

Notes on Kamp & Reyle 1993
Ch. 4 ‘The plural’

I. Review: Singular anaphora in DRT

(1) A man entered a bar. He had an umbrella

DRT *syntax* (a la Muskens 1995:154)

[x y z |
man x ,
bar y ,
enter xy ,
have xz ,
umbrella z] $:= K_1$

DRT *semantics* (a la Muskens 1995:154)

- DRT-models: $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$, as for PL
- DRT-contexts: partial assignments
 $G^M = \{g \mid \text{Dom } g \subseteq \mathbf{Var} \ \& \ \text{Ran } g \subseteq D^M\}$
- DRT-truth

$\models_M K_1$

iff $\exists g \in G^M$:

Dom g is the universe of K_1
 & g verifies every condition of K_1

iff $\exists g \in G^M$:

Dom $g = \{x, y, z\}$
 & $g(x) \in \llbracket \textit{man} \rrbracket^M$
 & $g(y) \in \llbracket \textit{bar} \rrbracket^M$
 & $\langle g(x), g(y) \rangle \in \llbracket \textit{enter} \rrbracket^M$
 & $\langle g(x), g(z) \rangle \in \llbracket \textit{have} \rrbracket^M$
 & $g(z) \in \llbracket \textit{umbrella} \rrbracket^M$

iff $\exists a, b, c \in D^M$:

$a \in \llbracket \textit{man} \rrbracket^M$
 & $b \in \llbracket \textit{bar} \rrbracket^M$
 & $\langle a, b \rangle \in \llbracket \textit{enter} \rrbracket^M$
 & $\langle a, c \rangle \in \llbracket \textit{have} \rrbracket^M$
 & $c \in \llbracket \textit{umbrella} \rrbracket^M$

II. Plurality and anaphora: Some examples

• Quantification (by *pl*) & summation

- (2) ¹John introduced Bill to Mary.
²*They* had been friends for a long time.

Salient reading(s)

rdg1: *they* = John + Bill
 rdg2: *they* = John + Mary

- (2') ¹John introduced Bill to Mary.
²*They* became friends.

they = Bill + Mary

- (2'') ¹John introduced Bill to Mary.
²Now *they* are *all* inseparable.

they all = John + Bill + Mary

- (3) ¹(Last month) *Fred* bought *three* donkeys.
²(Now) *they* are *all* unhappy.

rdg1: *they all* = the 3 donkeys
 rdg2: *they all* = Fred + the 3 donk's

• Quantification (by *most*) & abstraction

- (4a) *Most* farmers who own a donkey also own a cart.

proportion problem

- (4b) *Most* people who own *a dog* are fond of *it*.

strong a: fond of *each of* their dogs

- (4c) *Most* people who write *a paper* go on to develop *it* in another paper.

weak a: develop *one of* their papers

- (5) ¹Susan has found *most* of the books that Bill needs.
²*They* are on his desk.

they = the books Bill needs
 that Susan has found

• Collective & distributive predicates

- (6) ¹John *and* Mary are a nice couple.
²*They* met in Alaska.

collective

- (7) ¹(Last year) John *and* Mary bought *a* house.
²*They* both like *it* a lot.

collective rdg

- (8) ¹(Last year) John *and* Mary (both) bought *a* house.
²*They* both like *them* a lot.

distributive rdg

- (9) ¹(Today) Bill's students worked in groups.
²*They all* worked on *different* problems.
³*The* group with *the hardest* problem came up with *the best* solution.

distribution down to pluralities
 quantification over pluralities
 superlative quantification

• Dependent plurals

- (10) ¹*Most* of Bill's friends own *cars* with *automatic transmissions*.
²*They* like *them* a lot.

one or more cars per friend
 one transmission per car
 rdg1: *them* = their cars
 rdg2: *them* = their car's transmission

III. Dynamic Predicate Logic with Sums: (Debugged) DRL_{Plur} as an extension of DPL

DEFINITION 1.1 (DPL^Σ-basic terms)

• ₁ Con	=	{ <i>john, mary, ...</i> }	(individual) constants
Prd ¹	=	{ <i>SG, donkey, ..., PL, enter, ...</i> }	1-place predicates
Prd ²	=	{ <i>see, talk.to, ..., father.of, ...</i> }	2-place predicates
Prd ³	=	{ <i>give.to, introduce.to, ...</i> }	3-place predicates
Qnt	=	{ <i>ALL, MOST, SOME, NO, ...</i> }	quantifiers
• ₂ Var	=	{ <i>x, y, z, x', y', z', ..., z₁, z₂, ...</i> }	(individual) variables

DEFINITION 1.2 (DPL^Σ syntax).

t	$\alpha \in \mathbf{Trm}$	if $\alpha \in \mathbf{Con}$ or $\alpha \in \mathbf{Var}$	(individual) terms
R	$\alpha(\beta_1, \dots, \beta_n) \in \mathbf{Cnd}$	if $\alpha \in \mathbf{Prd}^n$ and $\beta_1, \dots, \beta_n \in \mathbf{Trm}$	relational conditions
=	$(\alpha = \beta) \in \mathbf{Cnd}$	if $\alpha, \beta \in \mathbf{Trm}$	identity
∅	$(\alpha \emptyset \beta) \in \mathbf{Cnd}$	if $\alpha, \beta \in \mathbf{Trm}$	non-overlap
 · 	$(\alpha = n) \in \mathbf{Cnd}$	if $\alpha \in \mathbf{Trm}$ and $n \in \{1, 2, \dots\}$	cardinality
≤	$(\alpha \leq \beta) \in \mathbf{Cnd}$	if $\alpha, \beta \in \mathbf{Trm}$	part-of
+	$(\alpha = \beta_1 + \dots + \beta_n) \in \mathbf{Cnd}$	if $\alpha, \beta_1, \dots, \beta_n \in \mathbf{Trm}$	summation
Σ	$(\alpha = \Sigma u. \Phi) \in \mathbf{Cnd}$	if $\alpha \in \mathbf{Trm}, u \in \mathbf{Var}$ and $\Phi \in \mathbf{Drs}$	abstraction
∩	$(\phi, \psi) \in \mathbf{Cnd}$	if $\phi, \psi \in \mathbf{Cnd}$	(static) conjunction
Q_u	$\Phi \alpha_u \Psi \in \mathbf{Cnd}$	if $\Phi, \Psi \in \mathbf{Drs}, \alpha \in \mathbf{Qnt}$, and $u \in \mathbf{Var}$	selective quant.
⇒	$(\Phi \Rightarrow \Psi) \in \mathbf{Cnd}$	if $\Phi, \Psi \in \mathbf{Drs}$	implication (unsel. ALL)
¬	$\neg \Phi \in \mathbf{Cnd}$	if $\Phi \in \mathbf{Drs}$	negation (unsel. NO)
[u	$[u_1 \dots u_n \phi] \in \mathbf{Drs}$	if $u_1, \dots, u_n \in \mathbf{Var}$ and $\phi \in \mathbf{Cnd}$	update drs (unsel. SOME)
[$[\phi] \in \mathbf{Drs}$	if $\phi \in \mathbf{Cnd}$	test drs

DEFINITION 2.1 (DPL^Σ-models and partial assignments)

