

Lecture 5

NOMINAL ANAPHORA IN ENGLISH: 2ND TRY

1. Discourse referents (drefs)

- PROBLEM for static theory (full-stop (.) as conjunction (\wedge):
prediction: order of antecedent & anaphor immaterial, e.g. (1) \equiv (2)
intuition: antecedent must precede the anaphor, e.g. (1) $\not\equiv$ (2)
- (1) Jim₁ came in. He₁ sat down. (antec. ref. z₁... anaphoric ref. z₁)
 (1'') $\models_{M,g} cm_{et} \text{ } \lambda x(x = z_1 \wedge z_1 = jim) \wedge sit_{et} z_1$
 iff $\llbracket cm_{et} \rrbracket^M(g(z_1)) = 1 \ \& \ g(z_1) = \llbracket jim \rrbracket^M \ \& \ \llbracket sit_{et} \rrbracket^M(g(z_1)) = 1$
- (2) He₁ sat down. Jim₁ came in. (anaphoric ref. z₁... antec. ref. z₁)
 (2'') $\models_{M,g} sit_{et} z_1 \wedge cm_{et} \text{ } \lambda x(x = z_1 \wedge z_1 = jim)$
 iff $\llbracket sit_{et} \rrbracket^M(g(z_1)) = 1 \ \& \ \llbracket cm_{et} \rrbracket^M(g(z_1)) = 1 \ \& \ g(z_1) = \llbracket jim \rrbracket^M$
- TOWARD A SOLUTION: antecedent *updates* the context, the anaphor *refers* in resulting ctx:
 - a. an antecedent npⁿ updates the *input* context to an *output* where an entity that satisfies the lexical content of the np is assigned to the *n*th entity variable, z_n. [Following Karttunen '76, we say that the antecedent sets up a *discourse referent (dref)*].
 - b. in discourse the *output* context of sentence 1 serves as the *input* for sentence 2.
 - c. an anaphoric pronoun, prn_n, refers to the entity assigned to z_n in the *input* context.
 - FORMALIZATION in *Discourse Representation Theory (DRT)*
 [original formulation: Kamp 1981 (formally correct but difficult to relate to other logics), formal clean-up: Groenendijk & Stokhof 1991, Muskens 1995, 1996 (this handout)]
- (1) Jim¹ came in. He₁ sat down. (antec. update [z₁ | ...]... ana. ref. z₁)
 (1') $\models_{M,g} ([z_1 | z_1 = jim]; [cm \ z_1]); [sit \ z_1]$
 iff $\llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M \ \& \ \llbracket jim \rrbracket^M \in \llbracket sit \rrbracket^M$
- (2) He₁ sat down. Jim¹ came in. (ana. ref. z₁... antec. update [z₁ | ...])
 (2') $\models_{M,g} [sit \ z_1]; ([z_1 | z_1 = jim]; [cm \ z_1])$
 iff $g(z_1) \in \llbracket cm \rrbracket^M \ \& \ \llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M$
- (3) Jim¹ has a² wife₁. She₂ is a doctor. (anaphora to *indefinite antecedent*,
 (3') $\models_{M,g} ([z_1 | z_1 = jim]; [z_2 | wife \ z_2 z_1]); [dr \ z_2]$
 iff $\exists d \in D^M(\langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M \ \& \ d \in \llbracket dr \rrbracket^M)$
- (4) Jim¹ doesn't have a² wife₁. She₂ is a doctor. ...interaction with *negation*,
 (4') $\models_{M,g} ([z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife \ z_2 z_1]]); [dr \ z_2]$
 iff $\neg \exists d \in D^M(\langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M) \ \& \ g(z_2) \in \llbracket dr \rrbracket^M$
- (5_{if}) If a¹ man marries a² woman he₁ gives her₂ a³ ring. ...conditionals,
 (5_{ev}) Every¹ man who marries a² woman gives her₂ a³ ring. ...and *quantifiers*)
 (5') $\models_{M,g} [[z_1 \ z_2 | man \ z_1, mrr \ z_1 z_2, wm \ z_2] \Rightarrow [z_3 | gvt \ z_1 z_3 z_2, rng \ z_3]]$
 iff $\forall b, c \in D^M(b \in \llbracket man \rrbracket^M \ \& \ \langle b, c \rangle \in \llbracket mrr \rrbracket^M \ \& \ c \in \llbracket wm \rrbracket^M$
 $\rightarrow \exists d \in D^M(\langle b, c, d \rangle \in \llbracket gvt \rrbracket^M \ \& \ d \in \llbracket rng \rrbracket^M)$

2. Discourse Representation Theory (DRT)

(a la Muskens 1995, 1996)

DEFINITION 1.1 (basic terms of DRT)

• Var	=	$\{z_1, z_2, \dots\}$	(entity) variables
• Con	=	$\{jim, ann, \dots\}$	(entity) constants
Con ¹	=	$\{man, wm, dr, rng, cm, sit, \dots\}$	1-place relation constants
Con ²	=	$\{hsb, wife, mrr, see, spk, \dots\}$	2-place relation constants
Con ³	=	$\{gvt, \dots\}$	3-place relation constants

DEFINITION 1.2 (DRT syntax: conditions and boxes).

