Notes on Link 1987
"Generalized quantifiers and plurals"

I. Type Theory with Pluralities (Typed LP)

DEFINITION 1.1 (Typed LP-types)

- $t, e \in \mathbf{Typ}$
- $(ab) \in \mathbf{Typ}$ if $a, b \in \mathbf{Typ}$

DEFINITION 1.2 (Basic Typed LP-terms)

- $\mathbf{Con}_e = \{john_e, mary_e, \ldots\}$
  $\mathbf{Con}_{(ee)} = \{fa_{(ee)}, ma_{(ee)}, \ldots\}$
  $\mathbf{Con}_{(et)} = \{enter_{(et)}, \ldots\}$
  $\mathbf{Con}_{(e(et))} = \{see_{(e(et))}, talk.to_{(e(et))}, \ldots\}$
  $\mathbf{Con}_{((et)(et))} = \{{}^{D}, *, \ldots\}$ (Link 1987, 1983)
  ⋮ ⋮ ⋮
- $\mathbf{Var}_e = \{x, y, z, x', y', z', \ldots, z_1, z_2, \ldots\}$
  $\mathbf{Var}_t = \{p, q, p', q', \ldots\}$
  $\mathbf{Var}_{(et)} = \{P, Q, P', Q', \ldots\}$

DEFINITION 1.3 (Typed LP syntax).

b	$\alpha \in \mathbf{Term}_a$	if $\alpha \in \mathbf{Con}_a$ or $\alpha \in \mathbf{Var}_a$	
a	$\alpha\beta \in \mathbf{Term}_a$	if $\alpha \in \mathbf{Term}_{(ba)}$ and $\beta \in \mathbf{Term}_b$	
$\boldsymbol{\lambda}$	$\lambda u[\alpha] \in \mathbf{Term}_{(ba)}$	if $u \in \mathbf{Var}_b$ and $\alpha \in \mathbf{Term}_a$	
$\boldsymbol{\sigma}$	$\sigma u.\alpha \in \mathbf{Term}_e$	if $u \in \mathbf{Var}_e$ and $\alpha \in \mathbf{Term}_t$	(Link 1987:152)
+	$(\alpha + \beta) \in \mathbf{Term}_e$	if $\alpha \in \mathbf{Term}_e$ and $\beta \in \mathbf{Term}_e$	(Link 1987:152)
∧	$(\alpha \wedge \beta) \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_t$ and $\beta \in \mathbf{Term}_t$	
¬	$\neg\alpha \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_t$	
=	$(\alpha = \beta) \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_a$ and $\beta \in \mathbf{Term}_a$	

DEFINITION 2.1 (Typed LP-frames).

- $D_t = \{1, 0\}$
  $D_e = \{X \mid \varnothing \subset X \subseteq A\}$, where A (*set of atoms*) is a non-empty set disjoint from D_t
- $D_{(ab)} = \{f \mid \text{Dom } f = D_a \ \&\ \text{Ran } f \subseteq D_b\}$

DEFINITION 2.2 (Typed LP-models and assignments)

- A Typed LP-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M \rangle$ such that:
  (i) $D^M = \{D_a^M : a \in \mathbf{Typ}\}$ is a Typed LP-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_a$ a *denotation* $[\![\alpha]\!]^M \in D_a^M$.
  (iii) for any $f \in D_{(et)}^M$, $d \in D_e^M$:
  $[\![{}^{D}]\!]^M(f)(d) = 1$ iff $\{d' \in D_e^M : |d'| = 1 \ \&\ d' \subseteq d\} \subseteq \{d' \in D_e^M : f(d') = 1\}$[1]
  $[\![*]\!]^M(f)(d) = 1$ iff $d \in \{\cup X : \varnothing \subset X \subseteq \{d' \in D_e^M : f(d') = 1\}\}$[2]
- An *M-assignment* is a function θ that assigns to any $u \in \mathbf{Var}_a$ a *value* $\theta(u) \in D_a^M$. If $d \in D_a^M$ then $\theta[u/d]$ is the M-assignment s.t. (i) $\theta[u/d](u) = d$, and (ii) $\theta[u/d](u') = \theta(u')$ if $u' \neq u$.

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[1] Link 1987:171: ${}^{D}\mathrm{VP} := \lambda x\, \forall y[y\ {}^{\bullet}\Pi x \rightarrow \mathrm{VP}(y)]$, where '$x\,{}^{\bullet}\Pi y$' stands for '$x$ is an atomic part of y'

[2] Link 1983:315: $[\![{*P}]\!] := \{x \in E \mid \exists X \subseteq [\![P]\!] \ \&\ X \neq \varnothing$ s.t. $x = \sup_i X\}$

