Belief change and anaphora in English
OU vs. DRT

- ***Kamp's example***

(1) *On Sunday Bill heard that Mary was in Paris.*
on Sunday Bill$_{\top}$ hear-PST that Mary$_{\top}$ be-PST.sg in Paris$_{\perp}$

(2) *On Tuesday he learned that on the previous day she had left.*
on Tuesday 3sm$_{\top}$ learn-PST that on the previous day 3sf$_{\top}$ have$_{\varepsilon\varepsilon}$-PST leave-PPF

1. DRT ANALYSIS: ANAPHORA VIA BELIEF-COPYING (Kamp et al 2003, Kamp 2005)

- *Final* DRS$_1$ *for* (1) (≡ (230) on p. 238 + **n** + '**on** Sunday')

DRS$_1$ [b m p **n** t_{11} t_{12} s_{11} s_{12} e_1|

(0) *Bill*(b), *Mary*(m), *Paris*(p),

(1) *Sun*(t_{11}), $t_{11} < n$, ¬[t| *Sun*(t), $t_{11} < t < n$],

(2) $e_1 \subseteq t_{12} \subseteq t_{11}$

(3) $s_{11} >\!< e_1 >\!< s_{12}$

(4) s_{11}: ¬[s| $s \subseteq s_{11}$,
s: Att(b, {⟨BEL, [s′| ($n \subseteq s'$), (s′: *in*(m, p))]⟩})]

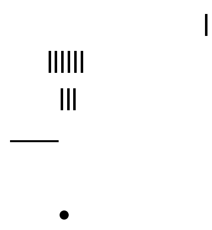
(5) s_{12}: Att(b, {⟨BEL, [s′ | ($n \subseteq s'$), (s′: *in*(m, p))]⟩})]

NOTATION:
plain font for variables (e.g. t_1, s_1,n, …), italics for their values (e.g. t_1, s_1, *n*,…)

$\vartheta_w\, s$	for	*w*-time of *s*
$M, w \models K$	for	DRS *K* is true in model *M* at world *w*
$M[n/\vartheta_w\, s]$	for	DRT-model like *M* except that *n* is replaced with the *w*-time of *s*

Model for DRS$_1$:

w_0



n: utterance time (determined by *M*)
t_{11}: last Sun. before *n*
$t_{12} \subseteq t_{11}$
s_{11}: there is no part $s \subseteq s_{11}$, s.t. in *s* Bill's beliefs include
$p_{\sigma 1}(s) \approx$ Mary in Paris at (w_0-)time of *s*
e_1:
s_{12}: Bill's beliefs include
$p_{\sigma 1}(s_{12}) \approx$ Mary in Paris at (w_0-)time of s_{12}

~~~~~~

$w_2 \in p_{\sigma 2}(s_{12})$ (one of s_{12}-beliefs)

s′: Mary in Paris,
$\vartheta_{w0}\, s_{12} \subseteq \vartheta_{w2}\, s'$ NB: temp. ***de re***, not *de se*
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• To interpret (2), Kamp requires some form of *belief-copying* (‘accommodation’). Prior to this operation, the final DRS_2 for (2) is *improper*: s_{31}, the prestate of the leaving event e_3, is free:

DRS_2 $[t_{21}\ t_{22}\ s_{21}\ s_{22}\ e_2|$ ≡ Kamp 2005:466

(1′) $Tue(t_{21}),\ t_{11} < t_{21},\ \neg[t|\ Tue(t),\ t_{11} < t < t_{21}]$, – superfluous drefs

(2′) $e_2 \subseteq t_{22} \subseteq t_{21},\ \ t_{22} < n$

(3′) $s_{21} >\!< e_2 >\!< s_{22}$

(4′) $s_{21}: \neg[s|\ s \subseteq s_{21},$
$s: Att(b, \{\langle \text{BEL}, \mathbf{K}\rangle\})]$

(5′) $s_{22}: Att(b, \{\langle \text{BEL}, \mathbf{K}\rangle\})$],

where
$\mathbf{K} := [l\ t_{31}\ t_{32}\ s_{32}\ e_3|$

(1_K) $day(t_{31}),\ t_{31} >\!< t_{21}$

(2_K) $e_3 \subseteq t_{32} \subseteq t_{31}$

(3_K) $s_{31} >\!< e_3 >\!< s_{32}$

(4_K) $e_3: leave(m, p)$

(5_K) $l \neq p,\ in(m, l)$]

• Kamp’s SOLUTION 1. On Tuesday Bill still believes**:** ‘*Mary is in Paris*’

DRS_2^A $[t_{21}\ t_{22}\ s_{21}\ s_{22}\ e_2|$

(1′) $Tue(t_{21}),\ t_{11} < t_{21},\ \neg[t|\ Tue(t),\ t_{11} < t < t_{21}]$,

(2′) $e_2 \subseteq t_{22} \subseteq t_{21},\ \ t_{22} < n$

(3′) $s_{21} >\!< e_2 >\!< s_{22}$

(4′) $s_{21}: \neg[s|\ s \subseteq s_{21},$
$s: Att(b, \{\langle \text{BEL}, (\mathbf{[\mathit{s}_{31}|\ n \subseteq s_{31},\ s_{31}: \mathit{in}(m, p)]\ ;\ K})\rangle\})]$

(5′) $s_{22}: Att(b, \{\langle \text{BEL}, (\mathbf{[\mathit{s}_{31}|\ n \subseteq s_{31},\ s_{31}: \mathit{in}(m, p)]\ ;\ K})\rangle\})$
(6′) $\mathbf{t_{22} \subseteq s_{12}}$]

where
$\mathbf{K} := [l\ t_{31}\ t_{32}\ s_{32}\ e_3|$

(1_K) $day(t_{31}),\ t_{31} >\!< t_{21}$

(2_K) $e_3 \subseteq t_{32} \subseteq t_{31}$

(3_K) $s_{31} >\!< e_3 >\!< s_{32}$

(4_K) $e_3: leave(m, p)$

(5_K) $l \neq p,\ in(m, l)$]

PROBLEM FOR SOLUTION 1

Just after the learning event (e_2), Bill is in a mental state with contradictory beliefs—to wit, "Mary is in Paris (now)" and "Mary left yesterday". Can he really be said to have *learned* that Mary had left. Not according to my intuitions.

• Kamp's SOLUTION 2. On Tuesday Bill still believes**:** '*Mary was in Paris on Sunday*'

DRS_2^{A2} $[t_{21}\ t_{22}\ s_{21}\ s_{22}\ e_2|$

(1′) $\mathit{Tue}(t_{21}),\ t_{11} < t_{21},\ \neg[t|\ \mathit{Tue}(t),\ t_{11} < t < t_{21}],$

(2′) $e_2 \subseteq t_{22} \subseteq t_{21},\ \ t_{22} < n$

(3′) $s_{21} >< e_2 >< s_{22}$