=	$u = \alpha \in \mathbf{Cnd}$	if	$u \in \mathbf{Var} \ \& \ \alpha \in \mathbf{Var} \cup \mathbf{Con}$	identity condition
R	$\beta u_1 \dots u_n \in \mathbf{Cnd}$	if	$\beta \in \mathbf{Con}^n \ \& \ u_1, \dots, u_n \in \mathbf{Var}$	relational condition
,	$(C_1, C_2) \in \mathbf{Cnd}$	if	$C_1, C_2 \in \mathbf{Cnd}$	conjoined condition
[$[C] \in \mathbf{Box}$	if	$C \in \mathbf{Cnd}$	test box
"	$[u_1 \dots u_n] \in \mathbf{Box}$	if	$u_1, \dots, u_n \in \mathbf{Var}$	update box
;	$(K_1; K_2) \in \mathbf{Box}$	if	$K_1, K_2 \in \mathbf{Box}$	sequence (of boxes)
n	not $K \in \mathbf{Cnd}$	if	$K \in \mathbf{Box}$	DRT negation
\Rightarrow	$K_1 \Rightarrow K_2 \in \mathbf{Cnd}$	if	$K_1, K_2 \in \mathbf{Box}$	DRT implication

DEFINITION 2.1 (DRT models and assignments)

- A DRT *model* is a pair $M = \langle D^M, \llbracket \cdot \rrbracket^M \rangle$ such that (i) D^M is a non-empty set (of M -entities), (ii) $\llbracket \cdot \rrbracket^M$ is an *interpretation function* that assigns to each $\alpha \in \mathbf{Con}$ an M -entity $\llbracket \alpha \rrbracket^M \in D^M$, and to each $\alpha \in \mathbf{Con}^n$, an n -place relation $\llbracket \alpha \rrbracket^M \subseteq (D^M)^n$.
- An M -*assignment* is a function g that assigns to each $u \in \mathbf{Var}$ an M -entity $g(u) \in D^M$.
- G^M is the set of all M -assignments. Moreover, for any $g, h \in G^M$ and $u_1, \dots, u_n \in \mathbf{Var}$: $g[u_1 \dots u_n]h$ iff $\forall v \in \mathbf{Var} \setminus \{u_1, \dots, u_n\}: g(v) = h(v)$.

DEFINITION 2.2 (DRT semantics). For any DRT-model M , a condition C denotes a subset of G^M , $\llbracket C \rrbracket^M \subseteq G^M$, and a box K , a binary input-output relation, $\llbracket K \rrbracket^M \subseteq (G^M)^2$, defined as follows: (Relational composition ($r_1 \circ r_2$) is defined by: $\langle x, y \rangle \in r_1 \circ r_2$ iff $\exists z (\langle x, z \rangle \in r_1 \ \& \ \langle z, y \rangle \in r_2)$.)

=	$\llbracket u = \alpha \rrbracket^M$	=	$\{g \mid g(u) = \llbracket \alpha \rrbracket^M\}$	if $\alpha \in \mathbf{Con}$
		=	$\{g \mid g(u) = g(\alpha)\}$	if $\alpha \in \mathbf{Var}$
R	$\llbracket \beta u_1 \dots u_n \rrbracket^M$	=	$\{g \mid \langle g(u_1), \dots, g(u_n) \rangle \in \llbracket \beta \rrbracket^M\}$	
,	$\llbracket (C_1, C_2) \rrbracket^M$	=	$(\llbracket C_1 \rrbracket^M \cap \llbracket C_2 \rrbracket^M)$	
[$\llbracket [C] \rrbracket^M$	=	$\{\langle g, h \rangle \mid g = h \ \& \ h \in \llbracket C \rrbracket^M\}$	
"	$\llbracket [u_1 \dots u_n] \rrbracket^M$	=	$\{\langle g, h \rangle \mid g[u_1 \dots u_n]h\}$	
;	$\llbracket (K_1; K_2) \rrbracket^M$	=	$(\llbracket K_1 \rrbracket^M \circ \llbracket K_2 \rrbracket^M)$	
n	$\llbracket \mathbf{not} \ K \rrbracket^M$	=	$\{g \mid \neg \exists h: \langle g, h \rangle \in \llbracket K \rrbracket^M\}$	
\Rightarrow	$\llbracket K_1 \Rightarrow K_2 \rrbracket^M$	=	$\{g \mid \forall h (\langle g, h \rangle \in \llbracket K_1 \rrbracket^M \rightarrow \exists k: \langle h, k \rangle \in \llbracket K_2 \rrbracket^M)\}$	

DEFINITION 3 (Truth).