- ₁ A DPL^Σ-*model* is a structure $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$ such that:
 - (i) $D^M = \{X \mid \emptyset \subset X \subseteq A\}$, where A is a non-empty set (of *atoms*)
 - (ii) $\llbracket \cdot \rrbracket^M$ is a function such that:
 - $\llbracket \alpha \rrbracket^M \in \{d \in D^M : |d| = 1\}$ if $\alpha \in \mathbf{Con}$
 - $\llbracket \alpha \rrbracket^M \subseteq (D^M)^n$ if $\alpha \in \mathbf{Prd}^n$
 - (iii) $\llbracket \mathbf{SG} \rrbracket^M = \{d \in D^M : |d| = 1\}$
 $\llbracket \textit{donkey} \rrbracket^M \subseteq \{d \in D^M : |d| = 1\}$
 \vdots
 $\llbracket \mathbf{PL} \rrbracket^M = \{d \in D^M : |d| > 1\}$
 $\llbracket \mathbf{ALL} \rrbracket^M = \{\langle X, Y \rangle \in (\mathcal{P}(D^M))^2 : X \subseteq Y\}$
 $\llbracket \mathbf{MOST} \rrbracket^M = \{\langle X, Y \rangle \in (\mathcal{P}(D^M))^2 : |X \cap Y| > |X - Y|\}$
 $\llbracket \mathbf{SOME} \rrbracket^M = \{\langle X, Y \rangle \in (\mathcal{P}(D^M))^2 : X \cap Y \neq \emptyset\}$
 $\llbracket \mathbf{NO} \rrbracket^M = \{\langle X, Y \rangle \in (\mathcal{P}(D^M))^2 : X \cap Y = \emptyset\}$
 \vdots
- ₂ (i) A *partial M-assignment* is a function g such that $\text{Dom } g \subseteq \mathbf{Var}$ and $\text{Ran } g \subseteq D^M$. We write Λ for the *empty M-assignment* (i.e. $\text{Dom } \Lambda = \{\}$), and G^M , for the set of all partial assignments, i.e. $G^M := \{g : \text{Dom } g \subseteq \mathbf{Var} \ \& \ \text{Ran } g \subseteq D^M\}$.
- (ii) For any $g, h \in G^M$ and $R \subseteq \mathbf{Var}$, h *extends* g to R , written $g \subseteq_R h$, iff $g \subseteq h$ and $\text{Dom } g = (\text{Dom } h - R)$

ABBREVIATION:

Given a drs Φ , the *universe* of Φ , written $U(\Phi)$, is defined as follows:

- $U(\Phi) := \{u_1, \dots, u_n\}$ if Φ is an update drs of the form $[u_1 \dots u_n | \phi]$
- $U(\Phi) := \{\}$ if Φ is a test drs of the form $[| \phi]$

DEFINITION 2.2 (DPL^Σ-semantics)

t	$\llbracket \alpha \rrbracket^{M,g}$	$= \llbracket \alpha \rrbracket^M$	$\text{if } \alpha \in \mathbf{Con}$
		$= g(\alpha)$	$\text{if } \alpha \in \text{Dom } g$
		$= \text{undefined}$	otherwise
R	$\llbracket \alpha(\beta_1, \dots, \beta_n) \rrbracket^M$	$= \{g \in G^M \mid \langle \llbracket \beta_1 \rrbracket^{M,g}, \dots, \llbracket \beta_n \rrbracket^{M,g} \rangle \in \llbracket \alpha \rrbracket^M\}$	
$=$	$\llbracket (\alpha = \beta) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} = \llbracket \beta \rrbracket^{M,g}\}$	
\emptyset	$\llbracket (\alpha \emptyset \beta) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} \cap \llbracket \beta \rrbracket^{M,g} = \emptyset\}$	
$ \cdot $	$\llbracket (\alpha = n) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} = n\}$	
\leq	$\llbracket (\alpha \leq \beta) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} \subseteq \llbracket \beta \rrbracket^{M,g}\}$	
$+$	$\llbracket (\alpha = \beta_1 + \dots + \beta_n) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} = \llbracket \beta_1 \rrbracket^{M,g} \cup \dots \cup \llbracket \beta_n \rrbracket^{M,g}\}$	
Σ	$\llbracket (\alpha = \Sigma u. \Phi) \rrbracket^M$	$= \{g \in G^M \mid \llbracket \alpha \rrbracket^{M,g} = \bigcup \{d \in D^M \mid \exists h \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \text{ \& } \langle g, h \rangle \in \llbracket \Phi \rrbracket^M\}\}$	
\cap	$\llbracket (\phi, \psi) \rrbracket^M$	$= \{g \in G^M \mid g \in \llbracket \phi \rrbracket^M \text{ \& } g \in \llbracket \psi \rrbracket^M\}$	
Q_u	$\llbracket \Phi \alpha_u \Psi \rrbracket^M$	$= \{g \in G^M \mid \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \alpha \rrbracket^M$	
		$\text{\& } X = \{d \in D^M \mid \exists h \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \text{ \& } \langle g, h \rangle \in \llbracket \Phi \rrbracket^M\}$	
		$\text{\& } Y = \{d \in D^M \mid (\exists h \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \text{ \& } \langle g, h \rangle \in \llbracket \Phi \rrbracket^M)$	
		$\text{\& } (\forall h \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \text{ \& } \langle g, h \rangle \in \llbracket \Phi \rrbracket^M$	
		$\text{\& } \rightarrow \exists k \in G^M: \langle h, k \rangle \in \llbracket \Psi \rrbracket^M\}$	
\Rightarrow	$\llbracket \Phi \Rightarrow \Psi \rrbracket^M$	$= \{g \in G^M \mid \forall h \in G^M: \langle g, h \rangle \in \llbracket \Phi \rrbracket^M \rightarrow \exists k \in G^M: \langle h, k \rangle \in \llbracket \Psi \rrbracket^M\}$	
\neg	$\llbracket \neg \Phi \rrbracket^M$	$= \{g \in G^M \mid \neg \exists h \in G^M: \langle g, h \rangle \in \llbracket \Phi \rrbracket^M\}$	
$[u]$	$\llbracket [u_1 \dots u_n \phi] \rrbracket^M$	$= \{\langle g, h \rangle \in (G^M)^2 \mid g \subseteq_{\{u_1, \dots, u_n\}} h \text{ \& } h \in \llbracket \phi \rrbracket^M\}$	
$[$	$\llbracket [\phi] \rrbracket^M$	$= \{\langle g, h \rangle \in (G^M)^2 \mid g = h \text{ \& } h \in \llbracket \phi \rrbracket^M\}$	

DEFINITION 3 (Truth).

- A condition ϕ is *true* in M under g , written $\models_{M,g} \phi$, iff $g \in \llbracket \phi \rrbracket^M$.
- A drs Φ is *true* in M under g , written $\models_{M,g} \Phi$, iff $\exists h \in G^M: \langle g, h \rangle \in \llbracket \Phi \rrbracket^M$
- A drs Φ is *true* in M , written $\models_M \Phi$, iff $\exists h \in G^M: \langle \Lambda, h \rangle \in \llbracket \Phi \rrbracket^M$

ABBREVIATIONS

A1.	$(\alpha \in \beta)$	$:= (\text{SG}(\alpha), (\alpha \leq \beta))$	$\alpha, \beta \in \mathbf{Trm}$
A2.	$\alpha^* \beta$	$:= [x \mid x \in \beta] \text{ ALL}_x [\alpha(x)]$	$\alpha \in \mathbf{Prd}^1, \beta \in \mathbf{Trm}$

IV. Analyzed examples

• Quantification (by *pl*) & summation

(3) ¹(Last month) Fred bought three donkeys.

²(Now) *they* are *all* unhappy.

rdg2: *they all* = Fred + the 3 donkeys

DPL^Σ syntax

$$\begin{aligned} & [x \ y \ x' \mid (x = \text{fred}), \text{buy}(x, y), (|y| = 3), \\ & [z \mid \text{SG}(z), (z \leq y)] \text{ALL}_z [\mid \text{donkey}(z) \mid], \\ & (x' = x + y), \\ & [z' \mid \text{SG}(z'), (z' \leq x')] \text{ALL}_{z'} [\mid \text{unhappy}(z') \mid]] \quad := K_{32} \end{aligned}$$