(4′) $s_{21}: \neg[s|\ s \subseteq s_{21},$
$\qquad s: \text{Att}(b, \{\langle \text{BEL}, (\mathbf{[}\boldsymbol{s'}|\ \mathbf{t_{21} < n,\ t_{21} \subseteq s',\ s': \boldsymbol{in}(m, p)]\ ;\ K})\})]$

(5′) $s_{22}: \text{Att}(b, \{\langle \text{BEL}, (\mathbf{[}\boldsymbol{s'}|\ \mathbf{t_{21} < n,\ t_{21} \subseteq s',\ s': \boldsymbol{in}(m, p)]\ ;\ K})\})$]

where

$\mathbf{K} := [l\ t_{31}\ t_{32}\ s_{32}\ e_3|$

(1_K) $\mathit{day}(t_{31}),\ t_{31} >< t_{21}$

(2_K) $e_3 \subseteq t_{32} \subseteq t_{31}$

(3_K) $s_{31} >< e_3 >< s_{32}$

(4_K) $e_3: \mathit{leave}(m, p)$

(5_K) $l \neq p,\ \mathit{in}(m, l)$]

PROBLEMS FOR SOLUTION 2

• This is not even straightforward belief-copying. What's the magic that derives this box??!….

• Also, this fails to account for the anaphoric link between *be in Paris* in (1) and *leave* in (2). On this view, after Sunday Bill maintains the belief "Mary was in Paris on Sunday". On Tuesday, he adds the belief "Mary left yesterday". But this still allows the prestate of leaving to be some state of being in Paris other than the Sunday state (e.g. if Mary commutes to Paris a few days a week).

PROBLEM FOR BOTH 'SOLUTIONS':

Consider the salient reading of (1′)–(2′), i.e. Sam = SC, Maria = MB, MB commutes from NYC.

(1′) *On Friday Sam heard that Maria was in New Brunswick.*

(2′) *On Tuesday he learned that on the previous days she had left.*

According to persistence assumption 1, when Sam gets the news on Tuesday he ought to think:
"What? She has already left. I better ***revise*** my mental state"

According to persistence assumption 2, he ought to think:
"Oh, I see. She left yesterday. I better ***update*** my mental state"

I don't think either reaction is likely. Instead, I expect SC to exclaim:
"What on Earth happened? Why didn't she go home on Friday like she normally does?"

2. OU ANALYSIS: DIRECT ANAPHORA TO MODAL DREFS (a la Bittner 2006, 2005)

• ***Preview***

(0)	*Speech start-up*	i-reality $^{\top}w_i$:	•	$^{\top}e_0$: e_0-agt speaks up
			|	$t_0 = \vartheta_{wi}\, e_0$: e_0-time (e.g. Th)
(1)	On Sunday…		‖	$t_1 \subseteq k^{\tau}{}_1$(last)-Su before e_0
	…Bill heard…		•	e_1: e_1-agt says p_1 to Bill
			–	$\text{RES}_{wi}\, e_1$: Bill believes p_1
			‖	$t_1{}' = \vartheta_{wi}\, \text{RES}_{wi}\, e_1$
		~~		
		$w_1 \in p_1$:		($\text{RES}_{wi}\, e_1$-belief)
	…that…		|	$\underline{t}_2 w_1 = \vartheta_{w1}\, e_1$
	…Mary was in Paris.		——	$\underline{s}_2 w_1$: Mary is in l_2 = Paris
				$\underline{t}_2 w_1 \subseteq \vartheta_{w1}\, \underline{s}_2 w_1$
				$^{\text{I}}(t_1{}' \subseteq \vartheta_{w1}\, \underline{s}_2 w_1 < \vartheta_{wi}\, e_0)$
(2)	On Tuesday…	i-reality $^{\top}w_i$:	‖	$t_3 \subseteq k^{\tau}{}_3$(last)-Tu before e_0
	…he learned…		•	e_3: Bill forms belief p_3
			–	$\text{RES}_{wi}\, e_3$: Bill believes p_3
			‖	$t_3{}' = \vartheta_{wi}\, \text{RES}_{wi}\, e_3$
		~~		
		$w_3 \in p_3 \subset p_1$:		($\text{RES}_{wi}\, e_3$-belief,
		$w_i \in p_3$		w_i-fact)