DEFINITION 2.3 (Typed LP-semantics)

b	$[\![\alpha]\!]^{M,\theta}$	=	$[\![\alpha]\!]^{M}$	if $\alpha \in \mathbf{Con}_\tau$
		=	$\theta(\alpha)$	if $\alpha \in \mathbf{Var}_\tau$
a	$[\![\alpha\beta]\!]^{M,\theta}$	=	$[\![\alpha]\!]^{M,\theta}([\![\beta]\!]^{M,\theta})$	
$\boldsymbol{\lambda}$	$[\![\lambda u[\alpha]]\!]^{M,\theta}$	=	$\langle [\![\alpha]\!]^{M,\theta[u/d]}: d \in D_\sigma^M \rangle$	
$\boldsymbol{\sigma}$	$[\![\sigma u.\alpha]\!]^{M,\theta}$	=	$\cup\{d \in D_e^M: [\![\alpha]\!]^{M,\theta[u/d]} = 1\}$	
+	$[\![(\alpha + \beta)]\!]^{M,\theta}$	=	$([\![\alpha]\!]^{M,\theta} \cup [\![\beta]\!]^{M,\theta})$	
∧	$[\![(\alpha \wedge \beta)]\!]^{M,\theta}$	=	1,	iff $[\![\alpha]\!]^{M,\theta} = 1$ & $[\![\beta]\!]^{M,\theta} = 1$
¬	$[\![\neg\alpha]\!]^{M,\theta}$	=	1,	iff $[\![\alpha]\!]^{M,\theta} = 0$
=	$[\![(\alpha = \beta)]\!]^{M,\theta}$	=	1,	iff $[\![\alpha]\!]^{M,\theta} = [\![\beta]\!]^{M,\theta}$
r	$[\![\alpha]\!]^{M,\theta}$	$\in$	D_τ^M	if $\alpha \in \mathbf{Term}_\tau$

DEFINITION 3 (Truth and equivalence).

- ϕ is *true* in M under θ, written $\models_{M,\theta} \phi$, iff $[\![\phi]\!]^{M,\theta} = 1$
- α is *equivalent* to β, written $\alpha \equiv \beta$, iff, for all M and θ, $[\![\alpha]\!]^{M,\theta} = [\![\beta]\!]^{M,\theta}$.

ABBREVIATIONS 1 (standard). We write:

•	$(\alpha \vee \beta)$	for	$\neg(\neg\alpha \wedge \neg\beta)$
	$(\alpha \rightarrow \beta)$	for	$\neg(\alpha \wedge \neg\beta)$
•	$\forall u\alpha$	for	$(\lambda u[\alpha] = \lambda u[(u = u)])$
	$\exists u\alpha$	for	$\neg\forall u\neg\alpha$

FACT 1:

•	$[\![(\alpha \vee \beta)]\!]^{M,\theta}$	=	0,	iff	$[\![\alpha]\!]^{M,\theta} = 0$ & $[\![\beta]\!]^{M,\theta} = 0$
	$[\![(\alpha \rightarrow \beta)]\!]^{M,\theta}$	=	0,	iff	$[\![\alpha]\!]^{M,\theta} = 1$ & $[\![\beta]\!]^{M,\theta} = 0$
•	$[\![\forall u\alpha]\!]^{M,\theta}$	=	1	iff	$\{d \in D_a^M: [\![\alpha]\!]^{M,\theta[u/d]} = 1\} = D_a^M$
	$[\![\exists u\alpha]\!]^{M,\theta}$	=	1	iff	$\{d \in D_a^M: [\![\alpha]\!]^{M,\theta[u/d]} = 1\} \neq \{\}$

ABBREVIATIONS 2 (plurality-based). We write:

				Read:•
•	$(\alpha \leq \beta)$	for	$\exists x(\alpha + x = \beta)$	α is *part of* β
	$(\alpha \bigcirc \beta)$	for	$\exists x(x \leq \alpha \wedge x \leq \beta)$	α *overlaps* β
	$\mathbf{sg}\ \alpha$	for	$\forall x(x \leq \alpha \rightarrow x = \alpha)$	α is an *atom*
•	$\mathbf{2}$	for	$\lambda x[\exists y\exists y'(x = y + y' \wedge \mathbf{sg}\ y \wedge \mathbf{sg}\ y' \wedge \neg y \bigcirc y')]$	*consists of two atoms*
	$\mathbf{3}$	for	$\lambda x[\exists y\exists y'(x = y + y' \wedge \mathbf{sg}\ y \wedge \mathbf{2}\ y' \wedge \neg y \bigcirc y')]$	*consists of three atoms*
	⋮	⋮	⋮	

FACT 2: For any $\alpha, \alpha', \beta \in \mathbf{Term}_e$, $d \in D_e^M$:

•	$[\![(\alpha \leq \beta)]\!]^{M,\theta}$	=	1	iff	$[\![\alpha]\!]^{M,\theta} \subseteq [\![\beta]\!]^{M,\theta}$
	$[\![(\alpha \bigcirc \beta)]\!]^{M,\theta}$	=	1	iff	$([\![\alpha]\!]^{M,\theta} \cap [\![\beta]\!]^{M,\theta}) \neq \{\}$
	$[\![\mathbf{sg}\ \alpha]\!]^{M,\theta}$	=	1	iff	$\lvert [\![\alpha]\!]^{M,\theta} \rvert = 1$
•	$[\![\mathbf{2}]\!]^{M,\theta}(d)$	=	1	iff	$\lvert d \rvert = 2$
	$[\![\mathbf{3}]\!]^{M,\theta}(d)$	=	1	iff	$\lvert d \rvert = 3$
	⋮				

II. Collectivity and distributivity

Examples:

(1)	[John and Mary] are a nice couple.	collective VP
(2)	[John and Mary] are students.	distributive VP
(3)	[John and Mary] bought a house	ambiguous VP
(4)	[John and Mary] invited their$_{dis}$ parents to their$_{coll}$ place.	mixed VP