DPL^Σ semantics

1. $\models_M K_{32}$
2. iff $\exists h \in G^M: \langle \Lambda, h \rangle \in \llbracket K_{32} \rrbracket^M$ D3
3. iff $\exists h \in G^M: \Lambda \subseteq_{\{x, y, z\}} h$ D2.2. [ul, ∩]
 - & $h \in \llbracket x = \text{fred} \rrbracket^M$
 - & $h \in \llbracket \text{buy}(x, y) \rrbracket^M$
 - & $h \in \llbracket |y| = 3 \rrbracket^M$
 - & $h \in \llbracket [z \mid \text{SG}(z), (z \leq y)] \text{ALL}_z [\mid \text{donkey}(z) \mid] \rrbracket^M$
 - & $h \in \llbracket x' = x + y \rrbracket^M$
 - & $h \in \llbracket [z' \mid \text{SG}(z'), (z' \leq x')] \text{ALL}_{z'} [\mid \text{unhappy}(z') \mid] \rrbracket^M$
4. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x'\}$ D2.1. •₂. Λ, ⊆_R
 - & $h(x) = \llbracket \text{fred} \rrbracket^M$ D2.2. =, t
 - & $|h(x)| = 1$ D2.1. Con
 - & $\langle h(x), h(y) \rangle \in \llbracket \text{buy} \rrbracket^M$ D2.2. R, t
 - & $|h(y)| = 3$ D2.2. !l, t
 - & $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$ D2.2. Q_u
 - & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\{z\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z \mid \text{SG}(z), (z \leq y)] \rrbracket^M\}$
 - & $Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\{z\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z \mid \text{SG}(z), (z \leq y)] \rrbracket^M\}$
 - & $(\forall h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\{z\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z \mid \text{SG}(z), (z \leq y)] \rrbracket^M\}$
 - $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [\mid \text{donkey}(z) \mid] \rrbracket^M\}$
 - & $h(x') = h(x) \cup h(y)$ D2.2. +, t
 - & $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$ D2.2. Q_u
 - & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{z'\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z' \mid \text{SG}(z'), (z' \leq x')] \rrbracket^M\}$
 - & $Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{z'\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z' \mid \text{SG}(z'), (z' \leq x')] \rrbracket^M\}$
 - & $(\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{z'\}} h'$
 - & $\langle h, h' \rangle \in \llbracket [z' \mid \text{SG}(z'), (z' \leq x')] \rrbracket^M\}$
 - $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [\mid \text{unhappy}(z') \mid] \rrbracket^M\}$

5. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x'\}$
 & $h(x) = \llbracket \text{fred} \rrbracket^M$ & $|h(x)| = 1$
 & $\langle h(x), h(y) \rangle \in \llbracket \text{buy} \rrbracket^M$ & $|h(y)| = 3$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid \exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\}$
 & $h \subseteq_{\{z\}} h' \text{ \& } |h'(z)| = 1 \text{ \& } h'(z) \subseteq h'(y)\}$
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\}$
 & $h \subseteq_{\{z\}} h' \text{ \& } |h'(z)| = 1 \text{ \& } h'(z) \subseteq h'(y))$
 & $(\forall h' \in G^M: h' = h \cup \{\langle z, d \rangle\}$
 & $h \subseteq_{\{z\}} h' \text{ \& } |h'(z)| = 1 \text{ \& } h'(z) \subseteq h'(y)$
 $\rightarrow \exists k \in G^M: h' = k \text{ \& } k(z) \in \llbracket \text{donkey} \rrbracket^M)\}$
 & $h(x') = h(x) \cup h(y)$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid \exists h' \in G^M: h' = h \cup \{\langle z', d \rangle\}$
 & $h \subseteq_{\{z'\}} h' \text{ \& } |h'(z')| = 1 \text{ \& } h'(z') \subseteq h'(x')\}$
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h' = h \cup \{\langle z', d \rangle\}$
 & $h \subseteq_{\{z'\}} h' \text{ \& } |h'(z')| = 1 \text{ \& } h'(z') \subseteq h'(x'))$
 & $(\forall h' \in G^M: h' = h \cup \{\langle z', d \rangle\}$
 & $h \subseteq_{\{z'\}} h' \text{ \& } |h'(z')| = 1 \text{ \& } h'(z') \subseteq h'(x')$
 $\rightarrow \exists k \in G^M: h' = k \text{ \& } k(z') \in \llbracket \text{unhappy} \rrbracket^M)\}$
 D2.1.ALL
 D2.1. \subseteq_R
 D2.2.[ul, \cap , \mathbf{R} , t , \leq
 D2.1.SG
6. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x'\}$
 & $h(x) = \llbracket \text{fred} \rrbracket^M$ & $|h(x)| = 1$
 & $\langle h(x), h(y) \rangle \in \llbracket \text{buy} \rrbracket^M$ & $|h(y)| = 3$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(y)\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(y)$
 & $(|d| = 1 \text{ \& } d \subseteq h(y) \rightarrow d \in \llbracket \text{donkey} \rrbracket^M)\}$
 & $h(x') = h(x) \cup h(y)$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')$
 & $(|d| = 1 \text{ \& } d \subseteq h(x') \rightarrow d \in \llbracket \text{unhappy} \rrbracket^M)\}$
 eliminate $\exists h'$
 elimin. $\exists h'$
 elimin. $\forall h', \exists k$
7. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x'\}$
 & $h(x) = \llbracket \text{fred} \rrbracket^M$ & $|h(x)| = 1$
 & $\langle h(x), h(y) \rangle \in \llbracket \text{buy} \rrbracket^M$ & $|h(y)| = 3$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(y)\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(y) \text{ \& } d \in \llbracket \text{donkey} \rrbracket^M\}$
 & $h(x') = h(x) \cup h(y)$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x') \text{ \& } d \in \llbracket \text{unhappy} \rrbracket^M\}$
 simplify
 simplify

8. iff $\exists d_1, d_2 \in D^M$:

eliminate \exists/h

& $d_1 = \llbracket fred \rrbracket^M$ & $|d_1| = 1$

& $\langle d_1, d_2 \rangle \in \llbracket buy \rrbracket^M$ & $|d_2| = 3$

& $\exists X, Y \subseteq D^M: X \subseteq Y$

& $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_2\}$

& $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_2 \text{ \& } d \in \llbracket donkey \rrbracket^M\}$

& $\exists X, Y \subseteq D^M: X \subseteq Y$

& $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_1 \cup d_2\}$

& $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_1 \cup d_2 \text{ \& } d \in \llbracket unhappy \rrbracket^M\}$

Verifying model:

e.g. $\models_M K_{32}$ for any $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$ such that:

- $\{a\}, \{b\}, \{b'\}, \{b''\} \in D^M$
- $\{a\} = \llbracket fred \rrbracket^M$
- $\{b\}, \{b'\}, \{b''\} \in \llbracket donkey \rrbracket^M$
- $\langle \{a\}, \{b, b', b''\} \rangle \in \llbracket buy \rrbracket^M$
- $\{a\}, \{b\}, \{b'\}, \{b''\} \in \llbracket unhappy \rrbracket^M$

- Quantification: Proportions (4a), strong rdg (4b), weak rdg (4c)

(4a) ¹Most farmers who own a donkey also own a cart.

DPL^Σ syntax

$$\llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \text{ MOST}_x [z \mid \text{own}(x, z), \text{cart}(z)] \rrbracket := K_{4a}$$

DPL^Σ semantics

1. $\models_M K_{4a}$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{y\}} h$ D3, D2.2.[ul
 $\& h \in \llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \text{ MOST}_x [z \mid \text{own}(x, z), \text{cart}(z)] \rrbracket^M$
3. iff $\Lambda \in \llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \text{ MOST}_x [z \mid \text{own}(x, z), \text{cart}(z)] \rrbracket^M$ eliminate $\exists h$
4. iff $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{MOST} \rrbracket^M$ D2.2.Q_u
 $\& X = \{d \in D^M \mid \exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \rrbracket^M$
 $\& (\forall h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \text{ y} \mid \text{frm}(x), \text{own}(x, y), \text{dnk}(y)] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h, k \rangle \in \llbracket [z \mid \text{own}(x, z), \text{cart}(z)] \rrbracket^M\}$
5. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ D2.1.MOST, Λ
 $\& X = \{d \in D^M \mid \exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$ D2.2.[ul, \cap , \mathbf{R} , \mathbf{t}
 $\& h(x) \in \llbracket \text{frm} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \& h(y) \in \llbracket \text{dnk} \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$
 $\& h(x) \in \llbracket \text{frm} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \& h(y) \in \llbracket \text{dnk} \rrbracket^M$
 $\& (\forall h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$
 $\& h(x) \in \llbracket \text{frm} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \& h(y) \in \llbracket \text{dnk} \rrbracket^M$
 $\rightarrow \exists k \in G^M: h \subseteq_{\{z\}} k \& \langle k(x), k(z) \rangle \in \llbracket \text{own} \rrbracket^M \& k(z) \in \llbracket \text{cart} \rrbracket^M\}$
6. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
 $\& X = \{d \in D^M \mid \exists d' \in D^M: d \in \llbracket \text{frm} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \& d' \in \llbracket \text{dnk} \rrbracket^M\}$ simplify
 $\& Y = \{d \in D^M \mid (\exists d' \in D^M: d \in \llbracket \text{frm} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \& d' \in \llbracket \text{dnk} \rrbracket^M)$
 $\& (\forall d' \in D^M: d \in \llbracket \text{frm} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \& d' \in \llbracket \text{dnk} \rrbracket^M$
 $\rightarrow \exists d'' \in D^M: \langle d, d'' \rangle \in \llbracket \text{own} \rrbracket^M \& d'' \in \llbracket \text{cart} \rrbracket^M\}$
7. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
 $\& X = \{d \in D^M \mid d \in \llbracket \text{frm} \rrbracket^M \& (\exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \& d' \in \llbracket \text{dnk} \rrbracket^M)\}$ rearr.
 $\& Y = \{d \in D^M \mid d \in \llbracket \text{frm} \rrbracket^M \& (\exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \& d' \in \llbracket \text{dnk} \rrbracket^M)$ simplify
 $\& (\exists d'' \in D^M: \langle d, d'' \rangle \in \llbracket \text{own} \rrbracket^M \& d'' \in \llbracket \text{cart} \rrbracket^M)\}$

