Notes on Asher & Wang 2003
'Ambiguity and anaphora with plurals in discourse'

I. The phenomenon

• *Collective* vs. *distributive* readings

(1)	[1]Two boys mowed a lawn (*together*). [2]They (*both*) had a good time.	collective distributive

• *Collective*/*dependent* ambiguity

(2)	[1](Last year) two scientists (*jointly*) wrote an important paper. [2]*They* presented *it* at a conference.	collective
(3)	[1](Last year) two scientists (*each*) wrote an important paper. [2]*They* presented *it* at two major conferences.	dependent sg
(4)	[1](Last year) two scientists (*each*) wrote an important paper. [2]*They* presented *them* at two major conferences.	dependent pl

• *Collective*/*cumulative* ambiguity (Scha 1981)

(5)	[1](Last year) three scientists (*jointly*) wrote five papers. [2]*They* presented *all five* at major conferences.	collective
(6)	[1](Last year) three scientists wrote (*a total of*) five papers (*between them*). (e.g. Dr. Smith wrote one, co-authored two more with Dr. Nelson, who co-authored two more papers with Dr. Slack.) [2]*They* presented *all five* at major conferences.	cumulative

• *Distributive* vs. *dependent* readings (Asher & Wang 2003)

(7) [1](This morning) *every* train was late.
[2]*It* was delayed by the snow.

(8) [1]*A* train leaves from this station *every* hour.
✓[2]*It* goes to New York.

• *Basic idea* of Asher & Wang 2003

The basic idea of A&W is that discourse participants keep track not only of the current discourse referents but also of their current organization, e.g. whether they are currently talked of as *collections* or *separate individuals*. Formally, they propose a system, DPL^+, where a state of information encodes information about all of these matters. So information can be updated by imposing a new collective or distributive structure—by updates such as 'COL *x*' and 'DIS *x*' in (1′)—and it is these updates that give rise to different readings of discourses with plurals.

(1) [1]Two boys mowed a lawn (*together*). [2]They (*both*) had a good time.

(1′) §; ∃*x*; **2***x*[*boy*(*x*)]; COL *x*; ∃*y*; *lawn*(*y*); *mow*(*x*, *y*); DIS *x*; *hgt*(*x*)

II. Augmented DPL: DPL$^+$ (attempt to make sense of A&W's definitions)

DEFINITION 1.1 (DPL$^+$-basic terms)

•$_1$	**Prd**^{1}	=	{*donkey*, …, *pair*, …, *enter*, …}	1-place predicates
	Prd^2	=	{*see*, *talk.to*, …, *father.of*, …}	2-place predicates
	Prd^3	=	{*give.to*, *introduce.to*, …}	3-place predicates
•$_2$	**Var**	=	{x, y, z}	(individual) variables

DEFINITION 1.2 (DPL$^+$ syntax).

§	§ ∈ **Drs**		
∃	$\exists u \in$ **Drs**	if	$u \in$ **Var**
R	$\alpha(u_1,\ldots u_n) \in$ **Drs**	if	$\alpha \in$ **Prd**^{n} and $u_1, \ldots, u_n \in$ **Var**
R′	SG(u) ∈ **Drs**	if	$u \in$ **Var**
D^1	DIS u, COL u, CUM u ∈ **Drs**	if	$u \in$ **Var**
D^2	DEP uu' ∈ **Drs**	if	$u, u' \in$ **Var**
n	***n***$u[\Phi]$, $^{\geq}$***n***$[\Phi]$ ∈ **Drs**	if	$u \in$ **Var**, ***n*** ∈ {**1**, **2**, …}, and $\Phi \in$ **Drs**
∀	$\forall u[\Phi] \in$ **Drs**	if	$u \in$ **Var** and $\Phi \in$ **Drs**
;	$(\Phi; \Psi) \in$ **Drs**	if	$\Phi, \Psi \in$ **Drs**
~	$\sim\Phi \in$ **Drs**	if	$\Phi \in$ **Drs**

DEFINITION 2.1 (DPL$^+$-models, atomic assignments, dependencies)

•$_1$ A DPL$^+$-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
- (i) $D^M = \{X \mid \varnothing \subset X \subseteq A\}$, where A is a non-empty set (of *atoms*)
  $[\![\alpha]\!]^M \subseteq (D^M)^n$ if $\alpha \in$ **Prd**^{n}
- (iii) $[\![donkey]\!]^M \subseteq \{d \in D^M: |d| = 1\}$
  ⋮
  $[\![pair]\!]^M \subseteq \{d \in D^M: |d| = 2\}$
  ⋮

•$_2$ (i) An *atomic M-assignment* is a function g such that Dom g = **Var** and for all $u \in$ Dom g, $g(u) \in \{d \in D^M: |d| = 1\}$.
- (ii) $G^M := \{d \in D^M: |d| = 1\}^{\mathbf{Var}}$ is the set of all atomic M-assignments.
- (iii) Given $g, h \in G^M$ and $u \in$ **Var**:
  h is a *u-alternative* to g, written $g[u]h$, iff $\forall u' \in (\mathbf{Var} - \{u\})$: $h(u') = g(u')$

•$_3$ (i) a *g-dependency* is a function f such that Dom f = **Var** and for all $u \in$ Dom f, $f(u) \in \{\langle H, d\rangle \in \mathcal{P}(G^M) \times D^M: g \in H\}$
- (ii) Given $f(u) = \langle H, d\rangle$,
  $^1(f(u)) := H$ is the *1st coordinate* of $f(u)$
  $^2(f(u)) := d$ is the *2nd coordinate* of $f(u)$

DEFINITION 2.1′ (information states)

•$_1$ A *M-information state* is a function S such that Dom $S \subseteq G^M$ and for all $g \in$ Dom S, $S(g)$ is a g-dependency.

•$_2$ **S**^{M} is the set of all M-information states. Λ is the empty information state (i.e. Dom Λ = ∅).

•$_3$ Given $S, S' \in$ **S**^{M} and $u \in$ **Var**:
S' is a *u-alternative* to S, written $S[u]S'$,
iff (i) $\forall g \in \text{Dom } S \exists h \in \text{Dom } S'$: $g[u]h$, and
(ii) $\forall h \in \text{Dom } S' \exists g \in \text{Dom } S$: $g[u]h$

DEFINITION 2.2 (DPL$^+$-semantics)

§ $\llbracket \S \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; S \subseteq S'$
$\quad\quad \& \; \forall h \in \text{Dom } S' \forall u \in \mathbf{Var}: {}^1(S'(h)(u)) = \text{Dom } S'$
$\quad\quad\quad\quad \& \; {}^2(S'(h)(u)) = h(u)$

∃ $\llbracket \exists u \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; S[u]S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': {}^2(S'(h)(u)) = h(u)$
$\quad\quad\quad\quad \& \; \forall u' \in (\mathbf{Var} - \{u\}) \; \exists g \in \text{Dom } S:$
$\quad\quad\quad\quad\quad g[u]h \; \& \; {}^2(S'(h)(u')) = {}^2(S(g)(u'))$
$\quad\quad\quad\quad \& \; \forall u'' \in \mathbf{Var}:$
$\quad\quad\quad\quad\quad {}^1(S'(h)(u'')) = \{k \in \text{Dom } S' | \; \exists g: g \in {}^1(S(g)(u))$
$\quad\quad\quad\quad\quad\quad \& \; g[u]k \; \& \; g[u]h\}$

R $\llbracket \alpha(u_1, \ldots u_n) \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; S = S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': \langle {}^2(S'(h)(u_1)), \ldots, {}^2(S'(h)(u_n))\rangle \in \llbracket \alpha \rrbracket^M\}$

R′ $\llbracket \text{SG}(u) \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; S = S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': |\{k(u): k \in {}^1(S'(h)(u))\}| = 1\}$

D^1 $\llbracket \text{DIS } u \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \text{Dom } S = \text{Dom } S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': {}^1(S'(h)(u)) = \text{Dom } S'$
$\quad\quad\quad\quad \& \; {}^2(S'(h)(u)) = h(u)$
$\quad\quad\quad\quad \& \; \forall u' \in (\mathbf{Var} - \{u\}): S'(h)(u') = S(h)(u)\}$

$\llbracket \text{COL } u) \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \text{Dom } S = \text{Dom } S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': {}^1(S'(h)(u)) = {}^1(S(h)(u))$
$\quad\quad\quad\quad \& \; {}^2(S'(h)(u)) = \cup\{g(u): g \in {}^1(S(h)(u))\}$
$\quad\quad\quad\quad \& \; \forall u' \in (\mathbf{Var} - \{u\}): S'(h)(u') = S(h)(u')\}$

$\llbracket \text{CUM } u) \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \text{Dom } S = \text{Dom } S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': {}^1(S'(h)(u)) = {}^1(S(h)(u))$
$\quad\quad\quad\quad \& \; {}^2(S'(h)(u)) \subseteq \cup\{g(u): g \in {}^1(S(h)(u))\}$
$\quad\quad\quad\quad \& \; \cup\{k(u): k \in {}^1(S'(h)(u))\}$
$\quad\quad\quad\quad\quad = \cup\{{}^2(S'(k)(u)): k \in {}^1(S'(h)(u))\}$
$\quad\quad\quad\quad \& \; \forall u' \in (\mathbf{Var} - \{u\}): S'(h)(u') = S(h)(u')\}$

D^2 $\llbracket \text{DEP } uu' \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \text{Dom } S = \text{Dom } S'$
$\quad\quad \& \; \forall h \in \text{Dom } S': {}^1(S'(h)(u')) = \{g \in {}^1(S(h)(u')) | \; g(u) = h(u)\}$
$\quad\quad\quad\quad \& \; {}^2(S'(h)(u')) = {}^2(S(h)(u'))$
$\quad\quad\quad\quad \& \; \forall u'' \in (\mathbf{Var} - \{u'\}): S'(h)(u'') = S(h)(u'')$

n $\llbracket \boldsymbol{n}u[\Phi] \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \langle S, S'\rangle \in \llbracket \Phi \rrbracket^M \; \& \; \forall h \in \text{Dom } S': |\{k(u): k \in {}^1(S'(h)(u))\}| = n\}$
$\llbracket {}^{\geq}\boldsymbol{n}u[\Phi] \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \langle S, S'\rangle \in \llbracket \Phi \rrbracket^M \; \& \; \forall h \in \text{Dom } S': |\{k(u): k \in {}^1(S'(h)(u))\}| \geq n\}$

∀ $\llbracket \forall u[\Phi] \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \langle S, S'\rangle \in \llbracket \Phi \rrbracket^M$
$\quad\quad \& \; \forall h \in \text{Dom } S': \{k(u): k \in {}^1(S'(h)(u))\}$
$\quad\quad\quad\quad = \{g(u): g \in \text{Dom } S \; \& \; \exists S'' \in \mathbf{S}^M: \langle S'', S\rangle \in \llbracket \Phi \rrbracket^M\}$

; $\llbracket (\Phi; \Psi) \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; \exists S'' \in \mathbf{S}^M: \langle S, S''\rangle \in \llbracket \Phi \rrbracket^M \; \& \; \langle S'', S\rangle \in \llbracket \Psi \rrbracket^M$
~ $\llbracket \sim \Phi \rrbracket^M = \{\langle S, S'\rangle \in (\mathbf{S}^M)^2 | \; S = S' \; \& \; \neg S'' \in \mathbf{S}^M: \langle S, S''\rangle \in \llbracket \Phi \rrbracket^M\}$

DEFINITION 3 (Truth).

