QPLA (1): Anaphora and information structure

I. Indefinites vs. names: Anaphoric islands

• *Anaphora without islands*: PLA *fine*

(1) [1]A man went into some bar. [2]*He* saw a *friend* (sitting alone). [3]*He* bought *her* a drink.

s_0	$=$	$\{\langle\,\rangle\}$	start-up state
s_1	$:=$	$s_0[\![\exists x(man\ x \wedge \exists y(bar\ y \wedge enter\ xy))]\!]^{M,g}$	upd. s_0 w. (1^1)
	$=$	$\langle \varepsilon' \cdot a \mid \varepsilon' \in s_0[\![man\ x \wedge \exists y(bar\ y \wedge enter\ xy)]\!]^{M,g[x/a]}\}$	sem PLA:∃
	$=$	$\{\varepsilon \cdot b \cdot a \mid \varepsilon \in s_0\ \&\ a \in [\![man]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a\rangle \mid a \in [\![man]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$	df. s_0, $\cdot$
s_2	$:=$	$s_1[\![male\ p_0 \wedge \exists x(frn\ xp_0 \wedge see\ p_0x)]\!]^{M,g}$	upd. s_1 w. (1^2)
	$=$	$\{\varepsilon \cdot c \mid \varepsilon \in s_1\ \&\ \varepsilon_{2-0} \in [\![male]\!]^M\ \&\ \langle c, \varepsilon_{2-0}\rangle \in [\![frn]\!]^M\ \&\ \langle \varepsilon_{2-0}, c\rangle \in [\![see]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a, c\rangle \mid a \in [\![man]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M$	df. s_1, $\cdot$,
		$\&\ \langle c, a\rangle \in [\![frn]\!]^M \&\ \langle a, c\rangle \in [\![see]\!]^M\}$	$[\![man]\!]^M \subseteq [\![male]\!]^M$
s_3	$:=$	$s_2[\![male\ p_1 \wedge fem\ p_0 \wedge \exists x(drnk\ x \wedge buy.for\ p_1xp_0)]\!]^{M,g}$	upd. s_2 w. (1^3)
	$=$	$\{\varepsilon \cdot d \mid \varepsilon \in s_2\ \&\ \varepsilon_{3-1} \in [\![male]\!]^M\ \&\ \varepsilon_{3-0} \in [\![fem]\!]^M$	
		$\&\ d \in [\![drink]\!]^M\ \&\ \langle \varepsilon_{3-1}, d, \varepsilon_{3-0}\rangle \in [\![buy.for]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a, c, d\rangle \mid a \in [\![man]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M$	df. s_2, $\cdot$,
		$\&\ \langle c, a\rangle \in [\![frn]\!]^M \&\ \langle a, c\rangle \in [\![see]\!]^M$	$[\![man]\!]^M \subseteq [\![male]\!]^M$
		$\&\ c \in [\![fem]\!]^M\ \&\ d \in [\![drink]\!]^M\ \&\ \langle a, d, c\rangle \in [\![buy.for]\!]^M\}$	

(2) [1]Adam went into some bar. [2]*He* saw Sue (sitting alone). [3]*He* bought *her* a drink.

s_0	$=$	$\{\langle\,\rangle\}$	start-up state
s_1	$:=$	$s_0[\![\exists x(x = adam \wedge \exists y(bar\ y \wedge enter\ xy))]\!]^{M,g}$	upd. s_0 w. (2^1)
	$=$	$\langle \varepsilon' \cdot a \mid a \in D^M\ \&\ \varepsilon' \in s_0[\![x = adam \wedge \exists y(bar\ y \wedge enter\ xy)]\!]^{M,g[x/a]}\}$	sem PLA:∃
	$=$	$\{\varepsilon \cdot b \cdot a \mid \varepsilon \in s_0\ \&\ a = [\![adam]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a\rangle \mid a = [\![adam]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$	df. s_0, $\cdot$
s_2	$:=$	$s_1[\![male\ p_0 \wedge \exists x(x = sue \wedge see\ p_0x)]\!]^{M,g}$	upd. s_1 w. (2^2)
	$=$	$\{\varepsilon \cdot c \mid \varepsilon \in s_1\ \&\ \varepsilon_{2-0} \in [\![male]\!]^M\ \&\ c = [\![sue]\!]^M\ \&\ \langle \varepsilon_{2-0}, c\rangle \in [\![see]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a, c\rangle \mid a = [\![adam]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M$	df. s_1, $\cdot$,
		$\&\ c = [\![sue]\!]^M \&\ \langle a, c\rangle \in [\![see]\!]^M\}$	$[\![adam]\!]^M \in [\![male]\!]^M$
s_3	$:=$	$s_2[\![male\ p_1 \wedge fem\ p_0 \wedge \exists x(drnk\ x \wedge buy.for\ p_1xp_0)]\!]^{M,g}$	upd. s_2 w. (2^3)
	$=$	$\{\varepsilon \cdot d \mid \varepsilon \in s_2\ \&\ \varepsilon_{3-1} \in [\![male]\!]^M\ \&\ \varepsilon_{3-0} \in [\![fem]\!]^M$	
		$\&\ d \in [\![drink]\!]^M\ \&\ \langle \varepsilon_{3-1}, d, \varepsilon_{3-0}\rangle \in [\![buy.for]\!]^M\}$	sem PLA
	$=$	$\{\langle b, a, c, d\rangle \mid a = [\![adam]\!]^M\ \&\ b \in [\![bar]\!]^M\ \&\ \langle a, b\rangle \in [\![enter]\!]^M$	df. s_2, $\cdot$,
		$\&\ c = [\![sue]\!]^M \&\ \langle a, c\rangle \in [\![see]\!]^M$	$[\![adam]\!]^M \in [\![male]\!]^M$
		$\&\ d \in [\![drink]\!]^M\ \&\ \langle a, d, c\rangle \in [\![buy.for]\!]^M\}$	$[\![sue]\!]^M \in [\![fem]\!]^M$

• *Negation and anaphora*: *Problems with names and pronouns*

(3) [1]A man went into some bar. [2]He did**n't** see {*any*, *a single*} friend. # [3]But *she* saw him.

$s_1 := s_0[\![\exists x(man\ x \wedge \exists y(bar\ y \wedge enter\ xy))]\!]^{M,g}$ — upd. s_0 w. (3^1)
$= \{\langle b, a\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$ — as for (1)

$s_2 := s_1[\![male\ p_0 \wedge \neg\exists x(frn\ xp_0 \wedge see\ p_0x)]\!]^{M,g}$ — upd. s_1 w. (3^2)
$= \{\varepsilon | \varepsilon \in s_1 \ \&\ \varepsilon_{2-0} \in [\![male]\!]^M \ \&\ \neg\exists c\text{: } \langle c, \varepsilon_{2-0}\rangle \in [\![frn]\!]^M \ \&\ \langle \varepsilon_{2-0}, c\rangle \in [\![see]\!]^M\}$ — sem PLA
$= \{\langle b, a\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M$ — df. s_1, $\cdot$,
$\ \&\ \neg\exists c\text{: } \langle c, a\rangle \in [\![frn]\!]^M \& \langle a, c\rangle \in [\![see]\!]^M\}$ — $[\![man]\!]^M \subseteq [\![male]\!]^M$

$s_3 := s_2[\![fem\ p_n \wedge male\ p_0 \wedge see\ p_np_0)]\!]^{M,g}$ — upd. s_2 w. (3^3)
$= \{\varepsilon | \varepsilon \in s_2 \ \&\ \varepsilon_{2-n} \in [\![fem]\!]^M \ \&\ \varepsilon_{2-0} \in [\![male]\!]^M \ \&\ \langle \varepsilon_{2-n}, \varepsilon_{2-0}\rangle \in [\![see]\!]^M)\}$ — sem PLA
$= \{\langle b, a\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M$ — df. s_2, $\cdot$,
$\ \&\ \neg\exists c\text{: } \langle c, a\rangle \in [\![frn]\!]^M \& \langle a, c\rangle \in [\![see]\!]^M$ — $[\![man]\!]^M \subseteq [\![male]\!]^M$
$\ \&\ \varepsilon_{2-n} \in [\![fem]\!]^M \ \&\ \langle \varepsilon_{2-n}, a\rangle \in [\![see]\!]^M)\}$

Good: no antecedent for p_n in this context, no matter how anaphora is resolved (i.e. how we choose n)

(4) [1]Adam went into some bar. [2]He did**n't** see *Sue*. ✓ [3]But *she* saw him.

$s_1 := s_0[\![\exists x(x = adam \wedge \exists y(bar\ y \wedge enter\ xy))]\!]^{M,g}$ — upd. s_0 w. (4^1)
$= \{\langle b, a\rangle | a = [\![adam]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$ — as for (2)

$s_2 := s'_1[\![male\ p_0 \wedge \neg\exists x(x = sue \wedge see\ p_0x)]\!]^{M,g}$ — upd. s'_1 w. (2^2)
$= \{\varepsilon | \varepsilon \in s_1 \ \&\ \varepsilon_{2-0} \in [\![male]\!]^M \ \&\ \neg\exists c\text{: } c = [\![sue]\!]^M \ \&\ \langle \varepsilon_{2-0}, c\rangle \in [\![see]\!]^M\}$ — sem PLA
$= \{\langle b, a\rangle | a = [\![adam]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M$ — df. s_1, $\cdot$
$\ \&\ \langle a, [\![sue]\!]^M\rangle \notin [\![see]\!]^M\}$ — $[\![adam]\!]^M \in [\![male]\!]^M$

$s_3 := s_2[\![fem\ p_n \wedge male\ p_0 \wedge see\ p_np_0)]\!]^{M,g}$ — upd. s''_2 w. (2^3)
$= \{\varepsilon | \varepsilon \in s_2 \ \&\ \varepsilon_{2-n} \in [\![fem]\!]^M \ \&\ \varepsilon_{2-0} \in [\![male]\!]^M \ \&\ \langle \varepsilon_{2-n}, \varepsilon_{2-0}\rangle \in [\![see]\!]^M)\}$ — sem PLA
$= \{\langle b, a\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M$ — df. s_2, $\cdot$,
$\ \&\ \langle a, [\![sue]\!]^M\rangle \notin [\![see]\!]^M$ — $[\![man]\!]^M \subseteq [\![male]\!]^M$
$\ \&\ \varepsilon_{2-n} \in [\![fem]\!]^M \ \&\ \langle \varepsilon_{2-n}, a\rangle \in [\![see]\!]^M)\}$

***Problem* 1**: no antecedent for p_n in this context either, no matter how anaphora is resolved

(5) [1]A man went into some bar. # [2]Chuck did**n't** see *her*.

$s_1 := s_0[\![\exists x(man\ x \wedge \exists y(bar\ y \wedge enter\ xy))]\!]^{M,g}$ — upd. s_0 w. (5^1)
$= \{\langle b, a\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$ — as for (1)

$s_2 := s_1[\![\exists x(x = chuck \wedge \neg(fem\ p_0 \wedge see\ xp_0)]\!]^{M,g}$ — upd. s_1 w. (1^2)
$= \{\varepsilon \cdot c | \varepsilon \in s_1 \ \&\ c = [\![chuck]\!]^M \ \&\ \neg(\varepsilon_{2-0} \in [\![fem]\!]^M \ \&\ \langle c, \varepsilon_{2-0}\rangle \in [\![see]\!]^M)\}$ — sem PLA
$= \{\langle b, a, c\rangle | \langle b, a\rangle \in s_1 \ \&\ c = [\![chuck]\!]^M \ \&\ \neg(a \in [\![fem]\!]^M \ \&\ \langle c, a\rangle \in [\![see]\!]^M)\}$ — df. s_1, ε_n
$= \{\langle b, a, [\![chuck]\!]^M\rangle | a \in [\![man]\!]^M \ \&\ b \in [\![bar]\!]^M \ \&\ \langle a, b\rangle \in [\![enter]\!]^M\}$ — df. s_1, $\cdot$, $a \notin [\![fem]\!]^M$

***Problem* 2**: Negated *her* predicted ok as an anaphor to a non-female antecedent

***Problem* 3:** Resolving negated *her* to a non-female antec. should suffice to make the negation true

II. QPLA: A theory of information structure and anaphora

• *Basic idea* (Dekker 2004:28)

"The basic idea is to split up meaning in a background and a focus component, where the background again is split up in an old and a new part. I will use 'presupposition', 'contribution' and 'assertion' as mere technical labels for the old, new, and focal dimension respectively, and let these labels apply to all the relevant categories and types of our language. Thus, for instance, formulas will have a presuppositional, contributional, and assertional dimension, which is stated as a satisfaction relation, $\models_p$, $\models_c$, and $\models_a$, respectively (MB: $\models_{\text{P}}$, $\models_{\text{N}}$, $\models_{\text{A}}$, for *presupposed background*, *new background*, and (*focal*) *assertion*, respectively); nouns and verbs will have sets of individuals as their presuppositional, contributional or assertional denotation, and terms and quantifiers sets of sets of individuals."

• e.g., in QPLA, discourse (4) will be analyzed as follows (see QPLA(2) for details):

(4) ^{1}Adam went into some bar. ^{2}He did**n't** see *Sue*. ✓ ^{3}But *she* saw him.

(4^1) $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$

• $M, g, aa'\delta \models_{\text{P}} (4^1)$ iff $a = [\![adam]\!]^M$	(4^1) ***presupposes*** a dref a for *adam*
$M, g, aa'\delta \models_{\text{N}} (4^1)$ iff $[\![bar]\!]^M(a') = 1$	(4^1) adds a ***novel dref*** a' for a bar
$M, g, aa'\delta \models_{\text{A}} (4^1)$ iff $[\![enter]\!]^M(a')(a) = 1$	(4^1) ***asserts*** that a entered a'

(4^2) $\ldots \wedge he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$

• $M, g, bb'aa'\delta \models_{\text{P}} (4^2)$ iff $b = a\ \&\ [\![male]\!]^M(a) = 1\ \&\ b' = [\![sue]\!]^M$	(4^2) ***presupposes*** a male antec. $b = a$ for he_1 & a dref b' for *sue*
$M, g, bb'aa'\delta \models_{\text{N}} (4^2)$	(4^2) adds ***no new background***
$M, g, bb'aa'\delta \models_{\text{A}} (4^2)$ iff $[\![see]\!]^M(b)(b') = 0$	(4^2) ***asserts*** that $b = a$ didn't see b'

(4^3) $\ldots \wedge she_2(\lambda x\ he_1(\lambda y\ see\ yx))$

• $M, g, cc'bb'aa'\delta \models_{\text{P}} (4^3)$ iff $c = b'\ \&\ [\![fem]\!]^M(b') = 1\ \&\ c' = b\ \&\ [\![male]\!]^M(b) = 1$	(4^3) ***presupposes*** a fem antec. $c = b'$ for she_3 & male antec. $c' = b$ for he_1
$M, g, cc'bb'aa'\delta \models_{\text{N}} (4^3)$	(4^3) adds ***no new background***
$M, g, cc'bb'aa'\delta \models_{\text{A}} (4^3)$ iff $[\![see]\!]^M(c')(c) = 1$	(4^3) ***asserts*** that $c = b'$ saw $c' = a$

III. QPLA (v. 1.1): Indefinites, names, and pronouns (one construal of Dekker 2003a, 2004)

DEFINITION 1.1 (QPLA-types)
- $e, t \in \mathbf{Typ}$
- $(\sigma\tau) \in \mathbf{Typ}$, if $\sigma, \tau \in \mathbf{Typ}$