In general:

- *Collective predicates* are predicated of pluralities
- *Distributive predicates* are distributed over pluralities (by ^{D} or *)
- *Mixed predicates* involve both collective predication and distribution

Typed LP translations:

- John and Mary $\rightsquigarrow$ $\lambda P[P(john + mary)]$ — Link 87:157
- are a nice couple $\rightsquigarrow$ $nice.couple$ — collective VP
  (1) $\rightsquigarrow$ $nice.couple(john + mary)$
- are students $\rightsquigarrow$ $^{D}student$ — distributive VP
  (2) $\rightsquigarrow$ $^{D}student(john + mary)$
- bought a house $\rightsquigarrow$ $\lambda x[\exists y(house\ y \wedge buy\ yx]$ — collective reading
  (3_{coll}) $\rightsquigarrow$ $\exists y(house\ y \wedge buy\ y\ (john + mary))$

  bought a house $\rightsquigarrow$ $^{D}\lambda x[\exists y(house\ y \wedge buy\ yx]$ — distributive reading
  (3_{dis}) $\rightsquigarrow$ $^{D}\lambda x[\exists y(house\ y \wedge buy\ yx](john + mary)$
- invited their$_{dis}$ parents to their$_{coll}$ place
  $\rightsquigarrow$ $\lambda x'[\exists z(plc.of\ x'z \wedge {}^{D}\lambda x[inv.to\ z\ (\sigma y.\ prnt.of\ xy)\ x]x')]$
  (4) — mixed VP
  $\rightsquigarrow$ $\exists z(plc.of\ (john + mary)z \wedge {}^{D}\lambda x[inv.to\ z\ (\sigma y.\ prnt.of\ xy)\ x](john + mary))$

Truth conditions and models:

(1) John and Mary are a nice couple

1. $\models_{M,\theta}\ nice.couple(john + mary)$
2. $[\![nice.couple(john + mary)]\!]^{M,\theta} = 1$ — D3
3. $[\![nice.couple]\!]^{M}([\![john]\!]^{M} \cup [\![mary]\!]^{M}) = 1$ — D2.3:***a*, +, *b***

e.g. true in any *M* s.t.:
$[\![john]\!]^{M} = \{J\}$ $[\![mary]\!]^{M} = \{M\}$ $[\![nice.couple]\!]^{M}(\{J, M\}) = 1$

(2) John and Mary are students.

1. $\models_{M,\theta}\ {}^{D}student(john + mary)$
2. $[\![{}^{D}student(john + mary)]\!]^{M,\theta} = 1$ — D3
3. $[\![{}^{D}]\!]^{M}([\![student]\!]^{M})([\![john]\!]^{M} \cup [\![mary]\!]^{M}) = 1$ — D2.3:***a*, +, *b***
4. $\{d \in D_e^{M}: |d| = 1\ \&\ d \subseteq [\![john]\!]^{M} \cup [\![mary]\!]^{M}\}$
   $\subseteq \{d \in D_e^{M}: [\![student]\!]^{M}(d) = 1\}$ — D2.2:^{D}

e.g. true in any *M* s.t.:
$[\![john]\!]^{M} = \{J\}$ $[\![mary]\!]^{M} = \{M\}$ $[\![student]\!]^{M}(\{J\}) = 1$ $[\![student]\!]^{M}(\{M\}) = 1$

(3_{coll}) John and Mary bought a house (jointly).

1. $\models_{M,\theta} \exists y(house\ y \wedge buy\ y(john + mary))$
2. $[\![\exists y(house\ y \wedge buy\ y(john + mary))]\!]^{M,\theta} = 1$ D3
3. $\{d \in D_e^M: [\![house]\!]^M(d) = 1$ $\&\ [\![buy]\!]^M(d)([\![john]\!]^M \cup [\![mary]\!]^M) = 1\} \neq \emptyset$ F1:**∃**, D2.3:∧, ***a***, **+**, ***b***, D2.2.θ[*u*/*d*]

e.g. true in any *M* s.t.:

$[\![john]\!]^M = \{J\}$ $[\![house]\!]^M(\{H\}) = 1$ $[\![buy]\!]^M(\{H\})(\{J, M\}) = 1$

$[\![mary]\!]^M = \{M\}$

(3_{dis}) John and Mary bought a house (each)

1. $\models_{M,\theta}\ {}^{D}\lambda x[\exists y(house\ y \wedge buy\ yx)](john + mary)$
2. $[\![{}^{D}\lambda x[\exists y(house\ y \wedge buy\ yx)](john + mary)]\!]^{M,\theta} = 1$ D3
3. $[\![{}^{D}]\!]^M([\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta})([\![john]\!]^M \cup [\![mary]\!]^M) = 1$ D2.2:***a***, **+**, ***b***
4. $\{d \in D_e^M: |d| = 1\ \&\ d \subseteq [\![john]\!]^M \cup [\![mary]\!]^M\}$ $\subseteq \{d \in D_e^M: [\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta}(d) = 1\}$ D2.2:^{D}
5. $\{d \in D_e^M: |d| = 1\ \&\ d \subseteq [\![john]\!]^M \cup [\![mary]\!]^M\}$ $\subseteq \{d \in D_e^M: \{d' \in D_e^M: [\![house]\!]^M(d') = 1$ $\&\ [\![buy]\!]^M(d')(d) = 1\} \neq \emptyset\}$ D2.3:**λ**, df. ⟨–:–⟩, F1:**∃**, D2.3:∧, ***a***, **+**, ***b***, D2.2.θ[*u*/*d*]

e.g. true in any *M* s.t.:

$[\![john]\!]^M = \{J\}$ $[\![house]\!]^M(\{H\}) = 1$ $[\![buy]\!]^M(\{H\})(\{J\}) = 1$

$[\![mary]\!]^M = \{M\}$ $[\![house]\!]^M(\{H'\}) = 1$ $[\![buy]\!]^M(\{H'\})(\{M\}) = 1$

(4_{dc}) John and Mary invited their$_{dis}$ parents to their$_{coll}$ place.

1. $\models_{M,\theta} \exists z(plc.of\ (john + mary)z \wedge {}^{D}\lambda x[inv.to\ z\ (\sigma y.\ prnt\ xy)\ x](john + mary))$
2. $[\![\exists z(plc.of\ (john + mary)z \wedge {}^{D}\lambda x[inv.to\ z\ (\sigma y.\ prnt\ xy)\ x](john + mary))]\!]^{M,\theta} = 1$ D3
3. $\{d'' \in D_e^M: [\![plc.of]\!]^M([\![john]\!]^M \cup [\![mary]\!]^M)(d'') = 1$ $\&\ [\![{}^{D}]\!]^M([\![\lambda x[inv.to\ z\ \sigma y.\ prnt\ xy\ x]]\!]^{M,\theta[z/d'']})([\![john]\!]^M \cup [\![mary]\!]^M) = 1\} \neq \emptyset$ F1:**∃**, D2.3:∧, ***a***, **+**, ***b***
4. $\{d'' \in D_e^M: [\![plc.of]\!]^M([\![john]\!]^M \cup [\![mary]\!]^M)(d'') = 1$ $\&\ \{d \in D_e^M: |d| = 1\ \&\ d \subseteq [\![john]\!]^M \cup [\![mary]\!]^M\}$ $\subseteq \{d \in D_e^M: [\![\lambda x[inv.to\ z\ \sigma y.\ prnt\ xy\ x]]\!]^{M,\theta[z/d'']}(d) = 1\}\} \neq \emptyset$ D2.2:^{D}
5. $\{d'' \in D_e^M: [\![plc.of]\!]^M([\![john]\!]^M \cup [\![mary]\!]^M)(d'') = 1$ $\&\ \{d \in D_e^M: |d| = 1\ \&\ d \subseteq [\![john]\!]^M \cup [\![mary]\!]^M\}$ $\subseteq \{d \in D_e^M: [\![inv.to]\!]^M(d'')(\cup\{d' \in D_e^M: [\![prnt]\!]^M(d)(d') = 1\})(d) = 1\}\} \neq \emptyset$ D2.2:**λ**, df. ⟨–:–⟩ D2.3:**λ**, ***a***, ***σ***, ***a***, ***b***, D2.2:θ[*u*/*d*]

e.g. true in any *M* s.t.:

$[\![john]\!]^M = \{J\}$ $\cup\{d' \in D_e^M: [\![prnt]\!]^M(\{J\})(d') = 1\} = \cup\{\{J'\}, \{J''\}\} = \{J', J''\}$

$[\![mary]\!]^M = \{M\}$ $[\![inv.to]\!]^M(\{H\})(\{J', J''\})(\{J\}) = 1$

$[\![plc.of]\!]^M(\{J, M\})(\{H\}) = 1$ $\cup\{d' \in D_e^M: [\![prnt]\!]^M(\{M\})(d') = 1\} = \cup\{\{M'\}, \{M''\}\} = \{M', M''\}$

$[\![inv.to]\!]^M(\{H\})(\{M', M''\})(\{M\}) = 1$

III. Distributivity (^{D}) vs. pluralization (*)

D2.2.iii

For any $f \in D_{(et)}{}^{M}$, $d \in D_e{}^{M}$:

- $[\![^{D}]\!]^{M}(f)(d) = 1$ iff $\{d' \in D_e{}^{M}: |d'| = 1 \;\&\; d' \subseteq d\} \subseteq \{d' \in D_e{}^{M}: f(d') = 1\}$
  [every atomic part of d satisfies f]
- $[\![*]\!]^{M}(f)(d) = 1$ iff $d \in \{\cup X: \varnothing \subset X \subseteq \{d' \in D_e{}^{M}: f(d') = 1\}\}$
  [d consists of parts that satisfy f]

Example:

(5) John and Mary and Tom [bought a house].