- A drs Φ is *true* in M given an information state S, written $\models_{M,S} \Phi$, iff $\exists S' \in \mathbf{S}^M$: $\langle S, S' \rangle \in [\![\Phi]\!]^M$.
- A drs Φ is *true* in M, written $\models_M \Phi$, iff $\exists S \in \mathbf{S}^M$: $\langle \Lambda, S \rangle \in [\![\Phi]\!]^M$

III. <u>From English syntax to DPL⁺ syntax</u>

(1) [1]Two boys mowed a lawn (*together*). — *collective*
[2]They (*both*) had a good time. — *distr. sub*

$\S; \exists x; \mathbf{2}x[boy(x)]; \textsc{col}\ x; \exists y; lawn(y); mow(x, y);$
$\textsc{dis}\ x; hgt(x)$ — see pp. 5-9

(2) [1](Last year) two scientists (*jointly*) wrote an important paper. — *collective*
[2]*They* presented *it* at a conference. — *collective*

$\S; \exists x; \mathbf{2}x[sci(x)]; \textsc{col}\ x; \exists y; \mathbf{1}y[ppr(y)]; wrt(x, y);$
$\textsc{sg}(y); \exists z; \mathbf{1}z[cnf(z)]; prs(x, y, z)$

(3) [1](Last year) two scientists (*each*) wrote an important paper. — *dependent* sg
[2]*They* presented *it* at two major conferences. — *dependent* sg

$\S; \exists x; \mathbf{2}x[sci(x)]; \textsc{dis}\ x; \exists y; \mathbf{1}y[ppr(y)]; \textsc{dep}\ xy; wrt(x, y);$ — see pp. 10-13
$\textsc{sg}(y); \exists z; \mathbf{2}z[cnf(z)]; \textsc{dep}\ yz; prs(x, y, z)$

or $\S; \exists x; \mathbf{2}x[sci(x)]; \exists y; \mathbf{1}y[ppr(y)]; wrt(x, y);$ — ***same meaning***
$\textsc{sg}(y); \exists z; \mathbf{2}z[cnf(z)]; prs(x, y, z)$ — W/O DIS or DEP!

(4) [1](Last year) two scientists (*each*) wrote an important paper. — *dependent* sg
[2]*They* presented *them* at two major conferences. — '*dependent*' pl

$\S; \exists x; \mathbf{2}x[sci(x)]; \exists y; \mathbf{1}y[ppr(y)]; wrt(x, y);$
$\exists z; \mathbf{2}z[cnf(z)]; prs(x, y, z)$

(6) [1](Last year) three scientists wrote (*a total of*) five papers (*between them*). — *cumulative*

$\S; \exists x; \exists y; \mathbf{3}x[sci(x)]; \textsc{cum}\ x; \mathbf{5}y[ppr(y)]; \textsc{cum}\ y; wrt(x, y)$ — A&W, p.5: ***wrong meaning***
$\S; \exists x; \mathbf{3}x[sci(x)]; \textsc{cum}\ x; \exists y; \mathbf{5}y[ppr(y)]; \textsc{cum}\ y; wrt(x, y)$ — see pp. 14-17: ***still wrong***

(9) Two students answered three questions. — *scope ambig.*

S > O ('surface scope'):
$\S; \exists x; \mathbf{2}x[std(x)]; \exists y; \mathbf{3}y[que(y)]; ans(x, y)$ — like (3), see pp. 10–13

O > S ('inverse scope'):
$\S; \exists y; \mathbf{3}y[que(y)]; \exists x; \mathbf{2}x[std(x)]; ans(x, y)$ — pp. 18–20

cf. $\S; \exists x; \exists y; \mathbf{2}x[std(x)]; \mathbf{3}y[que(y)]; \textsc{dep}\ yx; ans(x, y)$ — A&W, p. 11: ***wrong meaning***
$\S; \exists x; \exists y; \textsc{dep}\ yx; \mathbf{2}x[std(x)]; \mathbf{3}y[que(y)]; ans(x, y)$ — A&W, ftn. 14: ***wrong meaning***

IV. From DPL⁺ syntax to DPL⁺-semantics

• Collective antecedent. Distributive anaphor.

(1) [1]Two boys mowed a lawn.
[2]They had a good time.

DPL⁺ *syntax*

$\S$; $\exists x$; $\mathbf{2}x[boy(x)]$; COL x; $\exists y$; $lwn(y)$; $mow(x, y)$; DIS x; $hgt(x)$ $:= K_1$

DPL⁺ *semantics*

EXERCISE

Show that K_1 is true in a DPL⁺-model $M = \langle D^M, [\![\cdot]\!]^M\rangle$ s.t.

- $D^M = \{X \mid \varnothing \subset X \subset \{B, B', B'', L, L', C\}\}$
  $[\![boy]\!]^M = \{\{B\}, \{B'\}, \{B''\}\}$
  $[\![lawn]\!]^M = \{\{L\}, \{L'\}\}$
  $[\![hgt]\!]^M = \{\{B'\}, \{B''\}\}$
  $[\![mow]\!]^M = \{\langle\{B, B'\}, \{L\}\rangle, \langle\{B', B''\}, \{L'\}\rangle\}$

To show that, start from Λ and interpret each update in K_1 in turn, constructing a possible output and using that as the input for the next update (see below), until you get an output for $hgt(x)$. Since update in DPL⁺ is a *relation*, there may be more than one output for a given input.

Initial information state: Λ

- $\langle\Lambda, S_0\rangle \in [\![\S]\!]^M$

iff $\forall h \in \text{Dom } S_0 \forall u \in \textbf{Var}$: ${}^1(S_0(h)(u)) = \text{Dom } S_0$ D2.2.§, df. Λ
& ${}^2(S_0(h)(u)) = h(u)$

e.g. $S_0 = \{\langle g_1, S_0(g_1)\rangle, \langle g_2, S_0(g_2)\rangle\}$

where $g_1 = \{\langle x, \{B\}\rangle, \langle y, \{B'\}\rangle, \langle z, \{L\}\rangle\}$
$g_2 = \{\langle x, \{B'\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{B'\}\rangle\}$

$S_0(g_1)$: $x \to \langle\{g_1, g_2\}, \{B\}\rangle$
$y \to \langle\{g_1, g_2\}, \{B'\}\rangle$
$z \to \langle\{g_1, g_2\}, \{L\}\rangle$

$S_0(g_2)$: $x \to \langle\{g_1, g_2\}, \{B'\}\rangle$
$y \to \langle\{g_1, g_2\}, \{L'\}\rangle$
$z \to \langle\{g_1, g_2\}, \{B'\}\rangle$

- $\langle S_0, S_1\rangle \in [\![\exists x]\!]^M$

iff $\forall h \in \text{Dom } S_1 \exists g \in \text{Dom } S_0$: $g[x]h$ D2.2.∃,
& $\forall g \in \text{Dom } S_0 \exists h \in \text{Dom } S_1$: $g[x]h$ D2.1′.$S[u]S'$
& $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) = h(x)$
& $\forall u \in (\textbf{Var} - \{x\}) \exists g \in \text{Dom } S_0$: $g[x]h$ & ${}^2(S_1(h)(u)) = g(u)$
& $\forall u' \in \textbf{Var}$: ${}^1(S_1(h)(u')) = \{k \in \text{Dom } S_1 \mid \exists g: g \in {}^1(S_0(g)(x)) \;\&\; g[x]k \;\&\; g[x]h\}$

e.g. $S_1 = \ldots$

ONE SOLUTION TO EXERCISE

Initial state: Λ

- $\langle \Lambda, S_0 \rangle \in [\![\S]\!]^M$

iff $\forall h \in \text{Dom } S_0 \forall u \in \textbf{Var}$: ${}^1(S_0(h)(u)) = \text{Dom } S_0$ D2.2.§, df. Λ

$\& \ {}^2(S_0(h)(u)) = h(u)$

e.g. $S_0 = \{\langle g_1, S_0(g_1)\rangle, \langle g_2, S_0(g_2)\rangle\}$

where
$S_0(g_1)$: $x \rightarrow \langle \{g_1, g_2\}, \{\text{B}\}\rangle$
$y \rightarrow \langle \{g_1, g_2\}, \{\text{B}'\}\rangle$
$z \rightarrow \langle \{g_1, g_2\}, \{\text{L}\}\rangle$

$S_0(g_2)$: $x \rightarrow \langle \{g_1, g_2\}, \{\text{B}'\}\rangle$
$y \rightarrow \langle \{g_1, g_2\}, \{\text{L}'\}\rangle$
$z \rightarrow \langle \{g_1, g_2\}, \{\text{B}'\}\rangle$

- $\langle S_0, S_1 \rangle \in [\![\exists x]\!]^M$

iff $\forall h \in \text{Dom } S_1 \exists g \in \text{Dom } S_0$: $g[x]h$ D2.2.$\exists$,

& $\forall g \in \text{Dom } S_0 \exists h \in \text{Dom } S_1$: $g[x]h$ D2.1$'$.$S[u]S'$

& $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) = h(x)$

$\& \ \forall u \in (\textbf{Var} - \{x\}) \exists g \in \text{Dom } S_0$: $g[x]h \ \& \ {}^2(S_1(h)(u)) = g(u)$