Not true e.g. in the following model, where counting pairs would wrongly predict (4a) to be true:

100 farmers: $\{f_1\}, \dots, \{f_{100}\}$

$\{f_1\}$ owns 100 donkeys $\{d_{1.1}\}, \dots, \{d_{1.100}\}$ and a cart $\{c_1\}$

$\{f_2\}$ owns 100 donkeys $\{d_{2.1}\}, \dots, \{d_{2.100}\}$ and a cart $\{c_2\}$

$\{f_3\}, \dots, \{f_{100}\}$ own 1 donkey each ($\{f_3\}$ owns $\{d_3\}$, ..., $\{f_{100}\}$ owns $\{d_{100}\}$) and don't own any cart

Then $|X \cap Y| = |\{\{f_1\}, \{f_2\}\}| = 2$, but $|X - Y| = |\{\{f_3\}, \dots, \{f_{100}\}\}| = 98$.

(4b) ¹Most people who own a dog are fond of it.

DPL² *syntax*

$$\llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket \text{ MOST}_x \llbracket \text{fond.of}(x, y) \rrbracket := K_{4b}$$

DPL² *semantics*

1. $\models_M K_{4b}$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\emptyset} h$
& $h \in \llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket \text{ MOST}_x \llbracket \text{fond.of}(x, y) \rrbracket^M$ D3, D2.2.[ul]
3. iff $\Lambda \in \llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket \text{ MOST}_x \llbracket \text{fond.of}(x, y) \rrbracket^M$ eliminate $\exists h$
4. iff $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{MOST} \rrbracket^M$ D2.2.Q_u
& $X = \{d \in D^M \mid \exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
& $\langle \Lambda, h \rangle \in \llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket^M\}$
& $Y = \{d \in D^M \mid (\exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
& $\langle \Lambda, h \rangle \in \llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket^M\}$
& $(\forall h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
& $\langle \Lambda, h \rangle \in \llbracket [x \ y] \text{ pers}(x), \text{ own}(x, y), \text{ dog}(y) \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h, k \rangle \in \llbracket \llbracket \text{fond.of}(x, y) \rrbracket^M \rrbracket\}$
5. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ D2.1.MOST, Λ
& $X = \{d \in D^M \mid \exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \ \& \ \Lambda \subseteq_{\{x, y\}} h$ D2.2.[ul, \cap , \mathbf{R} , t]
& $h(x) \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ h(y) \in \llbracket \text{dog} \rrbracket^M\}$
& $Y = \{d \in D^M \mid (\exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \ \& \ \Lambda \subseteq_{\{x, y\}} h$
& $h(x) \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ h(y) \in \llbracket \text{dog} \rrbracket^M\}$
& $(\forall h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \ \& \ \Lambda \subseteq_{\{x, y\}} h$
& $h(x) \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle h(x), h(y) \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ h(y) \in \llbracket \text{dog} \rrbracket^M\}$
 $\rightarrow \exists k \in G^M: h \subseteq_{\emptyset} k \ \& \ \langle k(x), k(y) \rangle \in \llbracket \text{fond.of} \rrbracket^M\}$
6. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
& $X = \{d \in D^M \mid \exists d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M\}$ simplify
& $Y = \{d \in D^M \mid (\exists d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M\}$
& $(\forall d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \ \& \ \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M$
 $\rightarrow \langle d, d' \rangle \in \llbracket \text{fond.of} \rrbracket^M\}$
7. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
& $X = \{d \in D^M \mid d \in \llbracket \text{pers} \rrbracket^M \ \& \ (\exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M)\}$ rearr.
& $Y = \{d \in D^M \mid d \in \llbracket \text{pers} \rrbracket^M \ \& \ (\exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M)\}$ simplify
& $(\forall d' \in D^M: \langle d, d' \rangle \in \llbracket \text{own} \rrbracket^M \ \& \ d' \in \llbracket \text{dog} \rrbracket^M$
 $\rightarrow \langle d, d' \rangle \in \llbracket \text{fond.of} \rrbracket^M\}$

Paraphrase (strong rdg):

Most people who own one or more dogs are fond of **each** of their dogs.

This is correct for (4b), but unfortunately, wrong for (4c), whose salient reading is *weak*.

(4c) *Most* people who write *a* paper go on to develop *it* in another paper.

DPL^Σ syntax

$$\llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \text{MOST}_x [z \ \text{paper}(z), (z \ \emptyset \ y), \text{dev.in}(x, y, z)] \rrbracket := K_{4c}$$

DPL^Σ semantics

1. $\models_M K_{4c}$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{\}} h$ D3, D2.2.[ul
 $\& h \in \llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \text{MOST}_x [z \ \text{paper}(z), (z \ \emptyset \ y), \text{dev.in}(x, y, z)] \rrbracket^M$
3. iff $\Lambda \in \llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \text{MOST}_x [z \ \text{paper}(z), (z \ \emptyset \ y), \text{dev.in}(x, y, z)] \rrbracket^M$ eliminate $\exists h$
4. iff $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{MOST} \rrbracket^M$ D2.2.Q_u
 $\& X = \{d \in D^M \mid \exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \rrbracket^M$
 $\& (\forall h \in G^M: \Lambda \cup \{\langle x, d \rangle\} \subseteq_{\{y\}} h$
 $\& \langle \Lambda, h \rangle \in \llbracket [x \ y \mid \text{pers}(x), \text{wrt}(x, y), \text{paper}(y)] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h, k \rangle \in \llbracket [z \ \text{paper}(z), (z \ \emptyset \ y), \text{dev.in}(x, y, z)] \rrbracket^M\}$
5. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ D2.1.MOST, Λ
 $\& X = \{d \in D^M \mid \exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$ D2.2.[ul, \cap , \mathbf{R} , \emptyset , t
 $\& h(x) \in \llbracket \text{pers} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{wrt} \rrbracket^M \& h(y) \in \llbracket \text{paper} \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$
 $\& h(x) \in \llbracket \text{pers} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{wrt} \rrbracket^M \& h(y) \in \llbracket \text{paper} \rrbracket^M)$
 $\& (\forall h \in G^M: \{\langle x, d \rangle\} \subseteq_{\{y\}} h \& \Lambda \subseteq_{\{x, y\}} h$
 $\& h(x) \in \llbracket \text{pers} \rrbracket^M \& \langle h(x), h(y) \rangle \in \llbracket \text{wrt} \rrbracket^M \& h(y) \in \llbracket \text{paper} \rrbracket^M)$
 $\rightarrow \exists k \in G^M: h \subseteq_{\{z\}} k \& k(z) \in \llbracket \text{paper} \rrbracket^M \& k(z) \cap k(y) = \emptyset$
 $\& \langle k(x), k(y), k(z) \rangle \in \llbracket \text{dev.in} \rrbracket^M\}$
6. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ simplify
 $\& X = \{d \in D^M \mid \exists d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M)$
 $\& (\forall d' \in D^M: d \in \llbracket \text{pers} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M$
 $\rightarrow \exists d'' \in D^M: d'' \in \llbracket \text{paper} \rrbracket^M \& d' \cap d'' = \emptyset \& \langle d, d', d'' \rangle \in \llbracket \text{dev.in} \rrbracket^M\}$
7. iff $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ simplify
 $\& X = \{d \in D^M \mid d \in \llbracket \text{pers} \rrbracket^M \& \exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid d \in \llbracket \text{pers} \rrbracket^M \& (\exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M)$
 $\& (\forall d' \in D^M: \langle d, d' \rangle \in \llbracket \text{wrt} \rrbracket^M \& d' \in \llbracket \text{paper} \rrbracket^M$
 $\rightarrow \exists d'' \in D^M: d'' \in \llbracket \text{paper} \rrbracket^M \& d' \cap d'' = \emptyset$
 $\& \langle d, d', d'' \rangle \in \llbracket \text{dev.in} \rrbracket^M\}$

Predicted paraphrase (**strong** rdg):

Most people who write one or more papers develop **each** of their papers in another paper.

