  $\neg xRy \rightarrow [x]_R \cap [y]_R = \emptyset$ .

- Suppose  $z \in [x]_R$  &  $z \in [y]_R$ . Then  $xRz$  &  $yRz$ . Hence  $zRy$  (symm.),  $xRy$  (tr.), and  $[x]_R = [y]_R$  (by (c)). But then  $[x]_R \cap [y]_R = [x]_R \neq \emptyset$  (by (a)).

## • PARTITIONS ON DEMAND

Ex1:

$$\begin{array}{lll}
D_e = \{\llbracket aI \rrbracket, \llbracket ed \rrbracket\} & \langle w_0, \llbracket aI \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket & \langle w_0, \llbracket ed \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket \\
D_s = \{w_0, w_1, w_2, w_3\} & \langle w_1, \llbracket aI \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket & \langle w_1, \llbracket ed \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket \\
& \langle w_2, \llbracket aI \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket & \langle w_2, \llbracket ed \rrbracket \rangle \in \mathfrak{B}\llbracket sick \rrbracket \\
& \langle w_3, \llbracket aI \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket & \langle w_3, \llbracket ed \rrbracket \rangle \notin \mathfrak{B}\llbracket sick \rrbracket
\end{array}$$

cat phraseTy2-translation

$$\begin{array}{ll}
s & \text{whether AI is sick} \\
\text{(Q = Bittner) set} & = \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \\
& = \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al)] \wedge j \equiv_Q i)] \\
\text{(Q = Hamblin) set} & = \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al) \\
& \quad \vee p = \lambda k.\neg\text{sick}(k, al)] \wedge j \equiv_Q i)]
\end{array}$$

• GS partition

$$\begin{array}{ll}
\llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^{g[i/w_0]} & \\
= \times\{v \in D_s \mid \llbracket sick \rrbracket(v, \llbracket aI \rrbracket) = \llbracket sick \rrbracket(w_0, \llbracket aI \rrbracket)\} & \\
= \llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^{g[i/w_1]} & = \times\{w_0, w_1\} \\
\llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^{g[i/w_2]} & \\
= \llbracket \lambda j[\text{sick}(j, al) = \text{sick}(i, al)] \rrbracket^{g[i/w_3]} & = \times\{w_2, w_3\}
\end{array}$$

• B answer-set induced partition

$$\begin{array}{ll}
\llbracket \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al)] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_0]} & \\
= \times\{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \equiv_Q w_0)\} & \text{Ty2} \\
= \times\{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \in \mathfrak{B}_q\} & \\
= \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge w_0 \in \mathfrak{B}_q\} & \equiv_Q \\
= \times\{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \in \mathfrak{B}_q\} & \\
= \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} & w_0 \\
= \llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g & \text{Ty2} \\
= \llbracket \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al)] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_1]} & = \times\{w_0, w_1\} \\
\llbracket \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al)] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_2]} & \\
= \times\{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \equiv_Q w_2)\} & \text{Ty2} \\
= \times\{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \in \mathfrak{B}_q\} & \\
= \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge w_2 \in \mathfrak{B}_q\} & \equiv_Q \\
= \times\{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} \wedge v \in \mathfrak{B}_q\} & \\
= \{\} & w_2 \\
= \times\{v \in D_s \mid v \notin \mathfrak{B}\llbracket \lambda k[\text{sick}(k, al)] \rrbracket^g\} & \{-|- \} \\
= \llbracket \lambda j[\exists Q(Q = \lambda p[p = \lambda k.\text{sick}(k, al)] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_3]} & = \times\{w_0, w_1\}
\end{array}$$

H answer-set induced partition

$$\begin{aligned}
& \llbracket \lambda j [\exists Q(Q = \lambda p [p = \lambda k. \textit{sick}(k, al) \vee p = \lambda k. \neg \textit{sick}(k, al)] \\
& \quad \wedge j \equiv_Q i)] \rrbracket^{g[i/w_0]} \\
&= \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \equiv_Q w_0)\} \quad \text{Ty2} \\
&= \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \in \mathfrak{B}_q\} \\
& \quad = \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge w_0 \in \mathfrak{B}_q\} \quad \equiv_Q \\
&= \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \in \mathfrak{B}_q\} \\
& \quad = \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g\} \quad w_0 \\
&= \times \{v \in D_s \mid v \in \mathfrak{B} \llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g \wedge v \notin \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
&= \llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g \quad \text{Ty2} \\
&= \llbracket \lambda j [\exists Q(Q = \lambda p [p = \lambda k. \textit{sick}(k, al) \vee p = \lambda k. \neg \textit{sick}(k, al)] \\
& \quad \wedge j \equiv_Q i)] \rrbracket^{g[i/w_1]} \quad = \times \{w_0, w_1\}
\end{aligned}$$

$$\begin{aligned}
& \llbracket \lambda j [\exists Q(Q = \lambda p [p = \lambda k. \textit{sick}(k, al) \vee p = \lambda k. \neg \textit{sick}(k, al)] \\
& \quad \wedge j \equiv_Q i)] \rrbracket^{g[i/w_2]} \\
&= \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \equiv_Q w_2)\} \quad \text{Ty2} \\
&= \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \in \mathfrak{B}_q\} \\
& \quad = \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge w_0 \in \mathfrak{B}_q\} \quad \equiv_Q \\
&= \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g, \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
& \quad \wedge v \in \mathfrak{B}_q\} \\
& \quad = \{\llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \quad w_2 \\
&= \times \{v \in D_s \mid v \notin \mathfrak{B} \llbracket \lambda k [\textit{sick}(k, al)] \rrbracket^g \wedge v \in \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g\} \\
&= \llbracket \lambda k [\neg \textit{sick}(k, al)] \rrbracket^g \quad \text{Ty2} \\
&= \llbracket \lambda j [\exists Q(Q = \lambda p [p = \lambda k. \textit{sick}(k, al) \vee p = \lambda k. \neg \textit{sick}(k, al)] \\
& \quad \wedge j \equiv_Q i)] \rrbracket^{g[i/w_3]} \quad = \times \{w_2, w_3\}
\end{aligned}$$

| <u>cat</u> | <u>phrase</u> | <u>Ty2-translation</u>                                                                                                                                                                        |
|------------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <u>s</u>   | who is sick   | $\lambda j [\lambda x. \textit{sick}(j, x) = \lambda x. \textit{sick}(i, x)]$<br>$= \lambda j [\exists Q(Q = \lambda p [\exists x(p = \lambda k. \textit{sick}(k, x))] \wedge j \equiv_Q i)]$ |

- GS partition

$$\begin{aligned} & \llbracket \lambda j[\lambda x. \text{ sick}(j, x) = \lambda x. \text{ sick}(i, al)] \rrbracket^{g[i/w_0]} \\ & = \times \{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} \\ & \quad = \{d \mid \langle w_0, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} = \{\llbracket al \rrbracket, \llbracket ed \rrbracket\} \} = \times \{w_0\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\lambda x. \text{ sick}(j, x) = \lambda x. \text{ sick}(i, al)] \rrbracket^{g[i/w_1]} \\ & = \times \{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} \\ & \quad = \{d \mid \langle w_1, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} = \{\llbracket al \rrbracket\} \} = \times \{w_1\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\lambda x. \text{ sick}(j, x) = \lambda x. \text{ sick}(i, al)] \rrbracket^{g[i/w_2]} \\ & = \times \{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} \\ & \quad = \{d \mid \langle w_2, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} = \{\llbracket ed \rrbracket\} \} = \times \{w_2\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\lambda x. \text{ sick}(j, x) = \lambda x. \text{ sick}(i, al)] \rrbracket^{g[i/w_3]} \\ & = \times \{v \in D_s \mid \{d \mid \langle v, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} \\ & \quad = \{d \mid \langle w_3, d \rangle \in \mathfrak{B}[\llbracket \text{ sick} \rrbracket]\} = \{\} \} = \times \{w_3\} \end{aligned}$$

