QPLA (4): Toward a general theory of quantification & information structure

0. Research program (based on Dekker 1993ff)

• *Anaphora* & *strong readings* via *stack update* & *unselective quant*. (Sec. **1**, a la Dekker 1994)

(1) [1]A diver found a pearl. [2]She gave it to a friend.
(2) If farmer owns a donkey, he beats it with a stick.
(3) Every farmer who owns a donkey beats it with a stick. strong rdg. of (3) = (2)

• *Proportions* & *weak readings* via *selective quantification* (Sec. **2**, adapting Dekker 2003a)

(4) Most people who have *a* dime will put *it* in the meter. (Schubert & Pelletier)
(5) Most people who write *an* article (later) develop *it* in another article.

• *Uniqueness* via *referents for functions* (Sec. **3**, adapting Dekker 2003a)

(6) Most businessmen who sent a contribution to Gore sent a larger contribution to Bush.

• *Topical restriction* via *topical referents for sets* (**Paper topic 1**, adapting Dekker 2003b, a)

(7) [1]Tom was at a party (last night). [2]One girl was stunning, and [3]every boy went wild.

(8) [1]Every man (in this town) has a gun, [2]but most people don't use it.
[3]Only Bill uses it (to shoot deer). (after Sandu)

• *Negation* & *questions* via *other dimensions* (**Paper topic 2**, adapting Dekker 2003a, b)

(9) [1]Tom was at a party (last night). [2]He didn't talk to everybody, [3]but he had a good time.

(10) Q: [1]Who's coming?
A: [2]An undergraduate and a friend of his.

• *Presupposition* of else via *current possibility* (**Paper topic 3,** adapting Dekker 2003b)

($\Diamond$) Dekker 1993:154: $s[\![\Diamond\phi]\!]^{Mi,\,g} = \{i \in s \mid s[\![\phi]\!]^{M} \neq \varnothing\}$

(11) [1]Tom was at a party (last night). [2]He talked to a logician, a friend of hers & someone *else*.

(12) Q: [1]Who attacked Sue?
A: [2]A teenager, a friend of his, and somebody *else* (I didn't see).

(13) Q: [1]Who attacked Sue?
A: [2]A teenager and a friend of his. There was nobody *else*.

1. QPLA (v. 3.1): *Anaphora & strong rdgs* via *stack update & unsel. quant.* (a la Dekker 1994)

DEFINITION 1.1 (QPLA-types)
- $e, t \in \mathbf{Typ}$
- $(\sigma\tau) \in \mathbf{Typ}$, if $\sigma, \tau \in \mathbf{Typ}$

DEFINITION 1.2 (QPLA-basic terms)
- $\mathbf{Con}_e$ = {*bill*, *sue*, …}
  $\mathbf{Con}_{et}$ = {*diver*, *pearl*, …, *die*,…}
  $\mathbf{Con}_{eet}$ = {*friend*, …, *find*, *beat*, …}
  $\mathbf{Con}_{eeet}$ = {*give.to*, *beat.with*, …}
- $\mathbf{Var}_e$ = $\{x, y, z, \ldots\}$
- $\mathbf{Prn}_e$ = $\{p_1, p_2, \ldots\}$

DEFINITION 1.2 (QPLA-terms).

b	$\alpha \in \mathbf{Term}_\tau$,	if $\alpha \in \mathbf{Con}_\tau \cup \mathbf{Var}_\tau \cup \mathbf{Prn}_\tau$
=	$(\alpha = \beta) \in \mathbf{Term}_t$	if $\alpha, \beta \in \mathbf{Term}_e$
f	$\alpha\beta_1\ldots\beta_n \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{e\ldots et}$ (n times e) & $\beta_1,\ldots\beta_n \in \mathbf{Term}_e$
¬	$\neg\phi \in \mathbf{Term}_t$	if $\phi \in \mathbf{Term}_t$
∧	$(\phi \wedge \psi) \in \mathbf{Term}_t$	if $\phi, \psi \in \mathbf{Term}_t$
∃	$\exists u\phi \in \mathbf{Term}_t$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$

ABBREVIATIONS
- $(\phi \rightarrow \psi) := \neg(\phi \wedge \neg \psi)$
- $\forall u\phi := \neg\exists u\neg\phi$

DEFINITION 2.1 (frames). A QPLA-*frame* is a set of τ-domains, $\{D_\tau: \tau \in \mathbf{Typ}\}$ s.t.
- $D_t = \{1, 0\}$
  D_e is a non-empty set disjoint from D_t
- $D_{(\sigma\tau)} = \{f | \varnothing \subset \text{Dom } f \subseteq D_\sigma \text{ \& Ran } f \subseteq D_\tau\}$

DEFINITION 2.2 (models and assignments)
- A QPLA-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
  (i) $D^M = \{D_\tau^M: \tau \in \mathbf{Typ}\}$ is a QPLA-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_\tau$ a denotation $[\![\alpha]\!]^M \in D_\tau^M$
- An *M-assignment* is a function g that assigns to any $u \in \mathbf{Var}_\tau$ a value $g(u) \in D_\tau^M$. If $d \in D_\tau^M$, then $g[u/d]$ is the M-assignment s.t. (i) $g[u/d](u) = d$, and (ii) $g[u/d](u') = g(u')$ if $u' \neq u$.

DEFINITION 2.2′ (indices and information states)
- $i \in (D_e^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *n-stack* (aka *index*) of *M*-drefs
  $i_m := r_m$, if $i = \langle r_1, \ldots r_n\rangle \in (D_e^M)^n$ & $1 \leq m \leq n$
  $ij := \langle r_1,\ldots r_n, r'_1, \ldots r'_m\rangle$ if $i = \langle r_1, \ldots r_n\rangle \in (D_e^M)^n$ & $j = \langle r'_1, \ldots r'_n\rangle \in (D_e^M)^m$
- $s \in \mathcal{P}(D_e^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *M-state* of information about n *M*-drefs
  $S^M = \cup_{n \in \mathcal{N}} \mathcal{P}(D_e^M)^n$ is the set of *M*-information states

DEFINITION 2.3 (QPLA-semantics). For any M, g, $s \in S^M$, $i \in (D_e{}^M)^n$

b	$\{i\}\llbracket\alpha\rrbracket^{M,g}$	$=$	$\llbracket\alpha\rrbracket^M$	if $\alpha \in \mathbf{Con}_e$
	$\{i\}\llbracket\alpha\rrbracket^{M,g}$	$=$	$g(\alpha)$	if $\alpha \in \mathbf{Var}_e$
	$\{i\}\llbracket\alpha\rrbracket^{M,g}$	$=$	i_m	if $\alpha = p_m \in \mathbf{Prn}_e$ & $1 \leq m \leq n$
=	$s\llbracket\alpha = \beta\rrbracket^{M,g}$	$=$	$\{i \in s \mid \{i\}\llbracket\alpha\rrbracket^{M,g} = \{i\}\llbracket\beta\rrbracket^{M,g}\}$	
f	$s\llbracket\alpha\beta_1\ldots\beta_n\rrbracket^{M,g}$	$=$	$\{i \in s \mid \llbracket\alpha\rrbracket^M(\{i\}\llbracket\beta_1\rrbracket^{M,g})\ldots(\{i\}\llbracket\beta_n\rrbracket^{M,g}) = 1\}$	
¬	$s\llbracket\neg\phi\rrbracket^{M,g}$	$=$	$\{i \in s \mid \neg\exists j: ji \in s\llbracket\phi\rrbracket^{M,g}\}$	
∧	$s\llbracket(\phi \wedge \psi)\rrbracket^{M,g}$	$=$	$s\llbracket\phi\rrbracket^{M,g}\llbracket\psi\rrbracket^{M,g}$	
∃	$s\llbracket\exists u\phi\rrbracket^{M,g}$	$=$	$\{ri \mid i \in s\llbracket\phi\rrbracket^{M,g[u/r]}\}$	

DEFINITION 3 (Support and truth)