DEFINITION 1.2 (QPLA-basic terms)
- $\mathbf{Con}_e = \{bill, sue, \ldots\}$
  $\mathbf{Con}_{et} = \{bar, male, fem, \ldots\}$
  $\mathbf{Con}_{eet} = \{enter, see, \ldots\}$
- $\mathbf{Var}_e = \{x, y, z, \ldots\}$
- $\mathbf{Prn}_{(et)t} = \{he_1, he_2, \ldots, she_1, she_2, \ldots\}$

DEFINITION 1.2 (QPLA-terms and ∃-number). $n(\alpha)$ after Dekker 2004:23

b	$\alpha \in \mathbf{Term}_\tau$,	$n(\alpha) = 0$	if $\alpha \in \mathbf{Con}_\tau \cup \mathbf{Var}_\tau \cup \mathbf{Prn}_\tau$
f	$\alpha\beta \in \mathbf{Term}_\tau$	$n(\alpha\beta) = n(\alpha) + n(\beta)$	if $\alpha \in \mathbf{Term}_{(\sigma\tau)}$ & $\beta \in \mathbf{Term}_\sigma$
[]	$[\alpha] \in \mathbf{Term}_{(et)t}$	$n([\alpha]) = n(\alpha)$	if $\alpha \in \mathbf{Con}_e$
∃[	$\exists[\alpha] \in \mathbf{Term}_{(et)t}$	$n(\exists[\alpha]) = n(\alpha) + 1$	if $\alpha \in \mathbf{Term}_{et}$
∃	$\exists u\phi \in \mathbf{Term}_t$	$n(\exists u\phi) = n(\phi) + 1$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
λ	$(\lambda u\phi) \in \mathbf{Term}_{(et)}$	$n(\lambda u\phi) = n(\phi)$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
¬	$\neg\phi \in \mathbf{Term}_t$	$n(\neg\phi) = 0$	if $\phi \in \mathbf{Term}_t$
∧	$(\phi \wedge \psi) \in \mathbf{Term}_t$	$n(\phi \wedge \psi) = n(\phi) + n(\psi)$	if $\phi, \psi \in \mathbf{Term}_t$

DEFINITION 2.1 (frames). A *standard* QPLA-*frame* is a set of τ-domains, $\{D_\tau : \tau \in \mathbf{Typ}\}$ s.t.
- $_1$ $D_t = \{1, 0\}$
  D_e is a non-empty set disjoint from D_t
- $_2$ $D_{(\sigma\tau)} = \{f \mid \text{Dom } f = D_\sigma \text{ \& Ran } f \subseteq D_\tau\}$

DEFINITION 2.2 (models, assignments, and stacks)
- $_1$ A QPLA-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
  (i) $D^M = \{D_\tau^M : \tau \in \mathbf{Typ}\}$ is a standard QPLA-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_\tau$ a denotation $[\![\alpha]\!]^M \in D_\tau^M$
- $_2$ An *M-assignment* is a function g that assigns to any $u \in \mathbf{Var}_\tau$ a value $g(u) \in D_\tau^M$. If $d \in D_\tau^M$, then $g[u/d]$ is the M-assignment s.t. (i) $g[u/d](u) = d$, and (ii) $g[u/d](u') = g(u')$ if $u' \neq u$.
- $_3$ (i) $\delta \in (D_e^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *n-stack* of *e*-drefs
  $\delta_m := d_m$, if $\delta = \langle d_1, \ldots d_n\rangle \in (D_e^M)^n$ & $1 \leq m \leq n$
  $\delta\varepsilon := \langle d_1, \ldots d_n, \varepsilon_1, \ldots \varepsilon_m\rangle$ if $\delta = \langle d_1, \ldots d_n\rangle \in (D_e^M)^n$ & $\varepsilon = \langle e_1, \ldots e_n\rangle \in (D_e^M)^m$

  (ii) If $f \in D_{et}^M$, $d \in D_e^M$, $\varepsilon \in (D_e^M)^n$, $1 \leq m \leq n$, then:
  $[\![he_m]\!]^{M,d\varepsilon}_{\mathrm{P}}(f) = 1$, iff $d = \varepsilon_m$ & $[\![male]\!]^M(d) = 1$ & $f(d) = 1$ see Dekker 2003a:10
  $[\![he_m]\!]^{M,d\varepsilon}_{\mathrm{N}}(f) = 1$, iff $f(d) = 1$ on HE_i
  $[\![he_m]\!]^{M,d\varepsilon}_{\mathrm{A}}(f) = 1$, iff $f(d) = 1$

  $[\![she_m]\!]^{M,d\varepsilon}_{\mathrm{P}}(f) = 1$, iff $d = \varepsilon_m$ & $[\![fem]\!]^M(d) = 1$& $f(d) = 1$
  $[\![she_m]\!]^{M,d\varepsilon}_{\mathrm{N}}(f) = 1$, iff $f(d) = 1$
  $[\![she_m]\!]^{M,d\varepsilon}_{\mathrm{A}}(f) = 1$, iff $f(d) = 1$

DEFINITION 2.3 (QPLA-semantics)

r	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{X}} \in D_\tau^M$	if $\alpha \in \mathbf{Term}_\tau$ & X ∈ {P, N, A}	
b	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{X}} = [\![\alpha]\!]^M$	if $\alpha \in \mathbf{Con}_e$ & X ∈ {P, N, A}	
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{X}} = g(\alpha)$	if $\alpha \in \mathbf{Var}_e$ & X ∈ {P, N, A}	
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{X}} = [\![\alpha]\!]^{M,\varepsilon}_{\text{X}}$	if $\alpha \in \mathbf{Prn}_{(et)t}$ & X ∈ {P, N, A}	
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{P}}(d) = 1$	if $\alpha \in \mathbf{Con}_{et}$	
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{N}}(d) = 1$		
	$[\![\alpha]\!]^{M,g,\delta}_{\text{A}} = [\![\alpha]\!]^M$		
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{P}}(d')(d) = 1$	if $\alpha \in \mathbf{Con}_{eet}$	
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{N}}(d')(d) = 1$		
	$[\![\alpha]\!]^{M,g,\varepsilon}_{\text{A}} = [\![\alpha]\!]^M$		
f	$[\![\alpha\beta]\!]^{M,g,\gamma\delta\varepsilon}_{\text{X}} = [\![\alpha]\!]^{M,g,\gamma\varepsilon}_{\text{X}}([\![\beta]\!]^{M,g,\delta\varepsilon}_{\text{X}})$	if X ∈ {P, N, A}	see Dekker 2003a:11
[]	$[\![[\alpha]]\!]^{M,g,d\varepsilon}_{\text{P}}(f) = 1$	iff $d = [\![\alpha]\!]^M$ & $f(d) = 1$	see Dekker 2003a:10
	$[\![[\alpha]]\!]^{M,g,d\varepsilon}_{\text{N}}(f) = 1$	iff $f(d) = 1$	on MARY
	$[\![[\alpha]]\!]^{M,g,d\varepsilon}_{\text{A}}(f) = 1$	iff $f(d) = 1$	
∃[	$[\![\exists[\alpha]]\!]^{M,g,d\varepsilon}_{\text{P}}(f) = 1$	iff $[\![\alpha]\!]^{M,g,\varepsilon}_{\text{P}}(d) = 1$ & $f(d) = 1$	see Dekker 2003a:11
	$[\![\exists[\alpha]]\!]^{M,g,d\varepsilon}_{\text{N}}(f) = 1$	iff $[\![\alpha]\!]^{M,g,\varepsilon}_{\text{N}}(d) = 1$ & $[\![\alpha]\!]^{M,g,\varepsilon}_{\text{A}}(d) = 1$ & $f(d) = 1$	on SOME(π)
	$[\![\exists[\alpha]]\!]^{M,g,d\varepsilon}_{\text{A}}(f) = 1$	iff $f(d) = 1$	
∃	$[\![\exists u\phi]\!]^{M,g,d\varepsilon}_{\text{X}} = 1$	iff $[\![\phi]\!]^{M,g[u/d],\varepsilon}_{\text{X}} = 1$	see Dekker 2003a:6
λ	$[\![(\lambda u\phi)]\!]^{M,g,\varepsilon}_{\text{X}}(d) = 1$	iff $[\![\phi]\!]^{M,g[u/d],\varepsilon}_{\text{X}} = 1$	see Dekker 2003a:10
¬	$[\![\neg\phi]\!]^{M,g,\varepsilon}_{\text{P}} = 1$	iff $\exists\delta \in (D_e^M)^{n(\phi)}$: $[\![\phi]\!]^{M,g,\delta\varepsilon}_{\text{P}} = 1$	see Dekker 2004:23
	$[\![\neg\phi]\!]^{M,g,\varepsilon}_{\text{N}} = 1$	iff $\exists\delta \in (D_e^M)^{n(\phi)}$: $[\![\phi]\!]^{M,g,\delta\varepsilon}_{\text{N}} = 1$	see Dekker 2004:29
	$[\![\neg\phi]\!]^{M,g,\varepsilon}_{\text{A}} = 1$	iff $\neg\exists\delta \in (D_e^M)^{n(\phi)}$: $[\![\phi]\!]^{M,g,\delta\varepsilon}_{\text{N}} = 1$ & $[\![\phi]\!]^{M,g,\delta\varepsilon}_{\text{A}} = 1$	see Dekker 2004:29
∧	$[\![(\phi \wedge \psi)]\!]^{M,g,\delta\varepsilon}_{\text{X}} = 1$	iff $[\![\phi]\!]^{M,g,\varepsilon}_{\text{X}} = 1$ & $[\![\psi]\!]^{M,g,\delta\varepsilon}_{\text{X}} = 1$	see Dekker 2003a:6

DEFINITION 3 (Satisfaction, felicity and truth)

- ε satisfies the presupposed background of ϕ in M under g, written $M, g, \varepsilon \models_{\text{P}} \phi$, iff $[\![\phi]\!]^{M,g,\varepsilon}_{\text{P}} = 1$
  ε satisfies the new background of ϕ in M under g, written $M, g, \varepsilon \models_{\text{N}} \phi$, iff $[\![\phi]\!]^{M,g,\varepsilon}_{\text{N}} = 1$
- ε satisfies the background of ϕ (makes ϕ felicitous) in M under g, written $M, g, \varepsilon \models_{\text{B}} \phi$, iff $[\![\phi]\!]^{M,g,\varepsilon}_{\text{P}} = 1$ & $[\![\phi]\!]^{M,g,\varepsilon}_{\text{N}} = 1$
- ε satisfies the focal assertion of ϕ (makes ϕ true) in M under g, written $M, g, \varepsilon \models_{\text{A}} \phi$, iff $[\![\phi]\!]^{M,g,\varepsilon}_{\text{A}} = 1$

ABBREVIATIONS

A1.	$(\phi \rightarrow \psi)$	:=	$\neg(\phi \wedge \neg\psi)$
A2.	$\forall u\phi$	:=	$\neg\exists u\neg\phi$

QPLA (2): Names, pronouns, and negation

I. Name vs. indefinite: Presupposed vs. novel drefs

- Adam went into some bar.

(1) $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$ QPLA

$M, g, aa' \models_P (1^1)$ iff $a = [\![adam]\!]^M$ — (1^1) ***presupposes*** a dref a for $[adam]$

$M, g, aa' \models_N (1^1)$ iff $[\![bar]\!]^M(a') = 1$ — (1^1) adds a ***novel dref*** a' for a bar

$M, g, aa' \models_A (1^1)$ iff $[\![enter]\!]^M(a')(a) = 1$ — (1^1) ***asserts*** that a entered a'

Fact 1P: $M, g, aa' \models_P [adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$
iff $a = [\![adam]\!]^M$

Proof: (1) iff (11)

1. $M, g, aa' \models_P [adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$
2. $[\![[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, aa'\delta}{}_P = 1$ — D3
3. $[\![[adam]]\!]^{M, g, a}{}_P([\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_P) = 1$ — D2.3.***f***.γ/a. δ/a'. $\varepsilon/\langle\rangle$
4. $a = [\![adam]\!]^M$
 & $[\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_P(a) = 1$ — D2.3.**[]**.d/a. $\varepsilon/\langle\rangle$
5. $a = [\![adam]\!]^M$
 & $[\![\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], a'}{}_P = 1$ — D2.3.$\boldsymbol{\lambda}$.ε/a'. d/a
6. $a = [\![adam]\!]^M$
 & $[\![\exists[bar]]\!]^{M, g[x/a], a'}{}_P([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P) = 1$ — D2.3.***f***.γ/a'. $\delta/\langle\rangle$. $\varepsilon/\langle\rangle$
7. $a = [\![adam]\!]^M$
 & $[\![bar]\!]^{M, g[x/a], \langle\rangle}{}_P(a') = 1$
 & $[\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P(a') = 1$ — D2.3.$\boldsymbol{\exists}$**[**.d/a'. $\varepsilon/\langle\rangle$
8. $a = [\![adam]\!]^M$
 & $1 = 1$ — D2.3.***b***
 & $[\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P = 1$ — D2.3.$\boldsymbol{\lambda}$.$\varepsilon/\langle\rangle$. d/a'
9. $a = [\![adam]\!]^M$ — simplify
 & $[\![enter]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P(a')(a) = 1$ — D2.3.***f***, ***b***, D2.2.$g[u/d]$
10. $a = [\![adam]\!]^M$ — simplify
 & $1 = 1$ — D2.3.***b***
11. $a = [\![adam]\!]^M$ — simplify

□

Fact 1N: $M, g, aa' \models_{N}$ $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$
iff $[\![bar]\!]^{M}(a') = 1$

Proof: (1) iff (11)

1.	$M, g, aa' \models_{N} [adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$	
2.	$[\![[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, aa'}{}_{N} = 1$	D3
3.	$[\![[adam]]\!]^{M, g, a}{}_{N}([\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_{N}) = 1$	D2.3.$\boldsymbol{f}$.γ/a. δ/a'. $\varepsilon/\langle\rangle$
4.	$[\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_{N}(a) = 1$	D2.3.**[]**.d/a. $\varepsilon/\langle\rangle$
5.	$[\![\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], a'}{}_{N} = 1$	D2.3.$\boldsymbol{\lambda}$.ε/a'. d/a
6.	$[\![\exists[bar]]\!]^{M, g[x/a], a'}{}_{N}([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_{N}) = 1$	D2.3.$\boldsymbol{f}$.γ/a'. $\delta/\langle\rangle$. $\varepsilon/\langle\rangle$
7.	$[\![bar]\!]^{M, g[x/a], \langle\rangle}{}_{N}(a') = 1$ & $[\![bar]\!]^{M, g[x/a], \langle\rangle}{}_{A}(a') = 1$ & $[\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_{N}(a') = 1$	D2.3.$\boldsymbol{\exists}$[.d/a'. $\varepsilon/\langle\rangle$
8.	$1 = 1$ & $[\![bar]\!]^{M}(a') = 1$ & $[\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_{N} = 1$	D2.3.$\boldsymbol{b}$ D2.3.$\boldsymbol{\lambda}$.$\varepsilon/\langle\rangle$. d/a'
9.	$[\![bar]\!]^{M}(a') = 1$ & $[\![enter]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_{N}(a')(a) = 1$	simplify D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
10.	$[\![bar]\!]^{M}(a') = 1$ & $1 = 1$	 D2.3.$\boldsymbol{b}$
11.	$[\![bar]\!]^{M}(a') = 1$	simplify

□

Fact 1A: $M, g, aa' \models_{A}$ $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$
iff $[\![enter]\!]^{M}(a')(a) = 1$

Proof: (1) iff (10)