- Answer-set induced partition

$$\begin{aligned} & \llbracket \lambda j[\exists Q(Q = \lambda p[\exists x(p = \lambda k. \text{ sick}(k, x))] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_0]} \\ & = \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \equiv_Q w_0)\} \\ & = \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \in q\} \\ & \quad = \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge w_0 \in q\} \\ & \quad = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in \{\llbracket al \rrbracket, \llbracket ed \rrbracket\}\} \} = \times \{w_0\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\exists Q(Q = \lambda p[\exists x(p = \lambda k. \text{ sick}(k, x))] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_1]} \\ & = \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \equiv_Q w_1)\} \\ & = \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \in q\} \\ & \quad = \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge w_1 \in q\} \\ & \quad = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in \{\llbracket al \rrbracket\}\} \} = \times \{w_1\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\exists Q(Q = \lambda p[\exists x(p = \lambda k. \text{ sick}(k, x))] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_2]} \\ & = \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \equiv_Q w_2)\} \\ & = \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \in q\} \\ & \quad = \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge w_2 \in q\} \\ & \quad = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in \{\llbracket ed \rrbracket\}\} \} = \times \{w_2\} \end{aligned}$$

$$\begin{aligned} & \llbracket \lambda j[\exists Q(Q = \lambda p[\exists x(p = \lambda k. \text{ sick}(k, x))] \wedge j \equiv_Q i)] \rrbracket^{g[i/w_3]} \\ & = \times \{v \in D_s \mid \exists Q(Q = \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \equiv_Q w_3)\} \\ & = \times \{v \in D_s \mid \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge v \in q\} \\ & \quad = \{q \mid q \in \{\llbracket \lambda k[\text{ sick}(k, x)] \rrbracket^{g[x/d]} \mid d \in D_e\} \wedge w_3 \in q\} \\ & \quad = \{\} \} = \times \{w_3\} \end{aligned}$$

## 4 TOWARD INCREMENTAL TRANSLATION

## • TWO-SORTED UPDATE WITH CENTERING (UC2): Basic terms

| $a \in Typ$          | $\top Var_a$ | $\perp Var_a$ | $Con_a$                        | Name of objects                              |
|----------------------|--------------|---------------|--------------------------------|----------------------------------------------|
| $s$                  |              | $i, j$        |                                | indices ( $\top \perp$ -lists)               |
| $st$                 |              | $I, J$        |                                | infotention states                           |
| $e$                  | $\mathbf{x}$ | $y$           | $al, bill, \dots$              | entities                                     |
| $\omega$             | $\mathbf{w}$ | $v$           |                                | worlds                                       |
| $\omega t := \Omega$ | $\mathbf{p}$ | $q$           |                                | propositions (sets of worlds)                |
| $\Omega t$           | $\mathbf{Q}$ | $Q$           |                                | sets of propositions                         |
| $\omega et$          |              |               | $man, sick, \dots$             | $\omega et$ -relations                       |
| $\omega eet$         |              |               | $like, \dots$                  | $\omega eet$ -relations                      |
| $sa$                 |              |               | $\mathbf{da}_n, \mathbf{da}_n$ | a-projections, $a \in \{e, \omega, \Omega\}$ |

## • MATRIX IND WITH DEFAULT ORDER

Ex1:

$$p_0 = \{w_0, w_1\} =: {}_{01} \quad \langle w_0, \llbracket al \rrbracket \rangle \in {}^{\exists} \llbracket sick \rrbracket$$

$$*p_0 := \{ \langle \langle v, p_0 \rangle, \langle \rangle \rangle : v \in p_0 \} \quad \langle w_1, \llbracket al \rrbracket \rangle \notin {}^{\exists} \llbracket sick \rrbracket$$

\*<sub>(01)</sub>

$$\langle \langle w_0, {}_{01} \rangle, \langle \rangle \rangle$$

$$\langle \langle w_1, {}_{01} \rangle, \langle \rangle \rangle$$

(I<sup>2</sup>) (sentence 2 in inference (I))

$$Al \quad be.IND \quad sick \quad \cdot \text{ (prosody)}$$

$$[x | (x = al)^{\circ}]; \quad P[\mathbf{d}\omega \in \mathbf{d}\omega \{\}]; [sick\langle \mathbf{d}\omega, \mathbf{de} \rangle]; \quad [p]; [d\Omega = \mathbf{d}\omega \{\}]$$

$$C_1 \quad C_1 \quad C_2 \quad C_3 \text{ (}_0 := \{w_0\}\text{)}$$

$$\langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle, \langle \rangle \rangle \quad \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle, \langle \rangle \rangle \quad \langle \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle, \langle \rangle \rangle$$

$$\langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \rangle \rangle$$

(II<sup>2</sup>) (sentence 2 in inference (II))

$$Al \quad be.IND \quad not[$$

$$[x | (x = al)^{\circ}]; \quad P[\mathbf{d}\omega \in \mathbf{d}\omega \{\}]; [p \ w | \mathbf{d}\omega \notin p, w \in p];$$

$$C_1 \quad C_1 \quad C_2 \text{ (}_1 := \{w_1\}\text{)}$$

$$\langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle, \langle \rangle \rangle \quad \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle, \langle \langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle \rangle \rangle$$

$$\langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \rangle \rangle \quad \langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle \rangle \rangle$$

$$sick \quad ] \quad \cdot \text{ (prosody)}$$

$$[sick\langle \mathbf{d}\omega, \mathbf{de} \rangle]; \quad [d\Omega = \mathbf{d}\omega \{\mathbf{de}\}]; \quad [p]; [d\Omega = \mathbf{d}\omega \{\}]$$

$$C_3 \quad C_4 \quad C_5$$

$$\langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle \rangle \rangle \quad \langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle \rangle \rangle \quad \langle \langle \langle \llbracket al \rrbracket, w_1, {}_{01} \rangle, \langle \langle \langle \llbracket al \rrbracket, w_0, {}_{01} \rangle \rangle \rangle$$

• EMBEDDED [*that*...IND]Ex2:

$$\begin{array}{lll}
p_0 = \{w_0, w_1, w_2, w_3\} & \langle w_0, [aI] \rangle \in \mathfrak{B}[sick] & \langle w_0, [ed],_{01} \rangle \in \mathfrak{B}[bel] \\
=:_{0-3} & \langle w_1, [aI] \rangle \in \mathfrak{B}[sick] & \langle w_1, [ed],_{023} \rangle \in \mathfrak{B}[bel] \\
& \langle w_2, [aI] \rangle \notin \mathfrak{B}[sick] & \langle w_2, [ed],_{23} \rangle \in \mathfrak{B}[bel] \\
& \langle w_3, [aI] \rangle \notin \mathfrak{B}[sick] & \langle w_3, [ed],_{123} \rangle \in \mathfrak{B}[bel]
\end{array}$$

\*<sub>(0-3)</sub> $\langle \langle w_0,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle w_1,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle w_2,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle w_3,_{0-3} \rangle, \langle \rangle \rangle$ (I<sup>3</sup>) (sentence 3 in inference (I))

$$\begin{array}{lll}
Ed & know- & -IND[ \\
[x | (x = ed)^\circ]; & [p | (d\omega \in p)^\circ, bel\langle d\omega, de, p \rangle]; & P[d\omega \in d\omega \{\}]; \\
C_1 & C_2 & C_2
\end{array}$$

 $\langle \langle [ed], w_0,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle [ed], w_0,_{0-3} \rangle, \langle_{01} \rangle \rangle$  $\langle \langle [ed], w_0,_{0-3} \rangle, \langle_{012} \rangle \rangle$  $\langle \langle [ed], w_0,_{0-3} \rangle, \langle_{013} \rangle \rangle$  $\langle \langle [ed], w_0,_{0-3} \rangle, \langle_{0-3} \rangle \rangle$  $\langle \langle [ed], w_1,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle [ed], w_1,_{0-3} \rangle, \langle_{0-3} \rangle \rangle$  $\langle \langle [ed], w_2,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle [ed], w_2,_{0-3} \rangle, \langle_{23} \rangle \rangle$  $\langle \langle [ed], w_2,_{0-3} \rangle, \langle_{023} \rangle \rangle$  $\langle \langle [ed], w_2,_{0-3} \rangle, \langle_{123} \rangle \rangle$  $\langle \langle [ed], w_2,_{0-3} \rangle, \langle_{0-3} \rangle \rangle$  $\langle \langle [ed], w_3,_{0-3} \rangle, \langle \rangle \rangle$  $\langle \langle [ed], w_3,_{0-3} \rangle, \langle_{123} \rangle \rangle$  $\langle \langle [ed], w_3,_{0-3} \rangle, \langle_{0-3} \rangle \rangle$ 

$$\begin{array}{lll}
THAT[ & Al & be.IND \\
[w | (w \in d\Omega)^\circ]; & [x | (x = al)^\circ]; & P[d\omega \in d\omega \{\}]; \\
C_3 & C_4 & C_4
\end{array}$$

$$\begin{array}{ll}
sick & ]_{THAT} \\
[sick\langle d\omega, de \rangle]; & [x | (x = de_2)^\circ]; [w | (w = d\omega_2)^\circ]; \\
C_5 & C_6
\end{array}$$

$$\begin{array}{ll}
]_{know} & \cdot \text{ (prosody)} \\
[d\Omega = d\omega_2 \{d\omega, de\}]; [p]; [d\Omega = d\omega \{\}]; & \\
C_7 & C_8
\end{array}$$