$\& \ \forall u' \in \textbf{Var}$: ${}^1(S_1(h)(u')) = \{k \in \text{Dom } S_1 \mid \exists g\text{: } g \in {}^1(S_0(g)(x) \ \& \ g[x]k \ \& \ g[x]h\}$

e.g. $S_1 = \{\langle g'_1, S_1(g'_1)\rangle, \langle h_1, S_1(h_1)\rangle, \langle g_2, S_1(g_2)\rangle, \langle h_2, S_1(h_2)\rangle\}$

where
$g'_1 := g_1[x/\{\text{B}'\}] = \{\langle x, \{\text{B}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{L}\}\rangle\}$
$h_1 := g_1[x/\{\text{B}''\}] = \{\langle x, \{\text{B}''\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{L}\}\rangle\}$
$g_2 = \{\langle x, \{\text{B}'\}\rangle, \langle y, \{\text{L}'\}\rangle, \langle z, \{\text{B}'\}\rangle\}$
$h_2 := g_2[x/\{\text{B}''\}] = \{\langle x, \{\text{B}''\}\rangle, \langle y, \{\text{L}'\}\rangle, \langle z, \{\text{B}'\}\rangle\}$

$S_1(g'_1)$: $x \rightarrow \langle \{g'_1, h_1\}, \{\text{B}'\}\rangle$
$y \rightarrow \langle \{g'_1, h_1\}, \{\text{B}'\}\rangle$
$z \rightarrow \langle \{g'_1, h_1\}, \{\text{L}\}\rangle$

$S_1(h_1)$: $x \rightarrow \langle \{g'_1, h_1\}, \{\text{B}''\}\rangle$
$y \rightarrow \langle \{g'_1, h_1\}, \{\text{B}'\}\rangle$
$z \rightarrow \langle \{g'_1, h_1\}, \{\text{L}\}\rangle$

$S_1(g_2)$: $x \rightarrow \langle \{g_2, h_2\}, \{\text{B}'\}\rangle$
$y \rightarrow \langle \{g_2, h_2\}, \{\text{L}'\}\rangle$
$z \rightarrow \langle \{g_2, h_2\}, \{\text{B}'\}\rangle$

$S_1(h_2)$: $x \rightarrow \langle \{g_2, h_2\}, \{\text{B}''\}\rangle$
$y \rightarrow \langle \{g_2, h_2\}, \{\text{L}'\}\rangle$
$z \rightarrow \langle \{g_2, h_2\}, \{\text{B}'\}\rangle$

- $\langle S_1, S_1\rangle \in [\![\mathbf{2}x[boy(x)]]\!]^M$

iff $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) \in [\![boy]\!]^M$ D2.2.***n***, ***R***

$\& \ |\{k(x): k \in {}^1(S_1(h)(x))\}| = 2$

✓ since Dom $S_1 = \{g'_1, h_1, g_2, h_2\}$ and:

${}^2(S_1(g'_1)(x)) = \{\text{B}'\} \in [\![boy]\!]^M$

$\& \ |\{k(x): k \in {}^1(S_1(g'_1)(x))\}| = |\{k(x): k \in \{g'_1, h_1\}| = |\{\{\text{B}'\}, \{\text{B}''\}\}| = 2$

${}^2(S_1(h_1)(x)) = \{\text{B}''\} \in [\![boy]\!]^M$

$\& \ |\{k(x): k \in {}^1(S_1(g'_1)(x))\}| = |\{k(x): k \in \{g'_1, h_1\}| = |\{\{\text{B}'\}, \{\text{B}''\}\}| = 2$

${}^2(S_1(g_2)(x)) = \{\text{B}'\} \in [\![boy]\!]^M$

$\& \ |\{k(x): k \in {}^1(S_1(g_2)(x))\}| = |\{k(x): k \in \{g_2, h_2\}| = |\{\{\text{B}'\}, \{\text{B}''\}\}| = 2$

${}^2(S_1(h_2)(x)) = \{\text{B}''\} \in [\![boy]\!]^M$

$\& \ |\{k(x): k \in {}^1(S_1(h_2)(x))\}| = |\{k(x): k \in \{g_2, h_2\}| = |\{\{\text{B}'\}, \{\text{B}''\}\}| = 2$

- $\langle S_1, S_2\rangle \in [\![\text{COL } x]\!]^M$

iff Dom S_1 = Dom S_2 D2.2.**D**[1]

$\& \ \forall h \in \text{Dom } S_2$: ${}^1(S_2(h)(x)) = {}^1(S_1(h)(x))$

$\& \ {}^2(S_2(h)(x)) = \cup\{g(x): g \in {}^1(S_1(h)(x))\}$

$\& \ \forall u \in (\mathbf{Var} - \{x\}): S_2(h)(u) = S_1(h)(u)$

$S_2 = \{\langle g'_1, S_2(g'_1)\rangle, \langle h_1, S_2(h_1)\rangle, \langle g_2, S_2(g_2)\rangle, \langle h_2, S_2(h_2)\rangle\}$

where $g'_1 := g_1[x/\{\text{B}'\}] = \{\langle x, \{\text{B}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{L}\}\rangle\}$

$h_1 := g_1[x/\{\text{B}''\}] = \{\langle x, \{\text{B}''\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{L}\}\rangle\}$

$g_2 = \{\langle x, \{\text{B}'\}\rangle, \langle y, \{\text{L}'\}\rangle, \langle z, \{\text{B}'\}\rangle\}$

$h_2 := g_2[x/\{\text{B}''\}] = \{\langle x, \{\text{B}''\}\rangle, \langle y, \{\text{L}'\}\rangle, \langle z, \{\text{B}'\}\rangle\}$

$S_2(g'_1)$: $x \rightarrow \langle\{g'_1, h_1\}, \{\text{B}', \text{B}''\}\rangle$
$y \rightarrow \langle\{g'_1, h_1\}, \{\text{B}'\}\rangle$
$z \rightarrow \langle\{g'_1, h_1\}, \{\text{L}\}\rangle$

$S_2(h_1)$: $x \rightarrow \langle\{g'_1, h_1\}, \{\text{B}', \text{B}''\}\rangle$
$y \rightarrow \langle\{g'_1, h_1\}, \{\text{B}'\}\rangle$
$z \rightarrow \langle\{g'_1, h_1\}, \{\text{L}\}\rangle$

$S_2(g_2)$: $x \rightarrow \langle\{g_2, h_2\}, \{\text{B}', \text{B}''\}\rangle$
$y \rightarrow \langle\{g_2, h_2\}, \{\text{L}'\}\rangle$
$z \rightarrow \langle\{g_2, h_2\}, \{\text{B}'\}\rangle$

$S_2(h_2)$: $x \rightarrow \langle\{g_2, h_2\}, \{\text{B}', \text{B}''\}\rangle$
$y \rightarrow \langle\{g_2, h_2\}, \{\text{L}'\}\rangle$
$z \rightarrow \langle\{g_2, h_2\}, \{\text{B}'\}\rangle$

- $\langle S_2, S_3\rangle \in [\![\exists y]\!]^M$

iff $\forall h \in \text{Dom } S_3 \exists g \in \text{Dom } S_2$: $g[y]h$ — D2.2.∃,

& $\forall g \in \text{Dom } S_2 \exists h \in \text{Dom } S_3$: $g[y]h$ — D2.1′.$S[u]S'$

& $\forall h \in \text{Dom } S_3$: ${}^2(S_3(h)(y)) = h(y)$

& $\forall u \in (\mathbf{Var} - \{y\})\exists g \in \text{Dom } S_2$: $g[y]h$ & ${}^2(S_3(h)(u)) = {}^2(S_2(g)(u))$

& $\forall u' \in \mathbf{Var}$: ${}^1(S_3(h)(u')) = \{k \in \text{Dom } S_3 | \exists g\text{: } g \in {}^1(S_2(g)(y) \;\&\; g[y]k \;\&\; g[y]h\}$

$S_3 = \{\langle g''_1, S_3(g''_1)\rangle, \langle h'_1, S_3(h'_1)\rangle, \langle g_2, S_2(g_2)\rangle, \langle h_2, S_2(h_2)\rangle\}$

where
$g''_1 := g'_1[y/\{L'\}] = \{\langle x, \{B'\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{L\}\rangle\}$
$h'_1 := h_1[y/\{L'\}] = \{\langle x, \{B''\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{L\}\rangle\}$
$g_2 = \{\langle x, \{B'\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{B'\}\rangle\}$
$h_2 = \{\langle x, \{B''\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{B'\}\rangle\}$

$S_3(g''_1)$: $x \to \langle\{g''_1\}, \{B', B''\}\rangle$
$y \to \langle\{g''_1\}, \{L'\}\rangle$
$z \to \langle\{g''_1\}, \{L\}\rangle$

$S_3(h'_1)$: $x \to \langle\{h'_1\}, \{B', B''\}\rangle$
$y \to \langle\{h'_1\}, \{L'\}\rangle$
$z \to \langle\{h'_1\}, \{L\}\rangle$

$S_3(g_2)$: $x \to \langle\{g_2, h_2\}, \{B', B''\}\rangle$
$y \to \langle\{g_2, h_2\}, \{L'\}\rangle$
$z \to \langle\{g_2, h_2\}, \{B'\}\rangle$

$S_3(h_2)$: $x \to \langle\{g_2, h_2\}, \{B', B''\}\rangle$
$y \to \langle\{g_2, h_2\}, \{L'\}\rangle$
$z \to \langle\{g_2, h_2\}, \{B'\}\rangle$

- $\langle S_3, S_3\rangle \in [\![lawn(y); mow(x, y)]\!]^M$

iff $\forall h \in \text{Dom } S_3$: ${}^2(S_3(h)(y)) \in [\![lawn]\!]^M$ — D2.2.;, ***R***

& $\langle {}^2(S_3(h)(x)), {}^2(S_3(h)(y))\rangle \in [\![mow]\!]^M$ — simplify

✓ since Dom $S_3 = \{g''_1, h'_1, g_2, h_2\}$ and

$${}^2(S_3(g''_1)(y)) = \{L'\} \in [\![lawn]\!]^M$$
$$\& \; \langle {}^2(S_3(g''_1)(x)), {}^2(S_3(g''_1)(y))\rangle = \langle\{B', B''\}, \{L'\}\rangle \in [\![mow]\!]^M$$

$${}^2(S_3(h'_1)(y)) = \{L'\} \in [\![lawn]\!]^M$$
$$\& \; \langle {}^2(S_3(h'_1)(x)), {}^2(S_3(h'_1)(y))\rangle = \langle\{B', B''\}, \{L'\}\rangle \in [\![mow]\!]^M$$

$${}^2(S_3(g_2)(y)) = \{L'\} \in [\![lawn]\!]^M$$
$$\& \; \langle {}^2(S_3(g_2)(x)), {}^2(S_3(g_2)(y))\rangle = \langle\{B', B''\}, \{L'\}\rangle \in [\![mow]\!]^M$$

$${}^2(S_3(h_2)(y)) = \{L'\} \in [\![lawn]\!]^M$$
$$\& \; \langle {}^2(S_3(h_2)(x)), {}^2(S_3(h_2)(y))\rangle = \langle\{B', B''\}, \{L'\}\rangle \in [\![mow]\!]^M$$