(5) with distributive VP

1. $\models_{M,\theta}\ {}^{D}\lambda x[\exists y(house\ y \wedge buy\ yx)]((john + mary) + bill)$
2. $[\![{}^{D}\lambda x[\exists y(house\ y \wedge buy\ yx)]((john + mary) + bill)]\!]^{M,\theta} = 1$ — D3
3. $[\![^{D}]\!]^{M}([\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta})(([\![john]\!]^{M} \cup [\![mary]\!]^{M}) \cup [\![bill]\!]^{M}) = 1$ — D2.2:***a***, **+**, ***b***
4. $\{d \in D_e{}^{M}: |d| = 1 \;\&\; d \subseteq ([\![john]\!]^{M} \cup [\![mary]\!]^{M} \cup [\![bill]\!]^{M}\}$
   $\subseteq \{d \in D_e{}^{M}: [\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta}(d) = 1\}$ — D2.2:^{D}
5. $\{d \in D_e{}^{M}: |d| = 1 \;\&\; d \subseteq ([\![john]\!]^{M} \cup [\![mary]\!]^{M} \cup [\![bill]\!]^{M}\}$ — D2.3:**λ**, df. ⟨–:–⟩,
   $\subseteq \{d \in D_e{}^{M}: \{d' \in D_e{}^{M}: [\![house]\!]^{M}(d') = 1$ — F1:**∃**, D2.3:**∧**, ***a***, **+**, ***b***,
   $\&\ [\![buy]\!]^{M}(d')(d) = 1\} \neq \varnothing\}$ — D2.2.θ[u/d]

true if:

$[\![john]\!]^{M} = \{J\}$	$[\![house]\!]^{M}(\{H\}) = 1$	$[\![buy]\!]^{M}(\{H\})(\{J\}) = 1$
$[\![mary]\!]^{M} = \{M\}$	$[\![house]\!]^{M}(\{H'\}) = 1$	$[\![buy]\!]^{M}(\{H'\})(\{M\}) = 1$
$[\![bill]\!]^{M} = \{B\}$	$[\![house]\!]^{M}(\{H''\}) = 1$	$[\![buy]\!]^{M}(\{H''\})(\{B\}) = 1$

false if:

$[\![john]\!]^{M} = \{J\}$		$[\![buy]\!]^{M}(d)(\{J\}) = 0$ for all $d \in D_e{}^{M}$

(5) with plural VP

1. $\models_{M,\theta}\ {}^{*}\lambda x[\exists y(house\ y \wedge buy\ yx)]((john + mary) + bill)$
2. $[\![{}^{*}\lambda x[\exists y(house\ y \wedge buy\ yx)]((john + mary) + bill)]\!]^{M,\theta} = 1$ — D3
3. $[\![*]\!]^{M}([\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta})(([\![john]\!]^{M} \cup [\![mary]\!]^{M}) \cup [\![bill]\!]^{M}) = 1$ — D2.2:***a***, **+**, ***b***
4. $([\![john]\!]^{M} \cup [\![mary]\!]^{M}) \cup [\![bill]\!]^{M}$
   $\in \{\cup X: \varnothing \subset X \subseteq \{d \in D_e{}^{M}: [\![\lambda x[\exists y(house\ y \wedge buy\ yx]]\!]^{M,\theta}(d) = 1\}\}$ — D2.2:*
5. $\exists X: \cup X = ([\![john]\!]^{M} \cup [\![mary]\!]^{M}) \cup [\![bill]\!]^{M}$ — df. {-:-}, D2.3:**λ**, ⟨-:-⟩,
   $\&\ \varnothing \subset X \subseteq \{d \in D_e{}^{M}: \{d' \in D_e{}^{M}: [\![house]\!]^{M}(d') = 1$ — F1:**∃**, D2.3:**∧**, ***a***, **+**, ***b***,
   $\&\ [\![buy]\!]^{M}(d')(d) = 1\} \neq \varnothing\}$ — D2.2.θ[u/d]

true if:

$[\![john]\!]^{M} = \{J\}$		$[\![buy]\!]^{M}(d)(\{J\}) = 0$ for all $d \in D_e{}^{M}$
$[\![bill]\!]^{M} = \{B\}$	$[\![house]\!]^{M}(\{H\}) = 1$	$[\![buy]\!]^{M}(\{H\})(\{J, B\}) = 1$
$[\![mary]\!]^{M} = \{M\}$	$[\![house]\!]^{M}(\{H'\}) = 1$	$[\![buy]\!]^{M}(\{H'\})(\{M, B\}) = 1$

since the scope of $\exists X$ holds for $X = \{\{J, B\}, \{M, B\}\}$

IV. Definiteness and number (Link 1983 vs. Sharvy 1980)

Examples:

(7) the boys

(8) the boy

(7) according to **Link 1983**

1. $[\![\sigma x.{*}boy\ x]\!]^{M,\theta}$
2. $\cup\{d \in D_e^M: [\![{*}boy\ x]\!]^{M,\theta[x/d]} = 1\}$ D2.3:σ
3. $\cup\{d \in D_e^M: [\![*]\!]^M([\![boy]\!]^M)(d) = 1\}$ D2.3:***a***, ***b***, D2.2:θ[*u*/*d*]
4. $\cup\{d \in D_e^M: d \in \{\cup X: \varnothing \subset X \subseteq \{d' \in D_e^M: [\![boy]\!]^M(d') = 1\}\}\}$ D2.2:*
5. $\cup\{d' \in D_e^M: [\![boy]\!]^M(d') = 1\}$ simplify