1.	$M, g, aa' \models_{A} [adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$	
2.	$[\![[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, aa'}{}_{A} = 1$	D3
3.	$[\![[adam]]\!]^{M, g, a}{}_{A}([\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_{A}) = 1$	D2.3.$\boldsymbol{f}$.γ/a. δ/a'. $\varepsilon/\langle\rangle$
4.	$[\![(\lambda x\, \exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, a'}{}_{A}(a) = 1$	D2.3.**[]**.d/a. $\varepsilon/\langle\rangle$
5.	$[\![\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], a'}{}_{A} = 1$	D2.3.$\boldsymbol{\lambda}$.ε/a'. d/a
6.	$[\![\exists[bar]]\!]^{M, g[x/a], a'}{}_{A}([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_{A}) = 1$	D2.3.$\boldsymbol{f}$.γ/a'. $\delta/\langle\rangle$. $\varepsilon/\langle\rangle$
7.	$[\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_{A}(a') = 1$	D2.3.$\boldsymbol{\exists}$[.d/a', $\varepsilon/\langle\rangle$
8.	$[\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_{A} = 1$	D2.3.$\boldsymbol{\lambda}$.$\varepsilon/\langle\rangle$. d/a'
9.	$[\![enter]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_{A}(a')(a) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
10.	$[\![enter]\!]^{M}(a')(a) = 1$	D2.3.$\boldsymbol{b}$

□

II. ‘Standard’ (?) presupposition-preserving negation

- *English* negation

Adam didn’t go into some bar. — with positive polarity *some*

(∃¬) [*adam*](λx ∃[*bar*](λy ¬ *enter yx*)) — QPLA

$M, g, aa' \models_P$ (∃¬) iff $a = [\![adam]\!]^M$	(∃¬) ***presupposes*** a dref *a* for [*adam*]
$M, g, aa' \models_N$ (∃¬) iff $[\![bar]\!]^M(a') = 1$	(∃¬) adds a ***novel dref*** *a′* for a bar
$M, g, aa' \models_A$ (∃¬) iff $[\![enter]\!]^M(a')(a) = 0$	(∃¬) ***asserts*** that *a* did not enter *a′*

Proof: exercise

Adam didn’t go into a bar. — with ‘pragmatic fusion’ of O + tv

(¬∃) [*adam*](λx ¬∃*y*(*bar y* ∧ *enter yx*)) — QPLA

$M, g, a \models_P$ (¬∃) iff $a = [\![adam]\!]^M$	(¬∃) ***presupposes*** a dref *a* for [*adam*]
$M, g, a \models_N$ (¬∃)	(¬∃) has ***no new background***
$M, g, a \models_A$ (¬∃) iff $\neg\exists a': [\![bar]\!]^M(a') = 1$ & $[\![enter]\!]^M(a')(a) = 1$	(¬∃) ***asserts*** that there is no bar that *a* entered

Proof: see below

- *Kalaallisut* negation — with ‘what=or/even’ O + tv

Adam-ip	*Ukkusissa-ni*	*imirniartarvik*	*suna=luunniit*	*isir-vigi-nngi-la-a*
Adam-sg.ERG	Ukkusissat-LOC	bar.sg	what=or/even	enter-iv\tv-not-IRR-3s.3s

ROUGHLY: Adam didn’t go into a bar in Ukkusissat.
BUT: *infelicitous* in 1984, when there was no bar in Ukkusissat.

(¬[) [*adam*](λx ¬∃[*bar*](λy *enter yx*)) — QPLA

$M, g, a \models_P$ (¬[) iff $a = [\![adam]\!]^M$	(¬[) ***presupposes*** a dref *a* for [*adam*]
$M, g, a \models_N$ (¬[) iff $\boldsymbol{\exists a': [\![bar]\!]^M(a') = 1}$	(¬[) requires ***non-empty domain*** (there are bars), ***no novel drefs***
$M, g, a \models_A$ (¬[) iff $\neg\exists a': [\![bar]\!]^M(a') = 1$ & $[\![enter]\!]^M(a')(a) = 1$	(¬[) ***asserts*** that there is no bar that *a* entered

Proof: see below

Fact ¬∃P: $M, g, a \models_P [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$
iff $a = [\![adam]\!]^M$

Proof: (1) iff (8)

1. $M, g, a \models_P [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$

2. $a = [\![adam]\!]^M$
$\&\ [\![(\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))]\!]^{M, g, \langle\rangle}{}_P(a) = 1$ — D3, D2.3.***f*, []**

3. $a = [\![adam]\!]^M$
$\&\ [\![\neg\exists y(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P = 1$ — D2.3.**λ**

4. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![\exists y(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a], a'}{}_P = 1$ — D2.3.**¬**

5. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![bar\ y \wedge enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}) = 1$ — D2.3.**∃**

6. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![bar]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P(a') = 1\ \&\ [\![enter]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P(a')(a) = 1$ — D2.3.**∧**, ***f***, ***b***, D2.2.*g*[*u*/*d*]

7. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: 1 = 1\ \&\ 1 = 1$ — D2.3.***b***

8. $a = [\![adam]\!]^M$ — simplify

□

Fact ∃[P: $M, g, a \models_P [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$
iff $a = [\![adam]\!]^M$

Proof: (1) iff (8)

1. $M, g, a \models_P [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$

2. $a = [\![adam]\!]^M$
$\&\ [\![(\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, \langle\rangle}{}_P(a) = 1$ — D3, D2.3.***f*, []**

3. $a = [\![adam]\!]^M$
$\&\ [\![\neg\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P = 1$ — D2.3.**λ**

4. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![\exists[bar]]\!]^{M, g[x/a], a'}{}_P([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P) = 1$ — D2.3.¬, ***f***

5. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![bar]\!]^{M, g[x/a], \langle\rangle}{}_P(a') = 1$
$\&\ [\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_P(a') = 1$ — D2.3.**∃[**

6. $a = [\![adam]\!]^M$
$\&\ \exists a' \in D^M_e\!: [\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P = 1$ — D2.3.***b***, **λ**, simplify

7. $a = [\![adam]\!]^M$ — simplify
$\&\ \exists a' \in D^M_e\!: [\![enter]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_P(a')(a) = 1$ — D2.3.***f***, ***b***, D2.2.*g*[*u*/*d*]

8. $a = [\![adam]\!]^M$ — D2.3.***b***, simplify

□

Fact ¬∃N: $M, g, a \models_{\mathrm{N}} [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$

Proof: (1) iff (9)

1.	$M, g, a \models_{\mathrm{N}} [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$	
2.	$\llbracket [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))\rrbracket^{M, g, a}{}_{\mathrm{N}} = 1$	D3
3.	$\llbracket (\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))\rrbracket^{M, g, \langle\rangle}{}_{\mathrm{N}}(a) = 1$	D2.3.***f***, **[]**
4.	$\llbracket \neg\exists y(bar\ y \wedge enter\ yx))\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\lambda}$
5.	$\exists a' \in D_e{}^M$: $\llbracket \exists y(bar\ y \wedge enter\ yx))\rrbracket^{M, g[x/a], a'}{}_{\mathrm{N}} = 1$	D2.3.**¬**
6.	$\exists a' \in D^M{}_e$: $\llbracket bar\ y \wedge enter\ yx\rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.**∃**
7.	$\exists a' \in D^M{}_e$: $\llbracket bar\rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_{\mathrm{N}}(a') = 1$ & $\llbracket enter\ yx\rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.**∧**
8.	$\exists a' \in D^M{}_e$: 1 = 1 & 1 = 1	D2.3.***f***, ***b***, D2.2.*g*[*u*/*d*]
9.	1 = 1	

□

Fact ¬[N: $M, g, a \models_{\mathrm{P}} [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$
iff $\exists a' \in D_e{}^M$: $\llbracket bar\rrbracket^M(a') = 1$

Proof: (1) iff (11)

1.	$M, g, a \models_{\mathrm{N}} [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$	
2.	$\llbracket [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))\rrbracket^{M, g, a}{}_{\mathrm{N}} = 1$	D3
3.	$\llbracket (\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))\rrbracket^{M, g, \langle\rangle}{}_{\mathrm{N}}(a) = 1$	D2.3.***f***, **[]**
4.	$\llbracket \neg\exists[bar](\lambda y\ enter\ yx)\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\lambda}$
5.	$\exists a' \in (D_e{}^M)^1$: $\llbracket \exists[bar](\lambda y\ enter\ yx)\rrbracket^{M, g[x/a], a'}{}_{\mathrm{N}} = 1$	D2.3.**¬**
6.	$\exists a' \in D_e{}^M$: $\llbracket \exists[bar]\rrbracket^{M, g[x/a], a'\delta}{}_{\mathrm{N}}(\llbracket (\lambda y\ enter\ yx)\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{N}}) = 1$	D2.3.***f***, simplify
7.	$\exists a' \in D_e{}^M$: $\llbracket bar\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{N}}(a') = 1$ & $\llbracket bar\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{A}}(a') = 1$ & $\llbracket (\lambda y\ enter\ yx)\rrbracket^{M, g[x/a], \langle\rangle}{}_{\mathrm{N}}(a') = 1$	D2.3.**∃[**
8.	$\exists a' \in D_e{}^M$: 1 = 1 & $\llbracket bar\rrbracket^M(a') = 1$ & $\llbracket enter\ yx\rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.***b*** D2.3.$\boldsymbol{\lambda}$
9.	$\exists a' \in D_e{}^M$: $\llbracket bar\rrbracket^M(a') = 1$ & $\llbracket enter\rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_{\mathrm{N}}(a')(a) = 1$	simplify D2.3.***f***, ***b***, D2.2.*g*[*u*/*d*]
10.	$\exists a' \in D_e{}^M$: $\llbracket bar\rrbracket^M(a') = 1$ & 1 = 1	 D2.3.***b***
11.	$\exists a' \in D_e{}^M$: $\llbracket bar\rrbracket^M(a') = 1$	simplify

□

Fact ¬∃A: $M, g, a \models_A [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$
iff $\neg\exists a' \in D_e^M$: $[\![bar]\!]^M(a') = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$

Proof: (1) iff (8)

1. $M, g, a \models_A [adam](\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))$
2. $[\![(\lambda x\, \neg\exists y(bar\ y \wedge enter\ yx))]\!]^{M, g, a}{}_A(a) = 1$ — D3, D2.3.$\boldsymbol{f}$, []
3. $[\![\neg\exists y(bar\ y \wedge enter\ yx))]\!]^{M, g[x/a], \langle\rangle}{}_A = 1$ — D2.3.$\boldsymbol{\lambda}$
4. $\neg\exists a' \in D_e^M$: $[\![\exists y(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a], a'}{}_N = 1$
   & $[\![\exists y(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a], a'}{}_A = 1$ — D2.3.¬
5. $\neg\exists a' \in D_e^M$: $[\![(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$
   & $[\![(bar\ y \wedge enter\ yx)]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$ — D2.3.∃
6. $\neg\exists a' \in D_e^M$: $[\![bar\ y]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1\ \&\ [\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$
   & $[\![bar\ y]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1\ \&\ [\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$ — D2.3.∧
7. $\neg\exists a' \in D_e^M$: $1 = 1\ \&\ 1 = 1$
   & $[\![bar]\!]^M(a') = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$ — D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
8. $\neg\exists a' \in D_e^M$: $[\![bar]\!]^M(a') = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$ — simplify

□

Fact ¬[A: $M, g, a \models_A [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$
iff $\neg\exists a' \in D_e^M$: $[\![bar]\!]^M(a') = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$

Proof: (1) iff (9)

1. $M, g, a \models_A [adam](\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))$
2. $[\![(\lambda x\, \neg\exists[bar](\lambda y\ enter\ yx))]\!]^{M, g, \langle\rangle}{}_A(a) = 1$ — D3, D2.3.$\boldsymbol{f}$, []
3. $[\![\neg\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_A = 1$ — D2.3.$\boldsymbol{\lambda}$
4. $\neg\exists a' \in D_e^M$: $[\![\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], a'}{}_N = 1$
   & $[\![\exists[bar](\lambda y\ enter\ yx)]\!]^{M, g[x/a], a'}{}_A = 1$ — D2.3.¬
5. $\neg\exists a' \in D_e^M$: $[\![\exists[bar]]\!]^{M, g[x/a], a'}{}_N([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_N) = 1$
   & $[\![\exists[bar]]\!]^{M, g[x/a], a'}{}_A([\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_A) = 1$ — D2.3.$\boldsymbol{f}$
6. $\neg\exists a' \in D_e^M$: $[\![bar]\!]^{M, g[x/a], \langle\rangle}{}_N(a') = 1\ \&\ [\![bar]\!]^{M, g[x/a], \langle\rangle}{}_A(a') = 1$
   & $[\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_N(a') = 1$
   & $[\![(\lambda y\ enter\ yx)]\!]^{M, g[x/a], \langle\rangle}{}_A(a') = 1$ — D2.3.∃[
7. $\neg\exists a' \in D_e^M$: $[\![bar]\!]^{M, g[x/a], \langle\rangle}{}_N(a') = 1\ \&\ [\![bar]\!]^{M, g[x/a], \langle\rangle}{}_A(a') = 1$
   & $[\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$
   & $[\![enter\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$ — D2.3.$\boldsymbol{\lambda}$
8. $\neg\exists a' \in D_e^M$: $1 = 1\ \&\ [\![bar]\!]^M(a') = 1$
   & $1 = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$ — D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
9. $\neg\exists a' \in D_e^M$: $[\![bar]\!]^M(a') = 1\ \&\ [\![enter]\!]^M(a')(a) = 1$ — simplify

□

III. Presupposition of name preserved under negation

- [1]Adam went into some bar. [2]He didn't see Sue.

(2) $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx)) \wedge he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$ QPLA

(2^1) $[adam](\lambda x\, \exists[bar](\lambda y\ enter\ yx))$
(2^2) $he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$

In Section I we have shown the following facts about (2^1):

F1P $M, g, aa' \models_P (2^1)$ iff $a = [\![adam]\!]^M$ — (2^1) ***presupposes*** a dref a for *adam*
F1N $M, g, aa' \models_N (2^1)$ iff $[\![bar]\!]^M(a') = 1$ — (2^1) contributes a ***novel dref*** a' for a bar
F1A $M, g, aa' \models_A (2^1)$ iff $[\![enter]\!]^M(a')(a) = 1$ — (2^1) ***asserts*** that a entered a'

From D3 and D2.3.∧, we further infer, for any $X \in \{P, N, A\}$:

Fact 2: If $M, g, aa' \models_X (2^1)$ & $M, g, bb'aa'\delta \models_X (2^2)$
then $M, g, bb'aa' \models_X (2)$

We now show the following facts about (2^2):

F2P $M, g, bb'aa' \models_P (2^2)$ iff $b = a$ & $[\![male]\!]^M(a) = 1$ & $b' = [\![sue]\!]^M$ — (2^2) ***presupposes*** an old male dref $b = a$ for he_1 & a dref b' for *sue*
F2N $M, g, bb'aa' \models_N (2^2)$ — (2^2) has ***no new background***
F2A $M, g, bb'aa' \models_A (2^2)$ iff $[\![see]\!]^M(b')(b) = 0$ — (2^2) ***asserts*** that $b = a$ didn't see b'

Fact 2P: $M, g, bb'aa' \models_P he_1(\lambda x\, \neg([sue](\lambda y\; see\; yx)))$
iff $b = a$ & $\llbracket male \rrbracket^M(b) = 1$ & $b' = \llbracket sue \rrbracket^M$

Proof: (1) iff (14)