APPENDIX UC: *Update with Centering*

**D1.1 Definition** (Infotention states). Let  $D$  be a non-empty set of objects.

- $Z^{n,m} = D^n \times D^m$  is the set of structured stacks with  $n$  topical objects and  $m$  background objects, for all natural numbers  $n, m \in \mathbf{N}$
- $C^{n,m} = Pow(Z^{n,m})$  is the set of states of infotention about  $n$  topical objects and  $m$  background objects
- $C = \bigcup_{n,m \in \mathbf{N}} C^{n,m}$  is the set of states of infotention

**A1 Abbreviations** (Stacks, cardinality, extensions)

- For  $z = \langle z_1, z_2 \rangle \in Z^{n,m}$ ,  $\top z := z_1$  is the top stack of  $z$  and  $\perp z := z_2$  is the bottom stack of  $z$
- For  $z \in Z^{n,m}$  ( $c \in C^{n,m}$ ),  $|z|_{\top} := n$  ( $=: |c|_{\top}$ ) and  $|z|_{\perp} := m$  ( $=: |c|_{\perp}$ )
- $(x \cdot y) := \langle x_1, \dots, x_n, y_1, \dots, y_m \rangle \in D^{n+m}$ , for  $x \in D^n$  and  $y \in D^m$
- $y$  extends  $x$ ,  $x \leq y$ , iff  $\exists x' : y = (x' \cdot x)$

**D1.2 Definition** (Infotention update) State  $c'$  is an *update* of state  $c$ ,  $c \leq c'$ , iff  $|c|_{\top} \leq |c'|_{\top} \wedge |c|_{\perp} \leq |c'|_{\perp} \wedge \forall z' \in c' \exists z \in c (\top z \leq \top z' \wedge \perp z \leq \perp z')$

**D2.1 Definition** (UC types). The set of UC types,  $Typ$ , is the smallest set  $Y$  such that (i)  $\{\alpha, \beta, \varepsilon, \sigma, \tau, \omega, s, t\} \subseteq Y$ , and (ii)  $(ab) \in Y$  if  $a, b \in Y$ .

The subset  $DTyp := \{\alpha, \beta, \varepsilon, \sigma, \tau, \omega, (\omega\varepsilon), (\omega t), ((\omega t)t)\} \subseteq Typ$  is the set of UC types of discourse objects.

**D2.2 Definition** (UC frames). A UC frame is a set of sets  $\{D_a\}_{a \in Typ}$  where

- $D_{\alpha}, D_{\beta}, D_{\varepsilon}, D_{\sigma}, D_{\tau}, D_{\omega}$  and  $D_t$  are non-empty pairwise disjoint sets
- $D_{\alpha} = \{a \subseteq \mathbf{A} \mid a \neq \emptyset\}$  for some non-empty set  $\mathbf{A}$  (of  $\alpha$ -atoms)  
 $D_{\tau} = \{t \subseteq \mathbf{Z} \mid t \neq \emptyset \wedge \forall n, n' \in t \forall m \in \mathbf{Z} (n < m < n' \rightarrow m \in t)\}$
- $D_t = \{1, 0\}$
- $D_s = \bigcup_{n,m \in \mathbf{N}} (D^n \times D^m)$ , where  $D = \bigcup_{a \in DTyp} D_a$
- $D_{(ab)} = \{f \mid \emptyset \subset \text{Dom } f = D_a \wedge \text{Ran } f \subseteq D_b\}$  , if  $b = t$   
 $= \{f \mid \emptyset \subset \text{Dom } f \subseteq D_a \wedge \text{Ran } f \subseteq D_b\}$  , if  $b \neq t$

**A2 Abbreviations** (Basic terms of UC)

| $a \in Typ$                             | $\top Var_a$ | $\perp Var_a$ | $Con_a$                                                                                                        | Name of objects                                  |
|-----------------------------------------|--------------|---------------|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------|
| $s$                                     |              | $i, j$        |                                                                                                                | structured stacks                                |
| $st$                                    |              | $I, J$        |                                                                                                                | infotention states                               |
| $\alpha$                                | <b>a</b>     | $a$           | <i>john</i>                                                                                                    | animate (entities)                               |
| $\beta$                                 | <b>b</b>     | $b$           | <i>John</i>                                                                                                    | inanimate (entities)                             |
| $\varepsilon$                           | <b>e</b>     | $e$           |                                                                                                                | events                                           |
| $\sigma$                                | <b>s</b>     | $s$           |                                                                                                                | states (of entities)                             |
| $\tau$                                  | <b>t</b>     | $t$           |                                                                                                                | times                                            |
| $\omega$                                | <b>w, v</b>  | $w, v$        |                                                                                                                | worlds                                           |
| $\omega\varepsilon$                     | <b>e</b>     | $e$           |                                                                                                                | event concepts                                   |
| $\omega t =: \Omega$                    | <b>p, q</b>  | $p, q$        |                                                                                                                | propositions (sets of worlds)                    |
| $\Omega t$                              | <b>Q</b>     | $Q$           |                                                                                                                | sets of propositions                             |
| $\omega\tau\alpha t$                    |              |               | <i>man, ...</i>                                                                                                | $\omega\tau\alpha$ -relations                    |
| $\omega\sigma\alpha t$                  |              |               | <i>happy, ...</i>                                                                                              | $\omega\sigma\alpha$ -relations                  |
| $\omega\varepsilon\alpha t$             |              |               | <i>speak, ...</i>                                                                                              | $\omega\varepsilon\alpha$ -relations             |
| $\omega\sigma\alpha\Omega t$            |              |               | <i>hope, ...</i>                                                                                               | $\omega\sigma\alpha\Omega$ -relations            |
| $\omega\varepsilon\alpha\Omega t$       |              |               | <i>promise, ...</i>                                                                                            | $\omega\varepsilon\alpha\Omega$ -relations       |
| $\omega\varepsilon\alpha\alpha\Omega t$ |              |               | <i>direct.to, ...</i>                                                                                          | $\omega\varepsilon\alpha\alpha\Omega$ -relations |
| $\omega\varepsilon\alpha(\Omega t)t$    |              |               | <i>ask, ...</i>                                                                                                | $\omega\varepsilon\alpha(\Omega t)$ -relations   |
| $\varepsilon\alpha$                     |              |               | AGT                                                                                                            | $\varepsilon$ -dependent animates                |
| $\varepsilon\sigma$                     |              |               | CON                                                                                                            | $\varepsilon$ -dependent states                  |
| $\sigma\varepsilon$                     |              |               | BEG, END                                                                                                       | $\sigma$ -dependent events                       |
| $sa$                                    |              |               | <b>da</b> <sub>1</sub> , <b>da</b> <sub>2</sub> , ...<br><i>da</i> <sub>1</sub> , <i>da</i> <sub>2</sub> , ... | a-projections ( $a \in DTyp$ )                   |