Remark: The strong reading entails that most people live for ever, constantly writing a development of some paper they have written. I don't think (4c) can be read in this way.

Intuitive paraphrase (**weak** rdg):

Most people who write one or more papers develop (at least) **one** of their papers in another paper.

Cf. also (4c') modeled on an example due to Schubert & Pelletier:

(4c') *Most* people who have **a** quarter will put **it** in the meter.

Unsatisfactory fix:

We could derive the weak reading by adding an extra pair of rules to DPL^Σ, e.g.:

$$\begin{aligned}
 \mathbf{Q}_{u:\exists} \Phi \alpha_{u:\exists} \Psi \in \mathbf{Cnd} & \quad \text{if } \Phi, \Psi \in \mathbf{Drs}, \alpha \in \mathbf{Qnt}, \text{ and } u \in \mathbf{Var} \\
 \mathbf{Q}_{u:\exists} \llbracket \Phi \alpha_{u:\exists} \Psi \rrbracket^M & \\
 = \{g \in G^M \mid \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \alpha \rrbracket^M & \\
 \quad \& X = \{d \in D^M \mid \exists h \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \ \& \langle g, h \rangle \in \llbracket \Phi \rrbracket^M\} \\
 \quad \& Y = \{d \in D^M \mid \exists h, k \in G^M: g \cup \{\langle u, d \rangle\} \subseteq_{U(\Phi) - \{u\}} h \ \& \langle g, h \rangle \in \llbracket \Phi \rrbracket^M \\
 \quad \quad \& \langle h, k \rangle \in \llbracket \Psi \rrbracket^M\}
 \end{aligned}$$

But the resulting theory would still be unsatisfactory, for at least two reasons:

- i) it would greatly *overgenerate*, since for any sentence with a quantifier it would predict *both* a strong reading (derived by \mathbf{Q}_u) and a weak one (derived by $\mathbf{Q}_{u:\exists}$). Intuitively, it is rare for a sentence to allow both readings.
- ii) it would also *undergenerate*, since it would predict that multiple indefinites in the restriction of a quantifier are all weak or all strong. But intuitively, *mixed* readings are also possible—e.g. (4d) is naturally read as saying that most people who buy one or more pieces of furniture and have one or more credit cards use *one of* their cards to pay for *each* of their purchases (could be different cards for different types of furniture, e.g. home office furniture which is tax-deductible, vs. furniture which is not tax-deductible):

(4d) *Most* people who buy **a^{strong}** piece of furniture and have **a^{weak}** credit card use *it* to pay for *it*.

(See Brasoveanu 2006 for an analysis of mixed readings.)

• Quantification & abstraction

(5) ¹Susan has found *most* of the books that Bill needs. ²They are on his desk.

DPL^Σ syntax

$$\begin{aligned} & [x \ y \ x', y' \mid (x = \text{susan}), (y = \text{bill}), \\ & [z \mid \text{book}(z), \text{need}(y, z)] \text{MOST}_z [\mid \text{find}(x, z)], \\ & (x' = \Sigma z. [\mid \text{book}(z), \text{need}(y, z), \text{find}(x, z)]), \\ & \text{desk.of}(y', y), [\mid \text{SG}(z), (z \leq x')] \text{ALL}_z [\mid \text{on}(z, y')]] \end{aligned} := K_5$$

DPL^Σ semantics

1. $\models_M K_5$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{x, y, x', y'\}} h$ D3,
 $\& h \in \llbracket x = \text{susan} \rrbracket^M \& h \in \llbracket y = \text{bill} \rrbracket^M$ D2.2.[ul, \cap]
 $\& h \in \llbracket [z \mid \text{book}(z), \text{need}(y, z)] \text{MOST}_z [\mid \text{find}(x, z)] \rrbracket^M$
 $\& h \in \llbracket (x' = \Sigma z. [\mid \text{book}(z), \text{need}(y, z), \text{find}(x, z)]) \rrbracket^M$
 $\& h \in \llbracket \text{desk.of}(y', y) \rrbracket^M$
 $\& h \in \llbracket [\mid \text{SG}(z), (z \leq x')] \text{ALL}_z [\mid \text{on}(z, y')] \rrbracket^M$
3. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x', y'\}$ D2.1. Λ , \subseteq_R
 $\& h(x) = \llbracket \text{susan} \rrbracket^M \& h(y) = \llbracket \text{bill} \rrbracket^M$ D2.2.=, t
 $\& |h(x)| = 1 \& |h(y)| = 1$ D2.1.Con
 $\& \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{MOST} \rrbracket^M$ D2.2.Q_u
 $\& X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{book}(z), \text{need}(y, z)] \rrbracket^M \}$
 $\& Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{book}(z), \text{need}(y, z)] \rrbracket^M \}$
 $\& (\forall h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{book}(z), \text{need}(y, z)] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [\mid \text{find}(x, z)] \rrbracket^M \}$
 $\& h(x) = \cup \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$ D2.2. Σ , t
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{book}(z), \text{need}(y, z), \text{find}(x, z)] \rrbracket^M \}$
 $\& \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$ D2.2.Q_u
 $\& X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{SG}(z), (z \leq x')] \rrbracket^M \}$
 $\& Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{SG}(z), (z \leq x')] \rrbracket^M \}$
 $\& (\forall h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [\mid \text{SG}(z), (z \leq x')] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [\mid \text{on}(z, y')] \rrbracket^M \}$

4. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x', y'\}$
 & $h(x) = \llbracket \text{susan} \rrbracket^M$ & $h(y) = \llbracket \text{bill} \rrbracket^M$
 & $|h(x)| = 1$ & $|h(x)| = 1$
 & $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$ D2.1.MOST, \subseteq_R
 & $X = \{d \in D^M \mid \exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\} \text{ \& } h \subseteq_{\{z\}} h'\}$ D2.2. $[\ul, \cap,$
 & $h'(z) \in \llbracket \text{book} \rrbracket^M$ & $\langle h'(y), h'(z) \rangle \in \llbracket \text{need} \rrbracket^M$ **R, t**
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\} \text{ \& } h \subseteq_{\{z\}} h'$
 & $h'(z) \in \llbracket \text{book} \rrbracket^M$ & $\langle h'(y), h'(z) \rangle \in \llbracket \text{need} \rrbracket^M$
 & $(\forall h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h' \text{ \& } h \subseteq_{\{z\}} h'$
 & $h'(z) \in \llbracket \text{book} \rrbracket^M$ & $\langle h'(y), h'(z) \rangle \in \llbracket \text{need} \rrbracket^M$
 $\rightarrow \exists k \in G^M: h' = k \text{ \& } \langle k(x), k(z) \rangle \in \llbracket \text{find} \rrbracket^M$
 & $h(x') = \cup \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h' \text{ \& } h \subseteq_{\{z\}} h'$
 & $h'(z) \in \llbracket \text{book} \rrbracket^M$ & $\langle h'(y), h'(z) \rangle \in \llbracket \text{need} \rrbracket^M$
 & $\langle h'(x), h'(z) \rangle \in \llbracket \text{find} \rrbracket^M$
 & $\langle h(y'), h(y) \rangle \in \llbracket \text{desk.off} \rrbracket^M$
- & $\exists X, Y \subseteq D^M: X \subseteq Y$ D2.1.ALL, \subseteq_R
 & $X = \{d \in D^M \mid \exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\} \text{ \& } h \subseteq_{\{z\}} h'$
 & $|h'(z)| = 1$ & $h'(z) \subseteq h'(x')\}$ D2.2. $[\ul, \cap,$
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h' = h \cup \{\langle z, d \rangle\} \text{ \& } h \subseteq_{\{z\}} h'$
 & $|h'(z)| = 1$ & $h'(z) \subseteq h'(x')$ **R, \leq , t, D2.1.SG**
 & $(\forall h' \in G^M: h \cup \{\langle z, d \rangle\} \subseteq_{\emptyset} h' \text{ \& } h \subseteq_{\{z\}} h'$
 & $|h'(z)| = 1$ & $h'(z) \subseteq h'(x')$
 $\rightarrow \exists k \in G^M: h' = k \text{ \& } \langle k(z), k(y') \rangle \in \llbracket \text{on} \rrbracket^M$
 $\}$
5. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x', y'\}$ simplify
 & $h(x) = \llbracket \text{susan} \rrbracket^M$ & $h(y) = \llbracket \text{bill} \rrbracket^M$
 & $|h(x)| = 1$ & $|h(x)| = 1$
 & $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
 & $X = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M\}$
 & $Y = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M$
 & $(d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M \rightarrow \langle h(x), d \rangle \in \llbracket \text{find} \rrbracket^M)$
 & $h(x') = \cup \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M \text{ \& } \langle h(x), d \rangle \in \llbracket \text{find} \rrbracket^M\}$
 & $\langle h(y'), h(y) \rangle \in \llbracket \text{desk.off} \rrbracket^M$
- & $\exists X, Y \subseteq D^M: X \subseteq Y$ simplify
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')$
 & $(|d| = 1 \text{ \& } d \subseteq h(x') \rightarrow \langle d, h(y') \rangle \in \llbracket \text{on} \rrbracket^M)$