1.	$M, g, bb'aa'\delta \models_P he_1(\lambda x\, \neg([sue](\lambda y\; see\; yx)))$	
2.	$\llbracket he_1(\lambda x\, \neg[sue](\lambda y\; see\; yx)) \rrbracket^{M,g,bb'aa'}{}_P = 1$	D3
3.	$\llbracket he_1 \rrbracket^{M,g,baa'}{}_P(\llbracket(\lambda x\, \neg[sue](\lambda y\; see\; yx))\rrbracket^{M,g,b'aa'}{}_P) = 1$	D2.3.$\boldsymbol{f}$.γ/b. δ/b', ε/aa'
4.	$\llbracket he_1 \rrbracket^{M,baa'}{}_P(\llbracket(\lambda x\, \neg[sue](\lambda y\; see\; yx))\rrbracket^{M,g,b'aa'}{}_P) = 1$	D2.3.$\boldsymbol{b}$
5.	$b = (aa'\delta)_1$	D2.2.he_m
	& $\llbracket male \rrbracket^M(b) = 1$	
	& $\llbracket \lambda x\, \neg[sue](\lambda y\; see\; yx) \rrbracket^{M,g,b'aa'}{}_P(b) = 1$	
6.	$b = a$	D2.2.δ_m
	& $\llbracket male \rrbracket^M(b) = 1$	
	& $\llbracket \neg[sue](\lambda y\; see\; yx) \rrbracket^{M,g[x/b],b'aa'}{}_P = 1$	D2.3.$\boldsymbol{\lambda}$
7.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$	
	& $\exists\varepsilon \in (D_e{}^M)^0$: $\llbracket [sue](\lambda y\; see\; yx) \rrbracket^{M,g[x/b],b'aa'}{}_P = 1$	D2.3.$\neg$
8.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$	
	& $\llbracket [sue](\lambda y\; see\; yx) \rrbracket^{M,g[x/b],b'aa'}{}_P = 1$	$(D_e{}^M)^0 = \{\langle\,\rangle\}$
9.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$	
	& $\llbracket [sue] \rrbracket^{M,g[x/b],b'aa'}{}_P(\llbracket \lambda y\; see\; yx \rrbracket^{M,g[x/b],aa'}{}_P) = 1$	D2.3.$\boldsymbol{f}$.γ/b'. $\delta/\langle\rangle$. ε/aa'
10.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$	
	& $b' = \llbracket sue \rrbracket^M$ & $\llbracket \lambda y\; see\; yx \rrbracket^{M,g[x/b],aa'}{}_P(b') = 1$	D2.3.[]
11.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$ & $b' = \llbracket sue \rrbracket^M$	rearrange
	& $\llbracket see\; yx \rrbracket^{M,g[x/b][y/b'],aa'}{}_P = 1$	D2.3.$\boldsymbol{\lambda}$
12.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$ & $b' = \llbracket sue \rrbracket^M$	
	& $\llbracket see \rrbracket^{M,g[x/b][y/b'],aa'}{}_P(b')(b) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
13.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$ & $b' = \llbracket sue \rrbracket^M$	
	& $1 = 1$	D2.3.$\boldsymbol{b}$
14.	$b = a$ & $\llbracket male \rrbracket^M(b) = 1$ & $b' = \llbracket sue \rrbracket^M$	simplify

□

Fact 2N: $M, g, bb'aa' \models_N he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$

Proof: (1) iff (13)

1.	$M, g, bb'aa' \models_N he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$	
2.	$[\![he_1(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M,\, g,\, bb'aa'}{}_N = 1$	D3
3.	$[\![he_1]\!]^{M,\, g,\, baa'}{}_N([\![(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M,\, g,\, b'aa'}{}_N) = 1$	D2.3.$\boldsymbol{f}$.γ/b. δ/b'. ε/aa'
4.	$[\![he_1]\!]^{M,\, baa'}{}_N([\![(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M,\, g,\, b'aa'}{}_N) = 1$	D2.3.$\boldsymbol{b}$
5.	$[\![\lambda x\, \neg[sue](\lambda y\ see\ yx)]\!]^{M,\, g,\, b'aa'}{}_N(b) = 1$	D2.2.he_m
6.	$[\![\neg[sue](\lambda y\ see\ yx)]\!]^{M,\, g[x/b],\, b'aa'}{}_N = 1$	D2.3.$\boldsymbol{\lambda}$
7.	$\exists\varepsilon \in (D_e{}^M)^0$: $[\![[sue](\lambda y\ see\ yx)]\!]^{M,\, g[x/b],\, \varepsilon b'aa'}{}_N = 1$	D2.3.$\neg$
8.	$[\![[sue](\lambda y\ see\ yx)]\!]^{M,\, g[x/b],\, b'aa'}{}_N = 1$	$(D_e{}^M)^0 = \{\langle\,\rangle\}$
9.	$[\![[sue]]\!]^{M,\, g[x/b],\, b'aa'}{}_N([\![\lambda y\ see\ yx]\!]^{M,\, g[x/b],\, aa'}{}_N) = 1$	D2.3.$\boldsymbol{f}$.γ/b'. δ/⟨⟩. ε/aa'
10.	$[\![\lambda y\ see\ yx]\!]^{M,\, g[x/b],\, aa'}{}_N(b') = 1$	D2.3.**[]**
11.	$[\![see\ yx]\!]^{M,\, g[x/b][y/b'],\, aa'}{}_N = 1$	D2.3.$\boldsymbol{\lambda}$
12.	$[\![see]\!]^{M,\, g[x/b][y/b'],\, aa'}{}_N(b')(b) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
13.	$1 = 1$	D2.3.$\boldsymbol{b}$

□

Fact 2A: $M, g, bb'aa' \models_A he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$
iff $[\![see]\!]^M(b')(b) = 0$

Proof: (1) iff (15)

1.	$M, g, bb'aa' \models_A he_1(\lambda x\, \neg([sue](\lambda y\ see\ yx)))$	
2.	$[\![he_1(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M, g, bb'aa'}{}_A = 1$	D3
3.	$[\![he_1]\!]^{M, g, baa'}{}_A([\![(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M, g, b'aa'}{}_A) = 1$	D2.3.$\boldsymbol{f}$
4.	$[\![he_1]\!]^{M, baa'}{}_A([\![(\lambda x\, \neg[sue](\lambda y\ see\ yx))]\!]^{M, g, b'aa'}{}_A) = 1$	D2.3.$\boldsymbol{b}$
5.	$[\![\lambda x\, \neg[sue](\lambda y\ see\ yx)]\!]^{M, g, b'aa'}{}_A(b) = 1$	D2.2.he_m
6.	$[\![\neg[sue](\lambda y\ see\ yx)]\!]^{M, g[x/b], b'aa'}{}_A = 1$	D2.3.$\boldsymbol{\lambda}$
7.	$\neg\exists\varepsilon \in (D_e{}^M)^0$: $[\![[sue](\lambda y\ see\ yx)]\!]^{M, g[x/b], \varepsilon b'aa'}{}_N = 1$ & $[\![[sue](\lambda y\ see\ yx)]\!]^{M, g[x/b], \varepsilon b'aa'}{}_A = 1$	D2.3.$\boldsymbol{\neg}$
8.	$\neg([\![[sue](\lambda y\ see\ yx)]\!]^{M, g[x/b], b'aa'}{}_N = 1$ & $[\![[sue](\lambda y\ see\ yx)]\!]^{M, g[x/b], b'aa'}{}_A = 1)$	$(D_e{}^M)^0 = \{\langle\,\rangle\}$
9.	$\neg([\![[sue]]\!]^{M, g[x/b], b'aa'}{}_N([\![\lambda y\ see\ yx]\!]^{M, g[x/b], aa'}{}_N) = 1$ & $[\![[sue]]\!]^{M, g[x/b], b'aa'}{}_A([\![\lambda y\ see\ yx]\!]^{M, g[x/b], aa'}{}_A) = 1$)	D2.3.$\boldsymbol{f}$
10.	$\neg([\![\lambda y\ see\ yx]\!]^{M, g[x/b], aa'}{}_N(b') = 1$ & $[\![\lambda y\ see\ yx]\!]^{M, g[x/b], aa'}{}_A(b') = 1)$	D2.3.[]
11.	$\neg([\![see\ yx]\!]^{M, g[x/b][y/b'], aa'}{}_N = 1$ & $[\![see\ yx]\!]^{M, g[x/b][y/b'], aa'}{}_A = 1)$	D2.3.$\boldsymbol{\lambda}$
12.	$\neg([\![see]\!]^{M, g[x/b][y/b'], aa'}{}_N(b')(b) = 1$ & $[\![see]\!]^{M, g[x/b][y/b'], aa'}{}_A(b')(b) = 1)$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
13.	$\neg(1 = 1$ & $[\![see]\!]^M(b')(b) = 1)$	D2.3.$\boldsymbol{b}$
14.	$[\![see]\!]^M{}_A(b')(b) \neq 1$	simplify
15.	$[\![see]\!]^M{}_A(b')(b) = 0$	D2.2.$[\![\cdot]\!]^M$, $D_t = \{1, 0\}$

□

VI. Indefinites in the 'scope' of {other indefinites, pronouns}

- [1](First) one boy hit another boy.

  $(3^1) \;:=\; \exists[boy](\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))$ PROOFS

Fact 3^1P: $M, g, aa' \models_P (3^1)$ exercise

Fact 3^1N: $M, g, aa' \models_N (3^1)$ p. 17
iff $[\![boy]\!]^M(a) = 1$ & $[\![boy]\!]^M(a') = 1$ & $[\![\neq]\!]^M(a')(a) = 1$

Fact 3^1A: $M, g, aa' \models_A (3^1)$ p. 18
iff $[\![hit]\!]^M(a')(a) = 1$

- …[2](And then) he was hit by a yet another boy.

  $(3^2) \;:=\; he_2(\lambda y\, \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))$

Fact 3^2P: $M, g, a'baaa' \models_P (3^2)$ p. 19
iff $[\![male]\!]^M(a') = 1$ & $[\![male]\!]^M(a) = 1$

Fact 3^2N: $M, g, a'baaa' \models_N (3^2)$ p. 20
iff $[\![boy]\!]^M(b) = 1$ & $[\![\neq]\!]^M(b)(a') = 1$ & $[\![\neq]\!]^M(b)(a') = 1$

Fact 3^2A: $M, g, a'baaa' \models_A (3^2)$ exercise
iff $[\![hit]\!]^M(a')(b) = 1$

- [1]First one boy hit another boy. [2]And then he was hit by a yet another boy.

  $(3) \;:=\; ((3^1) \wedge (3^2))$

Fact 3P: $M, g, a'baaa' \models_P (3^2)$ exercise
iff $[\![male]\!]^M(a') = 1$ & $[\![male]\!]^M(a) = 1$

Fact 3N: $M, g, a'baaa' \models_N (3^2)$ exercise
iff $[\![boy]\!]^M(a) = 1$ & $[\![boy]\!]^M(a') = 1$ & $[\![\neq]\!]^M(a')(a) = 1$
& $[\![boy]\!]^M(b) = 1$ & $[\![\neq]\!]^M(a')(b) = 1$ & $[\![\neq]\!]^M(a)(b) = 1$

Fact 3A: $M, g, a'baaa' \models_A (3^2)$ exercise
iff $[\![hit]\!]^M(a')(a) = 1$
& $[\![hit]\!]^M(a')(b) = 1$

Fact 3^1N: $M, g, aa' \models_N \exists[boy](\lambda x\ \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))$
iff $\llbracket boy \rrbracket^M(a) = 1$ & $\llbracket boy \rrbracket^M(a') = 1$ & $\llbracket \neq \rrbracket^M(a')(a) = 1$

Proof:

1.	$M, g, aa' \models_N \exists[boy](\lambda x\ \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))$	
2.	$\llbracket \exists[boy](\lambda x\ \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx)) \rrbracket^{M, g, aa'}{}_N = 1$	D3
3.	$\llbracket \exists[boy] \rrbracket^{M, g, a}{}_N(\llbracket \lambda x\ \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx) \rrbracket^{M, g, a'}{}_N) = 1$	D2.3.***f***.γ/*a*. δ/*a′*. ε/⟨⟩
4.	$\llbracket boy \rrbracket^{M, g, \langle\rangle}{}_N(a) = 1$	D2.3.**∃[**.*d*/*a*. ε/⟨⟩
	& $\llbracket boy \rrbracket^{M, g, \langle\rangle}{}_A(a) = 1$	
	& $\llbracket \lambda x\ \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx) \rrbracket^{M, g, a'}{}_N(a) = 1$	
5.	$1 = 1$	D2.3.***b***
	& $\llbracket boy \rrbracket^M(a) = 1$	
	& $\llbracket \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx) \rrbracket^{M, g[x/a], a'}{}_N = 1$	D2.3.**λ**.ε/*a′*. *d*/*a*
6.	$\llbracket boy \rrbracket^M(a) = 1$	simplify
	& $\llbracket \exists[\lambda y(boy\ y \wedge \neq yx)] \rrbracket^{M, g[x/a], a'}{}_N(\llbracket \lambda y\ hit\ yx \rrbracket^{M, g[x/a], \langle\rangle}{}_N) = 1$	D2.3.***f***.γ/*a′*. δ/⟨⟩. ε/⟨⟩
7.	$\llbracket boy \rrbracket^M(a) = 1$	
	& $\llbracket \lambda y(boy\ y \wedge \neq yx) \rrbracket^{M, g[x/a], \langle\rangle}{}_N(a') = 1$	D2.3.**∃[**.*d*/*a′*. ε/⟨⟩
	& $\llbracket \lambda y(boy\ y \wedge \neq yx) \rrbracket^{M, g[x/a], \langle\rangle}{}_A(a') = 1$	
	& $\llbracket \lambda y\ hit\ yx \rrbracket^{M, g[x/a], \langle\rangle}{}_N(a') = 1$	
8.	$\llbracket boy \rrbracket^M(a) = 1$	
	& $\llbracket boy\ y \wedge \neq yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$	D2.3.**λ**.ε/⟨⟩. *d*/*a′*
	& $\llbracket boy\ y \wedge \neq yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$	
	& $\llbracket hit\ yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$	
9.	$\llbracket boy \rrbracket^M(a) = 1$	
	& $\llbracket boy\ y \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$ & $\llbracket \neq yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$	D2.3.**∧**.δ/⟨⟩. ε/⟨⟩
	& $\llbracket boy\ y \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$ & $\llbracket \neq yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$	
	& $\llbracket hit\ yx \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N = 1$	
10.	$\llbracket boy \rrbracket^M(a) = 1$	
	& $\llbracket boy \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N(a') = 1$ & $\llbracket \neq \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N(a')(a) = 1$	D2.3.***f*, *b*,** D2.2.*g*[*u*/*d*]
	& $\llbracket boy \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_A(a') = 1$ & $\llbracket \neq \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_A(a')(a) = 1$	
	& $\llbracket hit \rrbracket^{M, g[x/a][y/a'], \langle\rangle}{}_N(a')(a) = 1$	
11.	$\llbracket boy \rrbracket^M(a) = 1$	
	& $1 = 1$ & $1 = 1$	D2.3.***b***
	& $\llbracket boy \rrbracket^M(a') = 1$ & $\llbracket \neq \rrbracket^M(a')(a) = 1$	
	& $1 = 1$	
12.	$\llbracket boy \rrbracket^M(a) = 1$	simplify
	& $\llbracket boy \rrbracket^M(a') = 1$ & $\llbracket \neq \rrbracket^M(a')(a) = 1$	□

Fact 3[1]A: $M, g, aa' \models_A \exists[boy](\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))$
iff $[\![hit]\!]^M(a')(a) = 1$

Proof:

1.	$M, g, aa' \models_A \exists[boy](\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))$	
2.	$[\![\exists[boy](\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx))]\!]^{M, g, aa'}{}_A = 1$	D3
3.	$[\![\exists[boy]]\!]^{M, g, a}{}_A([\![\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx)]\!]^{M, g, a'}{}_A) = 1$	D2.3.$\boldsymbol{f}$.γ/a. δ/a'. $\varepsilon/\langle\rangle$
4.	$[\![\lambda x\, \exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx)]\!]^{M, g, a'}(a) = 1$	D2.3.**∃[**.d/a. $\varepsilon/\langle\rangle$
5.	$[\![\exists[\lambda y(boy\ y \wedge \neq yx)](\lambda y\ hit\ yx)]\!]^{M, g[x/a], a'}{}_A = 1$	D2.3.$\boldsymbol{\lambda}$.ε/a'. d/a
6.	$[\![\exists[\lambda y(boy\ y \wedge \neq yx)]]\!]^{M, g[x/a], a'}{}_A([\![\lambda y\ hit\ yx]\!]^{M, g[x/a], \langle\rangle}{}_A) = 1$	D2.3.$\boldsymbol{f}$.γ/a'. $\delta/\langle\rangle$. $\varepsilon/\langle\rangle$
7.	$[\![\lambda y\ hit\ yx]\!]^{M, g[x/a], \langle\rangle}{}_A(a') = 1$	D2.3.**∃[.**d/a'. $\varepsilon/\langle\rangle$
8.	$[\![hit\ yx]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A = 1$	D2.3.$\boldsymbol{\lambda}$.$\varepsilon/\langle\rangle$, d/a'
9.	$[\![hit]\!]^{M, g[x/a][y/a'], \langle\rangle}{}_A(a')(a) = 1$	D2.3.$\boldsymbol{f}$**,** $\boldsymbol{b}$**,** D2.2.$g[u/d]$
10.	$[\![hit]\!]^M(a')(a) = 1$	D2.3.$\boldsymbol{b}$

□

Fact 3^2P: $M, g, a'baaa' \models_N he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))$
iff $\llbracket male \rrbracket^M(a') = 1\ \&\ \llbracket male \rrbracket^M(a) = 1$

Proof:

1. $M, g, a'baaa'$
$\models_P he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))$

2. $\llbracket he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx)) \rrbracket^{M, g, a'baaa'}{}_P = 1$ — D3

3. $\llbracket he_2 \rrbracket^{M, g, a'aa'}{}_P$
$(\llbracket \lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx) \rrbracket^{M, g, baaa'}{}_P) = 1$ — D2.3.***f***.$\gamma/a'.\delta/ba.\ \varepsilon/aa'$

4. $\llbracket he_2 \rrbracket^{M, a'aa'}{}_P$ — D2.3.***b***
$(\llbracket \lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx) \rrbracket^{M, g, baaa'}{}_P) = 1$

5. $a' = (aa')_2\ \&\ \llbracket male \rrbracket^M(a') = 1$ — D2.2.$he_2.d/a'.\ \varepsilon/aa'$
$\&\ \llbracket \lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx) \rrbracket^{M, g, baaa'}{}_P(a') = 1$

6. $\llbracket male \rrbracket^M(a') = 1$ — D2.2.δ_m, simplify
$\&\ \llbracket \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx) \rrbracket^{M, g[y/a'], baaa'}{}_P = 1$ — D2.3.**λ**.$\varepsilon/baaa'.\ d/a'$

7. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))] \rrbracket^{M, g[y/a'], baaa'}{}_P$ — D2.3.***f***.$\gamma/ba.\ \delta/\langle\rangle.\varepsilon/aa'$
$(\llbracket \lambda x\ hit\ yx \rrbracket^{M, g[y/a'], aa'}{}_P) = 1$

8. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket \lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz)) \rrbracket^{M, g[y/a'], aaa'}{}_P(b) = 1$ — D2.3.**∃[**.$d/b.\ \varepsilon/aaa'$
$\&\ \llbracket \lambda x\ hit\ yx \rrbracket^{M, g[y/a'], aa'}{}_P(b) = 1$

9. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz) \rrbracket^{M, g[y/a'][x/b], aaa'}{}_P = 1$ — D2.3.**λ**.$d/b.\ \varepsilon/aaa'$
$\&\ \llbracket hit\ yx \rrbracket^{M, g[y/a'][x/b], aa'}{}_P = 1$

10. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket boy\ x \wedge \neq xy \rrbracket^{M, g[y/a'][x/b], aa'}{}_P = 1$ — D2.3.**∧**.$\delta/a.\ \varepsilon/aa'$
$\&\ \llbracket he_1(\lambda z \neq xz) \rrbracket^{M, g[y/a'][x/b], aaa'}{}_P = 1$
$\&\ \llbracket hit \rrbracket^{M, g[y/a'][x/b], aa'}{}_P(a')(a) = 1$ — D2.3.***f***, ***b***, D2.2.$g[u/d]$

11. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket boy\ x \rrbracket^{M, g[y/a'][x/b], aa'}{}_P = 1\ \&\ \llbracket \neq xy \rrbracket^{M, g[y/a'][x/b], aa'}{}_P = 1$ — D2.3.**∧**.$\delta/\langle\rangle.\ \varepsilon/aa'$
$\&\ \llbracket he_1 \rrbracket^{M, g[y/a'][x/b], aaa'}{}_P(\llbracket \lambda z \neq xz \rrbracket^{M, g[y/a'][x/b], aa'}{}_P) = 1$ — D2.3.***f***.$\gamma/a.\ \delta/\langle\rangle.\ \varepsilon/aa'$
$\&\ 1 = 1$ — D2.3.***b***

12. $\llbracket male \rrbracket^M(a') = 1$
$\&\ \llbracket boy \rrbracket^{M, g[y/a'][x/b], aa'}{}_P(b) = 1\ \&\ \llbracket \neq \rrbracket^{M, g[y/a'][x/b], aa'}{}_P(b)(a') = 1$ — D2.3.***f***, ***b***, D2.2.$g[u/d]$
$\&\ a = (aa')_1\ \&\ \llbracket male \rrbracket^M(a) = 1$ — D2.3.***b***, D2.2.he_n
$\&\ \llbracket \lambda z \neq xz \rrbracket^{M, g[y/a'][x/b], aa'}{}_P(a) = 1$

13. $\llbracket male \rrbracket^M(a') = 1$ — D2.3.***b***, simplify
$\&\ \llbracket male \rrbracket^M(a) = 1$ — D2.3.***b***, D2.2.he_n

D2.3.**λ**, ***f***, ***b***, D2.2.$g[u/d$
D2.3.***b***, simplify □

Fact 3^2N: $M, g, a'baaa' \models_{N} he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))$
iff $[\![boy]\!]^M(b) = 1\ \&\ [\![\neq]\!]^M(b)(a') = 1\ \&\ [\![\neq]\!]^M(b)(a') = 1$

Proof:

1. $M, g, a'baaa'$
$\models_{N} he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))$

2. $[\![he_2(\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx))]\!]^{M, g, a'baaa'}{}_{N} = 1$ D3

3. $[\![he_2]\!]^{M, g, a'aa'}{}_{N}$
$([\![\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx)]\!]^{M, g, baaa'}{}_{N}) = 1$ D2.3.$\boldsymbol{f}$.γ/a'.δ/ba. ε/aa'

4. $[\![he_2]\!]^{M, a'aa'}{}_{N}$ D2.3.$\boldsymbol{b}$
$([\![\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx)]\!]^{M, g, baaa'}{}_{N}) = 1$

5. $[\![\lambda y\ \exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx)]\!]^{M, g, baaa'}{}_{N}(a') = 1$ D2.2.he_2.d/a'. ε/aa'

6. $[\![\exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))](\lambda x\ hit\ yx)]\!]^{M, g[y/a'], baaa'}{}_{N} = 1$ D2.3.$\boldsymbol{\lambda}$.$\varepsilon/baaa'$. d/a'

7. $[\![\exists[\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))]]\!]^{M, g[y/a'], baaa'}{}_{N}$ D2.3.$\boldsymbol{f}$.γ/ba. $\delta/\langle\rangle$.ε/aa'
$([\![\lambda x\ hit\ yx]\!]^{M, g[y/a'], aa'}{}_{N}) = 1$

8. $[\![\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))]\!]^{M, g[y/a'], aaa'}{}_{N}(b) = 1$ D2.3.$\boldsymbol{\exists[}$.d/b. ε/aaa'
$\&\ [\![\lambda x(boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz))]\!]^{M, g[y/a'], aaa'}{}_{A}(b) = 1$
$\&\ [\![\lambda x\ hit\ yx]\!]^{M, g[y/a'], aa'}{}_{N}(b) = 1$

9. $[\![boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz)]\!]^{M, g[y/a'][x/b], aaa'}{}_{N} = 1$ D2.3.$\boldsymbol{\lambda}$.d/b. ε/aaa'
$\&\ [\![boy\ x \wedge \neq xy \wedge he_1(\lambda z \neq xz)]\!]^{M, g[y/a'][x/b], aaa'}{}_{A} = 1$ D2.3.$\boldsymbol{\lambda}$.d/b. ε/aaa'
$\&\ [\![hit\ yx]\!]^{M, g[y/a'][x/b], aa'}{}_{N} = 1$

10. $[\![boy\ x \wedge \neq xy]\!]^{M, g[y/a'][x/b], aa'}{}_{N} = 1$ D2.3.$\boldsymbol{\wedge}$.δ/a. ε/aa'
$\&\ [\![he_1(\lambda z \neq xz)]\!]^{M, g[y/a'][x/b], aaa'}{}_{N} = 1$
$\&\ [\![boy\ x \wedge \neq xy]\!]^{M, g[y/a'][x/b], aa'}{}_{A} = 1$
$\&\ [\![he_1(\lambda z \neq xz)]\!]^{M, g[y/a'][x/b], aaa'}{}_{A} = 1$
$\&\ [\![hit]\!]^{M, g[y/a'][x/b], aa'}{}_{N}(a')(a) = 1$ D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$

11. $[\![boy\ x]\!]^{M, g[y/a'][x/b], aa'}{}_{N} = 1\ \&\ [\![\neq xy]\!]^{M, g[y/a'][x/b], aa'}{}_{N} = 1$ D2.3.$\boldsymbol{\wedge}$.$\delta/\langle\rangle$. ε/aa'
$\&\ [\![he_1]\!]^{M, g[y/a'][x/b], aaa'}{}_{N}([\![\lambda z \neq xz]\!]^{M, g[y/a'][x/b], aa'}{}_{N}) = 1$ D2.3.$\boldsymbol{f}$.γ/a. $\delta/\langle\rangle$. ε/aa'
$\&\ [\![boy\ x]\!]^{M, g[y/a'][x/b], aa'}{}_{A} = 1\ \&\ [\![\neq xy]\!]^{M, g[y/a'][x/b], aa'}{}_{A} = 1$
$\&\ [\![he_1]\!]^{M, g[y/a'][x/b], aaa'}{}_{A}([\![\lambda z \neq xz]\!]^{M, g[y/a'][x/b], aa'}{}_{A}) = 1$
$\&\ 1 = 1$ D2.3.$\boldsymbol{b}$

12. $[\![boy]\!]^{M, g[y/a'][x/b], aa'}{}_{N}(b) = 1\ \&\ [\![\neq]\!]^{M, g[y/a'][x/b], aa'}{}_{N}(b)(a') = 1$ D2.3.$\boldsymbol{\lambda}$, $\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
$\&\ [\![\neq]\!]^{M, g[y/a'][x/b], aa'}{}_{N}(b)(a) = 1$
$\&\ [\![boy]\!]^{M, g[y/a'][x/b], aa'}{}_{A}(b) = 1\ \&\ [\![\neq]\!]^{M, g[y/a'][x/b], aa'}{}_{A}(b)(a') = 1$
$\&\ [\![\neq]\!]^{M, g[y/a'][x/b], aa'}{}_{A}(b)(a) = 1$

13. $[\![boy]\!]^M(b) = 1\ \&\ [\![\neq]\!]^M(b)(a') = 1$ D2.3.$\boldsymbol{b}$, simplify
$\&\ [\![\neq]\!]^M(b)(a) = 1$ □

QPLA (3): Quantifiers and higher order drefs

I. Reference to functions and (sub)domains

(1) Most businessmen who sent a contribution to Gore sent a larger contribution to Bush. (after Dekker 2003a, (16))

- $most[\lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))]$
  $(\lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y)](\lambda y\ [bush](\lambda z\ snd.to.\ zyx))))$

Abbreviation: Set $A := \{d \in D_e^M\!: A(d) = 1\}$

(1P) $M, g, Af'fbfa \models_P (1^1)$ a la Dekker 2003a:13
if $a = [\![al]\!]^M$ & $b = [\![bush]\!]^M$
& $\emptyset \subset$ Set $A = \{d \in D_e^M |\ [\![bmn]\!]^M(d) = 1$ & $\exists! c\!: [\![con]\!]^M(c) = 1$ & $[\![snd.to]\!]^M(a)(c)(d) = 1\}$
& Dom f = Set A & $\forall d \in$ Dom $f\!: [\![con]\!]^M(f(d)) = 1$ & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$

(1N) $M, g, Af'fbfa \models_N (1^1)$
if $\forall d \in$ Set $A\!: [\![con]\!]^M(f'(d)) = 1$ & $[\![lrg]\!]^M(f(d))(f'(d)) = 1$

(1A) $M, g, Af'fbfa \models_A (1^1)$
if $|\{d \in \text{Set } A|\ [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}| > |\{d \in \text{Set } A|\ [\![snd.to]\!]^M(b)(f'(d))(d) \neq 1\}|$

Good: Dekker's story explains the uniqueness requirement that native speakers often impose on domains of quantification—here, unique contribution ($\exists! c$) for every businessman in Set A.

Bad: According to Dekker's story, Set $A \subseteq$ Dom f'. So (1) should license not only anaphora to contributions by businessmen who do satisfy the nuclear scope, as in (2^2), but also by those who don't, as in (3^2). But intuitively, (1) only licenses anaphora of type (2^2), not (3^2).

(2) [1]Most businessmen who sent a contribution to Gore sent a larger contribution to Bush.
[2]Most of them asked Bush to spend *it* in their local area.

(3) [1]Most businessmen who sent a contribution to Gore sent a larger contribution to Bush.
\# [2]Most of the others didn't send *it*, because they disapproved of Bush's handling of Iraq.