- $\langle S_3, S_4\rangle \in [\![\text{DIS}\ x]\!]^M$

iff Dom S_4 = Dom S_3 D2.2.**D**[1]
& $\forall h \in$ Dom S_4: ${}^1(S_4(h)(x))$ = Dom S_4
& ${}^2(S_4(h)(x)) = h(x)$
& $\forall u \in (\mathbf{Var} - \{x\})$: $S_4(h)(u) = S_3(h)(u)$

$S_4 = \{\langle g''_1, S_4(g''_1)\rangle, \langle h'_1, S_4(h'_1)\rangle, \langle g_2, S_4(g_2)\rangle, \langle h_2, S_4(h_2)\rangle\}$

where
$g''_1 = \{\langle x, \{B'\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{L\}\rangle\}$
$h'_1 = \{\langle x, \{B''\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{L\}\rangle\}$
$g_2 = \{\langle x, \{B'\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{B'\}\rangle\}$
$h_2 = \{\langle x, \{B''\}\rangle, \langle y, \{L'\}\rangle, \langle z, \{B'\}\rangle\}$

$S_4(g''_1)$: $x \to \langle\{g''_1, h'_1, g_2, h_2\}, \{B'\}\rangle$
$y \to \langle\{g''_1\}, \{L'\}\rangle$
$z \to \langle\{g''_1\}, \{L\}\rangle$

$S_4(h'_1)$: $x \to \langle\{g''_1, h'_1, g_2, h_2\}, \{B''\}\rangle$
$y \to \langle\{h'_1\}, \{L'\}\rangle$
$z \to \langle\{h'_1\}, \{L\}\rangle$

$S_4(g_2)$: $x \to \langle\{g''_1, h'_1, g_2, h_2\}, \{B'\}\rangle$
$y \to \langle\{g_2, h_2\}, \{L'\}\rangle$
$z \to \langle\{g_2, h_2\}, \{B'\}\rangle$

$S_4(h_2)$: $x \to \langle\{g''_1, h'_1, g_2, h_2\}, \{B''\}\rangle$
$y \to \langle\{g_2, h_2\}, \{L'\}\rangle$
$z \to \langle\{g_2, h_2\}, \{B'\}\rangle$

- $\langle S_4, S_4\rangle \in [\![hgt(x)]\!]^M$ D2.2.***R***

iff $\forall h \in$ Dom S_4: ${}^2(S_4(h)(x)) \in [\![hgt]\!]^M$
iff $\forall h \in$ Dom S_4: $h(x) \in [\![hgt]\!]^M$

✓ since Dom $S_3 = \{g''_1, h'_1, g_2, h_2\}$ and

$g''_1(x) = \{B'\} \in [\![hgt]\!]^M$
$h'_1(x) = \{B''\} \in [\![hgt]\!]^M$
$g_2(x) = \{B'\} \in [\![hgt]\!]^M$
$h_2(x) = \{B''\} \in [\![hgt]\!]^M$

• Dependent sg antecedent. Dependent sg anaphor.

(3) [1]Two scientists (*each*) wrote *an* important paper.
[2]They presented *it* at two major conferences.

DPL$^+$ *syntax* ***with*** DIS and DEP

$\S$; $\exists x$; $\mathbf{2}x[sci(x)]$; DIS x; $\exists y$; $\mathbf{1}y[ppr(y)$; DEP xy; $wrt(x, y)$;
SG(y); $\exists z$; $\mathbf{2}z[cnf(z)]$; DEP yz; $prs(x, y, z)$ $:= K_3$

DPL$^+$-*semantics*

We show that K_3 is true in a DPL$^+$-model $M = \langle D^M, [\![\cdot]\!]^M\rangle$ s.t.

- $D^M = \{X | \varnothing \subset X \subset \{\text{A}, \text{A}', \text{B}, \text{B}', \text{C}, \text{C}', \text{D}, \text{D}'\}\}$
  $[\![sci]\!]^M = \{\{\text{A}\}, \{\text{A}'\}\}$
  $[\![ppr]\!]^M = \{\{\text{B}\}, \{\text{B}'\}\}$
  $[\![cnf]\!]^M = \{\{\text{C}\}, \{\text{C}'\}, \{\text{D}\}, \{\text{D}'\}\}$
  $[\![wrt]\!]^M = \{\langle\{\text{A}\}, \{\text{B}\}\rangle, \langle\{\text{A}'\}, \{\text{B}'\}\rangle\}$
  $[\![prs]\!]^M = \{\langle\{\text{A}\}, \{\text{B}\}, \{\text{C}\}\rangle, \langle\{\text{A}\}, \{\text{B}\}, \{\text{D}\}\rangle, \langle\{\text{A}'\}, \{\text{B}'\}, \{\text{C}'\}\rangle, \langle\{\text{A}'\}, \{\text{B}'\}, \{\text{D}'\}\rangle\}$

Initial state: Λ

- $\langle\Lambda, S_0\rangle \in [\![\S]\!]^M$

iff $\forall h \in \text{Dom } S_0 \forall u \in \textbf{Var}: {}^1(S_0(h)(u)) = \text{Dom } S_0$ D2.2.§
$\& \; {}^2(S_0(h)(u)) = h(u)$

e.g. $S_0 = \{\langle g_0, S_0(g_0)\rangle\}$

where $g_0 = \{\langle x, \{\text{C}\}\rangle, \langle y, \{\text{C}\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_0(g_0)$: $x \to \langle\{g_0\}, \{\text{C}\}\rangle$
$y \to \langle\{g_0\}, \{\text{C}\}\rangle$
$z \to \langle\{g_0\}, \{\text{C}\}\rangle$

- $\langle S_0, S_1\rangle \in [\![\exists x]\!]^M$

iff $\forall h \in \text{Dom } S_1 \exists g \in \text{Dom } S_0: g[x]h$ D2.2.∃,
& $\forall g \in \text{Dom } S_0 \exists h \in \text{Dom } S_1: g[x]h$ D2.1′.$S[u]S'$
& $\forall h \in \text{Dom } S_1: {}^2(S_1(h)(x)) = h(x)$
& $\forall u \in (\textbf{Var} - \{x\}) \exists g \in \text{Dom } S_0: g[x]h \;\&\; {}^2(S_1(h)(u)) = {}^2(S_0(g)(u))$
& $\forall u' \in \textbf{Var}: {}^1(S_1(h)(u')) = \{k \in \text{Dom } S_1 | \exists g: g \in {}^1(S_0(g)(u')) \;\&\; g[x]k \;\&\; g[x]h\}$

e.g. $S_1 = \{\langle g_1, S_1(g_1)\rangle, \langle h_1, S_1(h_1)\rangle\}$

where $g_1 := g_0[x/\{\text{A}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{C}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$h_1 := g_0[x/\{\text{A}'\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{C}\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_1(g_1)$: $x \to \langle\{g_1, h_1\}, \{\text{A}\}\rangle$
$y \to \langle\{g_1, h_1\}, \{\text{C}\}\rangle$
$z \to \langle\{g_1, h_1\}, \{\text{C}\}\rangle$

$S_1(h_1)$: $x \to \langle\{g_1, h_1\}, \{\text{A}'\}\rangle$
$y \to \langle\{g_1, h_1\}, \{\text{C}\}\rangle$
$z \to \langle\{g_1, h_1\}, \{\text{C}\}\rangle$

• $\langle S_1, S_1\rangle \in [\![\mathbf{2}x[sci(x)]]\!]^M$

iff $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) \in [\![sci]\!]^M$ D2.2.$\boldsymbol{n}$, $\boldsymbol{R}$

& $\forall h \in \text{Dom } S_1$: $|\{k(x): k \in {}^1(S_1(h)(x))\}| = 2$

✓ $S_1 = \{\langle g_1, S_1(g_1)\rangle, \langle h_1, S_1(h_1)\rangle\}$

since ${}^2(S_1(g_1)(x)) = \{\text{A}\} \in [\![sci]\!]^M$

& $|\{k(x): k \in {}^1(S_1(g_1)(x))\}| = |\{g_1(x), h_1(x)\}| = |\{\{\text{A}\}, \{\text{A}'\}\}| = 2$

${}^2(S_1(h_1)(x)) = \{\text{A}'\} \in [\![sci]\!]^M$

& $|\{k(x): k \in {}^1(S_1(h_1)(x))\}| = |\{g_1(x), h_1(x)\}| = |\{\{\text{A}\}, \{\text{A}'\}\}| = 2$

• $\langle S_1, S_2\rangle \in [\![\text{DIS } x]\!]^M$

iff Dom S_2 = Dom S_1 D2.2.D[1]

& $\forall h \in \text{Dom } S_2$: ${}^1(S_2(h)(x)) = \text{Dom } S_2$

& ${}^2(S_2(h)(x)) = h(x)$

& $\forall u \in (\mathbf{Var} - \{x\})$: $S_2(h)(u) = S_1(h)(u)$

iff $S_2 = S_1$ df. S_1

• $\langle S_2, S_3\rangle \in [\![\exists y]\!]^M$

iff $\forall h \in \text{Dom } S_3 \exists g \in \text{Dom } S_2$: $g[y]h$ D2.2.∃,

& $\forall g \in \text{Dom } S_2 \exists h \in \text{Dom } S_3$: $g[y]h$ D2.1′.$S[u]S'$

& $\forall h \in \text{Dom } S_3$: ${}^2(S_3(h)(y)) = h(y)$

& $\forall u \in (\mathbf{Var} - \{y\}) \exists g \in \text{Dom } S_2$: $g[y]h$ & ${}^2(S_3(h)(u)) = {}^2(S_2(g)(u))$

& $\forall u' \in \mathbf{Var}$: ${}^1(S_3(h)(u')) = \{k \in \text{Dom } S_3 | \exists g$: $g \in {}^1(S_2(g)(u'))$ & $g[y]k$ & $g[y]h\}$

e.g. $S_3 = \{\langle g_3, S_3(g_3)\rangle, \langle h_3, S_3(h_3)\rangle\}$

where $g_3 := g_1[y/\{\text{B}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$h_3 := h_1[y/\{\text{B}'\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_3(g_3)$:	$x \to$	$\langle\{g_3\}, \{\text{A}\}\rangle$	$S_3(h_3)$:	$x \to$	$\langle\{h_3\}, \{\text{A}'\}\rangle$
	$y \to$	$\langle\{g_3\}, \{\text{B}\}\rangle$		$y \to$	$\langle\{h_3\}, \{\text{B}'\}\rangle$
	$z \to$	$\langle\{g_3\}, \{\text{C}\}\rangle$		$z \to$	$\langle\{h_3\}, \{\text{C}\}\rangle$