(8) according to **Link 1983**

1. $[\![\sigma x.boy\ x]\!]^{M,\theta}$
2. $\cup\{d \in D_e^M: [\![boy\ x]\!]^{M,\theta[x/d]} = 1\}$ D2.3:σ
3. $\cup\{d \in D_e^M: [\![boy]\!]^M(d) = 1\}$ D2.3:***a***, ***b***, D2.2:θ[*u*/*d*]

Problem:

- if $\{d \in D_e^M: [\![boy]\!]^M(d) = 1\} = \{\{B\}, \{B'\}\}$, then $[\![\sigma x.{*}boy\ x]\!]^{M,\theta} = [\![\sigma x.\ boy\ x]\!]^{M,\theta} = \{B, B'\}$
- if $\{d \in D_e^M: [\![boy]\!]^M(d) = 1\} = \{\{B\}\}$, then $[\![\sigma x.{*}boy\ x]\!]^{M,\theta} = [\![\sigma x.\ boy\ x]\!]^{M,\theta} = \{B\}$

D4 (*generalized ι*)

Syn. $\iota u.\alpha \in$ **Term**$_e$ if $u \in$ **Var**$_e$ and $\alpha \in$ **Term**$_t$ (Sharvy 1980:610)

Sem.[1] $[\![\iota u.\alpha]\!]^{M,\theta}$ = $max_{\subseteq}\{d \in D_e^M: [\![\alpha]\!]^{M,\theta[u/d]} = 1\}$, if it exists

= undefined, otherwise

(7) according to **Sharvy 1980**

Suppose $\{d' \in D_e^M: [\![boy]\!]^M(d') = 1\} \neq \varnothing$

1. $[\![\iota x.{*}boy\ x]\!]^{M,\theta}$
2. $max_{\subseteq}\{d \in D_e^M: [\![{*}boy\ x]\!]^{M,\theta[x/d]} = 1\}$ D4:ι
3. $max_{\subseteq}\{d \in D_e^M: [\![*]\!]^M([\![boy]\!]^M)(d) = 1\}$ D2.3:***a***, ***b***, D2.2:θ[*u*/*d*]
4. $max_{\subseteq}\{d \in D_e^M: d \in \{\cup X: \varnothing \subset X \subseteq \{d' \in D_e^M: [\![boy]\!]^M(d') = 1\}\}\}$ D2.2:*
5. $\cup\{d' \in D_e^M: [\![boy]\!]^M(d') = 1\}$ simplify

(8) according to **Sharvy 1980**

1. $[\![\iota x.boy\ x]\!]^{M,\theta}$
2. $max_{\subseteq}\{d \in D_e^M: [\![boy\ x]\!]^{M,\theta[x/d]} = 1\}$ D2.3:σ
3. $max_{\subseteq}\{d \in D_e^M: [\![boy]\!]^M(d) = 1\}$ D2.3:***a***, ***b***, D2.2:θ[*u*/*d*]

Correct:

- if $\{d \in D_e^M: [\![boy]\!]^M(d) = 1\} = \{\{B\}, \{B'\}\}$, then $[\![\iota x.{*}boy\ x]\!]^{M,\theta} =$ **{B, B′}**
  but $max_{\subseteq}\{\{B\}, \{B'\}\}$ does not exist
  so $[\![\iota x.\ boy\ x]\!]^{M,\theta}$ is **undefined**
- if $\{d \in D_e^M: [\![boy]\!]^M(d) = 1\} = \{\{B\}\}$, then $[\![\iota x.\ boy\ x]\!]^{M,\theta} = max_{\subseteq}\{\{B\}\} =$ **{B}**

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[1] Let $\langle A, \leq\rangle$ be a partially ordered set , $B \subseteq A$, and $x \in A$. Then:

- x is the ≤-*maximum* of B, denoted by $x = max_{\leq}\ B$ iff (i) $x \in B$, and (ii) $\forall y \in B$: $y \leq x$
- x is the ≤-*minimum* of B, denoted by $x = min_{\leq}\ B$ iff (i) $x \in B$, and (ii) $\forall y \in B$: $x \leq y$
- x is the ≤-*supremum* of B, denoted by $x = sup_{\leq} B$, iff (i) $x = min_{\leq}\{z \in A: \forall y \in B: y \leq z\}$

V. Numerals and partitivity

Examples:

(9) Two students can carry this.
(cf. A first-year student can do this.)

(10) Last week two students presented a paper.
(cf. Last week a first-year student presented a paper.)

(11) Last week two of the students presented a paper.

Basic ideas:

- *Numerals* translate into cardinality predicates and combine with the head noun like intersective adjectives (e.g. two tables ~ four-legged table).
- A bare numeral+noun is associated with a contextually determined *quantificational force* (e.g. universal in (7), existential in (8)), like an indefinite (see Lewis 1975, Heim 1982)
- Partitive *of* translates into the part-of relation (≤).

