

Lecture 11 HAMBLIN SEMANTICS FOR QUESTIONS

1. From assertions to questions: English to Kalaallisut

All languages distinguish assertions and questions, but the grammatical means vary—e.g. English uses inversion, WH-words, and prosody (compare (1_E)–(3_E) vs. (4_E)–(7_E)), whereas Kalaallisut uses illocutionary mood inflection (compare (1_K)–(3_K) vs. (4_K)–(6_K), where DEC = declarative mood, QUE = interrogative mood, NG = negative, sth = something).

• Assertion

(1 _E) Ole is sick . Ole be.TNS sick	(1 _K) Ole naparsimavug. Ole naparsima- pu-q Ole sick-DEC.IV-3SG
(2 _E) Ole has bought a dog . Ole have.TNS buy.ASP A dog	(2 _K) Ole qimmisivug. Ole qimmiq-si- pu-q Ole dog-get-DEC.IV-3SG
(3 _E) Ole hasn't bought anything . Ole have.TNS=N'T buy.ASP anything	(3 _K) Ole susinngilaq. Ole su-si-nngit- la-q Ole sth-get-not-DEC.NG-3SG

• Polar question

(4 _E) is Ole sick ? be.TNS Ole sick	(4 _K) Ole naparsimava ? Ole naparsima- pi-a Ole sick-QUE-3SG
(5 _E) has Ole bought anything ? have.TNS Ole buy.ASP anything	(5 _K) Ole susiva ? (YN reading) Ole su-si- pi-a Ole sth-get-QUE-3SG

• Constituent question

(6 _E) who is sick ? who be.TNS sick	(6 _K) kina naparsimava ? kina naparsima- pi-a who sick-QUE-3SG
(7 _E) what has Ole bought ? what have.TNS Ole like.ASP	(5 _K) Ole susiva ? (WH reading) Ole su-si- pi-a Ole sth-get-QUE-3SG

BASIC IDEAS:

- At each point in the conversation the participants share partial information about the world, i.e. they locate themselves in the class of worlds where that information is true. This class of worlds—the current candidates for the speech world—is the current *common ground* (CG).
- Assertion* adds information—i.e. it eliminates those CG-worlds that are incompatible with the new information and thereby updates the common ground to a smaller set (Stalnaker 1978).
- Questions* do not add information—i.e. no CG-worlds are eliminated. Instead, to understand a question is to know what would count as an answer (Hamblin 1973). In terms of the common ground update, the speaker introduces a set of propositions (possible answers) compatible with the common ground, which raises the issue which of them are true in the speech world. A cooperative *answer* helps to resolve this issue (Groenendijk and Stokhof 1984 “On the semantics of questions and the pragmatics of answers”).

• POLAR QUESTION: YN-SENTENCE (Hamblin 1973) |– DU-*set of propositions*

(4_E) YN	be.TNS	Ole	sick
$s/\underline{s}: \lambda qp(p = q \vee p = \lambda j \neg qj)$	$\underline{s}/\text{prd}/\text{np}_N: \lambda xP(Px)$	$\text{np}_N: ole$	$\text{prd}: \lambda xj(\text{sick}_j x)$
	$\underline{s}/\text{prd}: \lambda P(P ole)$		
	$\underline{s}: \lambda j(\text{sick}_j ole)$		
$s: \lambda p(p = \lambda j(\text{sick}_j ole) \vee p = \lambda j(\neg \text{sick}_j ole))$			

(5_E) YN	have.TNS	Ole	buy.ASP anything
$s/\underline{s}: \lambda qp(p = q \vee p = \lambda j \neg qj)$	$\underline{s}/\text{vp}/\text{np}_N: \lambda xP(Px)$	$\text{np}_N: ole$	$\text{vp}: \lambda xj \exists y(\text{sth}_j y \wedge \text{buy}_j(x, y))$
	$\underline{s}/\text{vp}: \lambda P(P ole)$		
	$\underline{s}: \lambda j \exists y(\text{sth}_j y \wedge \text{buy}_j(ole, y))$		
$s: \lambda p(p = \lambda j \exists y(\text{sth}_j y \wedge \text{buy}_j(ole, y)) \vee p = \lambda j \neg \exists y(\text{sth}_j y \wedge \text{buy}_j(ole, y)))$			

