Lecture 14
ASSERTION AS INFORMATION UPDATE

0. Overview

• Speaking up focuses attention on the input CG, e.g.

$p_0 = {}^{\chi}\{w_0, w_1\}$ $\langle w_0, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$

$\langle w_1, [\![ole]\!]\rangle \notin {}^{\{\}}[\![sick]\!]$

• Specifically, speaking up focuses attention on p_0 (default Ω-topic) and the various p_0-worlds (default topic worlds). That is, it induces the following *default infotention state*:

$*p_0 = {}^{\chi}\{\langle\langle w, p_0\rangle, \langle\,\rangle\rangle | \, w \in {}^{\{\}}p_0\}$ D3.1

$= {}^{\chi}\{\langle\langle w_0, p_0\rangle, \langle\,\rangle\rangle$

$\langle\langle w_1, p_0\rangle, \langle\,\rangle\rangle\}$ $p_0 = {}^{\chi}\{w_0, w_1\}$

• $*p_0$ is then updated based on what is said in the speech act, e.g.:

Ole be.TNS sick . (prosody)

$*p_0[\![(([\mathbf{x}|\ (\mathbf{x} = ole)°];\ [sick_{\top\omega}\ \top\delta]);\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])]\!]^g$

$= *p_0[\![([\mathbf{x}|\ (\mathbf{x} = ole)°];\ [sick_{\top\omega}\ \top\delta])]\!]^g[\![[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}]]\!]^g$ D2.3.;

$= *p_0[\![[\mathbf{x}|\ (\mathbf{x} = ole)°]]\!]^g[\![[sick_{\top\omega}\ \top\delta]]\!]^g[\![[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}]]\!]^g$ D2.3.;

c_1 Sec. 1

c_2 Sec. 2

c_3 Sec. 3

1. <u>From default infotention state</u> $^*\mathtt{p}_0$ <u>to</u> $\mathtt{c}_1$

FACT 1[1]. $[\mathbf{x}|\ (\mathbf{x} = \mathit{ole})^\circ]$
$\equiv\ \lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))$

Proof: (1) ≡ (6)

1.	$[\mathbf{x}|\ (\mathbf{x} = \mathit{ole})^\circ]$	
2.	$\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole})^\circ i)$	A4.iv.$[u\ldots|\ C]$
3.	$\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge \lambda i'(\mathbf{x}^\circ i' = \mathit{ole}^\circ i')i)$	A4.iii.=
4.	$\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x}^\circ i = \mathit{ole}^\circ i))$	λ-cnv
5.	$\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\lambda i'(\mathbf{x})i = \lambda i'(\mathit{ole})i))$	A4.ii.$A_\delta{}^\circ$
6.	$\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))$	λ-cnv

FACT 1[2]. $^*\mathtt{p}_0[\![[\mathbf{x}|\ (\mathbf{x} = \mathit{ole})^\circ]]\!]^g$
$=\ {}^\chi\{\langle\langle[\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\}$ $\qquad$ $=:\ \mathtt{c}_1$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (18):