- condition C is true in M under g , written $\models_{M, g} C$, iff $g \in \llbracket C \rrbracket^M$
- box K is *true* in M given input g , written $\models_{M, g} K$, iff $\exists h: \langle g, h \rangle \in \llbracket K \rrbracket^M$

ABBREVIATIONS

- $[u_1 \dots u_n] C$:= $([u_1 \dots u_n]; [C])$ update with (output) test
- Parentheses may be omitted if they can be recovered from DRT-syntax (D1.1–1.2).

3. Update with output test

$$F \mid \llbracket [u_1 \dots u_n] C \rrbracket^M = \{ \langle g, h \rangle \mid g[u_1 \dots u_n]h \ \& \ h \in \llbracket C \rrbracket^M \}$$

Proof: (1) iff (7):

1. $\langle g, h \rangle \in \llbracket [u_1 \dots u_n] C \rrbracket^M$
2. $\langle g, h \rangle \in \llbracket ([u_1 \dots u_n]; [C]) \rrbracket^M$ df. [-|-]
3. $\langle g, h \rangle \in (\llbracket [u_1 \dots u_n] \rrbracket^M \circ \llbracket [C] \rrbracket^M)$ D2.2.;
4. $\exists k (\langle g, k \rangle \in \llbracket [u_1 \dots u_n] \rrbracket^M \ \& \ \langle k, h \rangle \in \llbracket [C] \rrbracket^M)$ df. \circ
5. $\exists k (\langle g, k \rangle \in \{ \langle g', k' \rangle \mid g' [u_1 \dots u_n] k' \}$ D2.2.^u
 $\ \& \ \langle k, h \rangle \in \{ \langle k', h' \rangle \mid k' = h' \ \& \ h' \in \llbracket [C] \rrbracket^M \})$ D2.2.:[
6. $\exists k (g[u_1 \dots u_n]k \ \& \ k = h \ \& \ h \in \llbracket [C] \rrbracket^M)$ df. { -|- }
7. $g[u_1 \dots u_n]h \ \& \ h \in \llbracket [C] \rrbracket^M$ elim. k

Hence F | (by definition of set abstraction)¹. □

4. From box² K , via denotation $\llbracket K \rrbracket^M$, to truth

(6) Jim¹ came in. (English)

(6') $[z_1 \mid z_1 = jim]; [cm \ z_1]$ (DRT)

$$F6^1 \ \langle g, h \rangle \in \llbracket [z_1 \mid z_1 = jim] \rrbracket^M$$

$$\text{iff } g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M$$

(box 1 in (6'))

Proof: (1) iff (5)

1. $\langle g, h \rangle \in \llbracket [z_1 \mid z_1 = jim] \rrbracket^M$
2. $\langle g, h \rangle \in \{ \langle g', h' \rangle \mid g' [z_1] h' \ \& \ h' \in \llbracket [z_1 = jim] \rrbracket^M \}$ F |
3. $g[z_1]h \ \& \ h \in \llbracket [z_1 = jim] \rrbracket^M$ df. { -|- }
4. $g[z_1]h \ \& \ h \in \{ h \mid h(z_1) = \llbracket jim \rrbracket^M \}$ D2.2.=
5. $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M$ df. { -|- }

□

$$F6^2 \ \langle g, h \rangle \in \llbracket [cm \ z_1] \rrbracket^M$$

$$\text{iff } g = h \ \& \ h(z_1) \in \llbracket [cm] \rrbracket^M$$

(box 2 in (6'))

Proof: (1) iff (6)

1. $\langle g, h \rangle \in \llbracket [cm \ z_1] \rrbracket^M$
2. $\langle g, h \rangle \in \{ \langle g', h' \rangle \mid g' = h' \ \& \ h' \in \llbracket [cm \ z_1] \rrbracket^M \}$ D2.2.:[
3. $g = h \ \& \ h \in \llbracket [cm \ z_1] \rrbracket^M$ df. { -|- }
4. $g = h \ \& \ h \in \{ h \mid \langle h'(z_1) \rangle \in \llbracket [cm] \rrbracket^M \}$ D2.2.:R¹
5. $g = h \ \& \ h \in \{ h \mid h'(z_1) \in \llbracket [cm] \rrbracket^M \}$ $\langle a \rangle := a$
6. $g = h \ \& \ h(z_1) \in \llbracket [cm] \rrbracket^M$ df. { -|- }

□

¹ $\langle a, b \rangle \in \{ \langle x, y \rangle \mid \phi \}$ iff $[a/x][b/y]\phi$

² Kamp's (1981) term for a DRT box is a *discourse representation structure* (DRS).