	…that…		|	$\underline{t}_4 w_3 = \vartheta_{w3}\, e_3$
	…on the previous day…		‖	$^{\top}\underline{t}_4{}' w_3 \subseteq$ the.day.before e_3-day
	…she had…		•	$^{f}\underline{ee}_3 w_3 = e_3$: e_3-point.of.view
	…left		——	$\underline{s}_2 w_3$: Mary is in l_2 = Paris
			•	$^{1}\underline{ee}_3 w_3 = \underline{e}_4 w_3$: end of $\underline{s}_2$, Mary leaves l_2 = Paris
			‖	$^{\top}\underline{t}_4{}'' w_3 = \vartheta_{w3}\, \text{RES}_{w3}\, {}^{1}\underline{ee}_3 w_3$

- ***Online update for*** (1–2), assuming $i_0 = \langle w_i, \langle\,\rangle, \langle bill, mary, paris\rangle\rangle$

(0) [**w**| **w** = *r*]; [**e**| **e**: AGT $speak.up_{\mathbf{d}\omega}$]; [**t**| **t** $=_{\mathbf{d}\omega}$ $\vartheta\mathbf{d}\varepsilon$];

(1) On[1] Sunday[2] *if*

$[\mathbf{t}\ k^\tau|\ \mathbf{t} \subseteq_{\mathbf{d}\omega} k^\tau\{\mathbf{d}\varepsilon\}]; [|\ d\kappa^\tau\ sunday.of\ \varepsilon^\circ];$

Bill ⊤ (male name) *ib*

$^{\mathrm{P}}[|\ d\alpha = bill]; [k^\alpha|\ k^\alpha \sim d\alpha]; [\mathbf{a}|\ \mathbf{a} =_{\mathbf{d}\omega} d\kappa^\alpha\{\mathbf{d}\varepsilon\}]; {}^{\mathrm{I}}[|\ 3sm_{\mathbf{d}\omega,\mathbf{d}\varepsilon}\ \mathbf{d}\alpha]$

hear-[3] *mf*

$[e\ p|\ (e\text{: AGT } say_{\mathbf{d}\omega}\ p), (\text{EXP } e =_{\mathbf{d}\omega} \text{EXP RES } e)];$

$^{\mathrm{I}}[|\ \text{EXP } d\varepsilon =_{\mathbf{d}\omega} \mathbf{d}\alpha]^4; {}^{\mathrm{I}}[|\ \text{RES } d\varepsilon\text{: EXP } believe_{\mathbf{d}\omega}\ d\Omega];$

-PST[5]

$^{\mathrm{P}}[|\ \mathbf{d}\tau <\vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon]; [|\ d\varepsilon \subseteq_{\mathbf{d}\omega} \mathbf{d}\tau]; [\mathbf{t}|\ \mathbf{t} =_{\mathbf{d}\omega} \vartheta\text{RES } d\varepsilon];$

that *fb*: *if*

$[\underline{\mathbf{t}}|\ \underline{\mathbf{t}} = (\vartheta d\varepsilon|\ d\Omega)];$

Mary ⊤ (female name) *fb*: *ib*

$^{\mathrm{P}}[|\ d\alpha_1 = mary]; [k^\alpha|\ k^\alpha \sim d\alpha_1]; [\mathbf{a}|\ \mathbf{a} =_{\mathbf{d}\omega} d\kappa^\alpha\{\mathbf{d}\varepsilon\}]; {}^{\mathrm{I}}[|\ 3sf_{\mathbf{d}\omega,\mathbf{d}\varepsilon}\ \mathbf{d}\alpha]$

be-[6] *fb*: *mf*

$[\underline{s}|\ \text{EXP } \underline{s} = \mathbf{d}\alpha];$

-PST.[7] .sg

$^{\mathrm{P}}[|\ \underline{\mathbf{d}}\tau < \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon]; [|\ \underline{\mathbf{d}}\tau \subseteq_{\mathbf{d}\underline{\tau}} d\underline{\sigma}]; {}^{\mathrm{I}}[|\ \mathbf{d}\tau \subseteq_{\mathbf{d}\underline{\tau}} d\underline{\sigma} <_{\mathbf{d}\underline{\tau}} \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon]; {}^{\mathrm{P}}[|\ sg\ \mathbf{d}\alpha]$

in[8] Paris ⊥ *fb*: *fb*

$[k^\pi|\ \Pi d\underline{\sigma} \subseteq k^\pi\{d\underline{\sigma}\}]; {}^{\mathrm{P}}[|\ d\pi = paris]; [|\ d\kappa^\pi \sim d\pi]; [|\ d\pi = d\kappa^\pi\{d\underline{\sigma}\}]$

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1 • In **d**ω, **t** is a subperiod of the k^τ-time instantiated in **d**ε