***Paper topic*:** Try to implement the "1st amendment" suggested by Dekker 2003a:10 so that you get an intuitively correct result for (1), e.g.:

(1P′) $M, g, BAf'fbfa \models_P (1)$
if $a = [\![al]\!]^M$ & $b = [\![bush]\!]^M$
& $\emptyset \subset$ Set $A = \{d \in D_e^M |\ [\![bmn]\!]^M(d) = 1$ & $\exists! c\!: [\![con]\!]^M(c) = 1$ & $[\![snd.to]\!]^M(a)(c)(d) = 1\}$
& Dom f = Set A & $\forall d \in$ Dom $f\!: [\![con]\!]^M(f(d)) = 1$ & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$

(1N′) $M, g, BAf'fbfa \models_N (1)$
if $\emptyset \subset$ Set $B = \{d \in \text{Set } A|\ \exists c'\!: [\![con]\!]^M(c') = 1$ & $[\![lrg]\!]^M(f(d))(c')) = 1$
& $[\![snd.to]\!]^M(b)(c')(d) = 1\}$
& Dom f' = Set B & $\forall d \in$ Dom $f'\!: [\![con]\!]^M(f'(d)) = 1$ & $[\![lrg]\!]^M(f(d))(f'(d)) = 1$

(1A′) $M, g, BAf'fbfa \models_A (1)$
if $|\text{Set } A \cap \text{Set } B| > |\text{Set } A - \text{Set } B|$

II. QPLA (v. 2): Fold in quantifiers & *et*- and *ee*-drefs (see Dekker 2003a:10–13)

DEFINITION 1.1 (QPLA-types)

- $e, t \in \mathbf{Typ}$
- $(\sigma\tau) \in \mathbf{Typ}$, if $\sigma, \tau \in \mathbf{Typ}$

DEFINITION 1.2 (QPLA-basic terms)

- $\mathbf{Con}_e = \{bill, sue, \ldots\}$
  $\mathbf{Con}_{et} = \{bar, male, fem, \ldots\}$
  $\mathbf{Con}_{eet} = \{enter, see, \ldots\}$
  $\mathbf{Con}_{eeet} = \{snd.to, \ldots\}$
  $\mathbf{Con}_{(et)(et)t} = \{all, most, \ldots\}$
- $\mathbf{Var}_e = \{x, y, z, \ldots\}$
- $\mathbf{Prn}_{(et)t} = \{he_1 \ldots, she_1, \ldots, it_1, \ldots\}$

DEFINITION 1.2 (QPLA-terms and ∃-number).

b	$\alpha \in \mathbf{Term}_\tau$,	$n(\alpha) = 0$	if $\alpha \in \mathbf{Con}_\tau \cup \mathbf{Var}_\tau \cup \mathbf{Prn}_\tau$
f	$\alpha\beta \in \mathbf{Term}_\tau$	$n(\alpha\beta) = n(\alpha) + n(\beta)$	if $\alpha \in \mathbf{Term}_{(\sigma\tau)}$ & $\beta \in \mathbf{Term}_\sigma$
!	$!\beta \in \mathbf{Term}_{et}$	$n(!\beta) = n(\beta)$	if $\beta \in \mathbf{Term}_{et}$
[]	$[\alpha] \in \mathbf{Term}_{(et)t}$	$n([\alpha]) = n(\alpha)$	if $\alpha \in \mathbf{Con}_e$
α[	$\alpha[\beta] \in \mathbf{Term}_{(et)t}$	$n(\alpha[\beta]) = n(\alpha) + n(\beta)$	if $\alpha \in \mathbf{Term}_{(et)(et)t}$ & $\beta \in \mathbf{Term}_{et}$
ι[	$\iota[\beta] \in \mathbf{Term}_{(et)t}$	$n(\iota[\beta]) = n(\beta)$	if $\beta \in \mathbf{Term}_{et}$
∃[	$\exists[\beta] \in \mathbf{Term}_{(et)t}$	$n(\exists[\beta]) = n(\beta) + 1$	if $\beta \in \mathbf{Term}_{et}$
∃	$\exists u\phi \in \mathbf{Term}_t$	$n(\exists u\phi) = n(\phi) + 1$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
λ	$(\lambda u\phi) \in \mathbf{Term}_{(et)}$	$n(\lambda u\phi) = n(\phi)$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
¬	$\neg\phi \in \mathbf{Term}_t$	$n(\neg\phi) = 0$	if $\phi \in \mathbf{Term}_t$
∧	$(\phi \wedge \psi) \in \mathbf{Term}_t$	$n(\phi \wedge \psi) = n(\phi) + n(\psi)$	if $\phi, \psi \in \mathbf{Term}_t$

DEFINITION 2.1 (frames). A QPLA-*frame* is a set of τ-domains, $\{D_\tau: \tau \in \mathbf{Typ}\}$ s.t.

- 1 $D_t = \{1, 0\}$
  D_e is a non-empty set disjoint from D_t
- 2 $D_{(\sigma\tau)} = \{f \mid \varnothing \subset \text{Dom } f \subseteq D_\sigma \ \&\ \text{Ran } f \subseteq D_\tau\}$ NB! partial functions

DEFINITION 2.2 (models, assignments, and stacks)

- 1 A QPLA-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
  (i) $D^M = \{D_\tau^M: \tau \in \mathbf{Typ}\}$ is a QPLA-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_\tau$ a denotation $[\![\alpha]\!]^M \in D_\tau^M$
  (iii) Set $F := \{d \in D_e^M \mid F(d) = 1\}$, for any $F \in D_{et}^M$. Moreover, if $F, F' \in D_{et}^M$,
  $[\![all]\!]^M(F)(F') = 1$ iff Set $F \subseteq$ Set F'
  $[\![most]\!]^M(F)(F') = 1$ iff $|\text{Set } F \cap \text{Set } F'| > |\text{Set } F - \text{Set } F'|$
  ⋮
- 2 An *M-assignment* is a function g that assigns to any $u \in \mathbf{Var}_\tau$ a value $g(u) \in D_\tau^M$. If $d \in D_\tau^M$, then $g[u/d]$ is the M-assignment s.t. (i) $g[u/d](u) = d$, and (ii) $g[u/d](u') = g(u')$ if $u' \neq u$.
- 3 (i) $R^M := D_e^M \cup D_{ee}^M \cup D_{et}^M$ is the *domain* of *drefs*
  $\rho \in (R^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *n-stack* of drefs
  $\rho_m := r_m$, if $\rho = \langle r_1, \ldots r_n\rangle \in (R^M)^n$ & $1 \leq m \leq n$
  $\rho\rho' := \langle r_1, \ldots r_n, r'_1, \ldots r'_m\rangle$ if $\rho = \langle r_1, \ldots r_n\rangle \in (R^M)^n$ & $\rho' = \langle r'_1, \ldots r'_n\rangle \in (R^M)^m$
  (ii) $\langle\rangle(d) := \langle\ \rangle$ if $d \in D_e^M$
  $c(d) := c$ if $c, d \in D_e^M$
  $\rho(d) := \langle r_1(d), \ldots r_n(d)\rangle$ if $\rho = \langle r_1, \ldots r_n\rangle \in (D_e^M \cup D_{ee}^M)^n$ & $d \in D_e^M$

(iii) $\llbracket he_m \rrbracket^{M, d\rho}{}_{\text{P}}(F) = 1$, iff $d = \rho_m$ & $\llbracket male \rrbracket^{M}(d) = 1$ & $F(d) = 1$

$\llbracket he_m \rrbracket^{M, d\rho}{}_{\text{N}}(F) = 1$, iff $F(d) = 1$

$\llbracket he_m \rrbracket^{M, d\rho}{}_{\text{A}}(F) = 1$, iff $F(d) = 1$

$\llbracket she_m \rrbracket^{M, d\rho}{}_{\text{P}}(F) = 1$, iff $d = \rho_m$ & $\llbracket fem \rrbracket^{M}(d) = 1$ & $F(d) = 1$

$\llbracket she_m \rrbracket^{M, d\rho}{}_{\text{N}}(F) = 1$, iff $F(d) = 1$

$\llbracket she_m \rrbracket^{M, d\rho}{}_{\text{A}}(F) = 1$, iff $F(d) = 1$

$\llbracket it_m \rrbracket^{M, d\rho}{}_{\text{P}}(F) = 1$, iff $d = \rho_m$ & $F(d) = 1$

$\llbracket it_m \rrbracket^{M, d\rho}{}_{\text{N}}(F) = 1$, iff $F(d) = 1$

$\llbracket it_m \rrbracket^{M, d\rho}{}_{\text{A}}(F) = 1$, iff $F(d) = 1$

DEFINITION 2.3 (QPLA-semantics). Let $c, d \in D_e{}^M$, $f \in D_{ee}{}^M$, $F \in D_{et}{}^M$, $r \in R^M$, $\rho, \rho', \rho'' \in (R^M)^n$:

r $\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{X}} \in D_\tau{}^M$ if $\text{X} \in \{\text{P, N, A}\}$

b $\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{X}} = \llbracket \alpha \rrbracket^M$ if $\alpha \in \mathbf{Con}_e$ & $\text{X} \in \{\text{P, N, A}\}$

$\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{X}} = g(\alpha)$ if $\alpha \in \mathbf{Var}_\tau$ & $\text{X} \in \{\text{P, N, A}\}$

$\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{X}} = \llbracket \alpha \rrbracket^{M, \rho}{}_{\text{X}}$ if $\alpha \in \mathbf{Prn}_\tau \cup$ & $\text{X} \in \{\text{P, N, A}\}$

$\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{P}}(b_n)\ldots(b_1) = 1$ if $\alpha \in \mathbf{Con}_{\sigma 1 \ldots \sigma n\, t}$, $b_1 \in D_{\sigma 1}{}^M, \ldots, b_n \in D_{\sigma n}{}^M$

$\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{N}}(b_n)\ldots(b_1) = 1$

$\llbracket \alpha \rrbracket^{M, g, \rho}{}_{\text{A}} = \llbracket \alpha \rrbracket^M$

f $\llbracket \alpha\beta \rrbracket^{M, g, \rho\rho'\rho''}{}_{\text{X}} = \llbracket \alpha \rrbracket^{M, g, \rho\rho''}{}_{\text{X}}(\llbracket \beta \rrbracket^{M, g, \rho'\rho''}{}_{\text{X}})$ if $\text{X} \in \{\text{P, N, A}\}$

! $\llbracket !\beta \rrbracket^{M, g, f\rho}(d) = 1$ iff $d \in \text{Dom}\, f$ see Dekker 2003a:10

& $\text{Dom}\, f = \{c \in D_e{}^M \mid \exists! c' \text{: } \llbracket \beta \rrbracket^{M, g, c'\rho(c)}{}_{\text{P}}(c) = 1$
& $\llbracket \beta \rrbracket^{M, g, c'\rho(c)}{}_{\text{N}}(c) = 1\}$
& $\llbracket \beta \rrbracket^{M, g, c'\rho(c)}{}_{\text{A}}(c) = 1\}$

& $\forall c \in \text{Dom}\, f$: $\llbracket \beta \rrbracket^{M, g, f(c)\rho(c)}{}_{\text{P}}(c) = 1$
& $\llbracket \beta \rrbracket^{M, g, f(c)\rho(c)}{}_{\text{N}}(c) = 1$
& $\llbracket \beta \rrbracket^{M, g, f(c)\rho(c)}{}_{\text{A}}(c) = 1$

[] $\llbracket [\alpha] \rrbracket^{M, g, r\rho}{}_{\text{P}}(F) = 1$ iff $r = \llbracket \alpha \rrbracket^M$ & $F(r) = 1$

$\llbracket [\alpha] \rrbracket^{M, g, r\rho}{}_{\text{N}}(F) = 1$ iff $F(r) = 1$

$\llbracket [\alpha] \rrbracket^{M, g, r\rho}{}_{\text{A}}(F) = 1$ iff $F(r) = 1$

α[$\llbracket \alpha[\beta] \rrbracket^{M, g, r\rho}{}_{\text{P}}(F) = 1$ iff $\varnothing \subset \text{Set}\, r = \text{Set}\, \llbracket !\beta \rrbracket^{M, g, \rho}$ see Dekker 2003a:12
& $\forall d \in \text{Set}\, r$: $F(d) = 1$ on DET(π)

$\llbracket \alpha[\beta] \rrbracket^{M, g, r\rho}{}_{\text{N}}(F) = 1$ iff $\forall d \in \text{Set}\, r$: $F(d) = 1$

$\llbracket \alpha[\beta] \rrbracket^{M, g, r\rho}{}_{\text{A}}(F) = 1$ iff $\llbracket \alpha \rrbracket^M(r)(F) = 1$

***ι*[** $\llbracket \iota[\beta] \rrbracket^{M, g, r\rho}{}_{\text{P}}(F) = 1$ iff $\exists\rho'$: $\rho'(r) = \rho$ & $\{r\} = \text{Set}\, \llbracket !\beta \rrbracket^{M, g, \rho'}$ see Dekker 2003a:11
& $F(r) = 1$ on THE(π)

$\llbracket \iota[\beta] \rrbracket^{M, g, r\rho}{}_{\text{N}}(F) = 1$ iff $F(r) = 1$

$\llbracket \iota[\beta] \rrbracket^{M, g, r\rho}{}_{\text{A}}(F) = 1$ iff $F(r) = 1$

∃[$\llbracket \exists[\beta] \rrbracket^{M, g, r\rho}{}_{\text{P}}(F) = 1$ iff $\llbracket \beta \rrbracket^{M, g, \rho}{}_{\text{P}}(r) = 1$ & $F(r) = 1$

$\llbracket \exists[\beta] \rrbracket^{M, g, r\rho}{}_{\text{N}}(F) = 1$ iff $\llbracket \beta \rrbracket^{M, g, \rho}{}_{\text{N}}(r) = 1$ & $\llbracket \beta \rrbracket^{M, g, \rho}{}_{\text{A}}(r) = 1$ & $F(r) = 1$

$\llbracket \exists[\beta] \rrbracket^{M, g, r\rho}{}_{\text{A}}(F) = 1$ iff $F(r) = 1$

$\exists$ $[\![\exists u\phi]\!]^{M, g, r\rho}{}_{X} = 1$ iff $[\![\phi]\!]^{M, g[u/r], \rho}{}_{X} = 1$

λ $[\![(\lambda u\phi)]\!]^{M, g, \rho}{}_{X}(d) = 1$ iff $[\![\phi]\!]^{M, g[u/d], \rho(d)}{}_{X} = 1$ $\qquad \rho \in (D_e{}^M \cup D_{ee}{}^M)^n$

$\neg$ $[\![\neg\phi]\!]^{M, g, \rho}{}_{P} = 1$ iff $\exists\delta \in (D_e{}^M)^{n(\phi)}$: $[\![\phi]\!]^{M, g, \delta\rho}{}_{P} = 1$
$[\![\neg\phi]\!]^{M, g, \rho}{}_{N} = 1$ iff $\exists\delta \in (D_e{}^M)^{n(\phi)}$: $[\![\phi]\!]^{M, g, \delta\rho}{}_{N} = 1$
$[\![\neg\phi]\!]^{M, g, \rho}{}_{A} = 1$ iff $\neg\exists\delta \in (D_e{}^M)^{n(\phi)}$: $[\![\phi]\!]^{M, g, \delta\rho}{}_{N} = 1$ & $[\![\phi]\!]^{M, g, \delta\rho}{}_{A} = 1$

$\wedge$ $[\![(\phi \wedge \psi)]\!]^{M, g, \rho'\rho}{}_{X} = 1$ iff $[\![\phi]\!]^{M, g, \rho}{}_{X} = 1$ & $[\![\psi]\!]^{M, g, \rho'\rho}{}_{X} = 1$

DEFINITION 3 (Satisfaction, felicity and truth)

- ρ satisfies the presupposed background of ϕ in *M* under *g*, written $M, g, \rho \models_P \phi$, iff $[\![\phi]\!]^{M, g, \rho}{}_{P} = 1$
  ρ satisfies the new background of ϕ in *M* under *g*, written $M, g, \rho \models_N \phi$, iff $[\![\phi]\!]^{M, g, \rho}{}_{N} = 1$
- ρ satisfies the background of ϕ (makes ϕ felicitous) in *M* under *g*, written $M, g, \rho \models_B \phi$, iff $[\![\phi]\!]^{M, g, \rho}{}_{P} = 1$ & $[\![\phi]\!]^{M, g, \rho}{}_{N} = 1$
- ρ satisfies the focal assertion of ϕ (makes ϕ true) in *M* under *g*, written $M, g, \rho \models_A \phi$, iff $[\![\phi]\!]^{M, g, \rho}{}_{A} = 1$

ABBREVIATIONS

A1. $(\phi \rightarrow \psi)$:= $\neg(\phi \wedge \neg\psi)$
A2. $\forall u\phi$:= $\neg\exists u\neg\phi$

III. Detailed analysis of (1)

(1) Most [businessmen who sent a contribution to Al Gore][sent a larger contribution to Bush]

- np := $(\lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx))))$ — proofs

  $[\![\text{np}]\!]^{M, g, fa}{}_{P}(d) = 1$ if $a = [\![al]\!]^M$ — p. 25
  $[\![\text{np}]\!]^{M, g, fa}{}_{N}(d) = 1$ if $[\![con]\!]^M(f(d)) = 1$ — p. 26
  $[\![\text{np}]\!]^{M, g, fa}{}_{A}(d) = 1$ if $[\![bmn]\!]^M(d) = 1$ & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$ — p. 27