- *s supports* ϕ wrt M under g, $s \models_{M,g} \phi$, iff $\forall i \in s \exists j: ji \in s\llbracket\phi\rrbracket^{M,g}$
- ϕ is *true* in M, $\models_M \phi$, iff $\forall g: \{\langle\rangle\}\llbracket\phi\rrbracket^{M,g} \neq \varnothing$

FACT A

→	$s\llbracket(\phi \rightarrow \psi)\rrbracket^{M,g}$	$=$	$\{i \in s \mid \forall j: ji \in s\llbracket\phi\rrbracket^{M,g} \rightarrow \exists k: kji \in s\llbracket\phi\rrbracket^{M,g}\llbracket\psi\rrbracket^{M,g}\}$
∀	$s\llbracket\forall u\phi\rrbracket^{M,g}$	$=$	$\{i \in s \mid \forall r \exists j: ji \in s\llbracket\phi\rrbracket^{M,g[u/r]}\}$

Proof:

→

1. $s\llbracket(\phi \rightarrow \psi)\rrbracket^{M,g}$
2. $s\llbracket\neg(\phi \wedge \neg\psi)\rrbracket^{M,g}$ A.→
3. $\{i \in s \mid \neg\exists j: ji \in s\llbracket(\phi \wedge \neg\psi)\rrbracket^{M,g}$ D2.3.¬
4. $\{i \in s \mid \neg\exists j: ji \in s\llbracket\phi\rrbracket^{M,g}\llbracket\neg\psi\rrbracket^{M,g}\}$ D2.3.∧
5. $\{i \in s \mid \neg\exists j: ji \in \{i' \in s\llbracket\phi\rrbracket^{M,g} \mid \neg\exists k: ki' \in s\llbracket\phi\rrbracket^{M,g}\llbracket\psi\rrbracket^{M,g}\}\}$ D2.3.¬
6. $\{i \in s \mid \neg\exists j: (ji \in s\llbracket\phi\rrbracket^{M,g}\ \&\ \neg\exists k: kji \in s\llbracket\phi\rrbracket^{M,g}\llbracket\psi\rrbracket^{M,g})\}$ df. {–:–}
7. $\{i \in s \mid \forall j: ji \in s\llbracket\phi\rrbracket^{M,g} \rightarrow \exists k: kji \in s\llbracket\phi\rrbracket^{M,g}\llbracket\psi\rrbracket^{M,g})\}$ metalg. ¬, ∃, ∀, →

∀

1. $s\llbracket\forall u\phi\rrbracket^{M,g}$
2. $s\llbracket\neg\exists u\neg\phi\rrbracket^{M,g}$ A.∀
3. $\{i \in s \mid \neg\exists j: ji \in s\llbracket\exists u\neg\phi\rrbracket^{M,g}\}$ D2.3.¬
4. $\{i \in s \mid \neg\exists j: ji \in \{ri' \mid i' \in s\llbracket\neg\phi\rrbracket^{M,g[u/r]}\}\}$ D2.3.∃
5. $\{i \in s \mid \neg\exists r, k: ki \in s\llbracket\neg\phi\rrbracket^{M,g[u/r]}\}$ df. {–:–}, D2.2′.*ij*
6. $\{i \in s \mid \neg\exists r, k: ki \in \{i' \in s \mid \neg\exists j: ji' \in s\llbracket\phi\rrbracket^{M,g[u/r]}\}\}$ D2.3.¬
7. $\{i \in s \mid \neg\exists r, k: (ki \in s\ \&\ \neg\exists j: jki \in s\llbracket\phi\rrbracket^{M,g[u/r]})\}$ df. {–:–}
8. $\{i \in s \mid \forall r, k: (ki \in s \rightarrow \exists j: jki \in s\llbracket\phi\rrbracket^{M,g[u/r]})\}$ metalg. –, ∃, ∀
9. $\{i \in s \mid \forall r \exists j: ji \in s\llbracket\phi\rrbracket^{M,g[u/r]}\}$ D2.2′.$s \in \mathcal{P}(D_e{}^M)^n$ implies $k = \langle\rangle$

(1) [1]A diver found a pearl. [2]She gave it to a friend.

(1′) $\exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge \exists z(F'zp_1 \wedge Gp_1p_2z)$ Dekker 2003a:3

Fact 1:

$\models_M$ (1′) iff

$\exists r, r', r'' \in D_e{}^M: \llbracket D\rrbracket^M(r) = 1\ \&\ \llbracket P\rrbracket^M(r') = 1\ \&\ \llbracket F\rrbracket^M(r)(r') = 1$

$\&\ \llbracket F'\rrbracket^M(r'')(r) = 1\ \&\ \llbracket G\rrbracket^M(r)(r')(r'') = 1$

Proof:

1. $\models_M \exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge \exists z(F'zp_1 \wedge Gp_1p_2z)$
2. $\forall g$: D3
   $\{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy)) \wedge \exists z(F'zp_1 \wedge Gp_1p_2z)\rrbracket^{M,g} \neq \varnothing$
3. $\forall g$: D2.3.$\wedge$
   $\{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}\llbracket \exists z(F'zp_1 \wedge Gp_1p_2z)\rrbracket^{M,g} \neq \varnothing$
4. $\forall g$: D2.3.$\exists$
   $\{r''i \mid i \in \{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}\llbracket F'zp_1 \wedge Gp_1p_2z\rrbracket^{M,g[z/r'']}\} \neq \varnothing$
5. $\forall g$: D2.3.$\wedge$
   $\{r''i \mid i \in \{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}\llbracket F'zp_1\rrbracket^{M,g[z/r'']}\llbracket Gp_1p_2z\rrbracket^{M,g[z/r'']}\} \neq \varnothing$
6. $\forall g$: D2.3.f
   $\{r''i \mid i \in \{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}\llbracket F'zp_1\rrbracket^{M,g[z/r'']}$
   $\&\ \llbracket G\rrbracket^{M}(\{i\}\llbracket p_1\rrbracket^{M,g[z/r'']})(\{i\}\llbracket p_2\rrbracket^{M,g[z/r'']})(\{i\}\llbracket z\rrbracket^{M,g[z/r'']}) = 1\} \neq \varnothing$
7. $\forall g$: D2.3.b, D2.2.$g[u/d]$
   $\{r''i \mid i \in \{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}\llbracket F'zp_1\rrbracket^{M,g[z/r'']}$
   $\&\ \llbracket G\rrbracket^{M}(i_1)(i_2)(r'') = 1\} \neq \varnothing$
8. $\forall g$: D2.3.f, b, D2.2.$g[u/d]$
   $\{r''i \mid i \in \{\langle\rangle\}\llbracket \exists x(Dx \wedge \exists y(Py \wedge Fxy))\rrbracket^{M,g}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(i_1) = 1\ \&\ \llbracket G\rrbracket^{M}(i_1)(i_2)(r'') = 1\} \neq \varnothing$
9. $\forall g$: D2.3.$\exists$
   $\{r''i \mid i \in \{rj \mid j \in \{\langle\rangle\}\llbracket Dx \wedge \exists y(Py \wedge Fxy)\rrbracket^{M,g[x/r]}\}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(i_1) = 1\ \&\ \llbracket G\rrbracket^{M}(i_1)(i_2)(r'') = 1\} \neq \varnothing$
10. $\forall g$: eliminate i using df. {–:–}, D2.2′.i_m, ij
   $\{r''rj \mid j \in \{\langle\rangle\}\llbracket Dx \wedge \exists y(Py \wedge Fxy)\rrbracket^{M,g[x/r]}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(j_1)(r'') = 1\} \neq \varnothing$
11. $\forall g$: D2.3.$\wedge$
   $\{r''rj \mid j \in \{\langle\rangle\}\llbracket Dx\rrbracket^{M,g[x/r]}\llbracket \exists y(Py \wedge Fxy)\rrbracket^{M,g[x/r]}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(j_1)(r'') = 1\} \neq \varnothing$
12. $\forall g$: D2.3.$\wedge$
   $\{r''rj \mid j \in \{r'i \mid i \in \{\langle\rangle\}\llbracket Dx\rrbracket^{M,g[x/r]}\llbracket Py \wedge Fxy\rrbracket^{M,g[x/r][y/r']}\}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(j_1)(r'') = 1\} \neq \varnothing$
13. $\forall g$: eliminate j using df. {–:–}, D2.2′.i_m, ij
   $\{r''rr'i \mid i \in \{\langle\rangle\}\llbracket Dx\rrbracket^{M,g[x/r]}\llbracket Py \wedge Fxy\rrbracket^{M,g[x/r][y/r']}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(r')(r'') = 1\} \neq \varnothing$
14. $\forall g$: D2.3.$\wedge$
   $\{r''rr'i \mid i \in \{\langle\rangle\}\llbracket Dx\rrbracket^{M,g[x/r]}\llbracket Py\rrbracket^{M,g[x/r][y/r']}\llbracket Fxy\rrbracket^{M,g[x/r][y/r']}$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(r')(r'') = 1\} \neq \varnothing$
15. $\forall g$: 3 × D2.3.f, b, D2.2.$g[u/d]$
   $\{r''rr'i \mid i \in \{\langle\rangle\}\ \&\ \llbracket D\rrbracket^{M}(r) = 1\ \&\ \llbracket P\rrbracket^{M}(r') = 1\ \&\ \llbracket F\rrbracket^{M}(r)(r') = 1$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \llbracket G\rrbracket^{M}(r)(r')(r'') = 1\} \neq \varnothing$
16. $\exists r, r', r'' \in D_e^M$: eliminate g, metalg. $\exists$, D2.2′, D2.2.$\llbracket\cdot\rrbracket^M$
   $\llbracket D\rrbracket^{M}(r) = 1\ \&\ \llbracket P\rrbracket^{M}(r') = 1\ \&\ \llbracket F\rrbracket^{M}(r)(r') = 1$
   $\&\ \llbracket F'\rrbracket^{M}(r'')(r) = 1\ \&\ \rrbracket G\rrbracket^{M}(r)(r')(r'') = 1$