6. iff $\exists h \in G^M: \text{Dom } h = \{x, y, x', y'\}$ simplify
 & $h(x) = \llbracket \text{susan} \rrbracket^M$ & $h(y) = \llbracket \text{bill} \rrbracket^M$
 & $|h(x)| = 1$ & $|h(y)| = 1$
 & $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
 & $X = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M\}$
 & $Y = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M \text{ \& } \langle h(x), d \rangle \in \llbracket \text{find} \rrbracket^M\}$
 & $h(x') = \cup \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle h(y), d \rangle \in \llbracket \text{need} \rrbracket^M \text{ \& } \langle h(x), d \rangle \in \llbracket \text{find} \rrbracket^M\}$
 & $\langle h(y'), h(y) \rangle \in \llbracket \text{desk.off} \rrbracket^M$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x')\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq h(x') \text{ \& } \langle d, h(y') \rangle \in \llbracket \text{on} \rrbracket^M\}$

7. iff $\exists d_1, d_2, d_3, d_4 \in D^M:$ simplify
 & $d_1 = \llbracket \text{susan} \rrbracket^M$ & $d_2 = \llbracket \text{bill} \rrbracket^M$
 & $|d_1| = 1$ & $|d_2| = 1$
 & $\exists X, Y \subseteq D^M: |X \cap Y| > |X - Y|$
 & $X = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle d_2, d \rangle \in \llbracket \text{need} \rrbracket^M\}$
 & $Y = \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle d_2, d \rangle \in \llbracket \text{need} \rrbracket^M \text{ \& } \langle d_1, d \rangle \in \llbracket \text{find} \rrbracket^M\}$
 & $d_3 = \cup \{d \in D^M \mid d \in \llbracket \text{book} \rrbracket^M \text{ \& } \langle d_2, d \rangle \in \llbracket \text{need} \rrbracket^M \text{ \& } \langle d_1, d \rangle \in \llbracket \text{find} \rrbracket^M\}$
 & $\langle d_4, d_2 \rangle \in \llbracket \text{desk.off} \rrbracket^M$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_3\}$
 & $Y = \{d \in D^M \mid |d| = 1 \text{ \& } d \subseteq d_3 \text{ \& } \langle d, d_4 \rangle \in \llbracket \text{on} \rrbracket^M\}$

Verifying model:

e.g. $\models_M K_5$ for any $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$ such that:

- $\{s\}, \{b\}, \{c\}, \{c'\}, \{c''\}, \{d\} \in D^M$
- $\{s\} = \llbracket \text{susan} \rrbracket^M$
- $\{b\} = \llbracket \text{bill} \rrbracket^M$
- $\{c\}, \{c'\}, \{c''\} \in \llbracket \text{book} \rrbracket^M$
- $\langle \{b\}, \{c\} \rangle, \langle \{b\}, \{c'\} \rangle, \langle \{b\}, \{c''\} \rangle \in \llbracket \text{need} \rrbracket^M$
- $\langle \{s\}, \{c\} \rangle, \langle \{s\}, \{c'\} \rangle \in \llbracket \text{find} \rrbracket^M$
- $\langle \{d\}, \{b\} \rangle \in \llbracket \text{desk.off} \rrbracket^M$
- $\langle \{c, c'\}, \{d\} \rangle \in \llbracket \text{on} \rrbracket^M$

- Collective & distributive predicates (cf. Link 1983, 1987)

(6) ¹John and Mary are a nice couple. ²They met in Alaska.

DPL^Σ syntax

$$\begin{aligned} & [x x' z y] \\ & (x = \text{john}), (x' = \text{mary}), \\ & (z = x + x'), \text{nice.couple}(z) \\ & (y = \text{alaska}), \text{meet.in}(z, y) \end{aligned} \quad := K_6$$

DPL^Σ semantics

1. $\models_M K_6$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{x, x', z, y\}} h$
 $\& h \in \llbracket x = \text{john} \rrbracket^M \& h \in \llbracket x' = \text{mary} \rrbracket^M$
 $\& h \in \llbracket z = x + x' \rrbracket^M$
 $\& h \in \llbracket \text{nice.couple}(z) \rrbracket^M$
 $\& h \in \llbracket y = \text{alaska} \rrbracket^M$
 $\& h \in \llbracket \text{meet.in}(z, y) \rrbracket^M$
D3,
D2.2.[ul, \cap]
3. iff $\exists h \in G^M: \text{Dom } h = \{x, x', z, y\}$
 $\& h(x) = \llbracket \text{john} \rrbracket^M \& h(x') = \llbracket \text{mary} \rrbracket^M$
 $\& |h(x)| = 1 \& |h(x')| = 1$
 $\& h(z) = h(x) \cup h(x')$
 $\& h(z) \in \llbracket \text{nice.couple} \rrbracket^M$
 $\& h(y) = \llbracket \text{alaska} \rrbracket^M \& |h(y)| = 1$
 $\& \langle h(z), h(y) \rangle \in \llbracket \text{meet.in} \rrbracket^M$
D2.1. Λ , \subseteq_R
D2.2.=, +, **R**, **t**
D2.1.**Con**
4. iff $\exists d_1, d_2, d_3 \in D^M:$
 $\& d_1 = \llbracket \text{john} \rrbracket^M \& d_2 = \llbracket \text{mary} \rrbracket^M \& d_3 = \llbracket \text{alaska} \rrbracket^M$
 $\& |d_1| = 1 \& |d_2| = 1 \& |d_3| = 1$
 $\& d_1 \cup d_2 \in \llbracket \text{nice.couple} \rrbracket^M$
 $\& \langle d_1 \cup d_2, d_3 \rangle \in \llbracket \text{meet.in} \rrbracket^M$
simplify

Verifying model:

e.g. $\models_M K_6$ for any $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$ such that:

- $\{j\}, \{m\}, \{a\} \in D^M$
- $\{j\} = \llbracket \text{john} \rrbracket^M$
 $\{m\} = \llbracket \text{mary} \rrbracket^M$
 $\{a\} = \llbracket \text{alaska} \rrbracket^M$
- $\{j, m\} \in \llbracket \text{nice.couple} \rrbracket^M$
 $\langle \{j, m\}, \{a\} \rangle \in \llbracket \text{meet.in} \rrbracket^M$

(7) ¹(Last year) John *and* Mary bought a house. ²They both like it a lot.