• CONSTITUENT QUESTION: WH-SENTENCE (Hamblin 1973) |– PL-*set of propositions*

Note: context represented by (variable) assignment (standard in static semantics)
world-variable i reserved for the *speech world* (a la Groenendijk & Stokhof 1982)

(6_E) who	be.TNS	sick
$s/(\underline{s}\backslash\text{np}_N): \lambda P\lambda p \exists x(\text{sbd}_i x \wedge p = Px)$	$\underline{s}\backslash\text{np}_N/\text{prd}: \lambda P(P)$	$\text{prd}: \lambda xj(\text{sick}_j x)$
	$\underline{s}\backslash\text{np}_N: \lambda xj(\text{sick}_j x)$	
$s: \lambda p \exists x(\text{sbd}_i x \wedge p = \lambda j(\text{sick}_j x))$		

(7_E) what	have.TNS	Ole	buy.ASP
$s/(\underline{s}\backslash\text{np}_A): \lambda Pp \exists y(\text{sth}_i y \wedge p = Py)$	$\underline{s}/\text{vp}/\text{np}_N: \lambda xP(Px)$	$\text{np}_N: ole$	$\text{vp}/\text{np}_A: \lambda yxj(\text{buy}_j(x, y))$
	$\underline{s}/\text{vp}: \lambda P(P ole)$		
	$\underline{s}/\text{np}_A \lambda yj(\text{buy}_j(ole, y))$		
$s: \lambda p \exists y(\text{sth}_i y \wedge p = \lambda j(\text{buy}_j(ole, y)))$			

Lectures 12–13
TOWARD DYNAMIC HAMBLIN SEMANTICS

1. Information vs. attention

BASIC IDEAS:

- at each point in discourse, Stalnaker's common ground (CG) is the current SET OF TOPIC WORLDS
- assertions update *information*: in the output the 1ry topic (τ_1) is a TOPICAL PROPOSITION—the output set of topic worlds (output CG), which is a subset of the input CG (Stalnaker's CG update)
- questions update *attention*: in the output the 1ry topic (τ_1) is a TOPICAL QUESTION—i.e. the characteristic set of answers (like Hamblin's, but different sets for *Is Ole sick?* vs. *Isn't Ole sick?*)

EXAMPLE 1: ASSERTION AS INFORMATION UPDATE

Speaking up focuses attention on the input CG, p_0 , e.g.

$$p_0 = \{w_0, w_1\} \quad \langle w_0, \llbracket ole \rrbracket \rangle \in \mathcal{B}\llbracket sick \rrbracket$$

$$\langle w_1, \llbracket ole \rrbracket \rangle \notin \mathcal{B}\llbracket sick \rrbracket$$

will induce a *default infotention state*, $*p_0$, i.e. default set of *topic* (τ)-*background* (\perp) *lists*. On each $\tau\perp$ -list the input CG, p_0 , is the TOPICAL PROPOSITION (1st proposition on τ -list); a CG-world, $w \in p_0$, is the TOPIC WORLD (1st world on τ -list); and there is nothing on the default \perp -list ($\langle \rangle$):

$$*p_0$$

$$\langle \langle w_0, p_0 \rangle, \langle \rangle \rangle$$

$$\langle \langle w_1, p_0 \rangle, \langle \rangle \rangle$$

The default infotention state, $*p_0$, is then updated based on what is said in the speech act, e.g.:

Ole [$\mathbf{x} \mid (\mathbf{x} = ole)^\circ$];	be.TNS sick [$sick_{\tau\omega} \tau\delta$];	. (prosody) [$\mathbf{p} \mid \mathbf{p} = \tau\omega\{\}\}$]
c_1 $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle \rangle \rangle$	c_2 $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle \rangle \rangle$	c_3 ($\tau_1 = \text{new CG}, \{w_0\}$) $\langle \langle \{w_0\}, \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle \rangle \rangle$

EXAMPLE 2: QUESTION AS ATTENTION UPDATE

Same input CG, p_0 , induces same default infotention state, $*p_0$, but what follows is a YN question:

YN be.TNS [$\mathbf{w} \mid \mathbf{w} \in \tau\omega\{\}\}$];	[Ole [$\mathbf{x} \mid (\mathbf{x} = ole)^\circ$];	sick [$sick_{\perp\omega} \tau\delta$];] [$\mathbf{p} \mid \mathbf{p} = \perp\omega\{\}\}$];
c_1 $\langle \langle w_0, p_0 \rangle, \langle w_0 \rangle \rangle$ $\langle \langle w_0, p_0 \rangle, \langle w_1 \rangle \rangle$ $\langle \langle w_1, p_0 \rangle, \langle w_0 \rangle \rangle$ $\langle \langle w_1, p_0 \rangle, \langle w_1 \rangle \rangle$	c_2 $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle w_0 \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle w_1 \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle w_0 \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle w_1 \rangle \rangle$	c_3 $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle w_0 \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle w_0 \rangle \rangle$	c_4 $\langle \langle \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle \{w_0\}, w_0 \rangle \rangle$ $\langle \langle \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle \{w_0\}, w_0 \rangle \rangle$
			? (prosody) [$\mathbf{Q} \mid \mathbf{Q} = \perp\omega t\{\}\}$] c_5 ($\tau_1 = \text{whether-}\{w_0\}$, no CG-update) $\langle \langle \{\{w_0\}\}, \llbracket ole \rrbracket, w_0, p_0 \rangle, \langle \{w_0\}, w_0 \rangle \rangle$ $\langle \langle \{\{w_0\}\}, \llbracket ole \rrbracket, w_1, p_0 \rangle, \langle \{w_0\}, w_0 \rangle \rangle$

2. 2-sorted Update with Centering (UC2)

DEFINITION 1.1 The set of UC2 types is the smallest set **Typ** such that:

- $t, \delta, \omega, s \in \mathbf{Typ}$
- $(ab) \in \mathbf{Typ}$, if $a, b \in \mathbf{Typ}$

The set of discourse object types is the subset $\mathbf{DTyp} = \{\delta, \omega, \omega t\} \cup \{(at) : a \in \{\delta, \omega, \omega t\}\}$

ABBREVIATIONS 1

$$\begin{aligned} a_1 \dots a_n b &:= (a_1 \dots (a_n b)) & [] &:= (st)st \\ \Omega &:= \omega t & [a] &:= a[] \end{aligned}$$

DEFINITION 1.2 (Basic UC2 terms). For each type $a \in \mathbf{Typ}$, there is a set of a -constants, \mathbf{Con}_a , a set of plain a -variables, ${}^{\perp}\mathbf{Var}_a$, and a set of topic-setting a -variables, ${}^{\top}\mathbf{Var}_a$, including:

$a \in \mathbf{Typ}$	${}^{\perp}\mathbf{Var}_a$	${}^{\top}\mathbf{Var}_a$	\mathbf{Con}_a	Name of objects
δ	x, y	\mathbf{x}, \mathbf{y}	<i>ole</i>	(ordinary) individuals
ω	w, v	\mathbf{w}, \mathbf{v}		worlds
Ω	p, q	\mathbf{p}, \mathbf{q}		propositions
δt	X, Y	\mathbf{X}, \mathbf{Y}		sets of individuals
Ωt	Q	\mathbf{Q}		sets of propositions
$\omega \delta t$			<i>sbd, dog, sick, ...</i>	world-dependent sets of individuals
s	i, j, k, h			indices (<i>aka</i> topic-background lists)
st	I, J			infotention states
$(st)st$	K			updates

DEFINITION 1.3 (UC2 syntax). For each type $a \in \mathbf{Typ}$ the set of a -terms is defined as follows:

b	$A \in \mathbf{Term}_a$	if $A \in \mathbf{Con}_a \cup {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$
a	$\top a_n, \perp a_n \in \mathbf{Term}_{sa}$	if $a \in \mathbf{DTyp}$ & $n \in \{1, 2, \dots\}$
$\{ \}$	$B\{A\} \in \mathbf{Term}_{at}$	if $a \in \mathbf{DTyp}$ & $B \in \mathbf{Term}_{sa}$ & $A \in \mathbf{Term}_{st}$
\cdot	$(u_a \cdot B) \in \mathbf{Term}_s$	if $a \in \mathbf{DTyp}$, $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_s$
$;$	$(A; B) \in \mathbf{Term}_{(st)st}$	if $A, B \in \mathbf{Term}_{(st)st}$
λ	$\lambda u_a(B) \in \mathbf{Term}_{ab}$	if $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_b$
\mathbf{A}	$BA \in \mathbf{Term}_b$	if $B \in \mathbf{Term}_{ab}$ & $A \in \mathbf{Term}_a$
\mathbf{C}	$\neg A, (A \rightarrow B), (A \wedge B), (A \vee B) \in \mathbf{Term}_t$	if $A, B \in \mathbf{Term}_t$
\mathbf{Q}	$\forall u_a B, \exists u_a B \in \mathbf{Term}_t$	if $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_t$
$=$	$(A = B) \in \mathbf{Term}_t$	if $A, B \in \mathbf{Term}_a$