$\mathbf{t} \subseteq_{\mathbf{d}\omega} k^\tau\{\mathbf{d}\varepsilon\}$:= $\lambda i.\ \mathbf{t} \subseteq k^\tau \mathbf{d}\omega_i \mathbf{d}\varepsilon_i$

2 • Any $d\kappa^\tau$-time is the Sunday before or after the instantiating eventuality, at the location of that eventuality
$d\kappa^\tau\ sunday.of\ \varepsilon^\bullet$:= $\lambda i.\ \forall w \in \text{Dom } d\kappa^\tau{}_i \forall e^\bullet \in \text{Dom } d\kappa^\tau{}_i w \exists t$:
$t = d\kappa^\tau{}_i w e^\bullet \wedge sunday.at_w\,(t, \Pi_w\, e^\bullet)$
$\wedge\ \forall t'(sunday.at_w(t', \Pi_w\, e^\bullet) \wedge t' \neq t \rightarrow \vartheta_w\, e < t < t' \vee t' < t < \vartheta_w\, e)$

3 • In **d**ω, *e* is a speech event whose agent says *p*.
e: AGT $say_{\mathbf{d}\omega}\ p$:= $\lambda i.\ say_{\mathbf{d}\omega}(e, \text{AGT}_{\mathbf{d}\omega i}\ e, p)$
• In **d**ω, in the result state of *d*ε, the experiencer believes *d*Ω
RES $d\varepsilon$: EXP $believe_{\mathbf{d}\omega}\ d\Omega$:= $\lambda i.\ believe_{\mathbf{d}\omega i}(\text{RES}_{\mathbf{d}\omega i}\ d\varepsilon_i, \text{EXP}_{\mathbf{d}\omega i}\ \text{RES}_{\mathbf{d}\omega i}\ d\varepsilon_i, d\Omega_i)$

4 See Manna 2006ms on thematic role assignment w/o agr as an implicature of v-stem (defeasible e.g. by passive)

5 • $\underline{\mathbf{t}}$ is the concept of the time of *d*ε restricted to *d*Ω
$\underline{\mathbf{t}} = (\vartheta d\varepsilon|\ d\Omega)$:= $\lambda i.\ \underline{\mathbf{t}} = \langle \vartheta_w\, d\varepsilon_i\text{: } w \in d\Omega_i\rangle$

6 • Any realization of $\underline{s}$ is a state experienced by **d**α
EXP $\underline{s} = \mathbf{d}\alpha$:= $\lambda i.\ \forall w \in \text{Dom } \underline{s}\text{: EXP}_w\ \underline{s}w = \mathbf{d}\alpha_i$

7 • In any $\mathbf{d}\underline{\tau}$-world, $\mathbf{d}\underline{\tau}$ is in the past of the **d**ω-real time of **d**ε
$\mathbf{d}\underline{\tau} < \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon$:= $\lambda i.\ \forall w \in \text{Dom } \mathbf{d}\underline{\tau}_i\text{: } \mathbf{d}\underline{\tau}_i w < \vartheta_{\mathbf{d}\omega i}\ \mathbf{d}\varepsilon_i$
• In any $\mathbf{d}\underline{\tau}$-world, $d\underline{\sigma}$ is realized as a state that holds at $\mathbf{d}\underline{\tau}$.
$\mathbf{d}\underline{\tau} \subseteq_{\mathbf{d}\underline{\tau}} d\underline{\sigma}$:= $\lambda i.\ \forall w \in \text{Dom } \mathbf{d}\underline{\tau}_i\text{: } \mathbf{d}\underline{\tau}_i w \subseteq \vartheta_w\ d\underline{\sigma}_i w$

8 • Any realization of $d\underline{\sigma}$ is a state located within the k^π-place instantiated in $d\underline{\sigma}$
$d\underline{\sigma} \subseteq k^\pi\{d\underline{\sigma}\}$:= $\lambda i.\ \forall w \in \text{Dom } d\underline{\sigma}_i\text{: } \Pi_w\ d\underline{\sigma}_i w \subseteq k^\pi w d\underline{\sigma}_i w$

(2) On Tuesday *if*