- vp := $(\lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y)](\lambda y\ [bush](\lambda z\ snd.to.\ zyx))))$

  $[\![\text{vp}]\!]^{M, g, f'fbfa}{}_{P}(d) = 1$ if $b = [\![bush]\!]^M$ — p. 28
  $[\![\text{vp}]\!]^{M, g, f'fbfa}{}_{N}(d) = 1$ if $[\![con]\!]^M(f'(d)) = 1$ & $[\![lrg]\!]^M(f(d))(f'(d)) = 1$ — p. 29
  $[\![\text{vp}]\!]^{M, g, f'fbfa}{}_{A}(d) = 1$ if $[\![snd.to]\!]^M(b)(f'(d))(d) = 1$ — p. 30

- (1′) := *most*[np]vp

(1P) $M, g, Af'fbfa \models_P (1^1)$ — p. 31
if $a = [\![al]\!]^M$ & $b = [\![bush]\!]^M$
& $\emptyset \subset$ Set $A = \{d \in D_e{}^M \mid [\![bmn]\!]^M(d) = 1$ & $\exists!c$: $[\![con]\!]^M(c) = 1$ & $[\![snd.to]\!]^M(a)(c)(d) = 1\}$
& Dom f = Set A & $\forall d \in$ Dom f: $[\![con]\!]^M(f(d)) = 1$ & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$

(1N) $M, g, Af'fbfa \models_N (1^1)$ — p. 32
if $\forall d \in$ Set A: $[\![con]\!]^M(f'(d)) = 1$ & $[\![lrg]\!]^M(f(d))(f'(d)) = 1$

(1A) $M, g, Af'fbfa \models_A (1^1)$ — p. 32
if $|\{d \in \text{Set } A \mid [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}|$
$> |\{d \in \text{Set } A \mid [\![snd.to]\!]^M(b)(f'(d))(d) \neq 1\}|$

Fact 1nP: Let $f \in D_{ee}{}^{M}$, a, $d \in D_{e}{}^{M}$. Then:
$[\![\lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))]\!]^{M,\,g,\,fa}{}_{P}\,(d) = 1$
if $a = [\![al]\!]^{M}$

Proof: (1) if (12):

1.	$[\![\lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))]\!]^{M,\,g,\,fa}{}_{P}\,(d) = 1$	
2.	$[\![bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx))]\!]^{M,\,g[x/d],\,f(d)a(d)}{}_{P} = 1$	D2.3.$\boldsymbol{\lambda}$
3.	$[\![bmn\ x]\!]^{M,\,g[x/d],\,\langle\rangle}{}_{P} = 1$	D2.3.$\boldsymbol{\wedge}$
	& $[\![\exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx))]\!]^{M,\,g[x/d],\,f(d)a}{}_{P} = 1$	D2.2.$c(d)$
4.	$[\![bmn]\!]^{M,\,g[x/d],\,\langle\rangle}{}_{P}(d) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
	& $[\![\exists[con]\,]\!]^{M,\,g[x/d],\,f(d)}{}_{P}([\![\lambda y\ [al](\lambda z\ snd.to\ zyx))]\!]^{M,\,g[x/d],\,a}{}_{P} = 1$	D2.3.$\boldsymbol{f}$.ρ/$f(d)$. ρ′/a. ρ″/⟨⟩
5.	1 = 1	D2.3.$\boldsymbol{b}$
	& $[\![con]\!]^{M,\,g[x/d],\,\langle\rangle}{}_{P}(f(d)) = 1$	D2.3.$\boldsymbol{\exists[}$
	& $[\![\lambda y[al](\lambda z\ snd.to\ zyx)]\!]^{M,\,g[x/d],\,a}{}_{P}(f(d)) = 1$	
7.	1 = 1	
	& 1 = 1	D2.3.$\boldsymbol{b}$
	& $[\![[al](\lambda z\ snd.to\ zyx)]\!]^{M,\,g[x/d][y/f(d)],\,a}{}_{P} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
8.	$[\![[al]]\!]^{M,\,g[x/a][y/f(d)],\,a}{}_{P}([\![\lambda z\ snd.to\ zyx]\!]^{M,\,g[x/d][y/f(d)],\,\langle\rangle}{}_{P}) = 1$	D2.3.$\boldsymbol{f}$, simplify
9.	$a = [\![al]\!]^{M}$	D2.3.**[]**
	& $[\![\lambda z\ snd.to\ zyx]\!]^{M,\,g[x/d][y/f(d)],\,\langle\rangle}{}_{P}(a) = 1$	
10.	$a = [\![al]\!]^{M}$	
	& $[\![snd.to\ zyx]\!]^{M,\,g[x/d][y/f(d)][z/a],\,\langle\rangle}{}_{P} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
11.	$a = [\![al]\!]^{M}$	
	& $[\![snd.to]\!]^{M,\,g[x/d][y/f(d)][z/a],\,\langle\rangle}{}_{P}(a)(f(d))(d) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
12.	$a = [\![al]\!]^{M}$	D2.3.$\boldsymbol{b}$, simplify

□

Fact 1nN: Let $f \in D_{ee}{}^{M}$, $a, d \in D_{e}{}^{M}$. Then:

$\llbracket \lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g,\, fa}{}_{\mathrm{N}}(d) = 1$

iff $\llbracket con\rrbracket^{M}(f(d)) = 1$

Proof:

1.	$\llbracket \lambda x\ (bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g,\, fa}{}_{\mathrm{N}}(d) = 1$	
2.	$\llbracket bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx))\rrbracket^{M,\, g[x/d],\, f(d)a}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
3.	$\llbracket bmn\ x\rrbracket^{M,\, g[x/d],\, \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\wedge}$
	& $\llbracket \exists[con](\lambda y[al](\lambda z\ snd.to\ zyx))\rrbracket^{M,\, g[x/d],\, f(d)a}{}_{\mathrm{N}} = 1$	
4.	$\llbracket bmn\rrbracket^{M,\, g[x/d],\, \langle\rangle}{}_{\mathrm{N}}(d) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
	& $\llbracket \exists[con]\ \rrbracket^{M,\, g[x/d],\, f(d)}{}_{\mathrm{N}}(\llbracket \lambda y\ [al](\lambda z\ snd.to\ zyx))\rrbracket^{M,\, g[x/d],\, a}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{f}$
5.	$1 = 1$	D2.3.$\boldsymbol{b}$
	& $\llbracket con\rrbracket^{M,\, g[x/d],\, \langle\rangle}{}_{\mathrm{N}}(f(d)) = 1$	D2.3.$\boldsymbol{\exists[}$
	& $\llbracket con\rrbracket^{M,\, g[x/d],\, \langle\rangle}{}_{\mathrm{A}}(f(d)) = 1$	
	& $\llbracket \lambda y\ [al](\lambda z\ snd.to\ zyx)\rrbracket^{M,\, g[x/d],\, a}{}_{\mathrm{N}}(f(d)) = 1$	
6.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	D2.3.$\boldsymbol{b}$, simplify
	& $\llbracket [al](\lambda z\ snd.to\ zyx)\rrbracket^{M,\, g[x/d][y/f(d)],\, a}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
7.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	
	& $\llbracket [al]\rrbracket^{M,\, g[x/a][y/f(d)],\, a}{}_{\mathrm{N}}(\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f(d)],\, \langle\rangle}{}_{\mathrm{N}}) = 1$	D2.3.$\boldsymbol{f}$
8.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	
	& $\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f(d)],\, \langle\rangle}{}_{\mathrm{N}}(a) = 1$	D2.3.**[]**
9.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	
	& $\llbracket snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f(d)][z/a],\, \langle\rangle}{}_{\mathrm{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
10.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	
	& $\llbracket snd.to\rrbracket^{M,\, g[x/d][y/f(d)][z/a],\, \langle\rangle}{}_{\mathrm{N}}(a)(f(d))(d) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
11.	$\llbracket con\rrbracket^{M}(f(d)) = 1$	D2.3.$\boldsymbol{b}$, simplify

□

Fact 1nA: Let $f \in D_{ee}{}^M$, $a, d \in D_e{}^M$. Then:

$[\![\lambda x\,(bmn\ x \wedge \exists[con](\lambda y[al](\lambda z\ snd.to\ zyx)))]\!]^{M,g,fa}{}_A(d) = 1$

if $[\![bmn]\!]^M(d) = 1$ & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$

Proof: (1) if (12)

1. $[\![\lambda x(bmn\ x \wedge \exists[con](\lambda y\ [al](\lambda z\ snd.to\ zyx)))]\!]^{M,g,fa}{}_A(d) = 1$

2. $[\![bmn\ x \wedge \exists[con](\lambda y[al](\lambda z\ snd.to\ zyx))]\!]^{M,g[x/d],f(d)a}{}_A = 1$ D2.3.**λ**, D2.2.*c*(*d*)

3. $[\![bmn\ x]\!]^{M,g[x/d],\langle\rangle}{}_A = 1$ D2.3.**∧**
   & $[\![\exists[con](\lambda y[al](\lambda z\ snd.to\ zyx))]\!]^{M,g[x/d],f(d)a}{}_A = 1$

4. $[\![bmn]\!]^{M,g[x/d],\langle\rangle}{}_A(d) = 1$ D2.3.***f***, ***b***, D2.2.*g*[*u*/*d*]
   & $[\![\exists[con]]\!]^{M,g[x/d],f(d)}{}_A([\![\lambda y[al](\lambda z\ snd.to\ zyx))]\!]^{M,g[x/d],a}{}_A = 1$ D2.3.***f***

5. $[\![bmn]\!]^M(d) = 1$ D2.3.***b***, **∃[**
   & $[\![\lambda y[al](\lambda z\ snd.to\ zyx)]\!]^{M,g[x/d],a}{}_A(f(d)) = 1$

7. $[\![bmn]\!]^M(d) = 1$
   & $[\![[al](\lambda z\ snd.to\ zyx)]\!]^{M,g[x/d][y/f(d)],a}{}_A = 1$ D2.3.**λ,** D2.2.*c*(*d*)

8. $[\![bmn]\!]^M(d) = 1$
   & $[\![[al]]\!]^{M,g[x/d][y/f(d)],a}{}_A([\![\lambda z\ snd.to\ zyx]\!]^{M,g[x/d][y/f(d)],\langle\rangle}{}_A) = 1$ D2.3.***f***

9. $[\![bmn]\!]^M(d) = 1$
   & $[\![\lambda z\ snd.to\ zyx]\!]^{M,g[x/d][y/f(d)],a}{}_A(a) = 1$ D2.3.**[]**

10. $[\![bmn]\!]^M(d) = 1$
    & $[\![snd.to\ zyx]\!]^{M,g[x/d][y/f(d)][z/a],\langle\rangle}{}_A = 1$ D2.3.**λ,** D2.2.*c*(*d*)

11. $[\![bmn]\!]^M(d) = 1$
    & $[\![snd.to]\!]^{M,g[x/d][y/f(d)][z/a],\langle\rangle}{}_A(a)(f(d))(d) = 1$ D2.3.***f***, ***b***, D2.2.*g*[*u*/*d*]

12. $[\![bmn]\!]^M(d) = 1$
    & $[\![snd.to]\!]^M(a)(f(d))(d) = 1$ D2.3.***b***

□

Fact 1VP: Let $f, f' \in D_{ee}{}^{M}$, $a, b, d \in D_{e}{}^{M}$. Then:

$\llbracket \lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g,\, f'fbfa}{}_{P}(d) = 1$

if $b = \llbracket bush \rrbracket^{M}$

Proof:

1.	$\llbracket \lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g,\, f'fbfa}{}_{P}(d) = 1$	
2.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))]$ $(\lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g[x/d],\, f'(d)f(d)bf(d)a}{}_{P} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
3.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))]\ \rrbracket^{M,\, g[x/d],\, f'(d)f(d)f(d)a}{}_{P}$ $(\llbracket \lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M,\, g[x/d],\, bf(d)a}{}_{P}) = 1$	D2.3.$\boldsymbol{f}$.$\rho/f'(d)f(d)$. ρ'/b. $\rho''/f(d)a$
4.	$\llbracket \lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))\rrbracket^{M,\, g[x/d],\, f(d)f(d)a}{}_{P}(f'(d)) = 1$ $\&\ \llbracket \lambda y[bush](\lambda z\ snd.to\ zyx)\rrbracket^{M,\, g[x/d],\, bf(d)a}{}_{P}(f'(d)) = 1$	D2.3.$\exists$[
5.	$\llbracket (con\ y \wedge it_1(\lambda y'\ lrg\ y'y))\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)f(d)a}{}_{P} = 1$ $\&\ \llbracket [bush](\lambda z\ snd.to\ zyx)\rrbracket^{M,\, g[x/d][y/f'(d)],\, bf(d)a}{}_{P} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
6.	$\llbracket con\ y\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)a}{}_{P} = 1$ $\&\ \llbracket it_1(\lambda y'\ lrg\ y'y)\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)f(d)a}{}_{P} = 1$ $\&\ \llbracket [bush]\rrbracket^{M,\, g[x/d][y/f'(d)],\, bf(d)a}{}_{P}(\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)a}{}_{P}) = 1$	D2.3.$\wedge$ D2.3.$\boldsymbol{f}$.ρ/b. $\rho'/\langle\rangle$. $\rho''/f(d)a$
7.	$\llbracket con\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)a}{}_{P}(f'(d)) = 1$ $\&\ \llbracket it_1\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)f(d)a}{}_{P}(\llbracket \lambda y'\ lrg\ y'y\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)a}{}_{P}) = 1$ $\&\ b = \llbracket bush\rrbracket^{M}$ $\&\ \llbracket \lambda z\ snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)a}{}_{P}(b) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$ D2.3.$\boldsymbol{f}$.$\rho/f(d)$. $\rho''/f(d)a$ D2.3.[]
8.	$1 = 1$ $\&\ f(d) = (f(d)a)_1$ $\&\ \llbracket \lambda y'\ lrg\ y'y\rrbracket^{M,\, g[x/d][y/f'(d)],\, f(d)f(d)a}{}_{P}(f(d)) = 1$ $\&\ b = \llbracket bush\rrbracket^{M}$ $\&\ \llbracket snd.to\ zyx\rrbracket^{M,\, g[x/d][y/f'(d)][z/b],\, f(d)a}{}_{P} = 1$	D2.3.$\boldsymbol{b}$ D2.3.$\boldsymbol{b}$, D2.2.it_n D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
9.	$f(d) = f(d)$ $\&\ \llbracket lrg\ y'y\rrbracket^{M,\, g[x/d][y/f'(d)][y'/f(d)],\, f(d)a}{}_{P} = 1$ $\&\ b = \llbracket bush\rrbracket^{M}$ $\&\ \llbracket snd.to\rrbracket^{M,\, g[x/d][y/f'(d)][z/b],\, f(d)f(d)a}{}_{P}(b)(f'(d))(d) = 1$	simplify, D2.2.ρ_m D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$ D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
10.	$\llbracket lrg\rrbracket^{M,\, g[x/d][y/f'(d)][y'/f(d)],\, f(d)a}{}_{P}(f(d))(f'(d)) = 1$ $\&\ b = \llbracket bush\rrbracket^{M}$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$ D2.3.$\boldsymbol{b}$, simplify
11.	$1 = 1$ $\&\ b = \llbracket bush\rrbracket^{M}$	D2.3.$\boldsymbol{b}$
12.	$b = \llbracket bush\rrbracket^{M}$	simplify

□

Fact 1vN: Let $f, f' \in D_{ee}{}^{M}$, $a, b, d \in D_{e}{}^{M}$. Then:

$\llbracket \lambda x\, \exists[\lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx))\rrbracket^{M, g, f'fbfa}{}_{\text{N}}(d) = 1$

iff $\llbracket con\rrbracket^{M}(f'(d)) = 1\ \&\ \llbracket lrg\rrbracket^{M}(f(d))(f'(d)) = 1$

Proof:

1.	$\llbracket \lambda x\, \exists[\lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx))\rrbracket^{M, g, f'fbfa}{}_{\text{N}}(d) = 1$	
2.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))]$ $(\lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g[x/d], f'(d)f(d)bf(d)a}{}_{\text{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
3.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))]\rrbracket^{M, g[x/d], f'(d)f(d)f(d)a}{}_{\text{N}}$ $(\llbracket \lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g[x/d], bf(d)a}{}_{\text{N}}) = 1$	D2.3.$\boldsymbol{f}$.$\rho/f'(d)f(d)$. ρ'/b. $\rho''/f(d)a$
4.	$\llbracket \lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))\rrbracket^{M, g[x/d], f(d)f(d)a}{}_{\text{N}}(f'(d)) = 1$	D2.3.$\boldsymbol{\exists}$[
	$\&\ \llbracket \lambda y(con\ y \wedge it_1(\lambda y'\, lrg\ y'y))\rrbracket^{M, g[x/d], f(d)f(d)a}{}_{\text{A}}(f'(d)) = 1$	
	$\&\ \llbracket \lambda y[bush](\lambda z\ snd.to\ zyx)\rrbracket^{M, g[x/d], bf(d)a}{}_{\text{N}}(f'(d)) = 1$	
5.	$\llbracket (con\ y \wedge it_1(\lambda y'\, lrg\ y'y))\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
	$\&\ \llbracket (con\ y \wedge it_1(\lambda y'\, lrg\ y'y))\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{A}} = 1$	
	$\&\ \llbracket [bush](\lambda z\ snd.to\ zyx)\rrbracket^{M, g[x/d][y/f'(d)], bf(d)a}{}_{\text{N}} = 1$	
6.	$\llbracket con\ y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}} = 1$	D2.3.$\wedge$
	$\&\ \llbracket it_1(\lambda y'\, lrg\ y'y)\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{N}} = 1$	
	$\&\ \llbracket con\ y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{A}} = 1$	D2.3.$\wedge$
	$\&\ \llbracket it_1(\lambda y'\, lrg\ y'y)\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{A}} = 1$	
	$\&\ \llbracket [bush]\rrbracket^{M, g[x/d][y/f'(d)], bf(d)a}{}_{\text{N}}(\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}}) = 1$	D2.3.$\boldsymbol{f}$.ρ/b. $\rho'/\langle\rangle$. $\rho''/f(d)a$
7.	$\llbracket con\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}}(f'(d)) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
	$\&\ \llbracket it_1\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{N}}(\llbracket \lambda y'\, lrg\ y'y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}}) = 1$	D2.3.$\boldsymbol{f}$.$\rho/f(d)$. $\rho''/f(d)a$
	$\&\ \llbracket con\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{A}}(f'(d)) = 1$	
	$\&\ \llbracket it_1\rrbracket^{M, g[x/d][y/f'(d)], f(d)f(d)a}{}_{\text{A}}(\llbracket \lambda y'\, lrg\ y'y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{A}}) = 1$	
	$\&\ \llbracket \lambda z\ snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}}(b) = 1$	D2.3.[]
8.	$1 = 1$	D2.3.$\boldsymbol{b}$
	$\&\ \llbracket \lambda y'\, lrg\ y'y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{N}}(f(d)) = 1$	D2.3.$\boldsymbol{b}$, D2.2.it_n
	$\&\ \llbracket con\rrbracket^{M}(f'(d)) = 1$	D2.3.$\boldsymbol{b}$
	$\&\ \llbracket \lambda y'\, lrg\ y'y\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\text{A}}(f(d)) = 1$	D2.3.$\boldsymbol{b}$, D2.2.it_n
	$\&\ \llbracket snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)][z/b], f(d)a}{}_{\text{N}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
9.	$\llbracket lrg\rrbracket^{M, g[x/d][y/f'(d)][y'/f(d)], f(d)a}{}_{\text{N}}(f(d))(f'(d)) = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
	$\&\ \llbracket con\rrbracket^{M}(f'(d)) = 1$	$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
	$\&\ \llbracket lrg\rrbracket^{M, g[x/d][y/f'(d)][y'/f(d)], f(d)a}{}_{\text{A}}(f(d))(f'(d)) = 1$	
	$\&\ \llbracket snd.to\rrbracket^{M, g[x/d][y/f'(d)][z/b], f(d)a}{}_{\text{N}}(b)(f'(d))(d) = 1$	
10.	$\llbracket con\rrbracket^{M}(f'(d)) = 1$	D2.3.$\boldsymbol{b}$, simplify
	$\&\ \llbracket lrg\rrbracket(f(d))(f'(d)) = 1$	

□

Fact 1vA: Let $f, f' \in D_{ee}{}^{M}$, $a, b, d \in D_{e}{}^{M}$. Then:

$$\llbracket \lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g, f'fbfa}{}_{\mathrm{A}}(d) = 1$$

iff $\llbracket snd.to\rrbracket^{M}(b)(f'(d))(d) = 1$

Proof:

1.	$\llbracket \lambda x\ \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y[bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g, f'fbfa}{}_{\mathrm{A}}(d) = 1$	
2.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))]$ $(\lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g[x/d], f'(d)f(d)bf(d)a}{}_{\mathrm{A}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
3.	$\llbracket \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))]\ \rrbracket^{M, g[x/d], f'(d)f(d)f(d)a}{}_{\mathrm{A}}$ $(\llbracket \lambda y\ [bush](\lambda z\ snd.to\ zyx)))\rrbracket^{M, g[x/d], bf(d)a}{}_{\mathrm{A}}) = 1$	D2.3.$\boldsymbol{f}$.$\rho/f'(d)f(d)$. ρ'/b. $\rho''/f(d)a$
4.	$\llbracket \lambda y[bush](\lambda z\ snd.to\ zyx)\rrbracket^{M, g[x/d], bf(d)a}{}_{\mathrm{A}}\ (f'(d)) = 1$	D2.3.$\boldsymbol{\exists}$[
5.	$\llbracket [bush](\lambda z\ snd.to\ zyx)\rrbracket^{M, g[x/d][y/f'(d)], bf(d)a}{}_{\mathrm{A}} = 1$	D2.3.$\boldsymbol{\lambda}$
6.	$\llbracket [bush]\rrbracket^{M, g[x/d][y/f'(d)], bf(d)a}{}_{\mathrm{A}}(\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\mathrm{A}}) = 1$	D2.3.$\boldsymbol{f}$.ρ/b. $\rho'/\langle\rangle$. $\rho''/f(d)a$
7.	$\llbracket \lambda z\ snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)], f(d)a}{}_{\mathrm{A}}\ (b) = 1$	D2.3.**[]**
8.	$\llbracket snd.to\ zyx\rrbracket^{M, g[x/d][y/f'(d)][z/b], f(d)a}{}_{\mathrm{A}} = 1$	D2.3.$\boldsymbol{\lambda}$, D2.2.$c(d)$
9.	$\llbracket snd.to\rrbracket^{M, g[x/d][y/f'(d)][z/b], f(d)f(d)a}{}_{\mathrm{A}}\ (b)(f'(d))(d) = 1$	D2.3.$\boldsymbol{f}$, $\boldsymbol{b}$, D2.2.$g[u/d]$
10.	$\llbracket snd.to\rrbracket^{M}(b)(f'(d))(d) = 1$	D2.3.$\boldsymbol{b}$

□

Fact 1P:

Let np $:= (\lambda x\,(bmn\ x \wedge \exists[con](\lambda y\,[al](\lambda z\ snd.to\ zyx))))$
vp $:= (\lambda x\,\exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y\,[bush](\lambda z\ snd.to.\ zyx)))$

Then $M, g, Af'fbfa \models_{\mathrm{P}} most[\mathrm{np}]\mathrm{vp}$
iff $a = \llbracket al \rrbracket^M$ & $b = \llbracket bush \rrbracket^M$
& Set $A = \{d \in D_e^M \mid \llbracket bmn \rrbracket^M(d) = 1$ & $\exists! c{:}\ \llbracket con \rrbracket^M(c) = 1$ & $\llbracket snd.to \rrbracket^M(a)(c)(d) = 1\} \neq \varnothing$
& Dom f = Set A & $\forall d \in$ Dom f: $\llbracket con \rrbracket^M(f(d)) = 1$ & $\llbracket snd.to \rrbracket^M(a)(f(d))(d) = 1$

Proof:

1. $M, g, Af'fbfa \models_{\mathrm{P}} most[\mathrm{np}]\mathrm{vp}$

2. $\llbracket most[\mathrm{np}] \rrbracket^{M,g,Afa}{}_{\mathrm{P}}(\llbracket \mathrm{vp} \rrbracket^{M,g,f'fbfa}{}_{\mathrm{P}}) = 1$ D3, D2.3.***f***

3. $\varnothing \subset$ Set A = Set $\llbracket !\mathrm{np} \rrbracket^{M,g,fa}$ D2.3.**α[**
& $\forall d \in$ Set A: $\llbracket \mathrm{vp} \rrbracket^{M,g,f'fbfa}{}_{\mathrm{P}}(d) = 1$

4. $\varnothing \subset$ Set A = Dom f D2.3.**!**,
& Dom $f = \{c \in D_e^M \mid \exists! c'{:}\ \llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{P}}(c) = 1$ D2.2.Set F, $c(d)$
& $\llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{N}}(c) = 1$ df. {–:–}
& $\llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{A}}(c) = 1\}$
& $\forall c \in$ Dom f: $\llbracket \mathrm{np} \rrbracket^{M,g,f(c)a}{}_{\mathrm{P}}(c) = 1$
& $\llbracket \mathrm{np} \rrbracket^{M,g,f(c)a}{}_{\mathrm{N}}(c) = 1$
& $\llbracket \mathrm{np} \rrbracket^{M,g,f(c)a}{}_{\mathrm{A}}(c) = 1$
& $\forall d \in$ Set A: $\llbracket \mathrm{vp} \rrbracket^{M,g,f'fbfa}{}_{\mathrm{P}}(d) = 1$

5. $\varnothing \subset$ Set $A = \{c \in D_e^M \mid \exists! c'{:}\ \llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{P}}(c) = 1$ rearrange
& $\llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{N}}(c) = 1$
& $\llbracket \mathrm{np} \rrbracket^{M,g,c'a}{}_{\mathrm{A}}(c) = 1\}$
& Dom f = Set A
& $\forall d \in$ Dom f: $\llbracket \mathrm{np} \rrbracket^{M,g,f(d)a}{}_{\mathrm{P}}(d) = 1$
& $\llbracket \mathrm{np} \rrbracket^{M,g,f(d)a}{}_{\mathrm{N}}(d) = 1$
& $\llbracket \mathrm{np} \rrbracket^{M,g,f(d)a}{}_{\mathrm{A}}(d) = 1$
& $\llbracket \mathrm{vp} \rrbracket^{M,g,f'fbfa}{}_{\mathrm{P}}(d) = 1$

6. $\varnothing \subset$ Set $A = \{c \in D_e^M \mid \exists! c'{:}\ a = \llbracket al \rrbracket^M$ 1′nP, 1′nN, 1′nA, 1′vP
& $\llbracket con \rrbracket^M(c') = 1$ ~ 1nP, 1nN, 1nA, 1vP
& $\llbracket bmn \rrbracket^M(c) = 1$ & $\llbracket snd.to \rrbracket^M(a)(c')(c) = 1\}$
& Dom f = Set A
& $\forall d \in$ Dom f: $a = \llbracket al \rrbracket^M$
& $\llbracket con \rrbracket^M(f(d)) = 1$
& $\llbracket bmn \rrbracket^M(d) = 1$ & $\llbracket snd.to \rrbracket^M(a)(f(d))(d) = 1\}$
& $b = \llbracket bush \rrbracket^M$

7. $a = \llbracket al \rrbracket^M$ & $b = \llbracket bush \rrbracket^M$ rearrange
$\varnothing \subset$ Set $A = \{c \in D_e^M \mid \llbracket bmn \rrbracket^M(c) = 1$
& $\exists! c' \in D_e^M{:}\ \llbracket con \rrbracket^M(c') = 1$ & $\llbracket snd.to \rrbracket^M(a)(c')(d) = 1\}$
& Dom f = Set A
& $\forall d \in$ Dom f: $\llbracket bmn \rrbracket^M(d) = 1$ & $\llbracket con \rrbracket^M(f(d)) = 1$ & $\llbracket snd.to \rrbracket^M(a)(f(d))(d) = 1\}$ □

Fact 1N:
Let np := $(\lambda x\, (bmn\ x \wedge \exists[con](\lambda y\, [al](\lambda z\ snd.to\ zyx))))$
vp := $(\lambda x\, \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y\, [bush](\lambda z\ snd.to.\ zyx))))$

Then $M, g, Af'fbfa \models_N most$[np]vp
iff $\forall d \in$ Set A: $[\![con]\!]^M(f'(d)) = 1\ \&\ [\![lrg]\!]^M(f(d))(f'(d)) = 1$

Proof:

1.	$M, g, Af'fbfa \models_N most$[np]vp	
2.	$[\![most[\text{np}]]\!]^{M,g,Afa}{}_N([\![\text{vp}]\!]^{M,g,f'fbfa}{}_N) = 1$	D3, D2.3.$\boldsymbol{f}$
3.	$\forall d \in$ Set A: $[\![\text{vp}]\!]^{M,g,f'fbfa}{}_N(d) = 1$	D2.3.**α[**
4.	$\forall d \in$ Set A: $[\![con]\!]^M(f'(d)) = 1\ \&\ [\![lrg]\!]^M(f(d))(f'(d)) = 1$	1vN

□

Fact 1A:
Let np := $(\lambda x\, (bmn\ x \wedge \exists[con](\lambda y\, [al](\lambda z\ snd.to\ zyx))))$
vp := $(\lambda x\, \exists[\lambda y(con\ y \wedge it_1(\lambda y'\ lrg\ y'y))](\lambda y\, [bush](\lambda z\ snd.to.\ zyx))))$

Then $M, g, Af'fbfa \models_A most$[np]vp
if $|\{d \in \text{Set } A |\ [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}| > |\{d \in \text{Set } A |\ [\![snd.to]\!]^M(b)(f'(d))(d) = 0\}|$

Proof:

1.	$M, g, Af'fbfa \models_A most$[np]vp	
2.	$[\![most[\text{np}]]\!]^{M,g,Afa}{}_A([\![\text{vp}]\!]^{M,g,f'fbfa}{}_A) = 1$	D3, D2.3.$\boldsymbol{f}$
3.	$[\![most]\!]^M(A)([\![\text{vp}]\!]^{M,g,f'fbfa}{}_A) = 1$	D2.3.**α[**
4.	$\lvert\text{Set } A \cap \text{Set } [\![\text{vp}]\!]^{M,g,f'fbfa}{}_A\rvert > \lvert\text{Set } A - \text{Set } [\![\text{vp}]\!]^{M,g,f'fbfa}{}_A\rvert$	D2.2.*most*
5.	$\lvert\text{Set } A \cap \{d \in D_e^M |\ [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}\rvert$ $> \lvert\text{Set } A - \{d \in D_e^M |\ [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}\rvert$	1vA
6.	$\lvert\{d \in \text{Set } A |\ [\![snd.to]\!]^M(b)(f'(d))(d) = 1\}\rvert$ $> \lvert\{d \in \text{Set } A |\ [\![snd.to]\!]^M(b)(f'(d))(d) \neq 1\}\rvert$	$\varnothing \subset \text{Set } A \subseteq D_e^M$

□