DEFINITION 2.1 (UC2 frames). A UC2 frame is a set of sets $\{D_a\}_{a \in \mathbf{Typ}}$ such that:

- i. $D_t = \{1, 0\}$, D_δ , and D_ω are non-empty pairwise disjoint sets
- ii. $D_s = \bigcup_{n, m \leq 0} (D^n \times D^m)$, where $D = \bigcup_{a \in \mathbf{DTyp}} D_a$
- iii. $D_{ab} = \{f \mid \emptyset \subset \text{Dom } f \subseteq D_a \text{ \& } \text{Ran } f \subseteq D_b\}$

ABBREVIATIONS 2 (lists, projections & extensions). Let $D = \bigcup_{a \in \mathbf{DTyp}} D_a$.

- For $\mathbf{i} = \langle \mathbf{i}_1, \mathbf{i}_2 \rangle \in D^n \times D^m$, $\top \mathbf{i} = \mathbf{i}_1$ is the *topic list* & $\perp \mathbf{i} = \mathbf{i}_2$ is the *background list* of \mathbf{i}
- For $\mathbf{x} \in D^{n+m}$, $(\mathbf{x})_n$ is \mathbf{x}_n (n th coordinate) & $(\mathbf{x})_a$ is the subsequence of type a coordinates
- For $d_0, \dots, d_n \in D$: $(d_0 \cdot \langle \rangle) = \langle d_0 \rangle$ & $(d_0 \cdot \langle d_1, \dots, d_n \rangle) := \langle d_0, d_1, \dots, d_n \rangle$ (d_0 -extensions)

DEFINITION 2.2 (UC2-models and assignments)

- A UC2-model is a pair, $M = \langle \{D_a\}_{a \in \mathbf{Typ}}, \llbracket \cdot \rrbracket \rangle$, s.t. (i) $\{D_a\}_{a \in \mathbf{Typ}}$ is a standard UC2-frame, (ii) $\llbracket \cdot \rrbracket$ is an *interpretation function* that assigns to each $\alpha \in \mathbf{Con}_a$ a denotation $\llbracket \alpha \rrbracket \in D_a$.
- An M -assignment is a function g that assigns $g(u) \in D_a$ to each $u \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$. If $d \in D_a$ then $g[u/d]$ is the M -assignment s.t. (i) $g[u/d](u') = g(u')$ for all $u' \neq u$, (ii) $g[u/d](u) = d$.

ABBREVIATIONS 3 (Functions & sets). For $f \in D_{a_1 \dots a_n t}$ and $\langle a_1, \dots, a_n \rangle \in D_{a_1} \times \dots \times D_{a_n}$:

- $f(a_1, \dots, a_n) := f(a_1) \dots (a_n)$
- $\{ \{ f \} = \{ \langle a_1, \dots, a_n \rangle : f(a_1, \dots, a_n) = 1 \} \}$ (set characterized by function f)
- $\chi(A)$ (characteristic function of set A) is the function f such that $\{ \{ f \} = A$

DEFINITION 2.3 (UC2 semantics). The value $\llbracket A \rrbracket^g$ of a term A on a model $M = \langle \{ D_a \}_{a \in \mathbf{Typ}}, \llbracket \cdot \rrbracket \rangle$ under an assignment g is defined as follows (Note: (i) we use von Neuman's definitions $0 := \{ \}$ & $1 := \{ \emptyset \}$, (ii) ' $X \doteq Y$ ' means ' X is Y if Y is defined, else X is undefined', and (iii) $a f \doteq f(a)$)