□

(3) Every farmer who owns a donkey beats it with a stick.
(3′) $\forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \rightarrow \exists z(Sz \wedge Bxp_1z))$ *strong rdg.* a la Dekker 2003a:3

Fact 3a:
$\{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket \exists y(Dy \wedge Oxy)\rrbracket^{M,\, g[x/r]}$
$= \{r\upharpoonleft \llbracket F\rrbracket^{M}(r) = 1\ \&\llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}$

Proof:
1. $\{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket \exists y(Dy \wedge Oxy)\rrbracket^{M,\, g[x/r]}$
2. $\{r'i|\ i \in \{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket Dy \wedge Oxy\rrbracket^{M,\, g[x/r][y/r']}\}$ D2.3.∃
3. $\{r'i|\ i \in \{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket Dy\rrbracket^{M,\, g[x/r][y/r']}\llbracket Oxy\rrbracket^{M,\, g[x/r][y/r']}\}$ D2.3.∧
4. $\{r'i|\ i \in \{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket Dy\rrbracket^{M,\, g[x/r][y/r']}$ D2.3.*f*, df. {–:–}
   $\&\ \llbracket O\rrbracket^{M}(\{i\}\llbracket x\rrbracket^{M,\, g[x/r][y/r']})(\{i\}\llbracket y\rrbracket^{M,\, g[x/r][y/r']}) = 1\}$
5. $\{r'i|\ i \in \{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}\llbracket Dy\rrbracket^{M,\, g[x/r][y/r']}$ D2.3.*b*, D2.2.*g*[*u*/*d*]
   $\&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}$
6. $\{r'i|\ i \in \{\langle\rangle\}\llbracket Fx\rrbracket^{M,\, g[x/r]}$ D2.3.*f*, df. {–:–}, D2.3.*b*, D2.2.*g*[*u*/*d*]
   $\&\ \llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}$
7. $\{r'i|\ i \in \{\langle\rangle\}$ D2.3.*f*, df. {–:–}, D2.3.*b*, D2.2.*g*[*u*/*d*]
   $\&\ \llbracket F\rrbracket^{M}(r) = 1\ \&\ \llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}$
8. $\{r\upharpoonleft \llbracket F\rrbracket^{M}(r) = 1\ \&\ \llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}$ eliminate ρ

Fact 3b:
$\{r\upharpoonleft \llbracket F\rrbracket^{M}(r) = 1\ \&\llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1\}\llbracket \exists z(Sz \wedge Bxp_1z)\rrbracket^{M,\, g[x/r]}$
$= \{r''r\upharpoonleft \llbracket F\rrbracket^{M}(r) = 1\ \&\llbracket D\rrbracket^{M}(r') = 1\ \&\ \llbracket O\rrbracket^{M}(r)(r') = 1$
$\&\ \llbracket S\rrbracket^{M}(r'') = 1\ \&\ \llbracket B\rrbracket^{M}(r)(r')(r'') = 1\}$

Proof:
Exercise.

Fact 3c:

$\models_M (3')$ iff

$\forall r, r' \in D_e^{\,M}$: $[\![F]\!]^M(r) = 1$ & $[\![D]\!]^M(r') = 1$ & $[\![O]\!]^M(r)(r') = 1$
$\qquad\qquad \to \exists r'' \in D_e^{\,M}$: $[\![S]\!]^M(r'') = 1$ & $[\![B]\!]^M(r)(r')(r'') = 1$

Proof:

1. $\models_M \forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \to \exists z(Sz \wedge Bxp_1z))$

2. $\forall g$: D3
   $\{\langle\rangle\}[\![\forall x((Fx \wedge \exists y(Dy \wedge Oxy)) \to \exists z(Sz \wedge Bxp_1z))]\!]^{M,\,g} \neq \varnothing$

3. $\forall g$: F.∀
   $\{i \in \{\langle\rangle\} \mid \forall r \exists j\colon ji \in \{\langle\rangle\}[\![(Fx \wedge \exists y(Dy \wedge Oxy)) \to \exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}\} \neq \varnothing$

4. $\forall g$: eliminate *i*
   $\forall r \exists j\colon j \in \{\langle\rangle\}[\![(Fx \wedge \exists y(Dy \wedge Oxy)) \to \exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}$

5. $\forall g$: rename *j*, F.→
   $\forall r \exists i\colon i \in \{\langle\rangle\}$
   $\quad$ & $\forall j\colon ji \in \{\langle\rangle\}[\![Fx \wedge \exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}$
   $\qquad \to \exists k\colon kji \in \{\langle\rangle\}[\![Fx \wedge \exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}[\![\exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}$

6. $\forall g$: eliminate *i*
   $\forall r \forall j\colon j \in \{\langle\rangle\}[\![Fx \wedge \exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}$
   $\quad \to \exists k\colon kj \in \{\langle\rangle\}[\![Fx \wedge \exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}[\![\exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}$

7. $\forall g$: D2.3.∧
   $\forall r \forall j\colon j \in \{\langle\rangle\}[\![Fx]\!]^{M,\,g[x/r]}[\![\exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}$
   $\quad \to \exists k\colon kj \in \{\langle\rangle\}[\![Fx]\!]^{M,\,g[x/r]}[\![\exists y(Dy \wedge Oxy)]\!]^{M,\,g[x/r]}[\![\exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}$

8. $\forall g$: F3a
   $\forall r \forall j\colon j \in \{r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1\}$
   $\quad \to \exists k\colon kj \in \{r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1\}[\![\exists z(Sz \wedge Bxp_1z)]\!]^{M,\,g[x/r]}$

9. $\forall g$: F3b
   $\forall r \forall j\colon j \in \{r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1\}$
   $\quad \to \exists k\colon kj \in \{r''r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$
   $\qquad\qquad \& \; [\![S]\!]^M(r'') = 1 \;\&\; [\![B]\!]^M(r)(r')(r'')\}$

10. $\forall r \forall j\colon j \in \{r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1\}$ eliminate *g*
   $\quad \to \exists k\colon kj \in \{r''r' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$
   $\qquad\qquad \& \; [\![S]\!]^M(r'') = 1 \;\&\; [\![B]\!]^M(r)(r')(r'')\}$