• $\langle S_3, S_3\rangle \in [\![\mathbf{1}y[ppr(y)]]\!]^M$

iff $\forall h \in \text{Dom } S_3$: ${}^2(S_3(h)(y)) \in [\![ppr]\!]^M$ D2.2.$\boldsymbol{n}$, $\boldsymbol{R}$

& $\forall h \in \text{Dom } S_3$: $|\{k(y): k \in {}^1(S_3(h)(y))\}| = 1$

✓ since ${}^2(S_3(g_3)(y)) = \{\text{B}\} \in [\![ppr]\!]^M$

& $|\{k(y): k \in {}^1(S_3(g_3)(y))\}| = |\{\{\text{B}\}\}| = 1$

${}^2(S_3(h_3)(y)) = \{\text{B}'\} \in [\![ppr]\!]^M$

& $|\{k(y): k \in {}^1(S_2(h_2)(y))\}| = |\{\{\text{B}'\}\}| = 1$

- $\langle S_3, S_4\rangle \in [\![\text{DEP } xy]\!]^M$

iff $\quad \text{Dom } S_4 = \text{Dom } S_3$ D2.2.$\mathbf{D}^1$

$\quad\&\quad \forall h \in \text{Dom } S_4{:}\ {}^1(S_4(h)(y)) = \{g \in {}^1(S_3(h)(y)) \mid g(x) = h(x)\}$

$\qquad\qquad \&\quad {}^2(S_4(h)(y)) = {}^2(S_3(h)(y))$

$\qquad\qquad \&\quad \forall u \in (\mathbf{Var} - \{y\}){:}\ S_4(h)(u) = S_3(h)(u)$

iff $S_4 = S_3$

i.e. $\quad S_4 = \{\langle g_3, S_4(g_3)\rangle, \langle h_3, S_4(h_3)\rangle\}$ $= S_3$

where $\quad g_3 = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$\qquad\quad h_3 = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_4(g_3){:}\quad x \rightarrow \langle\{g_3\}, \{\text{A}\}\rangle$ $\qquad S_4(h_3){:}\quad x \rightarrow \langle\{h_3\}, \{\text{A}'\}\rangle$

$\qquad\qquad y \rightarrow \langle\{g_3\}, \{\text{B}\}\rangle$ $\qquad\qquad\qquad y \rightarrow \langle\{h_3\}, \{\text{B}'\}\rangle$

$\qquad\qquad z \rightarrow \langle\{g_3\}, \{\text{C}\}\rangle$ $\qquad\qquad\qquad z \rightarrow \langle\{h_3\}, \{\text{C}\}\rangle$

- $\langle S_4, S_4\rangle \in [\![wrt(x, y)]\!]^M$

iff $\forall h \in \text{Dom } S_4{:}\ \langle {}^2(S_4(h)(x)), {}^2(S_4(h)(y))\rangle \in [\![wrt]\!]^M$ D2.2.$\boldsymbol{R}$

✓since: $\quad \langle {}^2(S_4(g_3)(x)), {}^2(S_4(g_3)(y))\rangle = \langle\{\text{A}\}, \{\text{B}\}\rangle \in [\![wrt]\!]^M$

$\qquad\qquad \langle {}^2(S_4(h_3)(x)), {}^2(S_4(h_3)(y))\rangle = \langle\{\text{A}'\}, \{\text{B}'\}\rangle \in [\![wrt]\!]^M$

- $\langle S_4, S_4\rangle \in [\![\text{SG}(y)]\!]^M$

iff $\forall h \in \text{Dom } S_4{:}\ |\{k(y){:}\ k \in {}^1(S_4(h)(y))\}| = 1$ D2.2.$\boldsymbol{R}'$

✓since $\quad |\{k(y){:}\ k \in {}^1(S_4(g_3)(y))\}| = |\{\{\text{B}\}\}| = 1$

$\qquad\qquad |\{k(y){:}\ k \in {}^1(S_4(h_3)(y))\}| = |\{\{\text{B}'\}\}| = 1$

- $\langle S_4, S_5\rangle \in [\![\exists z]\!]^M$

iff $\quad \forall h \in \text{Dom } S_5 \exists g \in \text{Dom } S_4{:}\ g[z]h$ D2.2.$\exists$,

$\quad\&\quad \forall g \in \text{Dom } S_4 \exists h \in \text{Dom } S_5{:}\ g[z]h$ D2.1′.$S[u]S'$

$\quad\&\quad \forall h \in \text{Dom } S_5{:}\ {}^2(S_5(h)(z)) = h(z)$

$\qquad\qquad \&\quad \forall u \in (\mathbf{Var} - \{z\}) \exists g \in \text{Dom } S_4{:}\ g[z]h\ \&\ {}^2(S_5(h)(u)) = {}^2(S_4(g)(u))$

$\qquad\qquad \&\quad \forall u' \in \mathbf{Var}{:}\ {}^1(S_5(h)(u')) = \{k \in \text{Dom } S_5 \mid \exists g{:}\ g \in {}^1(S_4(g)(u'))\ \&\ g[z]k\ \&\ g[z]h\}$

e.g. $S_5 = \{\langle g_5, S_5(g_5)\rangle, \langle g'_5, S_5(g'_5)\rangle, \langle h_5, S_5(h_5)\rangle, \langle h'_5, S_5(h'_5)\rangle\}$

where $\quad g_5 := g_3[z/\{\text{C}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$\qquad\quad g'_5 := g_3[z/\{\text{D}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{D}\}\rangle\}$

$\qquad\quad h_5 := h_3[z/\{\text{C}'\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}'\}\rangle\}$

$\qquad\quad h'_5 := h_3[z/\{\text{D}'\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{D}'\}\rangle\}$

$S_5(g_5){:}\quad x \rightarrow \langle\{g_5, g'_5\}, \{\text{A}\}\rangle$ $\qquad S_5(g'_5){:}\quad x \rightarrow \langle\{g_5, g'_5\}, \{\text{A}\}\rangle$

$\qquad\qquad y \rightarrow \langle\{g_5, g'_5\}, \{\text{B}\}\rangle$ $\qquad\qquad\qquad y \rightarrow \langle\{g_5, g'_5\}, \{\text{B}\}\rangle$

$\qquad\qquad z \rightarrow \langle\{g_5, g'_5\}, \{\text{C}\}\rangle$ $\qquad\qquad\qquad z \rightarrow \langle\{g_5, g'_5\}, \{\text{D}\}\rangle$

$S_5(h_5){:}\quad x \rightarrow \langle\{h_5, h'_5\}, \{\text{A}'\}\rangle$ $\qquad S_5(h'_5){:}\quad x \rightarrow \langle\{h_5, h'_5\}, \{\text{A}'\}\rangle$

$\qquad\qquad y \rightarrow \langle\{h_5, h'_5\}, \{\text{B}'\}\rangle$ $\qquad\qquad\qquad y \rightarrow \langle\{h_5, h'_5\}, \{\text{B}'\}\rangle$

$\qquad\qquad z \rightarrow \langle\{h_5, h'_5\}, \{\text{C}'\}\rangle$ $\qquad\qquad\qquad z \rightarrow \langle\{h_5, h'_5\}, \{\text{D}'\}\rangle$

- $\langle S_5, S_5\rangle \in [\![\mathbf{2}z[cnf(z)]]\!]^M$

iff $\forall h \in \text{Dom } S_5$: ${}^2(S_5(h)(z)) \in [\![cnf]\!]^M$ D2.2.***n***, ***R***

$\quad\quad \& \ |\{k(z)\text{: } k \in {}^1(S_5(h)(z))\}| = 2$ simplify

✓ since ${}^2(S_5(g_5)(z)) = \{\text{C}\} \in [\![cnf]\!]^M$

$\quad \& \ |\{k(z)\text{: } k \in {}^1(S_5(g_5)(z))\}| = |\{\{\text{C}\}, \{\text{D}\}\}| = 2$

${}^2(S_5(g'_5)(z)) = \{\text{D}\} \in [\![cnf]\!]^M$

$\quad \& \ |\{k(z)\text{: } k \in {}^1(S_5(g'_5)(z))\}| = |\{\{\text{C}\}, \{\text{D}\}\}| = 2$

${}^2(S_5(h_5)(z)) = \{\text{C}'\} \in [\![cnf]\!]^M$

$\quad \& \ |\{k(z)\text{: } k \in {}^1(S_5(h'_5)(z))\}| = |\{\{\text{C}'\}, \{\text{D}'\}\}| = 2$

${}^2(S_5(h'_5)(z)) = \{\text{D}'\} \in [\![cnf]\!]^M$

$\quad \& \ |\{k(z)\text{: } k \in {}^1(S_5(h'_5)(z))\}| = |\{\{\text{C}'\}, \{\text{D}'\}\}| = 2$

- $\langle S_5, S_6\rangle \in [\![\text{DEP } yz]\!]^M$

iff $\text{Dom } S_6 = \text{Dom } S_5$

$\& \ \forall h \in \text{Dom } S_6$: ${}^1(S_6(h)(z)) = \{g \in {}^1(S_5(h)(z)) |\ g(y) = h(y)\}$ D2.2.D^2

$\quad\quad \& \ {}^2(S_6(h)(z)) = {}^2(S_5(h)(z))$

$\quad\quad \& \ \forall u \in (\mathbf{Var} - \{z\})\text{: } S_6(h)(u) = S_5(h)(u)$

i.e. $S_6 = S_5 = \{\langle g_5, S_6(g_5)\rangle, \langle g'_5, S_6(g'_5)\rangle, \langle h_5, S_6(h_5)\rangle, \langle h'_5, S_6(h'_5)\rangle\}$

where
$g_5 = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$g'_5 = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{D}\}\rangle\}$
$h_5 = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}'\}\rangle\}$
$h'_5 = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{D}'\}\rangle\}$

$S_6(g_5)$: $x \rightarrow \langle\{g_5, g'_5\}, \{\text{A}\}\rangle$; $y \rightarrow \langle\{g_5, g'_5\}, \{\text{B}\}\rangle$; $z \rightarrow \langle\{g_5, g'_5\}, \{\text{C}\}\rangle$