b	$\llbracket A \rrbracket^g$	$= \llbracket A \rrbracket$	$\text{if } A \in \mathbf{Con}_a$
		$= g(A)$	$\text{if } A \in {}^T\mathbf{Var}_a \cup {}^\perp\mathbf{Var}_a$
a	$\llbracket \top a_n \rrbracket^g(i)$	$\doteq ((\top i)_{a_n})$	$\text{for any } i \in D_s$
	$\llbracket \perp a_n \rrbracket^g(i)$	$\doteq ((\perp i)_{a_n})$	$\text{for any } i \in D_s$
$\{ \}$	$\llbracket B \{ A \} \rrbracket^g$	$\doteq \chi\{ \llbracket B \rrbracket^g(i) \mid i \in \{ \{ A \} \} \}$	
\cdot	$\llbracket (u_a \cdot B) \rrbracket^g$	$\doteq \langle (g(u_a) \cdot \top \llbracket B \rrbracket^g), \perp \llbracket B \rrbracket^g \rangle$	$\text{for any } u_a \in {}^T\mathbf{Var}_a$
		$\doteq \langle \top \llbracket B \rrbracket^g, (g(u_a) \cdot \perp \llbracket B \rrbracket^g) \rangle$	$\text{for any } u_a \in {}^\perp\mathbf{Var}_a$
;	$c \llbracket (A; B) \rrbracket^g$	$\doteq c \llbracket A \rrbracket^g \llbracket B \rrbracket^g$	$\text{for any } c \in D_{st}$
λ	$\llbracket \lambda u_a(B) \rrbracket^g(d)$	$\doteq \llbracket B \rrbracket^{g[u/d]}$	$\text{for any } d \in D_a$
A	$\llbracket BA \rrbracket^g$	$\doteq \llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$	
C	$\llbracket \neg A \rrbracket^g$	$\doteq (1 - \llbracket A \rrbracket^g)$	
	$\llbracket (A \rightarrow B) \rrbracket^g$	$\doteq (1 - (\llbracket A \rrbracket^g - \llbracket B \rrbracket^g))$	
	$\llbracket (A \wedge B) \rrbracket^g$	$\doteq (\llbracket A \rrbracket^g \cap \llbracket B \rrbracket^g)$	
	$\llbracket (A \vee B) \rrbracket^g$	$\doteq (\llbracket A \rrbracket^g \cup \llbracket B \rrbracket^g)$	
Q	$\llbracket \forall u_a A \rrbracket^g$	$= 1$	$\text{if } \forall d \in D_a: \llbracket A \rrbracket^{g[u/d]} = 1$
		$= 0$	otherwise
	$\llbracket \exists u_a A \rrbracket^g$	$= 1$	$\text{if } \exists d \in D_a: \llbracket A \rrbracket^{g[u/d]} = 1$
		$= 0$	otherwise
$=$	$\llbracket (A = B) \rrbracket^g$	$= 1$	$\text{if } \llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_a \ \& \ \llbracket A \rrbracket^g = \llbracket B \rrbracket^g$
		$= 0$	$\text{if } \llbracket A \rrbracket^g, \llbracket B \rrbracket^g \in D_a \ \& \ \llbracket A \rrbracket^g \neq \llbracket B \rrbracket^g$

DEFINITION 3.1 (Initial context & default state). An *initial context* is a proposition $p_0 \in D_\Omega$ s.t. $\{ \{ p_0 \} \neq \emptyset$. This induces the *default infotention state* $*p_0 = \chi\{ \langle \langle w, p_0 \rangle, \langle \rangle \rangle \mid w \in \{ \{ p_0 \} \}$.

DEFINITION 3.2 (Topic, assertion, truth value). For a model M , initial context p_0 , and (*st*)*st*-term K :

- i. $\tau_{M, p_0} K := \{ (\top i)_1 \mid \forall g: i \in \{ \{ *p_0 \} \llbracket K \rrbracket^g \} \}$ (set of *primary topics* introduced by K wrt M and p_0)
- ii. Given M and p_0 , K can be used to *assert* a proposition $q \in D_\Omega$, iff $\tau_{M, p_0} K = \{ q \}$
- iii. K is *true* in world w wrt M and p_0 , iff $\exists q \in D_\Omega: \tau_{M, p_0} K = \{ q \} \ \& \ w \in \{ \{ q \}$
 K is *false* in world w wrt M and p_0 , iff $\exists q \in D_\Omega: \tau_{M, p_0} K = \{ q \} \ \& \ w \notin \{ \{ q \}$