11. $\forall r \forall r'\colon [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$ eliminate *j*
   $\quad \to \exists k\colon kr' \in \{r''t' \upharpoonleft [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(t') = 1 \;\&\; [\![O]\!]^M(r)(t') = 1$
   $\qquad\qquad \& \; [\![S]\!]^M(r'') = 1 \;\&\; [\![B]\!]^M(r)(t')(r'')\}$

12. $\forall r \forall r'\colon [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$ eliminate *k*
   $\quad \to \exists r''\colon [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$
   $\qquad \& \; [\![S]\!]^M(r'') = 1 \;\&\; [\![B]\!]^M(r)(r')(r'') = 1$

13. $\forall r, r'\colon [\![F]\!]^M(r) = 1 \;\&\; [\![D]\!]^M(r') = 1 \;\&\; [\![O]\!]^M(r)(r') = 1$ simplify
   $\quad \to \exists r''\colon [\![S]\!]^M(r'') = 1 \;\&\; [\![B]\!]^M(r)(r')(r'') = 1$

□

2. QPLA (v. 3.2): *Proportions & weak readings* via *selective quantification*

DEFINITION 1.1 (QPLA-types)
- $e, t \in \mathbf{Typ}$
- $(\sigma\tau) \in \mathbf{Typ}$, if $\sigma, \tau \in \mathbf{Typ}$

DEFINITION 1.2 (QPLA-basic terms)
- $\mathbf{Con}_e = \{bill, sue, \ldots\}$
  $\mathbf{Con}_{et} = \{diver, pearl, \ldots, die, \ldots\}$
  $\mathbf{Con}_{eet} = \{friend, \ldots, find, beat, \ldots\}$
  $\mathbf{Con}_{eeet} = \{give.to, beat.with, \ldots\}$
  $\mathbf{Con}_{(et)(et)t} = \{every, most, \ldots\}$
- $\mathbf{Var}_e = \{x, y, z, \ldots\}$
- $\mathbf{Prn}_e = \{p_1, p_2, \ldots\}$

DEFINITION 1.2 (QPLA-terms).

b	$\alpha \in \mathbf{Term}_\tau$,	if $\alpha \in \mathbf{Con}_\tau \cup \mathbf{Var}_\tau \cup \mathbf{Prn}_\tau$
=	$(\alpha = \beta) \in \mathbf{Term}_t$	if $\alpha, \beta \in \mathbf{Term}_e$
f	$\alpha\beta_1\ldots\beta_n \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{e\ldots et}$ (n times e) & $\beta_1,\ldots\beta_n \in \mathbf{Term}_e$
f$^\exists$	$\alpha\beta_1\beta_2 \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{(et)(et)t}$ & $\beta_1, \beta_2 \in \mathbf{Term}_{et}$
¬	$\neg\phi \in \mathbf{Term}_t$	if $\phi \in \mathbf{Term}_t$
∧	$(\phi \wedge \psi) \in \mathbf{Term}_t$	if $\phi, \psi \in \mathbf{Term}_t$
∃	$\exists u\phi \in \mathbf{Term}_t$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
λ	$(\lambda u\phi) \in \mathbf{Term}_{et}$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$

ABBREVIATIONS 1 (obj. lg)
- $(\phi \to \psi) := \neg(\phi \wedge \neg\, \psi)$
- $\forall u\phi := \neg\exists u\neg\phi$

ABBREVIATIONS 2 (meta-lg.)
- Set $F := \{r \in D|\, F(d) = 1\}$ for $F: D \to \{1, 0\}$
- ${}^\chi A = F$ iff $A =$ Set F & $A \subseteq D$

DEFINITION 2.1 (frames). A QPLA-*frame* is a set of τ-domains, $\{D_\tau: \tau \in \mathbf{Typ}\}$ s.t.
- $D_t = \{1, 0\}$
  D_e is a non-empty set disjoint from D_t
- $D_{(\sigma\tau)} = \{f|\, \varnothing \subset \text{Dom } f \subseteq D_\sigma \text{ \& Ran } f \subseteq D_\tau\}$

DEFINITION 2.2 (models and assignments)
- A QPLA-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
  (i) $D^M = \{D_\tau^M: \tau \in \mathbf{Typ}\}$ is a QPLA$_{3.2}$-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_\tau$ a denotation $[\![\alpha]\!]^M \in D_\tau^M$
- For any $F, F' \in D_{et}^M$,
  $[\![every]\!]^M(F)(F') = 1$ iff Set $F \subseteq$ Set F'
  $[\![most]\!]^M(F)(F') = 1$ iff $|\text{Set } F \cap \text{Set } F'| > |\text{Set } F - \text{Set } F'|$
- An *M-assignment* is a function g that assigns to any $u \in \mathbf{Var}_\tau$ a value $g(u) \in D_\tau^M$. If $d \in D_\tau^M$, then $g[u/d]$ is the M-assignment s.t. (i) $g[u/d](u) = d$, and (ii) $g[u/d](u') = g(u')$ if $u' \neq u$.

DEFINITION 2.2′ (indices and information states)
- $i \in (D_e^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *n-stack* (aka *index*) of M-drefs
  $i_m := r_m$, if $i = \langle r_1, \ldots r_n\rangle \in (D_e^M)^n$ & $1 \leq m \leq n$
  $ij := \langle r_1,\ldots r_n, r'_1, \ldots r'_m\rangle$ if $i = \langle r_1, \ldots r_n\rangle \in (D_e^M)^n$ & $j = \langle r'_1, \ldots r'_n\rangle \in (D_e^M)^m$
- $s \in \mathcal{P}(D_e^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *M-state* of information about n M-drefs
  $S^M = \cup_{n \in \mathcal{N}} \mathcal{P}(D_e^M)^n$ is the set of M-information states

DEFINITION 2.3 (QPLA-semantics). For any M, g, $s \in S^M$, $i \in (D_e^{\,M})^n$:

b	$\{i\}[\![\alpha]\!]^{M,g}$	$= [\![\alpha]\!]^M$	if $\alpha \in \mathbf{Con}_e$
	$\{i\}[\![\alpha]\!]^{M,g}$	$= g(\alpha)$	if $\alpha \in \mathbf{Var}_e$
	$\{i\}[\![\alpha]\!]^{M,g}$	$= i_m$	if $\alpha = p_m \in \mathbf{Prn}_e$ & $1 \leq m \leq n$
=	$s[\![\alpha = \beta]\!]^{M,g}$	$= \{i \in s \mid \{i\}[\![\alpha]\!]^{M,g} = \{i\}[\![\beta]\!]^{M,g}\}$	
f	$s[\![\alpha\beta_1\ldots\beta_n]\!]^{M,g}$	$= \{i \in s \mid [\![\alpha]\!]^M(\{i\}[\![\beta_1]\!]^{M,g})\ldots(\{i\}[\![\beta_n]\!]^{M,g}) = 1\}$	
f$^\exists$	$s[\![\alpha\beta_1\beta_2]\!]^{M,g}$	$= \{i \in s \mid [\![\alpha]\!]^M({}^\chi\{r \mid \exists j\text{: } ji \in \{i\}[\![\beta_1]\!]^{M,g}(r)\})$ $({}^\chi\{r \mid \exists j, k\text{: } ji \in \{i\}[\![\beta_1]\!]^{M,g}(r) \;\&\; kji \in \{ji\}[\![\beta_2]\!]^{M,g}(r)\}) = 1\}$	
¬	$s[\![\neg\phi]\!]^{M,g}$	$= \{i \in s \mid \neg\exists j\text{: } ji \in s[\![\phi]\!]^{M,g}\}$	
∧	$s[\![(\phi \wedge \psi)]\!]^{M,g}$	$= s[\![\phi]\!]^{M,g}[\![\psi]\!]^{M,g}$	
∃	$s[\![\exists u\phi]\!]^{M,g}$	$= \{ri \mid i \in s[\![\phi]\!]^{M,g[u/r]}\}$	
λ	$s[\![\lambda u\phi]\!]^{M,g}(r)$	$= s[\![\phi]\!]^{M,g[u/r]}$	if $r \in D_e^{\,M}$