$S_6(g'_5)$: $x \rightarrow \langle\{g_5, g'_5\}, \{\text{A}\}\rangle$; $y \rightarrow \langle\{g_5, g'_5\}, \{\text{B}\}\rangle$; $z \rightarrow \langle\{g_5, g'_5\}, \{\text{D}\}\rangle$

$S_6(h_5)$: $x \rightarrow \langle\{h_5, h'_5\}, \{\text{A}'\}\rangle$; $y \rightarrow \langle\{h_5, h'_5\}, \{\text{B}'\}\rangle$; $z \rightarrow \langle\{h_5, h'_5\}, \{\text{C}'\}\rangle$

$S_6(h'_5)$: $x \rightarrow \langle\{h_5, h'_5\}, \{\text{A}'\}\rangle$; $y \rightarrow \langle\{h_5, h'_5\}, \{\text{B}'\}\rangle$; $z \rightarrow \langle\{h_5, h'_5\}, \{\text{D}'\}\rangle$

- $\langle S_6, S_6\rangle \in [\![prs(x, y, z)]\!]^M$

iff $\forall h \in \text{Dom } S_6$: $\langle {}^2(S_6(h)(x)), {}^2(S_6(h)(y))\ {}^2(S_6(h)(z))\rangle \in [\![prs]\!]^M$ D2.2.***R***

✓ since: $\langle {}^2(S_6(g_5)(x)), {}^2(S_6(g_5)(y))\ {}^2(S_6(g_5)(z))\rangle = \langle\{\text{A}\}, \{\text{B}\}, \{\text{C}\}\rangle \in [\![prs]\!]^M$

$\langle {}^2(S_6(g'_5)(x)), {}^2(S_6(g'_5)(y))\ {}^2(S_6(g'_5)(z))\rangle = \langle\{\text{A}\}, \{\text{B}\}, \{\text{D}\}\rangle \in [\![prs]\!]^M$

$\langle {}^2(S_6(h_5)(x)), {}^2(S_6(h_5)(y))\ {}^2(S_6(h_5)(z))\rangle = \langle\{\text{A}'\}, \{\text{B}'\}, \{\text{C}'\}\rangle \in [\![prs]\!]^M$

$\langle {}^2(S_6(h'_5)(x)), {}^2(S_6(h'_5)(y))\ {}^2(S_6(h'_5)(z))\rangle = \langle\{\text{A}'\}, \{\text{B}'\}, \{\text{D}'\}\rangle \in [\![prs]\!]^M$

DPL^+ *syntax* ***without*** DIS and DEP

§; $\exists x$; $\mathbf{2}x[sci(x)]$; $\exists y$; $\mathbf{1}y[ppr(y)$; $wrt(x, y)$;

SG(y); $\exists z$; $\mathbf{2}z[cnf(z)]$; $prs(x, y, z)$ $:= K'_3$

EXERCISE: Show that K'_3 is likewise true in the above model. The same info states will do.

• Cumulative reading

(6′) Two scientists wrote (*a total of*) three papers (*between them*).

DPL⁺ *syntax*

$\S$; $\exists x$; $\mathbf{2}x[sci(x)]$; $\text{CUM } x$; $\exists y$; $\mathbf{3}y[ppr(y)$; $\text{CUM } y$; $wrt(x, y)$ $:= K_{6'}$

DPL⁺-*semantics*

Consider a DPL⁺-model $M = \langle D^M, [\![\cdot]\!]^M \rangle$ s.t.

- $D^M = \{X | \varnothing \subset X \subset \{\text{A, A}', \text{B, B}', \text{B}'', \text{C, C}', \text{C}'', \text{D}\}\}$
  $[\![sci]\!]^M = \{\{\text{A}\}, \{\text{A}'\}\}$
  $[\![ppr]\!]^M = \{\{\text{B}\}, \{\text{B}'\}, \{\text{B}''\}\}$
  $[\![cnf]\!]^M = \{\{\text{C}\}, \{\text{C}'\}, \{\text{C}''\}, \{\text{D}\}\}$
  $[\![wrt]\!]^M = \{\langle\{\text{A}\}, \{\text{B}\}\rangle, \langle\{\text{A}'\}, \{\text{B}'\}\rangle, \langle\{\text{A}'\}, \{\text{B}''\}\rangle\}$
  $[\![prs]\!]^M = \{\langle\{\text{A}\}, \{\text{B}\}, \{\text{C}\}\rangle, \langle\{\text{A}\}, \{\text{B}\}, \{\text{D}\}\rangle,$
  $\langle\{\text{A}'\}, \{\text{B}'\}, \{\text{C}'\}\rangle, \langle\{\text{A}'\}, \{\text{B}''\}, \{\text{C}''\}\rangle\}$

Intuitively (6′) is true in this model. But there is no way to verify $K_{6'}$.

Initial state: Λ

- $\langle \Lambda, S_0 \rangle \in [\![\S]\!]^M$

iff $\forall h \in \text{Dom } S_0 \forall u \in \mathbf{Var}$: $^1(S_0(h)(u)) = \text{Dom } S_0$ D2.2.§
$\& \ ^2(S_0(h)(u)) = h(u)$

e.g. $S_0 = \{\langle g_0, S_0(g_0)\rangle\}$

where $g_0 = \{\langle x, \{\text{C}\}\rangle, \langle y, \{\text{C}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$S_0(g_1)$: $x \to \langle\{g_1\}, \{\text{C}\}\rangle$
$y \to \langle\{g_1\}, \{\text{C}\}\rangle$
$z \to \langle\{g_1\}, \{\text{C}\}\rangle$

- $\langle S_0, S_1\rangle \in [\![\exists x]\!]^M$

iff $\forall h \in \text{Dom } S_1 \exists g \in \text{Dom } S_0$: $g[x]h$ D2.2.∃,
& $\forall g \in \text{Dom } S_0 \exists h \in \text{Dom } S_1$: $g[x]h$ D2.1′.$S[u]S'$
& $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) = h(x)$
& $\forall u \in (\mathbf{Var} - \{x\}) \exists g \in \text{Dom } S_0$: $g[x]h$ & ${}^2(S_1(h)(u)) = {}^2(S_0(g)(u))$
& $\forall u' \in \mathbf{Var}$: ${}^1(S_1(h)(u')) = \{k \in \text{Dom } S_1 | \exists g: g \in {}^1(S_0(g)(x))$ & $g[x]k$ & $g[x]h\}$

e.g. $S_1 = \{\langle g_1, S_1(g_1)\rangle, \langle h_1, S_1(h_1)\rangle\}$

where $g_1 := g_0[x/\{A\}] = \{\langle x, \{A\}\rangle, \langle y, \{C\}\rangle, \langle z, \{C\}\rangle\}$
$h_1 := g_0[x/\{A'\}] = \{\langle x, \{A'\}\rangle, \langle y, \{C\}\rangle, \langle z, \{C\}\rangle\}$

$S_1(g_1)$: $x \rightarrow \langle\{g_1, h_1\}, \{A\}\rangle$ $S_1(h_1)$: $x \rightarrow \langle\{g_1, h_1\}, \{A'\}\rangle$
$y \rightarrow \langle\{g_1, h_1\}, \{C\}\rangle$ $y \rightarrow \langle\{g_1, h_1\}, \{C\}\rangle$
$z \rightarrow \langle\{g_1, h_1\}, \{C\}\rangle$ $z \rightarrow \langle\{g_1, h_1\}, \{C\}\rangle$

- $\langle S_1, S_1\rangle \in [\![\mathbf{2}x[sci(x)]]\!]^M$

iff $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(x)) \in [\![sci]\!]^M$ D2.2.***n***, ***R***
& $\forall h \in \text{Dom } S_1$: $|\{k(x): k \in {}^1(S_1(h)(x))\}| = 2$

✓ since $g_1(x) = \{A\} \in [\![sci]\!]^M$
& $|\{k(x): k \in {}^1(S_1(g_1)(x))\}| = |\{\{A\}, \{A'\}\}| = 2$

& $h_1(x) = \{A'\} \in [\![sci]\!]^M$
& $|\{k(x): k \in {}^1(S_1(h_1)(x))\}| = |\{\{A\}, \{A'\}\}| = 2$

- $\langle S_1, S_2\rangle \in [\![\text{CUM } x]\!]^M$

iff $\text{Dom } S_2 = \text{Dom } S_1$ D2.2.$\mathbf{D}^1$
& $\forall h \in \text{Dom } S_2$: ${}^1(S_2(h)(x)) = {}^1(S_1(h)(x))$
& ${}^2(S_2(h)(x)) \subseteq \cup\{g(x): g \in {}^1(S_1(h)(x))\}$
& $\cup\{k(x): k \in {}^1(S_2(h)(x))\} = \cup\{{}^2(S_2(k)(x)): k \in {}^1(S_2(h)(x))\}$
& $\forall u \in (\mathbf{Var} - \{x\})$: $S_2(h)(u) = S_1(h)(u)$
iff $S_2 = S_1$ df. S_1

- $\langle S_2, S_3\rangle \in [\![\exists y]\!]^M$

iff $\forall h \in \text{Dom } S_3 \exists g \in \text{Dom } S_2\text{: } g[y]h$ D2.2.$\exists$,

& $\forall g \in \text{Dom } S_2 \exists h \in \text{Dom } S_3\text{: } g[y]h$ D2.1$'$.$S[u]S'$

& $\forall h \in \text{Dom } S_3\text{: } {}^2(S_3(h)(y)) = h(y)$

& $\forall u \in (\mathbf{Var} - \{y\})\exists g \in \text{Dom } S_2\text{: } g[y]h \ \&\ {}^2(S_3(h)(u)) = {}^2(S_2(g)(u))$