F6³ $\langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [cm z_1] \rrbracket^M$ (sequence (6'))
iff $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ h(z_1) \in \llbracket cm \rrbracket^M$

Proof: (1) iff (5)

1. $\langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [cm z_1] \rrbracket^M$
2. $\langle g, h \rangle \in (\llbracket [z_1 | z_1 = jim] \rrbracket^M \circ \llbracket [cm z_1] \rrbracket^M)$ D2.2.;
3. $\exists k (\langle g, k \rangle \in \llbracket [z_1 | z_1 = jim] \rrbracket^M \ \& \ \langle k, h \rangle \in \llbracket [cm z_1] \rrbracket^M)$ df. \circ
4. $\exists k (g[z_1]k \ \& \ k(z_1) = \llbracket jim \rrbracket^M \ \& \ (k = h \ \& \ h(z_1) \in \llbracket cm \rrbracket^M))$ F6¹, F6²
5. $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ h(z_1) \in \llbracket cm \rrbracket^M$ elim. k

□

F6⁴ $\models_{M,g} [z_1 | z_1 = jim]; [cm z_1]$ (truth in M for input g)
iff $\llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M$

Proof: (1) iff (4)

1. $\models_{M,g} [z_1 | z_1 = jim]; [cm z_1]$
2. $\exists h: \langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [cm z_1] \rrbracket^M$ D3:K
3. $\exists h (g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ h(z_1) \in \llbracket cm \rrbracket^M)$ F6³
4. $\llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M$ D2.1

□

(7) He₁ sat down. (English)

(7') $[sit z_1]$ (DRT)

F7¹ $\langle g, h \rangle \in \llbracket [sit z_1] \rrbracket^M$ ((7') like box 2 in (6'))
iff $g = h \ \& \ h(z_1) \in \llbracket sit \rrbracket^M$

F7² $\models_{M,g} [sit z_1]$ (truth in M for input g)
iff $g(z_1) \in \llbracket sit \rrbracket^M$

Proof: (1) iff (4)

1. $\models_{M,g} [sit z_1]$
2. $\exists h: \langle g, h \rangle \in \llbracket [sit z_1] \rrbracket^M$ D3:K
3. $\exists h (g = h \ \& \ h(z_1) \in \llbracket sit \rrbracket^M)$ F7¹
4. $g(z_1) \in \llbracket sit \rrbracket^M$ elim. h

□

Therefore:

(1) Jim¹ came in. He₁ sat down. (antec. update...ana. ref.)

(1') $\models_{M,g} ([z_1 | z_1 = jim]; [cm z_1]); [sit z_1]$
iff $\exists h \exists k (\langle g, k \rangle \in \llbracket [z_1 | z_1 = jim]; [cm z_1] \rrbracket^M \ \& \ \langle k, h \rangle \in \llbracket [sit z_1] \rrbracket^M)$ D3:K, D2.2.;;, \circ
iff $\exists h \exists k (g[z_1]k \ \& \ k(z_1) = \llbracket jim \rrbracket^M \ \& \ k(z_1) \in \llbracket cm \rrbracket^M$
 $\ \& \ k = h \ \& \ h(z_1) \in \llbracket sit \rrbracket^M)$ F6³, F7¹
iff $\llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M \ \& \ \llbracket jim \rrbracket^M \in \llbracket sit \rrbracket^M$ elim. h , D2.1

(2) He₁ sat down. Jim¹ came in. (ana. ref.... antec. update)

(2') $\models_{M,g} [sit z_1]; ([z_1 | z_1 = jim]; [cm z_1])$
iff $\exists h \exists k (\langle g, k \rangle \in \llbracket [sit z_1] \rrbracket^M \ \& \ \langle k, h \rangle \in \llbracket [z_1 | z_1 = jim]; [cm z_1] \rrbracket^M)$ D3, D2.2.;;, \circ
iff $g(z_1) \in \llbracket sit \rrbracket^M \ \& \ \llbracket jim \rrbracket^M \in \llbracket cm \rrbracket^M$ F7¹, F6³, elim h , D2.1

Lecture 6
INDEFINITES IN DRT

1. Overview: Data & DRT predictions

- *anaphora* to indefinite antecedent ((3') derived in Sec. 2)

(3) Jim¹ has a² wife₁. She₂ is a doctor.

(3') $\models_{M,g} ([z_1 | z_1 = jim]; [z_2 | wife\ z_2 z_1]); [dr\ z_2]$
iff $\exists d \in D^M (\langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M \ \& \ d \in \llbracket dr \rrbracket^M)$

- interaction with *negation* ((4') derived in Sec. 3)

(4) Jim¹ doesn't have a² wife₁. She₂ is a doctor.

(4') $\models_{M,g} ([z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]]); [dr\ z_2]$
iff $\neg \exists d \in D^M (\langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M \ \& \ g(z_2) \in \llbracket dr \rrbracket^M)$

- interaction with *conditionals & quantifiers* ((5') derived in Hwk 3)

(5c) If a¹ man marries a² woman he₁ gives her₂ a³ ring.

(5q) Every¹ man who marries a² woman gives her₂ a³ ring.