DPL^Σ syntax

$$\begin{aligned} & [x x' z y \mid (x = \text{john}), (x' = \text{mary}), (z = x + x'), \\ & \quad \text{buy}(z, y), \text{house}(y) \\ & [z \uparrow \text{SG}(z'), (z' \leq z)] \text{ALL}_{z'} [\mid \text{like}(z', y)]] \end{aligned} \quad := K_7$$

DPL^Σ semantics

1. $\models_M K_7$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{x, x', z, y\}} h$
 - & $h \in \llbracket x = \text{john} \rrbracket^M \& h \in \llbracket x' = \text{mary} \rrbracket^M$
 - & $h \in \llbracket z = x + x \rrbracket^M$
 - & $h \in \llbracket \text{buy}(z, y) \rrbracket^M \& h \in \llbracket \text{house}(y) \rrbracket^M$
 - & $\exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$
 - & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 - & $X = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 - & $Y = \{d \in D^M \mid \forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M \rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [\mid \text{like}(z', y)] \rrbracket^M\}$
3. iff $\exists h \in G^M: \text{Dom } h = \{x, x', z, y\}$
 - & $h(x) = \llbracket \text{john} \rrbracket^M \& h(x') = \llbracket \text{mary} \rrbracket^M$
 - & $|h(x)| = 1 \& |h(x')| = 1$
 - & $h(z) = h(x) \cup h(x')$
 - & $\langle h(z), h(y) \rangle \in \llbracket \text{buy} \rrbracket^M \& h(y) \in \llbracket \text{house} \rrbracket^M$
 - & $\exists X, Y \subseteq D^M: X \subseteq Y$
 - & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& h \subseteq_{\{z\}} h' \& |h'(z)| = 1 \& h'(z) \subseteq h(z)\}$
 - & $Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& h \subseteq_{\{z\}} h' \& |h'(z)| = 1 \& h'(z) \subseteq h(z)) \& (\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h' \& h \subseteq_{\{z\}} h' \& |h'(z)| = 1 \& h'(z) \subseteq h(z)) \rightarrow \exists k \in G^M: h' = k \& \langle k(z'), k(y) \rangle \in \llbracket \text{like} \rrbracket^M\}$
4. iff $\exists d_1, d_2, d_3 \in D^M:$
 - & $d_1 = \llbracket \text{john} \rrbracket^M \& d_2 = \llbracket \text{mary} \rrbracket^M$
 - & $|d_1| = 1 \& |d_2| = 1$
 - & $\langle d_1 \cup d_2, d_3 \rangle \in \llbracket \text{buy} \rrbracket^M \& d_3 \in \llbracket \text{house} \rrbracket^M$
 - & $\exists X, Y \subseteq D^M: X \subseteq Y$
 - & $X = \{d \in D^M \mid |d| = 1 \& d \subseteq d_1 \cup d_2\}$
 - & $Y = \{d \in D^M \mid |d| = 1 \& d \subseteq d_1 \cup d_2 \& \langle d, d_3 \rangle \in \llbracket \text{like} \rrbracket^M\}$

D3,
D2.2.[ul, ∩,
Q_u

D2.1.Λ, ⊆_R
D2.2.=, +, R,
[ul, ≤, t
D2.1.SG, ALL

simplify

Verifying model:

- $\{j\}, \{m\}, \{h\} \in D^M$
- $\{j\} = \llbracket \text{john} \rrbracket^M$ • $\{h\} \in \llbracket \text{house} \rrbracket^M$ • $\langle \{j, m\}, \{h\} \rangle \in \llbracket \text{buy} \rrbracket^M$
- $\{m\} = \llbracket \text{mary} \rrbracket^M$ • $\langle \{j\}, \{h\} \rangle, \langle \{m\}, \{h\} \rangle \in \llbracket \text{like} \rrbracket^M$

(8) ¹(Last year) John *and* Mary (both) bought *a* house. ²They both like *them* a lot.

DPL^Σ *syntax* (**semantic composition??!**...)

$$\begin{aligned} & [x x' z y \uparrow (x = \text{john}), (x' = \text{mary}), (z = x + x'), \\ & [z \uparrow \text{SG}(z'), (z' \leq z)] \text{ALL}_{z'}.[y \uparrow \text{buy}(z', y), \text{house}(y)], \\ & (y' = \Sigma y.[y z \uparrow \text{SG}(z'), (z' \leq z), \text{buy}(z', y), \text{house}(y)]), \\ & [z \uparrow \text{SG}(z'), (z' \leq z)] \text{ALL}_{z'}.[y \uparrow \text{buy}(z', y), \text{house}(y), \text{like}(z', y), (y \leq y')]] \end{aligned} \quad := K_8$$

DPL^Σ *semantics*

1. $\models_M K_8$
2. iff $\exists h \in G^M: \Lambda \subseteq_{\{x, x', z, y\}} h$ D3,
 $\& h \in \llbracket x = \text{john} \rrbracket^M \& h \in \llbracket x' = \text{mary} \rrbracket^M$ D2.2. $[ul, \cap$
 $\& h \in \llbracket z = x + x' \rrbracket^M$ Q_u, Σ
 $\& \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$
 $\& X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 $\& (\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [y \uparrow \text{buy}(z', y), \text{house}(y)] \rrbracket^M\}$
 $\& h(y') = \cup \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle y, d \rangle\} \subseteq_{\{z\}} h'$
 $\& \langle h, h' \rangle \in \llbracket [y z \uparrow \text{SG}(z'), (z' \leq z),$
 $\text{buy}(z', y), \text{house}(y)] \rrbracket^M\}$
 $\& \exists X, Y \subseteq D^M: \langle X, Y \rangle \in \llbracket \text{ALL} \rrbracket^M$
 $\& X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 $\& Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M\}$
 $\& (\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\emptyset} h'$
 $\& \langle h, h' \rangle \in \llbracket [z \uparrow \text{SG}(z'), (z' \leq z)] \rrbracket^M$
 $\rightarrow \exists k \in G^M: \langle h', k \rangle \in \llbracket [y \uparrow \text{buy}(z', y), \text{house}(y),$
 $\text{like}(z', y), (y \leq y')] \rrbracket^M\}$

3. iff $\exists h \in G^M: \text{Dom } h = \{x, x', z, y\}$ D2.1.Λ, \subseteq_R
 & $h(x) = \llbracket \text{john} \rrbracket^M \ \& \ h(x') = \llbracket \text{mary} \rrbracket^M$ D2.2.=, \mathbf{R} , ...
 & $|h(x)| = 1 \ \& \ |h(x')| = 1$ D2.1.SG, ALL
 & $h(z) = h(x) \cup h(x')$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z)\}$
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z))$
 & $(\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z))$
 → $\exists k \in G^M: h' \subseteq_{\{y\}} k$
 & $\langle k(z'), k(y) \rangle \in \llbracket \text{buy} \rrbracket^M$
 & $k(y) \in \llbracket \text{house} \rrbracket^M\}$
 & $h(y) = \cup \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle y, d \rangle\} \subseteq_{\{z\}} h' \ \& \ h \subseteq_{\{y, z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z)$
 & $\langle h'(z'), h'(y) \rangle \in \llbracket \text{buy} \rrbracket^M \ \& \ h'(y) \in \llbracket \text{house} \rrbracket^M\}$
 & $\exists X, Y \subseteq D^M: X \subseteq Y$
 & $X = \{d \in D^M \mid \exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z)\}$
 & $Y = \{d \in D^M \mid (\exists h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z))$
 & $(\forall h' \in G^M: h \cup \{\langle z', d \rangle\} \subseteq_{\{\}} h' \ \& \ h \subseteq_{\{z\}} h'$
 & $|h'(z')| = 1 \ \& \ h'(z') \subseteq h'(z))$
 → $\exists k \in G^M: h' \subseteq_{\{y\}} k$
 & $\langle k(z'), k(y) \rangle \in \llbracket \text{buy} \rrbracket^M$
 & $k(y) \in \llbracket \text{house} \rrbracket^M$
 & $\langle k(z'), k(y) \rangle \in \llbracket \text{like} \rrbracket^M$
 & $k(y) \subseteq k(y')\}$