**A3 Abbreviations** (Functions, projections,  $\tau$ -precedence and  $\tau$ -sum)

- For  $f \in D_{a_1 \dots a_n}$  and  $\langle a_1, \dots, a_n \rangle \subseteq D_{a_1} \times \dots \times D_{a_n}$ ,  
 $f(a_1, \dots, a_n) := f(a_1) \dots (a_n)$   
 $\mathfrak{B}(f) := \{\langle a_1, \dots, a_n \rangle : f(a_1, \dots, a_n) = 1\}$  is the set characterized by  $f$   
 $\chi(\mathbb{A}) := f$  is the characteristic function of  $\mathbb{A}$ , iff  $\mathfrak{B}(f) = \mathbb{A}$
- For  $\mathbf{x} \in D^{n+m}$ ,  $(\mathbf{x})_n$  is the  $n$ th coordinate of  $\mathbf{x}$ , and  ${}^a(\mathbf{x})$  (read: the  $a$ -subsequence of  $\mathbf{x}$ ) is the sequence of the  $D_a$ -coordinates of  $\mathbf{x}$
- $t$   $\tau$ -precedes  $t'$ ,  $t <_\tau t'$ , iff  $t, t' \in D_\tau \wedge \forall m \in t \forall n \in t' (m < n)$   
 $t \cup_\tau t' := \inf_{\subseteq} \{t'' \in D_\tau \mid t \subseteq t'' \wedge t' \subseteq t''\}$

**D2.3 Definition** (UC models). A UC model is a pair  $\langle \{D_a\}_a, \llbracket \cdot \rrbracket \rangle$  where  $\{D_a\}_a$  is a UC frame and  $\llbracket \cdot \rrbracket$  assigns  $\llbracket A \rrbracket \in D_a$  to each  $A \in Con_a$ . Moreover:

- i.  $Dom \llbracket da_n \rrbracket = \{z \in D_s \mid \exists m \in \mathbf{n}: {}^a(\top z) \in (D_a)^{n+m}\}$   
 $\wedge \forall z \in Dom \llbracket da_n \rrbracket: \llbracket da_n \rrbracket(z) = ({}^a(\top z))_n$   
 $Dom \llbracket da_n \rrbracket = \{z \in D_s \mid \exists m \in \mathbf{n}: {}^a(\perp z) \in (D_a)^{n+m}\}$   
 $\wedge \forall z \in Dom \llbracket da_n \rrbracket: \llbracket da_n \rrbracket(z) = ({}^a(\perp z))_n$
- ii.  $\emptyset \subset Dom \llbracket AGT \rrbracket = D_\varepsilon \setminus (Ran \llbracket BEG \rrbracket \cup Ran \llbracket END \rrbracket)$   
 $\emptyset \subset Dom \llbracket F \rrbracket \subseteq D_\varepsilon \cup D_\sigma \wedge Ran \llbracket F \rrbracket \subseteq D_\alpha$  for  $F \in \{EXP, CTR\}$   
 $\emptyset \subset Dom \llbracket CTR' \rrbracket \subseteq D_\varepsilon \cup D_\sigma \wedge Ran \llbracket CTR' \rrbracket \subseteq D_\beta$   
 $\forall e \in Dom \llbracket AGT \rrbracket: \llbracket CTR \rrbracket(e) = \llbracket AGT \rrbracket(e) = \llbracket EXP \rrbracket(\llbracket CON \rrbracket(e))$   
 $\forall e \in Dom \llbracket EXP \rrbracket \setminus Dom \llbracket AGT \rrbracket: \llbracket EXP \rrbracket(e) = \llbracket EXP \rrbracket(\llbracket CON \rrbracket(e))$   
 $\forall s \in Dom \llbracket EXP \rrbracket: \llbracket EXP \rrbracket(s) = \llbracket EXP \rrbracket(\llbracket BEG \rrbracket(s)) = \llbracket EXP \rrbracket(\llbracket END \rrbracket(s))$   
 $\forall e \in Dom \llbracket CTR \rrbracket \setminus Dom \llbracket AGT \rrbracket: \llbracket CTR \rrbracket(e) = \llbracket CTR \rrbracket(\llbracket CON \rrbracket(e))$   
 $\forall s \in Dom \llbracket CTR \rrbracket: \llbracket CTR \rrbracket(s) = \llbracket CTR \rrbracket(\llbracket BEG \rrbracket(s)) = \llbracket CTR \rrbracket(\llbracket END \rrbracket(s))$
- iii.  $\forall w \in D_\omega (\emptyset \subset Dom \llbracket \vartheta \rrbracket(w) \subseteq D_\varepsilon \cup D_\sigma \wedge Ran \llbracket \vartheta \rrbracket(w) \subseteq D_\tau$   
 $\wedge \forall e \in Dom \llbracket \vartheta \rrbracket(w) \exists n \in \mathbf{z} (\llbracket \vartheta \rrbracket(w, e) = \{n\}$   
 $\wedge \llbracket \vartheta \rrbracket(w, \llbracket BEG \rrbracket(\llbracket CON \rrbracket(e))) = \{(n+1)\}$   
 $\wedge (e \in Dom \llbracket AGT \rrbracket \rightarrow \llbracket \vartheta \rrbracket(w, \llbracket CON \rrbracket(e)) > 1)$   
 $\wedge \forall s \in Dom \llbracket \vartheta \rrbracket(w) (\llbracket \vartheta \rrbracket(w, \llbracket BEG \rrbracket(s)) = \{\inf_{<} \llbracket \vartheta \rrbracket(w, s)\}$   
 $\wedge \llbracket \vartheta \rrbracket(w, \llbracket END \rrbracket(s)) = \{\sup_{<} \llbracket \vartheta \rrbracket(w, s)\} \wedge (\llbracket \vartheta \rrbracket(w, s) > 1$   
 $\rightarrow \llbracket \vartheta \rrbracket(w, s) = \llbracket \vartheta \rrbracket(w, \llbracket BEG \rrbracket(s)) \cup \llbracket \vartheta \rrbracket(w, \llbracket CON \rrbracket(\llbracket BEG \rrbracket(s))))$

**D3 Definition** (UC syntax). For each UC type  $a \in Typ$ ,

- i.  $Con_a \cup {}^TVar_a \cup {}^+Var_a \subseteq Term_a$
- ii.  $BA \in Term_b$ , if  $A \in Term_a$  and  $B \in Term_{ab}$
- iii.  $(A = B) \in Term_t$ , if  $A, B \in Term_a$
- iv.  $\neg\phi, (\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi) \in Term_t$ , if  $\phi, \psi \in Term_t$
- v.  $\exists u\phi, \forall u\phi \in Term_t$ , if  $u \in {}^TVar_a \cup {}^+Var_a$  and  $\phi \in Term_t$
- vi.  $\lambda u(B) \in Term_{ab}$ , if  $u \in {}^TVar_a \cup {}^+Var_a$  and  $B \in Term_b$
- vii.  $(u \cdot B) \in Term_s$ , if  $u \in {}^TVar_a \cup {}^+Var_a$ ,  $a \in DTyp$  and  $B \in Term_s$ .
- viii.  $(A + B) \in Term_a$ , if  $A, B \in Term_a$  and  $a \in \{\tau, \alpha\} \cup \{b_1 \dots b_n t: b_1, \dots, b_n \in Typ\}$
- ix.  $(A \subset B), (A \emptyset B) \in Term_t$ , if  $A, B \in Term_a$  and  $a \in \{\tau, \alpha\} \cup \{b_1 \dots b_n t: b_1, \dots, b_n \in Typ\}$
- x.  $(A < B) \in Term_t$ , if  $A, B \in Term_\tau$
- xi.  $EXP A, CTR A \in Term_\alpha$  and  $CTR' A \in Term_\beta$ , if  $A \in Term_\varepsilon \cup Term_\sigma$
- xii.  $\vartheta(W, A) \in Term_\tau$ , if  $W \in Term_\omega$  and  $A \in Term_\varepsilon \cup Term_\sigma$

**D4 Definition** (UC semantics). The value  $\llbracket A \rrbracket^g \in D_a$  of a term  $A \in Term_a$  in a model  $M = \langle \{D_a\}_{a \in Typ}, \llbracket \cdot \rrbracket \rangle$  under an  $M$ -assignment  $g$  is defined as follows. (Meta-language  $\wedge, \vee, \rightarrow, \exists, \forall$ , etc, have their usual meaning.)