DEFINITION 3 (Support and truth)

- *s supports* ϕ wrt M under g, $s \models_{M,g} \phi$, iff $\forall i \in s \exists j\text{: } ji \in s[\![\phi]\!]^{M,g}$
- ϕ is *true* in M, $\models_M \phi$, iff $\forall g\text{: } \{\langle\rangle\}[\![\phi]\!]^{M,g} \neq \emptyset$

FACT A

→	$s[\![(\phi \rightarrow \psi)]\!]^{M,g}$	$= \{i \in s \mid \forall j\text{: } ji \in s[\![\phi]\!]^{M,g} \rightarrow \exists k\text{: } kji \in s[\![\phi]\!]^{M,g}[\![\psi]\!]^{M,g}\}$
∀	$s[\![\forall u\phi]\!]^{M,g}$	$= \{i \in s \mid \forall r \exists j\text{: } ji \in s[\![\phi]\!]^{M,g[u/r]}\}$

Proof:
As for QPLA (v. 3.1)

(5) Most people who write an article (later) develop it in another article.
(5′) $most(\lambda x(Px \wedge \exists y(Ay \wedge Wxy)))(\lambda x\, \exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z))$ a la Dekker 2003a

Fact 5a:
$\{j\}[\![\exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z)]\!]^{M,g[x/r]}$
$= \{r''j \mid [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq j_1 \;\&\; [\![D]\!]^M(r)(j_1)(r'') = 1\}$

Proof:

1. $\{j\}[\![\exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z)]\!]^{M,g[x/r]}$
2. $\{r''i \mid i \in \{j\}[\![Az]\!]^{M,g[x/r][z/r'']}[\![\neg(z = p_1)]\!]^{M,g[x/r][z/r'']}[\![Dxp_1z]\!]^{M,g[x/r][z/r'']}\}$ D2.3.∃, ∧
3. $\{r''i \mid i \in \{j\}[\![Az]\!]^{M,g[x/r][z/r'']}[\![\neg(z = p_1)]\!]^{M,g[x/r][z/r'']}$ D2.3.*f*, df. {–:–}, D2.3.*b*, D2.2.*g*[*u*/*d*]
   $\& [\![D]\!]^M(r)(i_1)(r'') = 1\}$
4. $\{r''i \mid i \in \{j\}[\![Az]\!]^{M,g[x/r][z/r'']}$ D2.3.–, df. {–:–}
   $\& \neg\exists k\text{: } (ki \in \{j\}[\![Az]\!]^{M,g[x/r][z/r'']}[\![z = p_1]\!]^{M,g[x/r][z/r'']})$
   $\& [\![D]\!]^M(r)(i_1)(r'') = 1\}$
5. $\{r''i \mid i \in \{j\} \;\&\; [\![A]\!]^M(r'') = 1$ 3 × D2.3.*f*, df. {–:–}, D2.3.=, *b*, D2.2.*g*[*u*/*d*]
   $\& \neg\exists k\text{: } (ki \in \{j\} \;\&\; [\![A]\!]^M(r'') = 1 \;\&\; \{ki\}[\![z]\!]^{M,g[x/r][z/r'']} = \{ki\}[\![p_1]\!]^{M,g[x/r][z/r'']})$
   $\& [\![D]\!]^M(r)(i_1)(r'') = 1\}$
6. $\{r''j \mid [\![A]\!]^M(r'') = 1$ eliminate *i*, *k*
   $\& \neg(j \in \{j\} \;\&\; [\![A]\!]^M(r'') = 1 \;\&\; \{j\}[\![z]\!]^{M,g[x/r][z/r'']} = \{j\}[\![p_1]\!]^{M,g[x/r][z/r'']})$
   $\& [\![D]\!]^M(r)(i_1)(r'') = 1\}$
7. $\{r''j \mid [\![A]\!]^M(r'') = 1 \;\&\; \neg(r'' = j_1) \;\&\; [\![D]\!]^M(r)(i_1)(r'') = 1\}$ simplify, D2.3.=, *b*, D2.2.*g*[*u*/*d*]

□

Fact 5b:

$\models_M$ (5′) iff

$$[\![most]\!]^M({}^{\chi}\{r \mid [\![P]\!]^M(r) = 1 \;\&\; \exists r'\colon [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\})$$
$$({}^{\chi}\{r \mid [\![P]\!]^M(r) = 1 \;\&\; \exists r', r''\colon [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1$$
$$\&\; [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq r' \;\&\; [\![D]\!]^M(r)(r')(r'') = 1\}) = 1$$

Proof:

1. $\models_M most(\lambda x(Px \wedge \exists y(Ay \wedge Wxy))(\lambda x\, \exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z))$

2. $\forall g$: D3

$$\{\langle\rangle\}[\![most(\lambda x(Px \wedge \exists y(Ay \wedge Wxy))(\lambda x\, \exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z))]\!]^{M,g} \neq \varnothing$$

3. $\forall g$: D2.3.$f^{\exists}$

$$\{i \in \{\langle\rangle\} \mid [\![most]\!]^M({}^{\chi}\{r \mid \exists j\colon ji \in \{i\}[\![\lambda x(Px \wedge \exists y(Ay \wedge Wxy))]\!]^{M,g}(r)\})$$
$$({}^{\chi}\{r \mid \exists j, k\colon ji \in \{i\}[\![\lambda x(Px \wedge \exists y(Ay \wedge Wxy))]\!]^{M,g}(r)$$
$$\&\; kji \in \{ji\}[\![\lambda x\, \exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z)]\!]^{M,g}(r))\}) = 1\} \neq \varnothing$$

4. $\forall g$: eliminate i

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists j\colon j \in \{\langle\rangle\}[\![\lambda x(Px \wedge \exists y(Ay \wedge Wxy))]\!]^{M,g}(r)\})$$
$$({}^{\chi}\{r \mid \exists j, k\colon j \in \{\langle\rangle\}[\![\lambda x(Px \wedge \exists y(Ay \wedge Wxy))]\!]^{M,g}(r)$$
$$\&\; kj \in \{j\}[\![\lambda x\, \exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z)]\!]^{M,g}(r))\}) = 1$$

5. $\forall g$: D2.3.λ

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists j\colon j \in \{\langle\rangle\}[\![Px \wedge \exists y(Ay \wedge Wxy)]\!]^{M,g[x/r]}\})$$
$$({}^{\chi}\{r \mid \exists j, k\colon j \in \{\langle\rangle\}[\![Px \wedge \exists y(Ay \wedge Wxy)]\!]^{M,g[x/r]}$$
$$\&\; kj \in \{j\}[\![\exists z(Az \wedge \neg(z = p_1) \wedge Dxp_1z)]\!]^{M,g[x/r]})\}) = 1$$

6. $\forall g$: D2.3.$\wedge$, F5a

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists j\colon j \in \{\langle\rangle\}[\![Px]\!]^{M,g[x/r]}[\![\exists y(Ay \wedge Wxy)]\!]^{M,g[x/r]}\})$$
$$({}^{\chi}\{r \mid \exists j, k\colon j \in \{\langle\rangle\}[\![Px]\!]^{M,g[x/r]}[\![\exists y(Ay \wedge Wxy)]\!]^{M,g[x/r]}$$
$$\&\; kj \in \{r''j \mid [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq j_1 \;\&\; [\![D]\!]^M(r)(j_1)(r'') = 1\}\}) = 1$$

7. $\forall g$: like F3a

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists j\colon j \in \{r' \mid [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\})$$
$$({}^{\chi}\{r \mid \exists j, k\colon j \in \{r' \mid [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\}$$
$$\&\; kj \in \{r''j \mid [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq j_1 \;\&\; [\![D]\!]^M(r)(j_1)(r'') = 1\}\}) = 1$$

8. $\forall g$: 2× eliminate j

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists r'\colon [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\})$$
$$({}^{\chi}\{r \mid \exists r', k\colon [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1$$
$$\&\; kr' \in \{r''r' \mid [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq r' \;\&\; [\![D]\!]^M(r)(r')(r'') = 1\}\}) = 1$$