(5') $\models_{M,g} [[z_1\ z_2 | man\ z_1, mrr\ z_1 z_2, wm\ z_2] \Rightarrow [z_3 | gvt\ z_1 z_3 z_2, rng\ z_3]]$
iff $\forall b, c \in D^M (b \in \llbracket man \rrbracket^M \ \& \ \langle b, c \rangle \in \llbracket mrr \rrbracket^M \ \& \ c \in \llbracket wm \rrbracket^M$
 $\rightarrow \exists d \in D^M (\langle b, d, c \rangle \in \llbracket gvt \rrbracket^M \ \& \ d \in \llbracket rng \rrbracket^M)$

2. Anaphora to indefinite antecedent

(8) Jim¹ has a² wife₁.

(English)

(8') $[z_1 | z_1 = jim]; [z_2 | wife\ z_2 z_1]$

(DRT)

F6¹ $\langle g, h \rangle \in \llbracket [z_1 | z_1 = jim] \rrbracket^M$
iff $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M$

(box 1 in (8'), same as in (6'))

F8² $\langle g, h \rangle \in \llbracket [z_2 | wife\ z_2 z_1] \rrbracket^M$
iff $g[z_2]h \ \& \ \langle h(z_2), h(z_1) \rangle \in \llbracket wife \rrbracket^M$

(box 2 in (8'))

Proof:

1. $\langle g, h \rangle \in \llbracket [z_2 | wife\ z_2 z_1] \rrbracket^M$
2. $\langle g, h \rangle \in \{ \langle g', h' \rangle | g'[z_2]h' \ \& \ h' \in \llbracket wife\ z_2 z_1 \rrbracket^M \}$
3. $g[z_2]h \ \& \ h \in \llbracket wife\ z_2 z_1 \rrbracket^M$
4. $g[z_1]h \ \& \ \langle h(z_2), h(z_1) \rangle \in \llbracket wife \rrbracket^M$

F|

df. $\{-|\}$

D2.2:R

□

3. Negated indefinite

(10) Jim¹ doesn't have a² wife₁.

(English)

(10') $[z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]]$

(DRT)

F10² $\langle g, h \rangle \in \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$
 iff $g = h \ \& \ \neg \exists d \in D^M (\langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M)$

(box 2 in (10'))

Proof: (1) iff (7)

1. $\langle g, h \rangle \in \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$
2. $\langle g, h \rangle \in \{ \langle g', h' \rangle \mid g' = h' \ \& \ h' \in \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M \}$ D2.2: [
3. $g = h \ \& \ h \in \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$ df. $\{-|\}$
4. $g = h \ \& \ h \in \{ h \mid \neg \exists k: \langle h', k \rangle \in \llbracket [z_2 | wife\ z_2 z_1] \rrbracket^M \}$ D2.2: **n**
5. $g = h \ \& \ \neg \exists k: \langle h, k \rangle \in \llbracket [z_2 | wife\ z_2 z_1] \rrbracket^M$ df. $\{-|\}$
6. $g = h \ \& \ \neg \exists k (h[z_2]k \ \& \ \langle k(z_2), k(z_1) \rangle \in \llbracket wife \rrbracket^M)$ F8²
7. $g = h \ \& \ \neg \exists d \in D^M (\langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M)$ D2.1

□

F10³ $\langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$ (sequence (9'))
 iff $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ \neg \exists d \in D^M (\langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M)$

Proof: (1) iff (5)

1. $\langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$
2. $\langle g, h \rangle \in (\llbracket [z_1 | z_1 = jim] \rrbracket^M \circ \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M)$ D2.2:;
3. $\exists k (\langle g, k \rangle \in \llbracket [z_1 | z_1 = jim] \rrbracket^M \ \& \ \langle k, h \rangle \in \llbracket [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M)$ df. \circ
4. $\exists k (g[z_1]k \ \& \ k(z_1) = \llbracket jim \rrbracket^M \ \& \ k = h \ \& \ \neg \exists d \in D^M: \langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M)$ F6¹, F10²
5. $g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ \neg \exists d \in D^M: \langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M$ elim k

□

F10⁴ $\models_{M, g} [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]]$ (truth in M for input g)
 iff $\neg \exists d \in D^M: \langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M$

Proof: (1) iff (4)

1. $\models_{M, g} [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]]$
2. $\exists h: \langle g, h \rangle \in \llbracket [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$ D3: $\models_{M, g}$
3. $\exists h (g[z_1]h \ \& \ h(z_1) = \llbracket jim \rrbracket^M \ \& \ \neg \exists d \in D^M (\langle d, h(z_1) \rangle \in \llbracket wife \rrbracket^M))$ F10³
4. $\neg \exists d \in D^M: \langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M$ D2.1

□

Therefore:

(4) Jim¹ doesn't have a² wife₁. She₂ is a doctor.