- i.  $\llbracket A \rrbracket^g = \llbracket A \rrbracket$  if  $A \in Con_a$   
 $\llbracket u \rrbracket^g = g(u)$  if  $u \in {}^TVar_a \cup {}^LVar_a$
- ii.  $\llbracket B_{ab}A_a \rrbracket^g = \llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$  if  $b \neq t \wedge \llbracket B \rrbracket^g(\llbracket A \rrbracket^g) \in D_b$   
 $= \llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$  if  $b = t \wedge \llbracket A \rrbracket^g \in Dom \llbracket B \rrbracket^g$   
 $= 0$  otherwise
- iii.  $\llbracket A = B \rrbracket^g = 1$  iff  $(\llbracket A \rrbracket^g = \llbracket B \rrbracket^g) \wedge \llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_a$
- iv.  $\llbracket \neg \phi \rrbracket^g = 1$  iff  $\llbracket \phi \rrbracket^g = 0$   
 $\llbracket (\phi \wedge \psi) \rrbracket^g = 1$  iff  $(\llbracket \phi \rrbracket^g = 1 \wedge \llbracket \psi \rrbracket^g = 1)$   
 $\llbracket (\phi \vee \psi) \rrbracket^g = 1$  iff  $(\llbracket \phi \rrbracket^g = 1 \vee \llbracket \psi \rrbracket^g = 1)$   
 $\llbracket (\phi \rightarrow \psi) \rrbracket^g = 1$  iff  $(\llbracket \phi \rrbracket^g = 1 \rightarrow \llbracket \psi \rrbracket^g = 1)$
- v.  $\llbracket \exists u_a \phi \rrbracket^g = 1$  iff  $\exists d \in D_a: \llbracket \phi \rrbracket^{g[u/d]} = 1$   
 $\llbracket \forall u_a \phi \rrbracket^g = 1$  iff  $\forall d \in D_a: \llbracket \phi \rrbracket^{g[u/d]} = 1$
- vi.  $\llbracket \lambda u_a (B_b) \rrbracket^g(d) = \llbracket B \rrbracket^{g[u/d]}$  if  $d \in D_a \wedge \llbracket B \rrbracket^{g[u/d]} \in D_b \wedge b \neq t$   
 $= \llbracket B \rrbracket^{g[u/d]}$  if  $d \in D_a \wedge \llbracket B \rrbracket^{g[u/d]} \in D_b \wedge b = t$   
 $= 0$  otherwise
- vii.  $\llbracket u_a \cdot B_s \rrbracket^g = \langle (g(u) \cdot \top \llbracket B \rrbracket^g), \perp \llbracket B \rrbracket^g \rangle$  if  $u \in {}^TVar_a \wedge \llbracket B \rrbracket^g \in D_s$   
 $= \langle \top \llbracket B \rrbracket^g, (g(u) \cdot \perp \llbracket B \rrbracket^g) \rangle$  if  $u \in {}^LVar_a \wedge \llbracket B \rrbracket^g \in D_s$
- viii.  $\llbracket A + B \rrbracket^g = \llbracket A \rrbracket^g \cup_\tau \llbracket B \rrbracket^g$  if  $\llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_\tau$   
 $= \llbracket A \rrbracket^g \cup \llbracket B \rrbracket^g$  if  $\llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_\alpha$   
 $= \chi(\cup \llbracket A \rrbracket^g \cup \cup \llbracket B \rrbracket^g)$  if  $\llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_{b1\dots bnt}$
- ix.  $\llbracket A \subset B \rrbracket^g = 1$  if  $\llbracket A \rrbracket^g \subset \llbracket B \rrbracket^g \wedge \llbracket A \rrbracket^g \in D_\tau \cup D_\alpha$   
 $= 1$  if  $\cup \llbracket A \rrbracket^g \subset \cup \llbracket B \rrbracket^g \wedge \llbracket A \rrbracket^g \in D_{b1\dots bnt}$   
 $= 0$  otherwise  
 $\llbracket A \emptyset B \rrbracket^g = 1$  if  $\llbracket A \rrbracket^g \cap \llbracket B \rrbracket^g = \emptyset$   
 $\wedge \llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_\tau \cup D_\alpha$   
 $= 1$  if  $\cup \llbracket A \rrbracket^g \cap \cup \llbracket B \rrbracket^g = \emptyset$   
 $\wedge \llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_{b1\dots bnt}$   
 $= 0$  otherwise
- x.  $\llbracket A < B \rrbracket^g = 1$  iff  $\llbracket A \rrbracket^g <_\tau \llbracket B \rrbracket^g$
- xi.  $\llbracket FA \rrbracket^g = \llbracket F \rrbracket(\llbracket A \rrbracket^g)$  if  $F \in \{EXP, CTR, CTR'\}$
- xii.  $\llbracket \vartheta(W, A) \rrbracket^g = \llbracket \vartheta \rrbracket(\llbracket W \rrbracket^g)(\llbracket A \rrbracket^g)$

**D5.1 Definition** (Initial contexts). An initial context is a stack  $\langle p_0, e_0 \rangle \in D_\Omega \times D_e$  where (i)  $\emptyset \neq p_0$ , (ii)  $\forall w \in \emptyset p_0 (\langle w, e_0, \llbracket \text{AGT} \rrbracket (e_0) \rangle \in \emptyset \llbracket \text{speak} \rrbracket)$  and (iii)  $\forall w, v \in \emptyset p_0 (\llbracket \emptyset \rrbracket (w, e_0) = \llbracket \emptyset \rrbracket (v, e_0))$

**D5.2 Definition** (Initial states).  $\langle p_0, e_0 \rangle$  induces the initial infotention state  $^* \langle p_0, e_0 \rangle := \times \{ \langle \langle t, w, p_0, e_0 \rangle, \langle \underline{e} \rangle \rangle \in D_s \mid w \in \emptyset p_0 \wedge t = \llbracket \emptyset \rrbracket (w, e_0) \wedge \underline{e} = \{ \langle v, e_0 \rangle : v \in \emptyset p_0 \} \}$

**D6 Definition** (truth & falsity). Said in an initial context  $\langle p_0, e_0 \rangle$ ,

$K$  is *true* in  $w$ , iff

$$\emptyset ((\tau z)_1 \in D_\Omega \wedge w \in \emptyset p_0 \wedge \exists z \forall g (z \in \emptyset \llbracket K \rrbracket^g (^* \langle p_0, e_0 \rangle) \wedge w \in \emptyset ((\tau z)_1))$$

$K$  is *false* in  $w$ , iff

$$\emptyset ((\tau z)_1 \in D_\Omega \wedge \neg (w \in \emptyset p_0 \wedge \exists z \forall g (z \in \emptyset \llbracket K \rrbracket^g (^* \langle p_0, e_0 \rangle) \wedge w \in \emptyset ((\tau z)_1)))$$

#### A4 Abbreviations (Set theory)

|                       |      |                                                                                                  |
|-----------------------|------|--------------------------------------------------------------------------------------------------|
| $B(A_1, \dots, A_n)$  | $:=$ | $BA_1 \dots A_n$                                                                                 |
| $(A \in B)$           | $:=$ | $B_{a_r} A_a$                                                                                    |
| $(A \supset B)$       | $:=$ | $(B \subset A)$                                                                                  |
| $(A \subseteq B)$     | $:=$ | $(A = B \vee A \subset B)$                                                                       |
| $(A \leq B)$          | $:=$ | $(A = B \vee A < B)$                                                                             |
| $(A <_B A')$          | $:=$ | $((\lambda u. u \in B \wedge A' \in u) \subset (\lambda u. u \in B \wedge A \in u))$             |
| $\{A_1, \dots, A_n\}$ | $:=$ | $\lambda u. u = A_1 \vee \dots \vee u = A_n$                                                     |
| $\cap(A)$             | $:=$ | $\lambda u. \forall v (v \in A \rightarrow u \in v)$                                             |
| $\text{SG} A_a$       | $:=$ | $\exists u_a (u \subseteq A) \wedge \neg \exists u_a (u \subset A) \quad a \in \{\tau, \alpha\}$ |
| $\text{PL} A_a$       | $:=$ | $\exists u_a \exists u'_a (u \subset A \wedge u' \subset A \wedge \neg (u = u'))$                |