& $\forall u' \in \mathbf{Var}\text{: } {}^1(S_3(h)(u')) = \{k \in \text{Dom } S_3 \mid \exists g\text{: } g \in {}^1(S_2(g)(y)) \ \&\ g[y]k \ \&\ g[y]h\}$

e.g. $S_3 = \{\langle g_{\text{B}}, S_3(g_{\text{B}})\rangle, \langle g_{\text{B}'}, S_3(g_{\text{B}'})\rangle, \langle g_{\text{B}''}, S_3(g_{\text{B}''})\rangle,$
$\langle h_{\text{B}}, S_3(h_{\text{B}})\rangle, \langle h_{\text{B}'}, S_3(h_{\text{B}'})\rangle, \langle h_{\text{B}''}, S_3(h_{\text{B}''})\rangle,$

where

$g_{\text{B}} := g_1[y/\{\text{B}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$g_{\text{B}'} := g_1[y/\{\text{B}'\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$g_{\text{B}''} := g_1[y/\{\text{B}''\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}''\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$h_{\text{B}} := h_1[y/\{\text{B}\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$h_{\text{B}'} := h_1[y/\{\text{B}'\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$h_{\text{B}''} := h_1[y/\{\text{B}''\}] = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}''\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_3(g_{\text{B}})$: $x \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}\}\rangle$
$z \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_3(g_{\text{B}'})$: $x \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}'\}\rangle$
$z \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_3(h_{\text{B}})$: $x \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{A}'\}\rangle$
$y \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{B}\}\rangle$
$z \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_3(h_{\text{B}'})$: $x \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{A}'\}\rangle$
$y \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{B}'\}\rangle$
$z \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_3(g_{\text{B}''})$: $x \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}''\}\rangle$
$z \to \langle\{g_{\text{B}}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_3(h_{\text{B}''})$: $x \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}'''}\}, \{\text{A}'\}\rangle$
$y \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}'''}\}, \{\text{B}''\}\rangle$
$z \to \langle\{h_{\text{B}}, h_{\text{B}'}, h_{\text{B}'''}\}, \{\text{C}\}\rangle$

- $\langle S_3, S_3\rangle \in [\![\mathbf{3}y[ppr(y)]]\!]^M$

& $\forall h \in \text{Dom } S_3\text{: } {}^2(S_3(h)(y)) \in [\![ppr]\!]^M$ D2.2.$\boldsymbol{n}$, $\boldsymbol{R}$

& $\forall h \in \text{Dom } S_3\text{: } |\{k(y)\text{: } k \in {}^1(S_3(h)(y))\}| = 3$

✓ since $g_{\text{B}}(y) = \{\text{B}\} \in [\![ppr]\!]^M$

& $|\{k(y)\text{: } k \in {}^1(S_3(g_{\text{B}})(y))\}| = |\{\{\text{B}\}, \{\text{B}'\}, \{\text{B}''\}\}| = 3$

⋮

& $h_{\text{B}''}(y) = \{\text{B}''\} \in [\![ppr]\!]^M$

& $|\{k(y)\text{: } k \in {}^1(S_3(g_{\text{B}''})(y))\}| = |\{\{\text{B}\}, \{\text{B}'\}, \{\text{B}''\}\}| = 3$

- $\langle S_3, S_4\rangle \in [\![\text{CUM } y]\!]^M$

iff Dom S_4 = Dom S_3 D2.2.**D**[1]

& $\forall h \in \text{Dom } S_4$: ${}^1(S_4(h)(y)) = {}^1(S_3(h)(y))$

& ${}^2(S_4(h)(y)) \subseteq \cup\{g(y): g \in {}^1(S_3(h)(y))\}$

& $\cup\{{}^2(S_4(k)(y)): k \in {}^1(S_4(h)(y))\} = \cup\{k(y): k \in {}^1(S_4(h)(y))\}$

& $\forall u \in (\textbf{Var} - \{y\})$: $S_4(h)(u) = S_3(h)(u)$

iff Dom S_4 = Dom S_3

& $\forall h \in \text{Dom } S_4$: ${}^1(S_4(h)(y)) = {}^1(S_3(h)(y))$

& ${}^2(S_4(h)(y)) \subseteq \cup\{g(y): g \in {}^1(S_3(h)(y))\}$

& $\cup\{{}^2(S_4(k)(y)): k \in {}^1(S_4(h)(y))\} = \{\text{B}, \text{B}', \text{B}''\}$ df. S_3

& $\forall u \in (\textbf{Var} - \{y\})$: $S_4(h)(u) = S_3(h)(u)$

- $\langle S_4, S_4\rangle \in [\![wrt(x, y)]\!]^M$

iff $\forall h \in \text{Dom } S_4$: $\langle {}^2(S_4(h)(x)), {}^2(S_4(h)(y))\rangle \in [\![wrt]\!]^M$ D2.2.***R***

Problem:

The semantic rule for 'CUM y' doesn't work correctly. In particular, requirement (a) constrains the wrong sum. Given our model, any state S_4 that satisfies (a) will fail test (b), and vice versa:

(a) $\forall h \in \text{Dom } S_4$: $\cup\{{}^2(S_4(k)(y)): k \in {}^1(S_4(h)(y))\} = \{\text{B}, \text{B}', \text{B}''\}$

(b) $\forall h \in \text{Dom } S_4$: $\langle {}^2(S_4(h)(x)), {}^2(S_4(h)(y))\rangle \in [\![wrt]\!]^M$

For instance, S_4 below will pass (b) in our model but fail (a):

$S_4 = \{\langle g_\text{B}, S_4(g_\text{B})\rangle, \langle g_{\text{B}'}, S_4(g_{\text{B}'})\rangle, \langle g_{\text{B}''}, S_4(g_{\text{B}''})\rangle,$
$\langle h_\text{B}, S_4(h_\text{B})\rangle, \langle h_{\text{B}'}, S_4(h_{\text{B}'})\rangle, \langle h_{\text{B}''}, S_4(h_{\text{B}''})\rangle,$

where
$g_\text{B} = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$g_{\text{B}'} = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$g_{\text{B}''} = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{B}''\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$h_\text{B} = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$h_{\text{B}'} = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}'\}\rangle, \langle z, \{\text{C}\}\rangle\}$
$h_{\text{B}''} = \{\langle x, \{\text{A}'\}\rangle, \langle y, \{\text{B}''\}\rangle, \langle z, \{\text{C}\}\rangle\}$

$S_4(g_\text{B})$: $x \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$, $y \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}\}\rangle$, $z \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_4(g_{\text{B}'})$: $x \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$, $y \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}\}\rangle$, $z \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_4(g_{\text{B}''})$: $x \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{A}\}\rangle$, $y \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{B}\}\rangle$, $z \to \langle\{g_\text{B}, g_{\text{B}'}, g_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_4(h_{\text{B}''})$: $x \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{A}'\}\rangle$, $y \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{B}''\}\rangle$, $z \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_4(h_\text{B})$: $x \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{A}'\}\rangle$, $y \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{B}'\}\rangle$, $z \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{C}\}\rangle$

$S_4(h_{\text{B}'})$: $x \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}'''}\}, \{\text{A}'\}\rangle$, $y \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{B}'\}\rangle$, $z \to \langle\{h_\text{B}, h_{\text{B}'}, h_{\text{B}''}\}, \{\text{C}\}\rangle$

Intuitively, cumulativity requires a two-place operator CUM uu' (cf. A&W's dependency operator DEP uu'). E.g. to get the right sum in (6′), we need to sum up the y-values over different x-values. I don't see how a one-place operator, like A&W's CUM u, can do this kind of summation.

• Inverse scope

(9) Two students answered three questions.

DPL$^+$ *syntax* for inverse scope rdg:

$\S$; $\exists y$; $\mathbf{3}y[que(y)]$; $\exists x$; $\mathbf{2}x[std(x)]$; $ans(x, y)$ $:= K_9$

DPL$^+$-*semantics*

Consider a DPL$^+$-model $M = \langle D^M, [\![\cdot]\!]^M\rangle$ s.t.

- $D^M = \{X |\ \varnothing \subset X \subset \{\text{A, A}', \text{B, B}', \text{Q, Q}', \text{Q}''\}\}$
  $[\![std]\!]^M = \{\{\text{A}\}, \{\text{A}'\}, \{\text{B}\}, \{\text{B}'\}\}$
  $[\![que]\!]^M = \{\{\text{Q}\}, \{\text{Q}'\}, \{\text{Q}''\}\}$
  $[\![ans]\!]^M = \{\langle\{\text{A}\}, \{\text{Q}\}\rangle, \langle\{\text{A}'\}, \{\text{Q}\}\rangle, \langle\{\text{A}\}, \{\text{Q}'\}\rangle, \langle\{\text{B}\}, \{\text{Q}'\}\rangle, \langle\{\text{B}\}, \{\text{Q}''\}\rangle, \langle\{\text{B}'\}, \{\text{Q}''\}\rangle\}$

Intuitively (9) is true in this model on the inverse scope reading. Let's see if $K_{9'}$ comes out true:

Initial state: Λ

- $\langle\Lambda, S_0\rangle \in [\![\S]\!]^M$

iff $\forall h \in \text{Dom } S_0 \forall u \in \mathbf{Var}$: ${}^1(S_0(h)(u)) = \text{Dom } S_0$ D2.2.§
$\quad\&\ {}^2(S_0(h)(u)) = h(u)$

e.g. $S_0 = \{\langle g_0,\ S_0(g_0)\rangle\}$

where $g_0 = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{A}\}\rangle, \langle z, \{\text{A}\}\rangle\}$

$S_0(g_0)$: $x \to \langle\{g_0\}, \{\text{A}\}\rangle$
$y \to \langle\{g_0\}, \{\text{A}\}\rangle$
$z \to \langle\{g_0\}, \{\text{A}\}\rangle$

- $\langle S_0, S_1\rangle \in [\![\exists y]\!]^M$

iff $\forall h \in \text{Dom } S_1 \exists g \in \text{Dom } S_0$: $g[y]h$ D2.2.$\exists$,
& $\forall g \in \text{Dom } S_0 \exists h \in \text{Dom } S_1$: $g[y]h$ D2.1$'$.$S[u]S'$
& $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(y)) = h(y)$
& $\forall u \in (\mathbf{Var} - \{y\}) \exists g \in \text{Dom } S_0$: $g[y]h$ & ${}^2(S_1(h)(u)) = {}^2(S_0(g)(u))$
& $\forall u' \in \mathbf{Var}$: ${}^1(S_1(h)(u')) = \{k \in \text{Dom } S_1 |\ \exists g$: $g \in {}^1(S_0(g)(y))$ & $g[y]k$ & $g[y]h\}$

e.g. $S_1 = \{\langle g_{\text{Q}}, S_1(g_{\text{Q}})\rangle, \langle g_{\text{Q}'}, S_1(g_{\text{Q}'})\rangle, \langle g_{\text{Q}''}, S_1(g_{\text{Q}''})\rangle,$

where $g_{\text{Q}} := g_0[y/\{\text{Q}\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{Q}\}\rangle, \langle z, \{\text{A}\}\rangle\}$
$g_{\text{Q}'} := g_0[y/\{\text{Q}'\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{Q}'\}\rangle, \langle z, \{\text{A}\}\rangle\}$
$g_{\text{Q}''} := g_0[y/\{\text{B}''\}] = \{\langle x, \{\text{A}\}\rangle, \langle y, \{\text{Q}''\}\rangle, \langle z, \{\text{A}\}\rangle\}$