(4') $\models_{M, g} ([z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]]); [dr\ z_2]$

² iff $\exists h \exists k (\langle g, k \rangle \in \llbracket [z_1 | z_1 = jim]; [\mathbf{not}[z_2 | wife\ z_2 z_1]] \rrbracket^M$ D3, D2.2:;, \circ
 $\ \& \ \langle k, h \rangle \in \llbracket [dr\ z_2] \rrbracket^M)$

³ iff $\exists h \exists k (g[z_1]k \ \& \ k(z_1) = \llbracket jim \rrbracket^M \ \& \ \neg \exists d \in D^M (\langle d, k(z_1) \rangle \in \llbracket wife \rrbracket^M)$ F10³, F9¹
 $\ \& \ k = h \ \& \ h(z_2) \in \llbracket [dr] \rrbracket^M)$

⁴ iff $\neg \exists d \in D^M (\langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M) \ \& \ g(z_2) \in \llbracket [dr] \rrbracket^M$ D2.1

Appendix: Two versions of DRT(3) Jim¹ has a² wife₁. She₂ is a doctor.**Kamp (1981)**

- DRT syntax

$$\boxed{\begin{array}{l} z_1 \quad z_2 \\ z_1 = jim \\ z_2 \text{ wife } z_1 \\ dr \ z_2 \end{array}} \quad (=: K_1)$$

- DRT semantics

- Kamp-models: as for PL
- Kamp-contexts: *partial* assignments
 $F^M = \{f \mid \exists g \in G^M: f \subseteq g\}$
- Kamp-truth

$$\begin{aligned} & \models_M K_1 \\ \text{iff } \exists f \in F^M: & \\ & \text{Dom } f \text{ is the universe of } K_1 \\ & \& f \text{ verifies every condition of } K_1 \\ \text{iff } \exists f \in F^M: & \\ & \text{Dom } f = \{z_1, z_2\} \\ & \& f(z_1) = \llbracket jim \rrbracket^M \\ & \& \langle f(z_2), f(z_1) \rangle \in \llbracket wife \rrbracket^M \\ & \& f(z_2) \in \llbracket dr \rrbracket^M \\ \text{iff } \exists d \in D^M: & \\ & \langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M \\ & \& d \in \llbracket dr \rrbracket^M \end{aligned}$$

Muskens (1995, 1996)

$$\begin{aligned} & ([z_1 \mid z_1 = jim]; [z_2 \mid wife \ z_2 z_1]); [dr \ z_2] \\ \equiv & [z_1 \ z_2 \mid z_1 = jim, wife \ z_2 z_1, dr \ z_2] \quad (=: K_1') \end{aligned}$$

- Muskens-models: as for PL
- Muskens-contexts: *total* assignments
 G^M
- Muskens-truth

$$\begin{aligned} & \models_{M, g} K_1' \\ \text{iff } \exists h \in G^M: & \\ & \langle g, h \rangle \in \llbracket K_1' \rrbracket^M \\ \text{iff } \exists h \in G^M: & \\ & g[z_1 \ z_2]h \\ & \& h(z_1) = \llbracket jim \rrbracket^M \\ & \& \langle h(z_1), h(z_2) \rangle \in \llbracket wife \rrbracket^M \\ & \& h(z_2) \in \llbracket dr \rrbracket^M \\ \text{iff } \exists d \in D^M: & \\ & \langle d, \llbracket jim \rrbracket^M \rangle \in \llbracket wife \rrbracket^M \\ & \& d \in \llbracket dr \rrbracket^M \end{aligned}$$

Homework 3

Complete and prove the following facts:

(11) (in this play) a¹ man marries a² woman. (English)

(11') $[z_1 z_2 | \text{man } z_1, (\text{crt } z_1 z_2, \text{wm } z_2)]$ (DRT)

F11¹ $\langle g, h \rangle \in \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \rrbracket^M$ (update)
iff ...

Proof: (1) iff ...

F11² $\models_{M,g} [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)]$ (truth cond.)
iff ...

Proof: (1) iff ...

(12) he₁ gives her₂ a³ ring. (English)

(12') $[z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3]$ (DRT)

F12¹ $\langle g, h \rangle \in \llbracket [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M$ (update)
iff ...

Proof: (1) iff ...

F12² $\models_{M,g} [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3]$ (truth cond.)
iff ...

Proof: (1) iff ...

(5) if a¹ man marries a² woman he₁ gives her₂ a³ ring. (English)

(5') $\llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \rrbracket \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3]$ (DRT)

F5¹ $\langle g, h \rangle \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \rrbracket \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M$ (test)
iff ...

Proof: (1) iff ...

F5² $\models_{M,g} \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \rrbracket \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3]$ (truth cond.)
iff $\forall b, c \in D^M (b \in \llbracket \text{man} \rrbracket^M \& \langle b, c \rangle \in \llbracket \text{mrr} \rrbracket^M \& c \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists d \in D^M (\langle b, d, c \rangle \in \llbracket \text{gvt} \rrbracket^M \& d \in \llbracket \text{rng} \rrbracket^M)$

Proof: (1) iff ...

Solution to Homework 3

(11) (in this play) a¹ man marries a² woman.