#### A5 Abbreviations (Modal, causal, and attitudinal relations)

|                   |      |                                                                                                                                                                               |
|-------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\text{MIN}$      | $:=$ | $\lambda Q \lambda p \lambda w. w \in p \wedge \neg \exists v (v \in p \wedge v <_Q w)$                                                                                       |
| $^+ \text{CON}$   | $:=$ | $\lambda w \lambda e \lambda s. \emptyset (w, s) = \emptyset (w, e) + \emptyset (w, \text{CON } e) \wedge \text{EXP } s = \text{EXP CON } e$                                  |
| $\text{BEL}$      | $:=$ | $\lambda w \lambda e \lambda p. \exists s (\emptyset (w, e) \subset \emptyset (w, s) \wedge \text{EXP } s = \text{EXP CON } e \wedge \text{believe}(w, s, \text{EXP } s, p))$ |
| $\text{DES}$      | $:=$ | $\lambda w \lambda e \lambda p. \exists s (\emptyset (w, e) \subset \emptyset (w, s) \wedge \text{EXP } s = \text{EXP CON } e \wedge \text{want}(w, s, \text{EXP } s, p))$    |
| $^{\text{BEG}} B$ | $:=$ | $\lambda w \lambda e \lambda p. \neg (\cap B(w, e) \subseteq p) \wedge (\cap B(w, \text{BEG CON } e) \subseteq p)$                                                            |

#### A6 Abbreviations (Dynamic expansions, local conditions, local updates)

|                 |      |                |                                             |
|-----------------|------|----------------|---------------------------------------------|
| $\text{da}, da$ | $:=$ | $da_1, da_1$   | if $a \in DTyp$                             |
| $A_a^\circ$     | $:=$ | $\lambda i. A$ | if $a \in DTyp$                             |
|                 | $:=$ | $A$            | if $a \in {}^s DTyp := \{sb : b \in DTyp\}$ |

$$\begin{aligned}
A_a^\circ &:= \lambda i \lambda w. A^\circ i && \text{if } a \in (DTyp \cup {}^sDTyp) \setminus \{\omega \varepsilon, s\omega \varepsilon\} \\
&:= \lambda i. A && \text{if } a = \omega \varepsilon \\
&:= A && \text{if } a = s\omega \varepsilon \\
(B(A_1, \dots, A_n))^\circ &:= \lambda i. B(A_1^\circ i, \dots, A_n^\circ i) \\
(B(\omega: A_1, \dots, A_n))^\circ &:= \lambda i \lambda w. B(w, A_1^\circ iw, \dots, A_n^\circ iw) \\
R\langle W: A_1, \dots, A_n \rangle &:= \lambda i. R(W^\circ i, A_1^\circ i W^\circ i, \dots, A_n^\circ i W^\circ i) \\
R\langle W, A: f_1, \dots, f_n \rangle &:= \lambda i. R(W^\circ i, A^\circ i W^\circ i, f_1(A^\circ i W^\circ i), \dots, f_n(A^\circ i W^\circ i)) \\
R\langle W, A: f_1, \dots, f_n: B \rangle &:= \lambda i. R(W^\circ i, A^\circ i W^\circ i, f_1(A^\circ i W^\circ i), \dots, f_n(A^\circ i W^\circ i), B^\circ i) \\
R\langle \omega, A: f_1, \dots, f_n \rangle &:= \lambda i. \forall w (\exists u_a (u = A^\circ iw) \rightarrow \\
&\quad R(w, A^\circ iw, f_1(A^\circ iw), \dots, f_n(A^\circ iw))) \\
R\langle \omega, A: f_1, \dots, f_n: B \rangle &:= \lambda i. \forall w (\exists u_a (u = A^\circ iw) \rightarrow \\
&\quad R(w, A^\circ iw, f_1(A^\circ iw), \dots, f_n(A^\circ iw), B^\circ i))
\end{aligned}$$

For  $\mathbf{R} \in \{=, \in, \subseteq, \subset, \supset, \emptyset, \leq, <\}$

$$\begin{aligned}
(A \mathbf{R} B)^\circ &:= \lambda i. A^\circ i \mathbf{R} B^\circ i \\
(A \mathbf{R}_\omega B) &:= \lambda i \forall w (\exists u \exists u' (u = A^\circ iw \wedge u' = B^\circ iw) \rightarrow A^\circ iw \mathbf{R} B^\circ iw) \\
(A \mathbf{R}_{\forall W} B) &:= \lambda i \forall w (w \in W^\circ i \rightarrow A^\circ iw \mathbf{R} B^\circ iw) \\
(A \mathbf{R}_{\exists W} B) &:= \lambda i. \exists (W^\circ i, A^\circ i W^\circ i) \mathbf{R} \exists (W^\circ i, B^\circ i W^\circ i) \\
\mathbf{R}\langle W: A, T \rangle &:= \lambda i. T^\circ i W^\circ i \mathbf{R} \exists (W^\circ i, A^\circ i W^\circ i) \\
[C] &:= \lambda I \lambda j. Ij \wedge Cj \\
[u_1 \dots u_n] &:= \lambda I \lambda j \exists u_1 \dots u_n \exists i (j = (u_1 \cdot \dots \cdot (u_n \cdot i)) \wedge Ii) \\
[u_1 \dots u_n | C] &:= \lambda I \lambda j \exists u_1 \dots u_n \exists i (j = (u_1 \cdot \dots \cdot (u_n \cdot i)) \wedge Ii \wedge Ci)
\end{aligned}$$

### A7 Abbreviations (global values, substates, global updates)

$$\begin{aligned}
A\{Z\} &:= \lambda u. \exists i (Zi \wedge A^\circ i = u) \\
Z_{(A_1: B_1, \dots, A_n: B_n)} &:= \lambda i. Zi \wedge A_1^\circ i = B_1 \wedge \dots \wedge A_n^\circ i = B_n \\
(K; K') &:= \lambda I. K'KI \\
({}^P K) &:= \lambda I \lambda j. KIj \wedge \mathbf{d}\omega \{KI\} = \mathbf{d}\omega \{I\} \\
[R\{\omega: A_1, \dots, A_n\}] &:= \lambda I \lambda j. Ij \wedge \forall w (w \in \mathbf{d}\omega \{I\} \rightarrow R(w, A_1^\circ jw, \dots, A_n^\circ jw)) \\
[\subset\{W: A, T\}] &:= \lambda I \lambda j. Ij \wedge (\exists i (Ii \wedge \text{PL } T^\circ i W^\circ i) \rightarrow \exists (W^\circ j, A^\circ j W^\circ j) \subset T^\circ j W^\circ j) \\
&\quad \wedge (\forall i (Ii \rightarrow \text{SG } T^\circ i W^\circ i) \rightarrow T^\circ j W^\circ j \subset \exists (W^\circ j, \text{CON } A^\circ j W^\circ j))
\end{aligned}$$

For  $\mathbf{R} \in \{=, \in, \subseteq, \subset, \supset, \emptyset\}$

$$\begin{aligned}
[A \mathbf{R} B \{\}] &:= \lambda I \lambda j. Ij \wedge A^\circ j \mathbf{R} B \{I\} \\
[A \mathbf{R} B \{\{C_1 \dots C_n\}\}] &:= \lambda I \lambda j. Ij \wedge A^\circ j \mathbf{R} B \{I\{C_1: C_1j, \dots, C_n: C_nj\}\}
\end{aligned}$$

APPENDIX E: *From English verbs to UC*

Sample basic meanings of English verbs. We write ‘ $a?$ ’ for an anaphoric presupposition in search of an antecedent of type  $a$  (or  $a$ -valued function).

## E1. main &amp; auxiliary verbs (sample)

- *go+out*, event  $v$
- $\varepsilon$   $[e | go.out \langle \mathbf{d}\omega, e: AGT \rangle]$
- *like*, state  $v$
- $\sigma$   $[s a | like \langle \mathbf{d}\omega, s: EXP: a \rangle, (EXP s = CTR s)^\circ]$
- *have ...-en*, state auxiliary +  $v \setminus v$ -suffix
- $\sigma$   $[s | (EXP s = CTR s)^\circ]; \dots; [(d\sigma = CON d\varepsilon)^\circ]$

E2. TNS marking: <sup>1</sup>tns.presup; <sup>2</sup>modal-tmp.upd; <sup>3</sup>temporal.attention.update