ABBREVIATIONS 4 (Syntactic sugar)

i.	$A_a \in B_{at}$	$:= BA$	(set theory)
ii.	$\top a, \perp a$	$:= \top a_1, \perp a_1$	$\text{if } a \in \mathbf{DTyp}$ (dynamic terms)
	A_a°	$:= \lambda i(A)$	$\text{if } a \in \mathbf{DTyp}$
		$:= A$	$\text{if } a \in \{ \{ sb \mid b \in \mathbf{DTyp} \} \}$
iii.	$(B_W A)$	$:= \lambda i(B \ W^\circ i \ A^\circ i)$	(conditions)
	$B_W(A_1, \dots, A_n)$	$:= \lambda i(B \ W^\circ i \ A_1^\circ i \ \dots \ A_n^\circ i)$	
	$(A = B)^\circ$	$:= \lambda i(A^\circ i = B^\circ i)$	
	(C_1, C_2)	$:= \lambda i(C_1 i \wedge C_2 i)$	
iv.	$[C]$	$:= \lambda I \lambda j(I j \wedge C j)$	(local drt-boxes)
	$[u_1 \dots u_n]$	$:= \lambda I \lambda j \exists u_1 \dots u_n \exists i(j = (u_1 \cdot \dots \cdot (u_n \cdot i)) \wedge I i)$	
	$[u_1 \dots u_n] C$	$:= \lambda I \lambda j \exists u_1 \dots u_n \exists i(j = (u_1 \cdot \dots \cdot (u_n \cdot i)) \wedge I i \wedge C i)$	
v.	$[u_{at} \mid u = A_{sa} \{ \{ \} \}]$	$:= \lambda I \lambda j \exists u \exists i(j = (u \cdot i) \wedge I i \wedge u = A \{ I \})$	$(\text{global drt-boxes})$
	$[u_a \mid u \in A_{sa} \{ \{ \} \}]$	$:= \lambda I \lambda j \exists u \exists i(j = (u \cdot i) \wedge I i \wedge u \in A \{ I \})$	

3. Topic-background lists & infotention states

Consider a UC2 model $M = \langle \{D_a\}_{a \in \mathbf{Typ}}, \llbracket \cdot \rrbracket \rangle$ such that $D_\delta = \{\Delta\}$ and $D_\omega = \{w_0, w_1\}$. Then:

- the domain of things that can be listed as discourse objects is

$$\begin{aligned}
 D &= \bigcup_{a \in \mathbf{DTyp}} D_a \\
 &= D_\delta \cup D_\omega \cup D_\Omega \cup D_{\delta t} \cup D_{\omega t} \cup D_{\Omega t} \\
 &= D_\delta \cup D_\omega \cup D_\Omega \cup D_{\delta t} \cup D_{\Omega t} \\
 &= \{\Delta\} \cup \{w_0, w_1\} \cup \{^x p \mid p \subseteq \{w_0, w_1\}\} \\
 &\quad \cup \{^x X \mid X \subseteq \{\Delta\}\} \cup \{^x Q \mid Q \subseteq \{^x p \mid p \subseteq \{w_0, w_1\}\}\}
 \end{aligned}$$

D1.1.DTyp

$\Omega := \omega t$

D2.1, df. D_δ, D_ω abv

- sample lists of (topical or background) discourse objects

$\langle \rangle$ (empty list)

$\langle \Delta, w_0, ^x \{w_0, w_1\} \rangle$

- sets of possible lists of n (topical or background) discourse objects

$$D^0 = \{\langle \rangle\}$$

$$D^1 = \{\langle d \rangle \mid d \in D\}$$

$$D^n = \{\langle d_1, \dots, d_n \rangle \mid d_i \in D\}$$

- domain of topic-background lists

$$\begin{aligned}
 D_s &= \bigcup_{n, m \geq 0} (D^n \times D^m) \\
 &= (D^0 \times D^0) \cup (D^1 \times D^0) \cup (D^0 \times D^1) \cup (D^1 \times D^1) \cup \dots \\
 &= \{\langle \rangle, \langle \rangle\} \cup \{\langle d \rangle, \langle \rangle\} \mid d \in D\} \cup \{\langle \rangle, \langle d \rangle\} \mid d \in D\} \\
 &\quad \cup \{\langle d \rangle, \langle d' \rangle\} \mid d, d' \in D\} \cup \dots
 \end{aligned}$$

D2.1. D_s

df. $\bigcup_{m, n \geq 0}$

df. D^n, \times

- an infotention state, $c \in D_{st}$, is (the characteristic function of) a set of topic-background lists.