(English)

(11') $[z_1 z_2 | man z_1, (crt z_1 z_2, grl z_2)]$

(DRT)

F11¹ $\langle g, h \rangle \in \llbracket [z_1 z_2 | man z_1, (mrr z_1 z_2, wm z_2)] \rrbracket^M$

(update)

iff $g[z_1 z_2]h \ \& \ h(z_1) \in \llbracket man \rrbracket^M \ \& \ \langle h(z_1), h(z_2) \rangle \in \llbracket mrr \rrbracket^M \ \& \ h(z_2) \in \llbracket wm \rrbracket^M$

Proof: (1) iff (4)

1. $\langle g, h \rangle \in \llbracket [z_1 z_2 | man z_1, (mrr z_1 z_2, wm z_2)] \rrbracket^M$

2. $g[z_1 z_2]h \ \& \ h \in \llbracket man z_1, (mrr z_1 z_2, wm z_2) \rrbracket^M$

F|

3. $g[z_1 z_2]h \ \& \ h \in \llbracket man z_1 \rrbracket^M \ \& \ h \in \llbracket mrr z_1 z_2 \rrbracket^M \ \& \ h \in \llbracket wm z_2 \rrbracket^M$

D2.2:., \cap

4. $g[z_1 z_2]h \ \& \ h(z_1) \in \llbracket man \rrbracket^M \ \& \ \langle h(z_1), h(z_2) \rangle \in \llbracket mrr \rrbracket^M \ \& \ h(z_2) \in \llbracket wm \rrbracket^M$

D2.2:R

□

F11² $\models_{M,g} [z_1 z_2 | man z_1, (mrr z_1 z_2, wm z_2)]$

(truth cond.)

iff $\exists c, d \in D^M (c \in \llbracket man \rrbracket^M \ \& \ \langle c, d \rangle \in \llbracket mrr \rrbracket^M \ \& \ d \in \llbracket wm \rrbracket^M)$

Proof: (1) iff (4)

1. $\models_{M,g} [z_1 z_2 | man z_1, (mrr z_1 z_2, wm z_2)]$

2. $\exists h: \langle g, h \rangle \in \llbracket [z_1 z_2 | man z_1, (mrr z_1 z_2, wm z_2)] \rrbracket^M$

D3

3. $\exists h (g[z_1 z_2]h \ \& \ h(z_1) \in \llbracket man \rrbracket^M \ \& \ \langle h(z_1), h(z_2) \rangle \in \llbracket mrr \rrbracket^M \ \& \ h(z_2) \in \llbracket wm \rrbracket^M)$

F11¹

4. $\exists b, c \in D^M (b \in \llbracket man \rrbracket^M \ \& \ \langle b, c \rangle \in \llbracket mrr \rrbracket^M \ \& \ c \in \llbracket wm \rrbracket^M)$

D2.1

□

(12) he₁ gives her₂ a³ ring.

(English)

(12') $[z_3 | gvt z_1 z_3 z_2, rng z_3]$

(DRT)

F12¹ $\langle g, h \rangle \in \llbracket [z_3 | gvt z_1 z_3 z_2, rng z_3] \rrbracket^M$

(update)

iff $g[z_3]h \ \& \ \langle h(z_1), h(z_3), h(z_2) \rangle \in \llbracket gvt \rrbracket^M \ \& \ h(z_3) \in \llbracket rng \rrbracket^M$

Proof: (1) iff (4)

1. $\langle g, h \rangle \in \llbracket [z_3 | gvt z_1 z_3 z_2, rng z_3] \rrbracket^M$

2. $g[z_3]h \ \& \ h \in \llbracket gvt z_1 z_3 z_2, rng z_3 \rrbracket^M$

F|

3. $g[z_3]h \ \& \ h \in \llbracket gvt z_1 z_3 z_2 \rrbracket^M \ \& \ h \in \llbracket rng z_3 \rrbracket^M$

D2.2:., \cap

4. $g[z_3]h \ \& \ \langle h(z_1), h(z_3), h(z_2) \rangle \in \llbracket gvt \rrbracket^M \ \& \ h(z_3) \in \llbracket rng \rrbracket^M$

D2.2:R

□

F12² $\models_{M,g} [z_3 | gvt z_1 z_3 z_2, rng z_3]$

(truth cond.)

iff $\exists d \in D^M (\langle g(z_1), d, g(z_2) \rangle \in \llbracket gvt \rrbracket^M \ \& \ d \in \llbracket rng \rrbracket^M)$

Proof: (1) iff (4)

1. $\models_{M,g} [z_3 | gvt z_1 z_3 z_2, rng z_3]$

2. $\exists h: \langle g, h \rangle \in \llbracket [z_3 | gvt z_1 z_3 z_2, rng z_3] \rrbracket^M$

D3

3. $\exists h (g[z_3]h \ \& \ \langle h(z_1), h(z_3), h(z_2) \rangle \in \llbracket gvt \rrbracket^M \ \& \ h(z_3) \in \llbracket rng \rrbracket^M)$