- -PST (e.g. *was*), on event  $v$   
 $^1(P[(\mathbf{d}\tau_n < \vartheta(\mathbf{d}\omega, \mathbf{d}\varepsilon))^\circ]); ^2[\supset \langle \omega?, d\varepsilon, \mathbf{d}\tau_n \rangle, (CTR d\varepsilon = \alpha?)^\circ];$   
 $^3[\mathbf{t} | \mathbf{t} \subset \vartheta(\mathbf{d}\omega, CON d\varepsilon)^\circ, (\mathbf{t} < \vartheta(\omega?, \mathbf{d}\varepsilon)^\circ)]$
- -NPST (e.g. *is*), on event  $v$   
 $^1(P[(\vartheta(\mathbf{d}\omega, \mathbf{d}\varepsilon) \leq \mathbf{d}\tau_n)^\circ]); ^2[\supset \langle \omega?, d\varepsilon, \mathbf{d}\tau_n \rangle, (CTR d\varepsilon = \alpha?)^\circ];$   
 $^3[\mathbf{t} | \mathbf{t} \subset \vartheta(\omega?, CON d\varepsilon)^\circ]$
- FUT (e.g. *will*), on event  $v$   
 $^1(P[(\vartheta(\mathbf{d}\omega, \mathbf{d}\varepsilon) \leq \mathbf{d}\tau_n)^\circ]); ^2([e]; [\supset \langle \omega?, d\varepsilon, \mathbf{d}\tau_n \rangle, (CTR d\varepsilon = \alpha?)^\circ]);$   
 $^3[\mathbf{t} | \mathbf{t} \subset \vartheta(\omega?, CON d\varepsilon)^\circ]$

E3. MODAL marking: (fused w. TNS) <sup>1</sup>modal.presup; <sup>3</sup>modal.attn.update

- .IND (e.g. *is*)  
 $^P[\omega? \in \mathbf{d}\omega \{\}] ]$
- .VIV (e.g. *will*), doxastic rdg.  
 $\lambda D. (^P[\mathbf{d}\Omega \subseteq \mathbf{d}\omega \{\}] ); [Q | (Q \subset BEL(\omega?, \varepsilon?))^\circ]; D;$   
 $[MIN(d\Omega t, \mathbf{d}\Omega) \subseteq d\omega \{_{|\omega?, d\Omega t}\} ]$
- $\circ$  .REM (e.g. *would*)  
 $\lambda D. [Q | (Q \subset BEL(\omega?, \varepsilon?))^\circ]; D; [MIN(d\Omega t, \mathbf{d}\Omega) \subseteq d\omega \{_{|\omega?, d\Omega t}\} ]$

## E4. MOOD marking prosody

- .  
 $[\mathbf{p}]; [\mathbf{d}\Omega = \mathbf{d}\omega \{\}] ]$
- ?  
 $[q]; [d\Omega = d\omega \{ \dots \}]; [\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{\}] ]$
- !  
 $[d\varepsilon =_{\vartheta d\omega} END CON \mathbf{d}\varepsilon, (CTR d\varepsilon = EXP \mathbf{d}\varepsilon)^\circ]; [q]; [d\Omega = d\omega \{\}];$   
 $[\mathbf{t} | (\mathbf{t} \subset \vartheta(d\omega, CON d\varepsilon))^\circ]$

APPENDIX K: *From Kalaallisut verbs to UC*

Sample basic meanings of Kalaallisut verbs. We write ‘ $a?$ ’ for an anaphoric presupposition in search of an antecedent of type  $a$  (or  $a$ -valued function).

## K1. Verbal roots and derivational suffixes (sample)

- *ani-* (go.out-), event  $v$ -root  
 $\varepsilon$  [ $e$ ] *go.out* $\langle \mathbf{d}\omega, e: \text{AGT} \rangle$
- *anniar-* (in.pain-), state  $v$ -root  
 $\sigma$  [ $s$ ] *in.pain* $\langle \mathbf{d}\omega, s: \text{EXP} \rangle$ , ( $\text{EXP } s = \text{CTR } s$ ) $^\circ$ ]
- *niriug-* (hope-), state  $v$ -root  
 $\sigma$  [ $s$   $p$ ] *hope* $\langle \mathbf{d}\omega, s: \text{EXP}: p \rangle$ , ( $\text{EXP } s = \text{CTR } s$ ) $^\circ$ ]
- *-tit* (-cause), event  $v \setminus v$ -suffix on state  $v$   
 $\varepsilon \setminus \varepsilon$  [ $e$ ] ( $\text{CON } e = d\sigma$ ) $^\circ$ , ( $\text{AGT } e = \text{CTR } e$ ) $^\circ$ , ( $\text{CTR } e = \text{CTR } d\sigma$ ) $^\circ$ ]
- *-sima* (-prf), state  $v \setminus v$ -suffix on event  $v$   
 $\varepsilon \setminus \sigma$  [ $s$ ] ( $s = \text{CON } d\varepsilon$ ) $^\circ$ , ( $\text{CTR } s = \text{CTR } d\varepsilon$ ) $^\circ$ ]
- *-ssa* (-exp/des), state  $v \setminus v$ -suffix on event  $v$   
 $\varepsilon \setminus \sigma$   $^P$ [( $? \Omega \in ? \Omega t$ ) $^\circ$ , ( $? \Omega t \subseteq \text{BEL} + \text{DES}(\mathbf{d}\omega, ? \varepsilon)$ ) $^\circ$ ]; [ $\mathbb{C} \{d\omega: d\varepsilon, d\tau\}$ ];  
 $[s$ ]  $s \in_{\mathbf{V}\{\mathbf{d}\omega, d\omega\}} \text{CON}(\omega: ? \varepsilon)$ , ( $\text{CTR } s = \text{CTR } d\varepsilon$ ) $^\circ$ ,  $d\varepsilon <_{\vartheta d\omega} \text{END } s$ ,  
 $(\text{MIN}(? \Omega t, ?' \Omega) \in \text{BEG BEL}(d\omega, \text{END } s))$ ) $^\circ$ ];  
 $[\text{MIN}(? \Omega t, ?' \Omega) \subseteq d\omega \{ |_{d\omega, d\sigma, ? \Omega t, ? \varepsilon} \}]$
- *-nngit* (-not), state  $v \setminus v$  suffix on event  $v$  ( $\varepsilon \setminus \sigma$ ) state  $v$  ( $\sigma \setminus \sigma$ )  
 $\varepsilon \setminus \sigma$  [ $\mathbb{C} \{d\omega: d\varepsilon, ? \tau\}$ ]; [ $s$ ] ( $\vartheta(\mathbf{d}\omega, s) = ? \tau$ ) $^\circ$ , ( $\text{CTR } s = \text{CTR } d\varepsilon$ ) $^\circ$ ];  
 $[p]$ ; [ $d\Omega = d\omega \{ |_{d\sigma, ? \tau} \}$ ]; [ $(\{\mathbf{d}\omega\} \emptyset d\Omega)$ ] $^\circ$ ]
- *-galuar* (-rem), aspect-preserving  $v \setminus v$ -suffix  
 $\sigma \setminus \sigma$  [ $\mathbb{C} \{d\omega: \text{BEG } d\sigma, ? \tau\}$ ]; [ $s$ ] ( $\vartheta(\mathbf{d}\omega, s) = ? \tau$ ) $^\circ$ , ( $\text{CTR } s = \text{CTR } d\sigma$ ) $^\circ$ ];  
 $[p]$ ; [ $d\Omega = d\omega \{ |_{d\sigma, ? \tau} \}$ ]; [ $(\{\mathbf{d}\omega\} \emptyset d\Omega)$ ] $^\circ$ ]
- *-la* (-rem), aspect-preserving  $v \setminus v$ -suffix  
 $\sigma \setminus \sigma$  [ $p$ ] ( $\{\mathbf{d}\omega\} \emptyset p$ ) $^\circ$ , ( $p \in \text{BEL} + \text{DES}(\mathbf{d}\omega, ? \varepsilon)$ ) $^\circ$ ]

## K2. Illocutionary moods for matrix verbs (sample, general form:

$^1$ illoc.presup;  $^2$ modal.tmp.upd;  $^3$ modal.att.upd;  $^4$ illoc.declaration)