9. $\forall g$: eliminate k

$$[\![most]\!]^M({}^{\chi}\{r \mid \exists r'\colon [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\})$$
$$({}^{\chi}\{r \mid \exists r', r''\colon [\![P]\!]^M(r) = 1 \;\&\; [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1$$
$$\&\; [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq r' \;\&\; [\![D]\!]^M(r)(r')(r'') = 1\}) = 1$$

10. $[\![most]\!]^M({}^{\chi}\{r \mid [\![P]\!]^M(r) = 1 \;\&\; \exists r'\colon [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1\})$ eliminate g, rearrange

$$({}^{\chi}\{r \mid [\![P]\!]^M(r) = 1 \;\&\; \exists r', r''\colon [\![A]\!]^M(r') = 1 \;\&\; [\![W]\!]^M(r)(r') = 1$$
$$\&\; [\![A]\!]^M(r'') = 1 \;\&\; r'' \neq r' \;\&\; [\![D]\!]^M(r)(r')(r'') = 1\}) = 1$$

□

3. <u>QPLA (v. 3.3): *Uniqueness* via *referents for functions*</u>

DEFINITION 1.1 (QPLA-types)
- $e, t \in \mathbf{Typ}$
- $(\sigma\tau) \in \mathbf{Typ}$, if $\sigma, \tau \in \mathbf{Typ}$

DEFINITION 1.2 (QPLA-basic terms)
- $\mathbf{Con}_e$ = {*bill*, *sue*, …}
  $\mathbf{Con}_{et}$ = {*diver*, *pearl*,…, *die*, …}
  $\mathbf{Con}_{eet}$ = {*friend*, …, *find*, *beat*, …}
  $\mathbf{Con}_{eeet}$ = {*give.to*, *beat.with*, …}
  $\mathbf{Con}_{(et)(et)t}$ = {*every*, *most*, …}
- $\mathbf{Var}_e$ = $\{x, y, z, \ldots\}$
- $\mathbf{Prn}_e$ = $\{p_1, p_2, \ldots\}$

DEFINITION 1.2 (QPLA-terms).

b	$\alpha \in \mathbf{Term}_\tau$,	if $\alpha \in \mathbf{Con}_\tau \cup \mathbf{Var}_\tau \cup \mathbf{Prn}_\tau$
=	$(\alpha = \beta) \in \mathbf{Term}_t$	if $\alpha, \beta \in \mathbf{Term}_e$
f	$\alpha\beta_1\ldots\beta_n \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{e\ldots et}$ (n times e) & $\beta_1,\ldots\beta_n \in \mathbf{Term}_e$
$\boldsymbol{f}^{\exists}$	$\alpha\beta_1\beta_2 \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{(et)(et)t}$ & $\beta_1, \beta_2 \in \mathbf{Term}_{et}$
$\boldsymbol{f}^{!}$	$\alpha^{!}\beta_1\beta_2 \in \mathbf{Term}_t$	if $\alpha \in \mathbf{Term}_{(et)(et)t}$ & $\beta_1, \beta_2 \in \mathbf{Term}_{et}$
¬	$\neg\phi \in \mathbf{Term}_t$	if $\phi \in \mathbf{Term}_t$
∧	$(\phi \wedge \psi) \in \mathbf{Term}_t$	if $\phi, \psi \in \mathbf{Term}_t$
∃	$\exists u\phi \in \mathbf{Term}_t$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
λ	$(\lambda u\phi) \in \mathbf{Term}_{et}$	if $u \in \mathbf{Var}_e$ & $\phi \in \mathbf{Term}_t$
!	$!\beta \in \mathbf{Term}_{et}$	if $\beta \in \mathbf{Term}_{et}$

ABBREVIATIONS 1 (obj. lg)
- $(\phi \rightarrow \psi) := \neg(\phi \wedge \neg\psi)$
- $\forall u\phi := \neg\exists u\neg\phi$

ABBREVIATIONS 2 (meta-lg.)
- Set $F := \{r \in D \mid F(d) = 1\}$ for $F: D \rightarrow \{1, 0\}$
- ${}^{\chi}A = F$ iff A = Set F & $A \subseteq D$

DEFINITION 2.1 (frames). A QPLA-*frame* is a set of τ-domains, $\{D_\tau: \tau \in \mathbf{Typ}\}$ s.t.
- $D_t = \{1, 0\}$
  D_e is a non-empty set disjoint from D_t
- $D_{(\sigma\tau)} = \{f \mid \varnothing \subset \text{Dom } f \subseteq D_\sigma \text{ \& Ran } f \subseteq D_\tau\}$

ABBREVIATIONS 2 (meta-language)
- Set $F := \{r \in D \mid F(d) = 1\}$ for $F: D_e \rightarrow \{1, 0\}$
- ${}^{\chi}A = F$ iff A = Set F for $F: D_e \rightarrow \{1, 0\}, A \subseteq D_e$

DEFINITION 2.2 (models and assignments)
- A QPLA-*model* is a structure $M = \langle D^M, [\![\cdot]\!]^M\rangle$ such that:
  (i) $D^M = \{D_\tau^M: \tau \in \mathbf{Typ}\}$ is a $\text{QPLA}_{3.2}$-frame.
  (ii) $[\![\cdot]\!]^M$ is a function that assigns to any $\alpha \in \mathbf{Con}_\tau$ a denotation $[\![\alpha]\!]^M \in D_\tau^M$
- For all $F, F' \in D_{et}^M$:
  $[\![every]\!]^M(F)(F') = 1$ iff Set $F \subseteq$ Set F'
  $[\![most]\!]^M(F)(F') = 1$ iff $|\text{Set } F \cap \text{Set } F'| > |\text{Set } F - \text{Set } F'|$
- An *M-assignment* is a function g that assigns to any $u \in \mathbf{Var}_\tau$ a value $g(u) \in D_\tau^M$. If $d \in D_\tau^M$, then $g[u/d]$ is the *M*-assignment s.t. (i) $g[u/d](u) = d$, and (ii) $g[u/d](u') = g(u')$ if $u' \neq u$.

DEFINITION 2.2′ (QPLA indices and information states)

- $R^M = D_e^M \cup D_{ee}^M$ is the *domain* of *M*-drefs.
  $i \in (R^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *n-stack* (aka *index*) of *M*-drefs
  $i_m := r_m$, if $i = \langle r_1, \ldots r_n \rangle \in (R^M)^n$ & $1 \leq m \leq n$
  $ij := \langle r_1, \ldots r_n, r'_1, \ldots r'_m \rangle$ if $i = \langle r_1, \ldots r_n \rangle \in (R^M)^n$ & $j = \langle r'_1, \ldots r'_n \rangle \in (R^M)^m$
- $\langle\rangle(d) := \langle\rangle$ if $d \in D_e^M$
  $r(d) := r$ if $r, d \in D_e^M$
  $i(d) := \langle r_1(d), \ldots r_n(d) \rangle$ if $i = \langle r_1, \ldots r_n \rangle \in (R^M)^n$, $d \in D_e^M$
- $s \in \mathcal{P}(R^M)^n$, where $n \in \mathcal{N} = \{0, 1, 2, \ldots\}$, is an *M-state* of information about *n M*-drefs
  $S^M = \cup_{n \in \mathcal{N}} \mathcal{P}(R^M)^n$ is the set of *M*-information states

DEFINITION 2.3 (QPLA-semantics). For any M, g, $s \in S^M$, $i \in (D_e^M)^n$:

b	$\{i\}[\![\alpha]\!]^{M,g}$	$=$	$[\![\alpha]\!]^M$	if $\alpha \in \mathbf{Con}_e$
	$\{i\}[\![\alpha]\!]^{M,g}$	$=$	$g(\alpha)$	if $\alpha \in \mathbf{Var}_e$
	$\{i\}[\![\alpha]\!]^{M,g}$	$=$	i_m	if $\alpha = p_m \in \mathbf{Prn}_e$ & $1 \leq m \leq n$