Typed LP translations:

• students	⇝	$*student$	
two	⇝	$\lambda Q\lambda x[\mathbf{2}x \wedge Qx]$	
[∀]	⇝	$\lambda Q\lambda P[\forall x(Qx \rightarrow Px)]$	
[∃]	⇝	$\lambda Q\lambda P[\exists x(Qx \wedge Px)]$	
the	⇝	$\lambda Q[\sigma x.Qx]$	(ok with bare plurals)
of	⇝	$\lambda y\lambda x[x \leq y]$	

• [∀] two std.	⇝	$\lambda P[\forall x(\mathbf{2}x \wedge *student\ x \rightarrow Px]$	[∀] + numeral + noun
can carry this$_z$	⇝	$\lambda x[can.carry\ zx]$	generic aspect
(7)	⇝	$\forall x(\mathbf{2}x \wedge *student\ x \rightarrow can.carry\ zx)$	

• [∃] two std.	⇝	$\lambda P[\exists x(\mathbf{2}x \wedge *student\ x \wedge Px]$	[∃] + numeral + noun
prsntd a paper	⇝	$\lambda x[\exists y(paper\ y \wedge present\ yx)]$	episodic aspect, coll. VP
		$^{D}\lambda x[\exists y(paper\ y \wedge present\ yx)]$	dis. VP
(8_{coll})	⇝	$\exists x(\mathbf{2}x \wedge *student\ x \wedge \exists y(paper\ y \wedge present\ yx))$	
(8_{dis})	⇝	$\exists x(\mathbf{2}x \wedge *student\ x \wedge {}^{D}\lambda z[\exists y(paper\ y \wedge present\ yz)]x)$	

• of the students	⇝	$\lambda x[x \leq \sigma z.*student\ z]$	partitive *of*-NP
[∃] two of …	⇝	$\lambda P[\exists x(\mathbf{2}x \wedge x \leq \sigma z.*student\ z \wedge Px)]$	[∃] + numeral + *of*-NP
prsntd a paper	⇝	$\lambda x[\exists y(paper\ y \wedge present\ yx)]$	episodic aspect, coll. VP
		$^{D}\lambda x[\exists y(paper\ y \wedge present\ yx)]$	dis. VP
(9_{coll})	⇝	$\exists x(\mathbf{2}x \wedge x \leq \sigma z.*student\ z \wedge \exists y(paper\ y \wedge present\ yx))$	
(9_{dis})	⇝	$\exists x(\mathbf{2}x \wedge x \leq \sigma z.*student\ z \wedge {}^{D}\lambda x'[\exists y(paper\ y \wedge present\ yx')]x)$	

Truth conditions and models:

(9) Two students can carry this (together).

1. $\models_{M,\theta} \forall x(\mathbf{2}x \wedge {*student}\ x \rightarrow can.carry\ zx)$

2. $[\![\forall x(\mathbf{2}x \wedge {*student}\ x \rightarrow can.carry\ zx)]\!]^{M,\theta} = 1$ — D3

3. $\{d \in D_e^M\colon [\![\mathbf{2}x \wedge {*student}\ x]\!]^{M,\theta[x/d]} = 1$
   $\&\ [\![can.carry\ zx]\!]^{M,\theta[x/d]} = 0\} = \varnothing$ — F1:∀, →, df. {–:–}

3. $\{d \in D_e^M\colon [\![\mathbf{2}]\!]^{M,\theta}(d) = 1\ \&\ [\![*]\!]^M([\![student]\!]^M)(d) = 1$
   $\&\ [\![can.carry]\!]^M(\theta(z))(d) = 0\} = \varnothing$ — D2.3:∧, ***a***, ***b***, D2.2.θ[*u*/*d*]

4. $\{d \in D_e^M\colon |d| = 2$
   $\&\ d \in \{\cup X\colon \varnothing \subset X \subseteq \{d' \in D_e^M\colon [\![student]\!]^M(d') = 1\}\}$
   $\&\ [\![can.carry]\!]^M(\theta(z))(d) = 0\} = \varnothing$ — F2:**2**, D2.2:*

Consider *M* s.t.:
$\{d \in D_e^M\colon [\![student]\!]^M(d) = 1\} = \{\{J\}, \{M\}, \{B\}\}$

Then: $\{\cup X\colon \varnothing \subset X \subseteq \{d' \in D_e^M\colon [\![student]\!]^M(d') = 1\}\}$
$= \{\{J\}, \{M\}, \{B\}, \{J, M\}, \{J, B\}, \{M, B\}, \{J, M, B\}\}$

So the condition in 4. is true if *M* and θ further satisfy:
$[\![can.carry]\!]^M(\theta(z))(\{J, M\}) = 1$
$[\![can.carry]\!]^M(\theta(z))(\{J, B\}) = 1$
$[\![can.carry]\!]^M(\theta(z))(\{M, B\}) = 1$

(10_{coll}) Two students presented a paper (together).

1. $\models_{M,\theta} \exists x(\mathbf{2}x \wedge {*student}\ x$
   $\wedge \exists y(paper\ y \wedge present\ yx))$

2. $[\![\exists x(\mathbf{2}x \wedge {*student}\ x$
   $\wedge \exists y(paper\ y \wedge present\ yx))]\!]^{M,\theta} = 1$ — D3

3. $\{d \in D_e^M\colon [\![\mathbf{2}]\!]^{M,\theta[u/d]}(d) = 1\ \&\ [\![*]\!]^M([\![student]\!]^M(d) = 1$
   $\&\ [\![\exists y(paper\ y \wedge present\ yx)]\!]^{M,\theta[x/d]} = 1\} \neq \varnothing$ — F1:∃, D2.3:∧, ***a***, ***b***, D2.2:θ[*u*/*d*]