- *-pu* (-DEC), on event  $v$  ( $\varepsilon$ ), or state  $v$  ( $\sigma$ )  
 $\varepsilon$   $^1$ ( $^P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon]$ );  $^2$ ( $[\mathbb{C} \{\mathbf{d}\omega: d\varepsilon, d\tau\}$ ]; [ $d\varepsilon <_{\vartheta d\omega} d\omega\varepsilon$ , ( $\text{CTR } d\varepsilon = \mathbf{d}\alpha$ ) $^\circ$ ]);  
 $^3$ ( $[\mathbf{p}]$ ; [ $\mathbf{d}\Omega = \mathbf{d}\omega \{ | \}$ ])
- *-la* (-DEC $_{\neq}$ ), on state  $v$   
 $\sigma$   $^1$ ( $^P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon$ , ( $\{\mathbf{d}\omega\} \emptyset d\Omega$ ) $^\circ$ ]);  $^2$ ( $[\mathbb{C} \{\mathbf{d}\omega: d\sigma, d\tau\}$ ]; [ $\text{BEG } d\sigma \leq_{\vartheta d\omega} d\omega\varepsilon$ , ( $\text{CTR } d\sigma = \mathbf{d}\alpha$ ) $^\circ$ ]);  
 $^3$ ( $[\mathbf{p}]$ ; [ $\mathbf{d}\Omega = \mathbf{d}\omega \{ | \}$ ])

- *-pi* (-QUE), on event v  
 $\varepsilon$   $^1(P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \mathbf{d}\tau\}]; [d\varepsilon <_{\vartheta d\omega} d\omega\varepsilon, (\text{CTR } d\varepsilon = \mathbf{d}\alpha)^{\circ}]);$   
 $^3([\mathbf{p}]; [d\Omega = d\omega \{\}\}); [\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{\}\}); ^4[ask\langle\omega, d\omega\varepsilon: \text{AGT: } \mathbf{d}\Omega t\rangle]$
- *-li* (-QUE<sub>ε</sub>), on event v  
 $\varepsilon$   $^1(P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon, (\{d\omega\} \emptyset d\Omega)^{\circ}]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \mathbf{d}\tau\}]; [d\varepsilon <_{\vartheta d\omega} d\omega\varepsilon,$   
 $(\text{CTR } d\varepsilon = \mathbf{d}\alpha)^{\circ}]); ^3([\mathbf{p}]; [d\Omega = d\omega \{\}\}); [\mathbf{Q}]; [\mathbf{d}\Omega t = d\Omega \{\}\});$   
 $^4[ask\langle\omega, d\omega\varepsilon: \text{AGT: } \mathbf{d}\Omega t\rangle]$
- *-gi* (-IMP), on event v  
 $\varepsilon$   $^1(P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \vartheta(\omega: \text{CON } \mathbf{d}\varepsilon)\}]; [\text{CON } d\omega\varepsilon <_{\vartheta d\omega} \text{CON } d\varepsilon,$   
 $(\text{CTR } d\varepsilon = \text{EXP } \mathbf{d}\varepsilon)^{\circ}]); ^3([\mathbf{p}]; [d\Omega = d\omega \{\}\}); ^4[dir.do\langle\omega, d\omega\varepsilon: \text{AGT, EXP: } d\Omega\rangle]$
- *-qina* (-IMP<sub>ε</sub>), on event v  
 $\varepsilon$   $^1(P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \vartheta(\omega: \text{CON } \mathbf{d}\varepsilon)\}]; [d\varepsilon <_{\vartheta d\omega} \text{END CON } d\omega\varepsilon,$   
 $(\text{CTR } d\varepsilon = \text{EXP } \mathbf{d}\varepsilon)^{\circ}]); ^3([\mathbf{p}]; [d\Omega = d\omega \{\}\}); ^4[dir.to\langle\omega, d\omega\varepsilon: \text{AGT, EXP: } \mathbf{d}\Omega \setminus d\Omega\rangle]$
- *-li* ~ *-la* (-OPT), on event v  
 $\varepsilon$   $^1(P[d\omega\varepsilon =_{\omega} \mathbf{d}\varepsilon]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \vartheta(\omega: \text{CON } \mathbf{d}\varepsilon)\}]; [\text{CON } d\omega\varepsilon <_{\vartheta d\omega} \text{CON } d\varepsilon,$   
 $(\text{CTR } d\varepsilon = \text{EXP } \mathbf{d}\varepsilon)^{\circ}]); ^3([\mathbf{p}]; [d\Omega = d\omega \{\}\}); ^4[wish.for\langle\omega, d\omega\varepsilon: \text{AGT: } d\Omega\rangle]$

K3. Subject-centering moods for dependent verbs (sample, general form:  
 $^1\alpha$ -recentering.presup;  $^2$ modal-tmp.upd;  $^3$ modvtmp.att.upd)

- *-ga* (-FCT<sub>τ</sub>), on event v  
 $\varepsilon$   $^1(P[\mathbf{a} | \mathbf{a} = d\alpha]); ^2([\mathbb{C}\{\mathbf{d}\omega: d\varepsilon, \mathbf{d}\tau\}]; [(\vartheta\varepsilon =_{\vartheta d\omega} d\varepsilon)^{\circ}, d\varepsilon <_{\vartheta d\omega} d\omega\varepsilon,$   
 $(\text{CTR } d\varepsilon = \mathbf{d}\alpha)^{\circ}]); ^3[\mathbf{t} | (\mathbf{t} \subset \vartheta(\mathbf{d}\omega, \text{CON } d\varepsilon))^{\circ}, (\mathbf{t} < \vartheta(\mathbf{d}\omega, d\omega\varepsilon))^{\circ}]$
- *-mm* (-FCT<sub>⊥</sub>), on event v  
 $\varepsilon$   $^1(P[a | a = \mathbf{d}\alpha]); ^2([\mathbb{C}\{\mathbf{d}\omega: d\varepsilon, \mathbf{d}\tau\}]; [(\vartheta\varepsilon =_{\vartheta d\omega} d\varepsilon)^{\circ}, d\varepsilon <_{\vartheta d\omega} d\omega\varepsilon,$   
 $(\text{CTR } d\varepsilon = d\alpha)^{\circ}]); ^3[\mathbf{t} | (\mathbf{t} \subset \vartheta(\mathbf{d}\omega, \text{CON } d\varepsilon))^{\circ}, (\mathbf{t} < \vartheta(\mathbf{d}\omega, d\omega\varepsilon))^{\circ}]$
- *-gu* (-HYP<sub>τ</sub>), on event v  
 $\varepsilon$   $^1(P[\mathbf{a} | \mathbf{a} = d\alpha]); ^2([\mathbb{C}\{d\omega: d\varepsilon, \vartheta\tau\}]; [(\vartheta\varepsilon <_{\vartheta d\omega} d\varepsilon)^{\circ}, (\text{CTR } d\varepsilon = \mathbf{d}\alpha)^{\circ}]);$   
 $^3([\mathbf{p}]; [\mathbf{d}\Omega = d\omega \{d\alpha, \vartheta\tau\}]; [\mathbf{t} | (\mathbf{t} \subset \vartheta(d\omega, \text{CON } d\varepsilon))^{\circ}])$
- *-pp* (-HYP<sub>⊥</sub>), on state v  
 $\sigma$   $^1(P[a | a = \mathbf{d}\alpha]); ^2([\mathbb{C}\langle d\omega: d\sigma, \vartheta\tau \rangle]; [(\vartheta\varepsilon <_{\vartheta d\omega} \text{END } d\sigma)^{\circ}, (\text{CTR } d\sigma = d\alpha)^{\circ}]);$   
 $^3([\mathbf{p}]; [\mathbf{d}\Omega = d\omega \{d\alpha, \vartheta\tau\}])$
- *-llu*, (-ELA<sub>τ</sub>), on event v, after main V  
 $\varepsilon$   $^1(P[\mathbf{a} | \mathbf{a} = d\alpha]); ^2([\mathbb{C}\{\mathbf{d}\omega: d\varepsilon, \vartheta\tau\}]; [(\vartheta\varepsilon = d\varepsilon)^{\circ}, (\text{CTR } d\varepsilon = \mathbf{d}\alpha)^{\circ}]);$   
 $^3([\mathbf{p}]; [\mathbf{d}\Omega = \mathbf{d}\omega \{\}\})$
- *-tu*, (-ELA<sub>⊥.IV</sub>), on state v, after main V  
 $\sigma$   $^1(P[a | a = \mathbf{d}\alpha]); ^2([\mathbb{C}\langle d\omega: d\sigma, \vartheta\tau \rangle]; [(\vartheta\sigma =_{\vartheta d\omega} d\sigma)^{\circ}, (\text{CTR } d\sigma = d\alpha)^{\circ}]);$   
 $^3([\mathbf{p}]; [\mathbf{d}\Omega = \mathbf{d}\omega \{\}\})$