4. iff $\exists h \in G^M: \text{Dom } h = \{x, x', z, y'\}$ simplify

$$\begin{aligned}
 & \& h(x) = \llbracket \text{john} \rrbracket^M \& h(x') = \llbracket \text{mary} \rrbracket^M \\
 & \& |h(x)| = 1 \& |h(x')| = 1 \\
 & \& h(z) = h(x) \cup h(x') \\
 & \& \exists X, Y \subseteq D^M: X \subseteq Y \\
 & \quad \& X = \{d \in D^M \mid |d| = 1 \& d \subseteq h(z)\} \\
 & \quad \& Y = \{d \in D^M \mid (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \& (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \rightarrow \exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{buy} \rrbracket^M \& d' \in \llbracket \text{house} \rrbracket^M\} \\
 & \& h(y') = \cup \{d \in D^M \mid \exists d' \in D^M: |d'| = 1 \& d' \subseteq h(z) \\
 & \quad \& \langle d', d \rangle \in \llbracket \text{buy} \rrbracket^M \& d \in \llbracket \text{house} \rrbracket^M\} \\
 & \& \exists X, Y \subseteq D^M: X \subseteq Y \\
 & \quad \& X = \{d \in D^M \mid |d| = 1 \& d \subseteq h(z)\} \\
 & \quad \& Y = \{d \in D^M \mid (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \& (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \rightarrow \exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{buy} \rrbracket^M \& d' \in \llbracket \text{house} \rrbracket^M\} \\
 & \quad \quad \& \langle d, d' \rangle \in \llbracket \text{like} \rrbracket^M \& d' \subseteq h(y')\}
 \end{aligned}$$

5. iff $\exists d_1, d_2, d_3 \in D^M:$ simplify

$$\begin{aligned}
 & \& d_1 = \llbracket \text{john} \rrbracket^M \& d_2 = \llbracket \text{mary} \rrbracket^M \\
 & \& |d_1| = 1 \& |d_2| = 1 \\
 & \& \exists X, Y \subseteq D^M: X \subseteq Y \\
 & \quad \& X = \{d \in D^M \mid |d| = 1 \& d \subseteq d_1 \cup d_2\} \\
 & \quad \& Y = \{d \in D^M \mid |d| = 1 \& d \subseteq d_1 \cup d_2 \\
 & \quad \quad \& \exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{buy} \rrbracket^M \& d' \in \llbracket \text{house} \rrbracket^M\} \\
 & \& d_3 = \cup \{d \in D^M \mid \exists d' \in D^M: |d'| = 1 \& d' \subseteq d_1 \cup d_2 \\
 & \quad \& \langle d', d \rangle \in \llbracket \text{buy} \rrbracket^M \& d \in \llbracket \text{house} \rrbracket^M\} \\
 & \& \exists X, Y \subseteq D^M: X \subseteq Y \\
 & \quad \& X = \{d \in D^M \mid |d| = 1 \& d \subseteq h(z)\} \\
 & \quad \& Y = \{d \in D^M \mid (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \& (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \rightarrow \exists d' \in D^M: \langle d, d' \rangle \in \llbracket \text{buy} \rrbracket^M \& d' \in \llbracket \text{house} \rrbracket^M\} \\
 & \quad \quad \& \langle d, d' \rangle \in \llbracket \text{like} \rrbracket^M \& d' \subseteq h(y')\}
 \end{aligned}$$

6. iff $\exists d_1, d_2 \in D^M:$ simplify

$$\begin{aligned}
 & \& d_1 = \llbracket \text{john} \rrbracket^M \& d_2 = \llbracket \text{mary} \rrbracket^M \\
 & \& |d_1| = 1 \& |d_2| = 1 \\
 & \& \exists X, Y \subseteq D^M: X \subseteq Y \\
 & \quad \& X = \{d \in D^M \mid |d| = 1 \& d \subseteq h(z)\} \\
 & \quad \& Y = \{d \in D^M \mid (|d| = 1 \& d \subseteq h(z)) \\
 & \quad \quad \& \exists d' \in D^M: d' \in \llbracket \text{house} \rrbracket^M \& \langle d, d' \rangle \in \llbracket \text{buy} \rrbracket^M \\
 & \quad \quad \& \langle d, d' \rangle \in \llbracket \text{like} \rrbracket^M\}
 \end{aligned}$$

Verifying model:

- $\{j\}, \{m\}, \{h\}, \{h'\} \in D^M$
- $\{j\} = \llbracket \text{john} \rrbracket^M$ • $\{h\} \in \llbracket \text{house} \rrbracket^M$ • $\langle \{j\}, \{h\} \rangle, \langle \{m\}, \{h'\} \rangle \in \llbracket \text{buy} \rrbracket^M$
- $\{m\} = \llbracket \text{mary} \rrbracket^M$ • $\{h'\} \in \llbracket \text{house} \rrbracket^M$ • $\langle \{j\}, \{h'\} \rangle, \langle \{m\}, \{h'\} \rangle \in \llbracket \text{like} \rrbracket^M$

(9¹) ¹(Today) Bill's students worked in groups. distribution down to pluralities

DPL^Σ *syntax*

[$x \ y \mid (x = \textit{bill}),$
 $(y = \Sigma y'. [y \uparrow \textit{std.of}(y', x)]),$
 $[y \uparrow \text{SG}(y), y' \leq y] \text{ ALL}_{y'} [y \uparrow y'' \leq y, y'' \emptyset y', \textit{wrk.tg}(y' + y'')]$]

(9²) ¹(Today) Bill's students worked in groups.

²*They all* worked on *different* problems.

quantification over pluralities

DPL^Σ *syntax* (**semantic composition?!...**)

[$x \ y \mid (x = \textit{bill}),$
 $(y = \Sigma y'. [y \uparrow \textit{std.of}(y', x)]),$
 $[y \uparrow \text{SG}(y), y' \leq y] \text{ ALL}_{y'} [y \uparrow y'' \leq y, y'' \emptyset y', \textit{wrk.tg}(y' + y'')],$
 $[y \uparrow y' \leq y, \textit{wrk.tg}(y')]$
 $\text{ ALL}_{y'} [z \mid (z = \Sigma z' [z \uparrow \textit{prb}(z'), \textit{wrk.on}(y', z')]),$
 $[y'' z \uparrow y'' \leq y, y'' \emptyset y', \textit{wrk.tg}(y''), z'' = \Sigma z'' [z'' \uparrow \textit{prb}(z''), \textit{wrk.on}(y'', z'')]$
 $\text{ ALL}_{z''} [z'' \emptyset z]]]$

(9³) ¹(Today) Bill's students worked in groups.

²*They all* worked on *different* problems.

³*The group with the hardest* problem

superlative quantification

came up with *the best* solution.

DPL^Σ *syntax* (**semantic composition??!!!!...**)

[$x \ y \ y_1 \ z_1 \ z_2 \mid (x = \textit{bill}),$
 $(y = \Sigma y'. [y \uparrow \textit{std.of}(y', x)]),$
 $[y \uparrow \text{SG}(y), y' \leq y] \text{ ALL}_{y'} [y \uparrow y'' \leq y, y'' \emptyset y', \textit{wrk.tg}(y' + y'')],$
 $[y \uparrow y' \leq y, \textit{wrk.tg}(y')]$
 $\text{ ALL}_{y'} [z \mid (z = \Sigma z' [z \uparrow \textit{prb}(z'), \textit{wrk.on}(y', z')]),$
 $[y'' z \uparrow y'' \leq y, y'' \emptyset y', \textit{wrk.tg}(y''), z'' = \Sigma z'' [z'' \uparrow \textit{prb}(z''), \textit{wrk.on}(y'', z'')]$
 $\text{ ALL}_{z''} [z'' \emptyset z]]]$
 $y_1 \leq y, \textit{wrk.tg}(y_1), \textit{wrk.on}(y_1, z_1), \textit{prb}(z_1),$
 $[y' z \uparrow y' \leq y, y' \emptyset y_1, \textit{wrk.tg}(y'), \textit{wrk.on}(y', z'), \textit{prb}(z')] \text{ ALL}_{z'} [z' \uparrow \textit{harder}(z_1, z')],$
 $\textit{come.up.w}(y_1, z_2), \textit{sol.to}(z_2, z_1),$
 $[y' z' z \uparrow y' \leq y, y' \emptyset y_1, \textit{wrk.tg}(y'), \textit{wrk.on}(y', z'), \textit{prb}(z'), \textit{come.up.w}(y', z''), \textit{sol.to}(z'', z')] \text{ ALL}_{z''} [z'' \uparrow \textit{better}(z_2, z'')]$

Remark: I won't work out the semantics of this monster, because at this point I would start looking for an alternative theory, with a simpler semantic representation and more transparent syntax-semantics interface. We'll see if other theories to be considered in this course can do any better.