$[\mathbf{t}\ k^{\tau}|\ \mathbf{t} \subseteq_{\mathbf{d}\omega} k^{\tau}\{\mathbf{d}\varepsilon\}]; [|\ d\kappa^{\tau}\ \textit{tuesday.of}\ \varepsilon^{\bullet}];$

3sm $\top$ *ib*

$^{P}[|\ \textit{3sm}_{\mathbf{d}\omega,\,\mathbf{d}\varepsilon}\ \mathbf{d}\alpha_1]; [\mathbf{a}|\ \mathbf{a} = \mathbf{d}\alpha_1];$

learn-[1] *mf*

$^{P}[|\ \text{RES}\ d\varepsilon\text{: EXP}\ \textit{believe}_{\mathbf{d}\omega}\ d\Omega];$

$[e\ p|\ (\text{EXP RES}\ d\varepsilon =_{\mathbf{d}\omega} \text{EXP}\ e =_{\mathbf{d}\omega} \text{EXP RES}\ e), (\text{RES}\ e\text{: EXP}\ \textit{believe}_{\mathbf{d}\omega}\ p), (\mathbf{d}\omega \in p \subset d\Omega)];$

$^{I}[|\ \text{EXP}\ d\varepsilon =_{\mathbf{d}\omega} \mathbf{d}\alpha]$

-PST

$^{P}[|\ \mathbf{d}\tau < \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon]; [|\ d\varepsilon \subseteq_{\mathbf{d}\omega} \mathbf{d}\tau]; [\mathbf{t}|\ \mathbf{t} =_{\mathbf{d}\omega} \vartheta\text{RES}\ d\varepsilon];$

that on[2] *fb*: *if*

$[\underline{\mathbf{t}}|\ \underline{\mathbf{t}} = (\vartheta d\varepsilon|\ d\Omega)];\ {}^{A}[\underline{\mathbf{t}}|\ \text{Dom}\ \underline{\mathbf{t}} = \text{Dom}\ \mathbf{d}\underline{\tau}]; [k^{\tau}|\ \mathbf{d}\underline{\tau} \subseteq_{\mathbf{d}\underline{\tau}} k^{\tau}\{d\varepsilon\}];$

the previous day[3]

$[t_{\varepsilon}|\ d\kappa^{\tau}\{d\varepsilon\} =_{\mathbf{d}\underline{\tau}} t_{\varepsilon}(d\varepsilon)]; [|\ d\varepsilon\tau\ \textit{yst.of}_{\mathbf{d}\underline{\tau}}\ \varepsilon]$

3sf $\top$ *fb*: *ib*

$^{P}[|\ \textit{3sf}_{\mathbf{d}\omega,\,\mathbf{d}\varepsilon}\ \mathbf{d}\alpha_1]; [\mathbf{a}|\ \mathbf{a} = \mathbf{d}\alpha_1];$

have$_{\varepsilon\varepsilon}$-[4] *fb*: *mf*

$[\underline{ee}|\ \textit{process}\ \underline{ee}, ({}^{f}\underline{ee} =_{\mathbf{d}\underline{\tau}} d\varepsilon), (\text{EXP}\ {}^{1}\underline{ee} =_{\mathbf{d}\underline{\tau}} \mathbf{d}\alpha)]$

-PST[5]

$^{P}[|\ \mathbf{d}\underline{\tau} < \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon]; [|\ {}^{1}d\underline{\varepsilon\varepsilon} \subseteq_{\mathbf{d}\underline{\tau}} \mathbf{d}\underline{\tau}]; [\underline{\mathbf{t}}|\ \underline{\mathbf{t}} = (\vartheta\text{RES}\ {}^{1}d\underline{\varepsilon\varepsilon}|\ \text{Dom}\ \mathbf{d}\underline{\tau})];$

leave-[6]

$^{P}[|\ d\underline{\sigma} \subseteq_{\mathbf{d}\underline{\tau}} d\pi]; [\underline{e}|\ (\text{EXP}\ d\underline{\sigma} = \text{EXP}\ \underline{e}), \textit{end}_{\mathbf{d}\underline{\tau}}\{\underline{e}, d\underline{\sigma}\}, \neg(\Pi\text{RES}\ \underline{e} \circ d\pi)];$

-PPF

$[|\ {}^{1}d\underline{\varepsilon\varepsilon} = d\underline{\varepsilon}];$

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[1] This representation is meant to capture the non-agentive reading of *learn*, similar to *hear*.

[2] • Any realization of $\mathbf{d}\underline{\tau}$ is a subperiod of the k^{τ}-time instantiated in $d\varepsilon$