4. $\{d \in D_e^M\colon |d| = 2$
   $\&\ d \in \{\cup X\colon \varnothing \subset X \subseteq \{d' \in D_e^M\colon [\![student]\!]^M(d') = 1\}\}$
   $\&\ \{d' \in D_e^M\colon [\![paper]\!]^M(d') = 1$
   $\&\ [\![present]\!]^M(d')(d) = 1\} \neq \varnothing\} \neq \varnothing$ — F2:**2**, D2.2:*, F1:∃, D2.3:∧, ***a***, ***b***, D2.2:θ[*u*/*d*]

e.g. true in any *M* s.t.:
$\{d \in D_e^M\colon [\![student]\!]^M(d) = 1\} = \{\{J\}, \{M\}, \{B\}\}$
$[\![paper]\!]^M(\{P\}) = 1$
$[\![present]\!]^M(\{P\})(\{J, B\}) = 1$

(10_{dis}) Two students (each) presented a paper.

1. $\models_{M,\theta} \exists x(\mathbf{2}x \wedge {*student}\ x$
$\qquad \wedge\ {}^{D}\lambda x'[\exists y(paper\ y \wedge present\ yx')]x)$

2. $[\![\exists x(\mathbf{2}x \wedge {*student}\ x$
$\qquad \wedge\ {}^{D}\lambda x'[\exists y(paper\ y \wedge present\ yx')]x)]\!]^{M,\theta} = 1$ — D3

3. $\{d \in D_e^M\!: [\![\mathbf{2}]\!]^{M,\theta[u/d]}(d) = 1\ \&\ [\![*]\!]^M([\![student]\!]^M(d) = 1$ — F1:∃, D2.3:∧, ***a***, ***b***,
$\qquad \&\ [\![{}^{D}]\!]^M([\![\lambda x'[\exists y(paper\ y \wedge present\ yx')]]\!]^{M,\theta[x/d]}(d) = 1\} \neq \varnothing$ — D2.2:θ[*u*/*d*]

4. $\{d \in D_e^M\!: |d| = 2$ — F2:**2**
$\qquad \&\ d \in \{\cup X\!: \varnothing \subset X \subseteq \{d' \in D_e^M\!: [\![student]\!]^M(d') = 1\}\}$ — D2.2:*, ^{D}
$\qquad \&\ \{d' \in D_e^M\!: |d'| = 1\ \&\ d' \subseteq d\}$
$\qquad\quad \subseteq \{d' \in D_e^M\!: [\![\lambda x'[\exists y(paper\ y \wedge present\ yx')]]\!]^{M,\theta[x/d]}(d') = 1\}\} \neq \varnothing$

5. $\{d \in D_e^M\!: |d| = 2$ — D2.3.**λ**, df. ⟨–:–⟩
$\qquad \&\ d \in \{\cup X\!: \varnothing \subset X \subseteq \{d' \in D_e^M\!: [\![student]\!]^M(d') = 1\}\}$
$\qquad \&\ \{d' \in D_e^M\!: |d'| = 1\ \&\ d' \subseteq d\}$
$\qquad\quad \subseteq \{d' \in D_e^M\!: [\![\exists y(paper\ y \wedge present\ yx')]\!]^{M,\theta[x/d][x'/d']} = 1\}\} \neq \varnothing$

6. $\{d \in D_e^M\!: |d| = 2$ — F1:∃, D2.3:∧, ***a***, ***b***
$\qquad \&\ d \in \{\cup X\!: \varnothing \subset X \subseteq \{d' \in D_e^M\!: [\![student]\!]^M(d') = 1\}\}$ — D2.2:θ[*u*/*d*]
$\qquad \&\ \{d' \in D_e^M\!: |d'| = 1\ \&\ d' \subseteq d\}$
$\qquad\quad \subseteq \{d' \in D_e^M\!: \{d'' \in D_e^M\!: [\![paper]\!]^M(d') = 1$
$\qquad\qquad\qquad \&\ [\![present]\!]^M(d'')(d') = 1\} \neq \varnothing\}\} \neq \varnothing$

e.g. true in any M s.t.:
- $\{d \in D_e^M\!: [\![student]\!]^M(d) = 1\} = \{\{J\}, \{M\}, \{B\}\}$
- $\{d \in D_e^M\!: [\![paper]\!]^M(d) = 1\} = \{\{P\}, \{P'\}, \{P''\}\}$
- $[\![present]\!]^M(\{P\})(\{J\}) = 1$
  $[\![present]\!]^M(\{P'\})(\{B\}) = 1$
  $[\![present]\!]^M(\{P''\})(\{M, J\}) = 1$

since we then have:
- $|\{J, B\}| = 2$
- $\{J, B\} \in \{\cup X\!: \varnothing \subset X \subseteq \{d' \in D_e^M\!: [\![student]\!]^M(d') = 1\}\}$
- $\{d' \in D_e^M\!: |d'| = 1\ \&\ d' \subseteq \{J, B\}\}$
  $= \{\{J\}, \{B\}\}$
  $\subseteq \{d' \in D_e^M\!: \{d'' \in D_e^M\!: [\![paper]\!]^M(d') = 1\ \&\ [\![present]\!]^M(d'')(d') = 1\} \neq \varnothing\}\}$
  $\qquad = \{\{J\}, \{M, J\}, \{B\}\}$