F12¹

4. $\exists d \in D^M (\langle g(z_1), d, g(z_2) \rangle \in \llbracket gvt \rrbracket^M \ \& \ d \in \llbracket rng \rrbracket^M)$

D2.1

□

(5_{if}) if a¹ man marries a² woman he₁ gives her₂ a³ ring. (English)

(5') $\llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M$ (DRT)

F5¹ $\langle g, h \rangle \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M \rrbracket^M$ (test)

iff $g = h$

$\& \forall k (h[z_1 z_2]k \& k(z_1) \in \llbracket \text{man} \rrbracket^M \& \langle k(z_1), k(z_2) \rangle \in \llbracket \text{mrr} \rrbracket^M \& k(z_2) \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists k' (k[z_3]k' \& \langle k'(z_1), k'(z_3), k'(z_2) \rangle \in \llbracket \text{gvt} \rrbracket^M \& k'(z_3) \in \llbracket \text{rng} \rrbracket^M)$)

Proof: (1) iff (6)

- 1 $\langle g, h \rangle \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M \rrbracket^M$
- 2 $g = h \& h \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M \rrbracket^M$ D2.2:[
3. $g = h$ D2.2: \Rightarrow
 $\& \forall k (\langle h, k \rangle \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \rrbracket^M \rrbracket^M$
 $\rightarrow \exists k' (\langle k, k' \rangle \in \llbracket \llbracket [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M \rrbracket^M)$)
4. $g = h$ F11¹, F12¹
 $\& \forall k (h[z_1 z_2]k \& k(z_1) \in \llbracket \text{man} \rrbracket^M \& \langle k(z_1), k(z_2) \rangle \in \llbracket \text{mrr} \rrbracket^M \& k(z_2) \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists k' (k[z_3]k' \& \langle k'(z_1), k'(z_3), k'(z_2) \rangle \in \llbracket \text{gvt} \rrbracket^M \& k'(z_3) \in \llbracket \text{rng} \rrbracket^M)$)
5. $g = h$ D2.1
 $\& \forall k (h[z_1 z_2]k \& k(z_1) \in \llbracket \text{man} \rrbracket^M \& \langle k(z_1), k(z_2) \rangle \in \llbracket \text{mrr} \rrbracket^M \& k(z_2) \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists k' (k[z_3]k' \& \langle k'(z_1), k'(z_3), k'(z_2) \rangle \in \llbracket \text{gvt} \rrbracket^M \& k'(z_3) \in \llbracket \text{rng} \rrbracket^M)$)

□

F5² $\models_{M, g} \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket$ (truth cond.)

iff $\forall b, c \in D^M (b \in \llbracket \text{man} \rrbracket^M \& \langle b, c \rangle \in \llbracket \text{mrr} \rrbracket^M \& c \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists d \in D^M (\langle b, d, c \rangle \in \llbracket \text{gvt} \rrbracket^M \& d \in \llbracket \text{rng} \rrbracket^M)$)

Proof: (1) iff (5)

1. $\models_{M, g} \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket$
2. $\exists h (\langle g, h \rangle \in \llbracket \llbracket [z_1 z_2 | \text{man } z_1, (\text{mrr } z_1 z_2, \text{wm } z_2)] \Rightarrow [z_3 | \text{gvt } z_1 z_3 z_2, \text{rng } z_3] \rrbracket^M \rrbracket^M$ D3
3. $\exists h (g = h$ F5¹
 $\& \forall k (h[z_1 z_2]k \& k(z_1) \in \llbracket \text{man} \rrbracket^M \& \langle k(z_1), k(z_2) \rangle \in \llbracket \text{mrr} \rrbracket^M \& k(z_2) \in \llbracket \text{wm} \rrbracket^M$
 $\rightarrow \exists k' (k[z_3]k' \& \langle k'(z_1), k'(z_3), k'(z_2) \rangle \in \llbracket \text{gvt} \rrbracket^M \& k'(z_3) \in \llbracket \text{rng} \rrbracket^M)$)
4. $\forall k (g[z_1 z_2]k \& k(z_1) \in \llbracket \text{man} \rrbracket^M \& \langle k(z_1), k(z_2) \rangle \in \llbracket \text{mrr} \rrbracket^M \& k(z_2) \in \llbracket \text{wm} \rrbracket^M$ elim. h
 $\rightarrow \exists k' (k[z_3]k' \& \langle k'(z_1), k'(z_3), k'(z_2) \rangle \in \llbracket \text{gvt} \rrbracket^M \& k'(z_3) \in \llbracket \text{rng} \rrbracket^M)$)
5. $\forall b, c \in D^M (b \in \llbracket \text{man} \rrbracket^M \& \langle b, c \rangle \in \llbracket \text{mrr} \rrbracket^M \& c \in \llbracket \text{wm} \rrbracket^M$ D2.1
 $\rightarrow \exists d \in D^M (\langle b, d, c \rangle \in \llbracket \text{gvt} \rrbracket^M \& d \in \llbracket \text{rng} \rrbracket^M)$)

□