$=$ $\quad s[\![\alpha = \beta]\!]^{M,g} = \{i \in s \mid \{i\}[\![\alpha]\!]^{M,g} = \{i\}[\![\beta]\!]^{M,g}\}$

f $\quad s[\![\alpha\beta_1\ldots\beta_n]\!]^{M,g} = \{i \in s \mid [\![\alpha]\!]^M(\{i\}[\![\beta_1]\!]^{M,g})\ldots(\{i\}[\![\beta_n]\!]^{M,g}) = 1\}$

$\boldsymbol{f}^{\exists}$ $\quad s[\![\alpha\beta_1\beta_2]\!]^{M,g} = \{i \in s \mid [\![\alpha]\!]^M({}^{\chi}\{r \mid \exists j\text{: } ji \in \{i\}[\![\beta_1]\!]^{M,g}(r)\})$
$\qquad ({}^{\chi}\{r \mid \exists j, k\text{: } ji \in \{i\}[\![\beta_1]\!]^{M,g}(r) \text{ \& } kji \in \{ji\}[\![\beta_2]\!]^{M,g}(r)\}) = 1\}$

$\boldsymbol{f}^{!}$ $\quad s[\![\alpha^{!}\beta_1\beta_2]\!]^{M,g} = \{kji \mid i \in s \text{ \& } [\![\alpha]\!]^M({}^{\chi}\{r \mid ji \in \{i\}[\![!\beta_1]\!]^{M,g}(r)\})$
$\qquad ({}^{\chi}\{r \mid [kji](r) \in \{[ji](r)\}[\![\beta_2]\!]^{M,g}(r)\}) = 1\}$

$\neg$ $\quad s[\![\neg\phi]\!]^{M,g} = \{i \in s \mid \neg\exists j\text{: } ji \in s[\![\phi]\!]^{M,g}\}$

$\wedge$ $\quad s[\![(\phi \wedge \psi)]\!]^{M,g} = s[\![\phi]\!]^{M,g}[\![\psi]\!]^{M,g}$

$\exists$ $\quad s[\![\exists u\phi]\!]^{M,g} = \{di \mid i \in s[\![\phi]\!]^{M,g[u/d]}\}$

$\boldsymbol{\lambda}$ $\quad s[\![\lambda u\phi]\!]^{M,g}(d) = \{i(d) \mid i(d) \in s[\![\phi]\!]^{M,g[u/d]}\}$

! $\quad s[\![!\beta]\!]^{M,g}(d) = \{fi \mid d \in \text{Dom } f$
$\qquad$ & $\text{Dom } f = \{c \in D_e^M \mid \exists!c' \in D_e^M\text{: } c'i(c) \in s[\![\beta]\!]^{M,g}(c)\}$
$\qquad$ & $\forall c \in \text{Dom } f\text{: } f(c)i(c) \in s[\![\beta]\!]^{M,g}(c)\}$

DEFINITION 3 (Support and truth)

- *s supports* ϕ wrt *M* under *g*, $s \models_{M,g} \phi$, iff $\forall i \in s \exists j\text{: } ji \in s[\![\phi]\!]^{M,g}$
- ϕ is *true* in *M*, $\models_M \phi$, iff $\forall g\text{: } \{\langle\rangle\}[\![\phi]\!]^{M,g} \neq \emptyset$

FACT A

$\rightarrow$ $\quad s[\![(\phi \rightarrow \psi)]\!]^{M,g} = \{i \in s \mid \forall j\text{: } ji \in s[\![\phi]\!]^{M,g} \rightarrow \exists k\text{: } kji \in s[\![\phi]\!]^{M,g}[\![\psi]\!]^{M,g}\}$

$\forall$ $\quad s[\![\forall u\phi]\!]^{M,g} = \{i \in s \mid \forall r \exists j\text{: } ji \in s[\![\phi]\!]^{M,g[u/r]}\}$

Proof: As for QPLA (v. 3.1)

(6) Most businessmen who sent a contribution to Gore sent a larger contribution to Bush.

(6′) $most^{!}(\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))(\lambda x\, \exists z(Cz \wedge Lzp_1 \wedge Sxzb))$

Fact 6a:
$\{\langle\rangle\}\llbracket \lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,g}(d)$
$= \{f(d) | \llbracket B\rrbracket^M(d) = 1 \;\&\; \llbracket C\rrbracket^M(f(d)) = 1 \;\&\; \llbracket S\rrbracket^M(d)(f(d))(\llbracket a\rrbracket^M) = 1\}$

Proof.

1. $\{\langle\rangle\}\llbracket \lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,g}(d)$
2. $\{i(d) | i(d) \in \{\langle\rangle\}\llbracket Bx \wedge \exists y(Cy \wedge Sxya)\rrbracket^{M,g[x/d]}\}$ — D2.3.λ
3. $\{i(d) | i(d) \in \{\langle\rangle\}\llbracket Bx \wedge \exists y(Cy \wedge Sxya)\rrbracket^{M,g[x/d]}\}$ — eliminate i
4. $\{i(d) | i(d) \in \{\langle\rangle\}\llbracket Bx\rrbracket^{M,\,g[x/d]}\llbracket \exists y(Cy \wedge Sxya)\rrbracket^{M,g[x/d]}\}$ — D2.3.$\wedge$
5. $\{i(d) | i(d) \in \{rj | j \in \{\langle\rangle\}\llbracket Bx\rrbracket^{M,\,g[x/d]}\llbracket Cy \wedge Sxya\rrbracket^{M,g[x/d][y/r]}\}$ — D2.3.$\exists$
6. $\{i(d) | i(d) \in \{rj | j \in \{\langle\rangle\}\llbracket Bx\rrbracket^{M,\,g[x/d]}\llbracket Cy\rrbracket^{M,\,g[x/d][y/r]}\llbracket Sxya\rrbracket^{M,g[x/d][y/r]}\}$ — D2.3.$\wedge$
7. $\{i(d) | i(d) \in \{rj | j \in \{\langle\rangle\}$ — $3 \times$ D2.3.f, df. $\{–:–\}$, D2.3.b, D2.2.$g[u/d]$
   $\&\; \llbracket B\rrbracket^M(d) = 1 \;\&\; \llbracket C\rrbracket^M(r) = 1 \;\&\; \llbracket S\rrbracket^M(d)(r)(\llbracket a\rrbracket^M = 1\}$
8. $\{i(d) | i(d) \in \{r | \llbracket B\rrbracket^M(d) = 1 \;\&\; \llbracket C\rrbracket^M(r) = 1 \;\&\; \llbracket S\rrbracket^M(d)(r)(\llbracket a\rrbracket^M = 1\}$ — eliminate j
9. $\{f(d) | \llbracket B\rrbracket^M(d) = 1 \;\&\; \llbracket C\rrbracket^M(f(d)) = 1 \;\&\; \llbracket S\rrbracket^M(d)(f(d))(\llbracket a\rrbracket^M = 1\}$ — simplify, rename i

□

Fact 6b:
$\{\langle\rangle\}\llbracket\, !\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,g}(d)$
$= \{f | d \in \text{Dom } f$
$\;\&\; \text{Dom } f = \{c \in D_e{}^M | \llbracket B\rrbracket^M(c) = 1 \;\&\; \exists! c' \in D_e{}^M\!: \llbracket C\rrbracket^M(c') = 1 \;\&\; \llbracket S\rrbracket^M(c)(c')(\llbracket a\rrbracket^M) = 1\}$
$\;\&\; \forall c \in \text{Dom } f\!: \llbracket C\rrbracket^M(f(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f(c))(\llbracket a\rrbracket^M) = 1\}$

Proof:

1. $\{\langle\rangle\}\llbracket\, !\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,\,g}(d)$
2. $\{fi | d \in \text{Dom } f$ — D2.3.$f^!$
   $\;\&\; \text{Dom } f = \{c \in D_e{}^M | \exists! c' \in D_e{}^M\!: c'i(c) \in \{\langle\rangle\}\llbracket \lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,\,g}(c)\}$
   $\;\&\; \forall c \in \text{Dom } f\!: f(c)i(c) \in \{\langle\rangle\}\llbracket \lambda x(Bx \wedge \exists y(Cy \wedge Sxya))\rrbracket^{M,\,g}(c)\}$
3. $\{fi | d \in \text{Dom } f$ — Fact 6a
   $\;\&\; \text{Dom } f = \{c \in D_e{}^M | \exists! c' \in D_e{}^M\!:$
   $c'i(c) \in \{f'(c) | \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f'(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f'(c))(\llbracket a\rrbracket^M = 1\}$
   $\;\&\; \forall c \in \text{Dom } f\!: f(c)i(c) \in \{f''(c) | \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f''(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f''(c))(\llbracket a\rrbracket^M = 1\}$
4. $\{f | d \in \text{Dom } f$ — eliminate i
   $\;\&\; \text{Dom } f = \{c \in D_e{}^M | \exists! c' \in D_e{}^M\!:$
   $c' \in \{f'(c) | \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f'(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f'(c))(\llbracket a\rrbracket^M = 1\}$
   $\;\&\; \forall c \in \text{Dom } f\!: f(c) \in \{f''(c) | \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f''(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f''(c))(\llbracket a\rrbracket^M = 1\}$
5. $\{f | d \in \text{Dom } f$ — simplify
   $\;\&\; \text{Dom } f = \{c \in D_e{}^M | \exists! c' \in D_e{}^M\!: \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(c') = 1 \;\&\; \llbracket S\rrbracket^M(c)(c')(\llbracket a\rrbracket^M = 1\}$
   $\;\&\; \forall c \in \text{Dom } f\!: \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f(c))(\llbracket a\rrbracket^M = 1\}$
6. $\{f | d \in \text{Dom } f$ — rearrange
   $\;\&\; \text{Dom } f = \{c \in D_e{}^M | \llbracket B\rrbracket^M(c) = 1 \;\&\; !\exists c' \in D_e{}^M\!: \llbracket C\rrbracket^M(c') = 1 \;\&\; \llbracket S\rrbracket^M(c)(c')(\llbracket a\rrbracket^M = 1\}$
   $\;\&\; \forall c \in \text{Dom } f\!: \llbracket B\rrbracket^M(c) = 1 \;\&\; \llbracket C\rrbracket^M(f(c)) = 1 \;\&\; \llbracket S\rrbracket^M(c)(f(c))(\llbracket a\rrbracket^M = 1\}$ □

Fact 6c:
$\{f(d)\}\llbracket \lambda x\, \exists z(Cz \wedge Lzp_1 \wedge Sxzb))\rrbracket^{M,g}(d)$
$= \{f'(d)f(d) | \llbracket C\rrbracket^M(f'(d)) = 1 \;\&\; \llbracket L\rrbracket^M(f'(d))(f(d)) = 1 \;\&\; \llbracket S\rrbracket^M(d)(f'(d))(\llbracket b\rrbracket^M) = 1\}$

Proof: Exercise

Fact 6d:

$$\{\langle\rangle\}[\![most^!(\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))(\lambda x\, \exists z(Cz \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}$$
$$= \{f'f \mid \text{Dom } f = \{c \in D_e^M \mid [\![B]\!]^M(c) = 1 \;\&\; \exists! c' \in D_e^M\colon [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1$$
$$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$$
$$\&\; [\![most]\!]^M({}^{\chi}\text{Dom } f)$$
$$({}^{\chi}\{d \mid [\![C]\!]^M(f'(d)) = 1 \;\&\; [\![L]\!]^M(f'(d))(f(d)) = 1 \;\&\; [\![S]\!]^M(d)(f'(d))([\![b]\!]^M) = 1\}) = 1\}$$

Proof:

1. $\{\langle\rangle\}[\![most^!(\lambda x(Px \wedge \exists y(Cy \wedge Sxya))(\lambda x\, \exists z(Cz \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}$

2. $\{kji \mid i \in \{\langle\rangle\}$ D2.3.$f^!$
$\&\; [\![most]\!]^M({}^{\chi}\{r \mid ji \in \{i\}[\![\,!\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))]\!]^{M,g}(r)\})$
$({}^{\chi}\{r \mid [kji](r) \in \{[ji](r)\}[\![\lambda x\, \exists z(Cy \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}(r)\}) = 1\}$

3. $\{kj \mid [\![most]\!]^M({}^{\chi}\{r \mid j \in \{\langle\rangle\}[\![\,!\lambda x(Bx \wedge \exists y(Cy \wedge Sxya))]\!]^{M,g}(r)\})$ eliminate i
$({}^{\chi}\{r \mid [kj](r) \in \{j(r)\}[\![\lambda x\, \exists z(Cy \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}(r)\}) = 1\}$

4. $\{kj \mid [\![most]\!]^M({}^{\chi}\{r \mid j \in \{f \mid r \in \text{Dom } f$ Fact 6b
$\&\; \text{Dom } f = \{c \in D_e^M \mid \exists! c' \in D_e^M\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1\}$
$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$
$({}^{\chi}\{r \mid [kj](r) \in \{j(r)\}[\![\lambda x\, \exists z(Cy \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}(r)\}) = 1\}$

5. $\{kf \mid [\![most]\!]^M({}^{\chi}\{r \mid r \in \text{Dom } f$ rename j, simplify, rearrange
$\&\; \text{Dom } f = \{c \in D_e^M \mid [\![B]\!]^M(c) = 1 \;\&\; \exists! c' \in D_e^M\colon [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1\}$
$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$
$({}^{\chi}\{r \mid [kf](r) \in \{f(r)\}[\![\lambda x\, \exists z(Cy \wedge Lzp_1 \wedge Sxzb))]\!]^{M,g}(r)\}) = 1\}$

6. $\{kf \mid [\![most]\!]^M({}^{\chi}\{r \mid r \in \text{Dom } f$ Fact 6c
$\&\; \text{Dom } f = \{c \in D_e^M \mid [\![B]\!]^M(c) = 1 \;\&\; \exists! c' \in D_e^M\colon [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1\}$
$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$
$({}^{\chi}\{r \mid [kf](r) \in \{f'(r)f(r) \mid [\![C]\!]^M(f'(r)) = 1 \;\&\; [\![L]\!]^M(f'(r))(f(r)) = 1$
$\&\; [\![S]\!]^M(r)(f'(r))([\![b]\!]^M) = 1\}\}) = 1\}$

7. $\{f'f \mid [\![most]\!]^M({}^{\chi}\{r \mid r \in \text{Dom } f$ simplify, rename k
$\&\; \text{Dom } f = \{c \in D_e^M \mid [\![B]\!]^M(c) = 1 \;\&\; \exists! c' \in D_e^M\colon [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1\}$
$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$
$({}^{\chi}\{r \mid [\![C]\!]^M(f'(r)) = 1 \;\&\; [\![L]\!]^M(f'(r))(f(r)) = 1 \;\&\; [\![S]\!]^M(r)(f'(r))([\![b]\!]^M) = 1\}) = 1\}$

8. $\{f'f \mid \text{Dom } f = \{c \in D_e^M \mid [\![B]\!]^M(c) = 1$ rearrange, rename r
$\&\; \exists! c' \in D_e^M\colon [\![C]\!]^M(c') = 1 \;\&\; [\![S]\!]^M(c)(c')([\![a]\!]^M) = 1$
$\&\; \forall c \in \text{Dom } f\colon [\![B]\!]^M(c) = 1 \;\&\; [\![C]\!]^M(f(c)) = 1 \;\&\; [\![S]\!]^M(c)(f(c))([\![a]\!]^M = 1\})$
$\&\; [\![most]\!]^M({}^{\chi}\text{Dom } f)$
$({}^{\chi}\{d \mid [\![C]\!]^M(f'(d)) = 1 \;\&\; [\![L]\!]^M(f'(d))(f(d)) = 1 \;\&\; [\![S]\!]^M(d)(f'(d))([\![b]\!]^M) = 1\}) = 1\}$

□

Remark: This improves on Dekker 2003a. Dom f' now only needs to include those businessmen from Dom f who sent a larger contribution to Bush. The problem noted in Topic 4 is solved.