$\mathbf{d}\underline{\tau} \subseteq_{\mathbf{d}\underline{\tau}} k^{\tau}\{d\varepsilon\}$:= $\lambda i.\ \forall w \in \mathbf{d}\underline{\tau}_i\text{: } \mathbf{d}\underline{\tau}_i w \subseteq k^{\tau}wd\varepsilon_i$

[3] • In any $\mathbf{d}\underline{\tau}$-world, $\forall e \in \text{Dom}\ t_{\varepsilon}$, the k^{τ}-time instantiated in $d\varepsilon$ is the t_{ε}-time of $d\varepsilon$

$d\kappa^{\tau}\{d\varepsilon\} =_{\mathbf{d}\underline{\tau}} t_{\varepsilon}(d\varepsilon)$:= $\lambda i.\ \forall w \in \text{Dom}\ \mathbf{d}\underline{\tau}_i\text{: } d\kappa^{\tau}{}_i wd\varepsilon_i = t_{\varepsilon}d\varepsilon_i$

In any $\mathbf{d}\underline{\tau}$-world, $\forall e \in \text{Dom}\ d\varepsilon\tau$, the $d\varepsilon\tau$-time of e is the yesterday of e

$d\varepsilon\tau\ \textit{yst.of}_{\mathbf{d}\underline{\tau}}\ \varepsilon$:= $\lambda i.\ \forall w \in \text{Dom}\ \mathbf{d}\underline{\tau}_i \exists t, t'\text{: } \textit{day.at}_w(t, \Pi_w\ e) \wedge \textit{day.at}_w(t', \Pi_w\ e) \wedge \vartheta_w\ e \subseteq t'$
$\wedge\ t < t' \wedge \neg\exists t''(\textit{day.at}_w(t'', \Pi_w\ e) \wedge t < t'' < t')$

[4] • In any $\mathbf{d}\underline{\tau}$-world, the end of $\underline{ee}$ is $d\varepsilon$

${}^{f}\underline{ee} =_{\mathbf{d}\underline{\tau}} d\varepsilon$:= $\lambda i.\ \forall w \in \text{Dom}\ \mathbf{d}\underline{\tau}\text{: } {}^{f}\underline{ee}w = d\varepsilon_i$

[5] • Any realization of $\mathbf{d}\underline{\tau}$ is in the past of the $\mathbf{d}\omega$-real time of $\mathbf{d}\varepsilon$

$\mathbf{d}\underline{\tau} < \vartheta_{\mathbf{d}\omega}\ \mathbf{d}\varepsilon$:= $\lambda i.\ \forall w \in \text{Dom}\ \mathbf{d}\underline{\tau}_i\text{: } \mathbf{d}\underline{\tau}_i w < \vartheta_{\mathbf{d}\omega i}\ \mathbf{d}\varepsilon_i$

• In any $\mathbf{d}\underline{\tau}$-world, $d\underline{\varepsilon\varepsilon}$ begins within $\mathbf{d}\underline{\tau}$

${}^{1}d\underline{\varepsilon\varepsilon} \subseteq_{\mathbf{d}\underline{\tau}} \mathbf{d}\underline{\tau}$:= $\lambda i.\ \forall w \in \mathbf{d}\underline{\tau}_i\text{: } \vartheta_w\ {}^{1}d\underline{\varepsilon\varepsilon}_i w \subseteq \mathbf{d}\underline{\tau}_i w$

[6] • In any $\mathbf{d}\underline{\tau}$-world, $\underline{e}$ terminates $d\underline{\sigma}$ (i.e. $\underline{e}$ is a change of state during $d\underline{\sigma}$-time, resulting in a state of $d\underline{\sigma}$-exp. which does not temporally overlap $d\underline{\sigma}$) and results in a state of the experiencer outside of $d\pi$

$\textit{end}_{\mathbf{d}\underline{\tau}}\{\underline{e}, d\underline{\sigma}\}$:= $\lambda i.\ \forall w \in \text{Dom}\ \mathbf{d}\underline{\tau}\text{:}$
$\vartheta_w\ \underline{e}w \subseteq \vartheta_w\ d\underline{\sigma}_i \wedge \neg(\vartheta_w\ \text{RES}_w\ \underline{e}w \circ \vartheta_w\ d\underline{\sigma}_i) \wedge \text{EXP}_w\ \text{RES}_w\ \underline{e}w = \text{EXP}_w\ d\underline{\sigma}_i w$