$S_1(g_{\text{Q}})$: $x \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{Q}\}\rangle$
$z \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$

$S_1(g_{\text{Q}'})$: $x \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{Q}'\}\rangle$
$z \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$

$S_1(g_{\text{Q}''})$: $x \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$
$y \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{Q}''\}\rangle$
$z \to \langle\{g_{\text{Q}}, g_{\text{Q}'}, g_{\text{Q}''}\}, \{\text{A}\}\rangle$

- $\langle S_1, S_1\rangle \in [\![\mathbf{3}y[que(y)]]\!]^M$

iff $\forall h \in \text{Dom } S_1$: ${}^2(S_1(h)(y)) \in [\![que]\!]^M$ D2.2.***n***, ***R***
& $\forall h \in \text{Dom } S_1$: $|\{k(y): k \in {}^1(S_1(h)(y))\}| = 3$

✓ since ${}^2(S_1(g_Q)(y)) = \{Q\} \in [\![que]\!]^M$
& $|\{k(y): k \in {}^1(S_1(g_Q)(y))\}| = |\{\{Q\}, \{Q'\}, \{Q''\}\}| = 3$
⋮
& ${}^2(S_1(g_{Q''})(y)) = \{Q''\} \in [\![que]\!]^M$
& $|\{k(y): k \in {}^1(S_1(g_{Q''})(y))\}| = |\{\{Q\}, \{Q'\}, \{Q''\}\}| = 3$

- $\langle S_1, S_2\rangle \in [\![\exists x]\!]^M$

iff $\forall h \in \text{Dom } S_2 \exists g \in \text{Dom } S_1$: $g[x]h$ D2.2.∃,
& $\forall g \in \text{Dom } S_1 \exists h \in \text{Dom } S_2$: $g[x]h$ D2.1′.$S[u]S'$
& $\forall h \in \text{Dom } S_2$: ${}^2(S_2(h)(x)) = h(x)$
& $\forall u \in (\mathbf{Var} - \{x\}) \exists g \in \text{Dom } S_1$: $g[x]h$ & ${}^2(S_2(h)(u)) = {}^2(S_1(g)(u))$
& $\forall u' \in \mathbf{Var}$: ${}^1(S_2(h)(u')) = \{k \in \text{Dom } S_2 |\ \exists g: g \in {}^1(S_1(g)(x))$ & $g[x]k$ & $g[x]h\}$

e.g. $S_2 = \{\langle g_A, S_2(g_A)\rangle, \langle h_A, S_2(h_A)\rangle, \langle k_B, S_2(k_B)\rangle,$
$\langle g_{A'}, S_2(g_{A'})\rangle, \langle h_B, S_2(h_B)\rangle, \langle k_{B'}, S_2(k_{B'})\rangle,$

where
$g_A := g_Q[x/\{A\}] = \{\langle x, \{A\}\rangle, \langle y, \{Q\}\rangle, \langle z, \{A\}\rangle\}$
$g_{A'} := g_Q[x/\{A'\}] = \{\langle x, \{A'\}\rangle, \langle y, \{Q\}\rangle, \langle z, \{A\}\rangle\}$
$h_A := g_{Q'}[x/\{A\}] = \{\langle x, \{A\}\rangle, \langle y, \{Q'\}\rangle, \langle z, \{A\}\rangle\}$
$h_B := g_{Q'}[x/\{B\}] = \{\langle x, \{B\}\rangle, \langle y, \{Q'\}\rangle, \langle z, \{A\}\rangle\}$
$k_B := g_{Q''}[x/\{B\}] = \{\langle x, \{B\}\rangle, \langle y, \{Q''\}\rangle, \langle z, \{A\}\rangle\}$
$k_{B'} := g_{Q''}[x/\{B'\}] = \{\langle x, \{B'\}\rangle, \langle y, \{Q''\}\rangle, \langle z, \{A\}\rangle\}$

$S_2(g_A)$: $x \to \langle\{g_A, g_{A'}\}, \{A\}\rangle$, $y \to \langle\{g_A, g_{A'}\}, \{Q\}\rangle$, $z \to \langle\{g_A, g_{A'}\}, \{A\}\rangle$

$S_2(g_{A'})$: $x \to \langle\{g_A, g_{A'}\}, \{A'\}\rangle$, $y \to \langle\{g_A, g_{A'}\}, \{Q\}\rangle$, $z \to \langle\{g_A, g_{A'}\}, \{A\}\rangle$

$S_2(h_A)$: $x \to \langle\{h_A, h_B\}, \{A\}\rangle$, $y \to \langle\{h_A, h_B\}, \{Q'\}\rangle$, $z \to \langle\{h_A, h_B\}, \{A\}\rangle$

$S_2(h_B)$: $x \to \langle\{h_A, h_B\}, \{B\}\rangle$, $y \to \langle\{h_A, h_B\}, \{Q'\}\rangle$, $z \to \langle\{h_A, h_B\}, \{A\}\rangle$

$S_2(k_B)$: $x \to \langle\{k_B, k_{B'}\}, \{B\}\rangle$, $y \to \langle\{k_B, k_{B'}\}, \{Q''\}\rangle$, $z \to \langle\{k_B, k_{B'}\}, \{A\}\rangle$

$S_2(k_{B'})$: $x \to \langle\{k_B, k_{B'}\}, \{B'\}\rangle$, $y \to \langle\{k_B, k_{B'}\}, \{Q''\}\rangle$, $z \to \langle\{k_B, k_{B'}\}, \{A\}\rangle$

- $\langle S_2, S_2\rangle \in [\![\mathbf{2}x[std(x)]]\!]^M$

iff $\forall h \in \text{Dom } S_2$: ${}^2(S_2(h)(x)) \in [\![std]\!]^M$ D2.2.***n***, ***R***
& $\forall h \in \text{Dom } S_2$: $|\{k(x): k \in {}^1(S_2(h)(x))\}| = 2$

✓ since ${}^2(S_2(g_A)(x)) = \{A\} \in [\![std]\!]^M$
& $|\{k(x): k \in {}^1(S_2(g_A)(x))\}| = |\{g_A(x), g_{A'}(x)\}| = |\{\{A\}, \{A'\}\}| = 2$
⋮
& ${}^2(S_2(k_{B'})(x)) = \{B'\} \in [\![std]\!]^M$
& $|\{k(x): k \in {}^1(S_2(k_{B'})(x))\}| = |\{k_B(x), k_{B'}(x)\}| = |\{\{B\}, \{B'\}\}| = 2$

- $\langle S_2, S_2\rangle \in [\![ans(x, y)]\!]^M$

iff $\forall h \in \text{Dom } S_2$: $\langle {}^2(S_2(h)(x)), {}^2(S_2(h)(y))\rangle \in [\![ans]\!]^M$ D2.2.***R***

✓ since $\langle {}^2(S_2(g_A)(x)), {}^2(S_2(g_A)(y))\rangle = \langle\{A\}, \{Q\}\rangle \in [\![ans]\!]^M$
& $\langle {}^2(S_2(g_{A'})(x)), {}^2(S_2(g_{A'})(y))\rangle = \langle\{A'\}, \{Q\}\rangle \in [\![ans]\!]^M$
& $\langle {}^2(S_2(h_A)(x)), {}^2(S_2(h_A)(y))\rangle = \langle\{A\}, \{Q'\}\rangle \in [\![ans]\!]^M$
& $\langle {}^2(S_2(h_B)(x)), {}^2(S_2(h_B)(y))\rangle = \langle\{B\}, \{Q'\}\rangle \in [\![ans]\!]^M$
& $\langle {}^2(S_2(k_B)(x)), {}^2(S_2(k_B)(y))\rangle = \langle\{B\}, \{Q''\}\rangle \in [\![ans]\!]^M$
& $\langle {}^2(S_2(k_{B'})(x)), {}^2(S_2(k_{B'})(y))\rangle = \langle\{B'\}, \{Q''\}\rangle \in [\![ans]\!]^M$

Remark 1:
The final output S_2 encodes (i) the students (A, A′, B, B′), (ii) the questions (Q, Q′, Q″), as well as (iii) the distributive dependency between them (⟨A, Q⟩, etc). The standard semantics for sequencing, which A&W adopt, then explains how distributive dependencies of this sort carry over to subsequent discourse, e.g. in (10).

(10) [1]Three questions were (*each*) answered by two students.
[2]The students then (*all*) presented their answers to the class.

§; ∃*y*; **3***y*[*que*(*y*)]; ∃*x*; **2***x*[*std*(*x*)]; *ans*(*x*, *y*);
prs(*x*, *y*)

But as this example illustrates, this useful result has nothing to do with A&W's 'transition updates': DEP, DIS, etc. These don't seem to do any useful work, contrary to A&W's claims.

Remark 2:
A&W claim that in DPL⁺ quantifier scope can be updated online. If this were true, it would be very exciting and useful for direct online interpretation, but unfortunately it is not. A&W's dependency updates cannot reverse scope relations previously established by existential updates. E.g., in (9) the relative scope is determined by the order of the updates '∃*x*' and '∃*y*' (cf. PTQ):

(9) Two students answered three questions. *scope ambig.*

S > O ('surface scope'):
(a) §; ∃*x*; **2***x*[*std*(*x*)]; ∃*y*; **3***y*[*que*(*y*)]; *ans*(*x*, *y*) like (3), see pp. 10–13

O > S ('inverse scope'):
(b) §; ∃*y*; **3***y*[*que*(*y*)]; ∃*x*; **2***x*[*std*(*x*)]; *ans*(*x*, *y*) pp. 18–20

Contrary to A&W's claims, 'DEP *yx*' cannot reverse the '*x*-over-*y*' relation established by '∃*x*;…∃*y*;…'. Thus, neither (c) nor (d) is equivalent to (b)—e.g., (b) is true but both (c) and (d) are false in the above model for the inverse reading of (9). Given the semantics of DPL⁺, (c) and (d) mean something strange, which bears no resemblance to English sentences like (9) or (10[1]). I leave it as an EXERCISE for the reader to figure out what these formulas of DPL⁺ mean:

(c) §; ∃*x*; ∃*y*; **2***x*[*std*(*x*)]; **3***y*[*que*(*y*)]; DEP *yx*; *ans*(*x*, *y*) A&W, p. 11
(d) §; ∃*x*; ∃*y*; DEP *yx*; **2***x*[*std*(*x*)]; **3***y*[*que*(*y*)]; *ans*(*x*, *y*) A&W, ftn. 14