1.	$\mathtt{j} \in {}^{\{\}}(^*\mathtt{p}_0[\![[\mathbf{x}|\ (\mathbf{x} = \mathit{ole})^\circ]]\!]^g)$	
2.	$\mathtt{j} \in {}^{\{\}}(^*\mathtt{p}_0[\![\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^g)$	F1[1]
3.	$\mathtt{j} \in {}^{\{\}}([\![\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^g(^*\mathtt{p}_0))$	$\mathtt{af} := \mathtt{f(a)}$
4.	$\mathtt{j} \in {}^{\{\}}([\![\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^{g[I/^*\mathtt{p0}]})$	D2.3.λ
5.	$[\![\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^{g[I/^*\mathtt{p0}]}(\mathtt{j}) = 1$	A3.$^{\{\}}\mathtt{f}$
6.	$[\![\exists\mathbf{x}\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}]} = 1$	D2.3.λ
7.	$\exists\mathtt{d} \in D_\delta$: $[\![\exists i(j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole}))]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}]} = 1$	D2.3.Q$\exists$
8.	$\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$: $[\![j = (\mathbf{x}\cdot i) \wedge Ii \wedge (\mathbf{x} = \mathit{ole})]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = 1$	D2.3.Q$\exists$
9.	$\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$:	D2.3.$\wedge$, D2.1
	$[\![j]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = [\![\mathbf{x}\cdot i]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}$	D2.3.=
	& $[\![I]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}([\![i]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}) = 1$	D2.3.A
	& $[\![\mathbf{x}]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = [\![\mathit{ole}]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}$	D2.3.=
10.	$\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$:	
	$\mathtt{j} = \langle(g[I/^*\mathtt{p}_0][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}](\mathbf{x})\cdot\top[\![i]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}), \bot[\![i]\!]^{g[I/^*\mathtt{p0}][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}\rangle$	D2.3.$\cdot$, b, D2.2
	& $^*\mathtt{p}_0(\mathtt{i}) = 1$ & $\mathtt{d} = [\![\mathit{ole}]\!]$	D2.3.b, D2.2
11.	$\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle(\mathtt{d}\cdot\top\mathtt{i}, \bot\mathtt{i}\rangle$ & $^*\mathtt{p}_0(\mathtt{i}) = 1$ & $\mathtt{d} = [\![\mathit{ole}]\!]$	D2.3.b, D2.2
12.	$\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle$ & $^*\mathtt{p}_0(\mathtt{i}) = 1$	elim. $\exists\mathtt{d}$
13.	$\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle$ & $\mathtt{i} \in {}^{\{\}}(^*\mathtt{p}_0)$	A3.$^\chi\mathtt{f}$
14.	$\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle$ & $\mathtt{i} \in \{\langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\}$	D3.1, A3
15.	$\exists\mathtt{i} \in D_s\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle$ & $\mathtt{i} = \langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle$	df. {–|–}
16.	$\exists\mathtt{i} \in D_s\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\langle\mathtt{w}, \mathtt{p}_0\rangle), \langle\ \rangle\rangle$ & $\mathtt{i} = \langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle$	A2.$\top$, $\bot$
17.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![\mathit{ole}]\!]\cdot\langle\mathtt{w}, \mathtt{p}_0\rangle), \langle\ \rangle\rangle$	elim. $\exists\mathtt{i}$
18.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle$	A2.$\cdot$

FACT 1[3]. If $\mathtt{p}_0 = {}^\chi\{\mathtt{w}_0, \mathtt{w}_1\}$,
$^*\mathtt{p}_0[\![[\mathbf{x}|\ (\mathbf{x} = \mathit{ole})^\circ]]\!]^g$
$=\ {}^\chi\{\langle\langle[\![\mathit{ole}]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\ \rangle\rangle,$
$\langle\langle[\![\mathit{ole}]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\}$

2. <u>From state $\mathtt{c}_1$ to $\mathtt{c}_2$</u>

FACT 2^1. $[sick_{\top\omega}\ \top\delta]$

$\equiv\ \lambda I\lambda j\,(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)$

Proof: (1) ≡ (5)

1.	$[sick_{\top\omega}\ \top\delta]$	
2.	$\lambda I\lambda j(Ij \wedge (sick_{\top\omega}\ \top\delta)j)$	A4.iv.[C]
3.	$\lambda I\lambda j(Ij \wedge \lambda i(sick\ \top\omega^\circ i\ \top\delta^\circ i\ )j)$	A4.iii.$B_w A$
4.	$\lambda I\lambda j(Ij \wedge sick\ \top\omega^\circ j\ \top\delta^\circ j)$	λ-cnv
5.	$\lambda I\lambda j(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)$	A4.ii

FACT 2^2. $\mathtt{c}_1[\![[sick_{\top\omega}\ \top\delta]]\!]^g$

$= \ {}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{w}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$ $=: \mathtt{c}_2$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (15):

1.	$\mathtt{j} \in {}^{\{\}}(\mathtt{c}_1[\![[sick_{\top\omega}\ \top\delta]]\!]^g)$	
2.	$\mathtt{j} \in {}^{\{\}}(\mathtt{c}_1[\![\lambda I\lambda j(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)]\!]^g)$	F2^1
3.	$\mathtt{j} \in {}^{\{\}}([\![\lambda I\lambda j(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)]\!]^g(\mathtt{c}_1))$	$\mathtt{af} := \mathtt{f(a)}$
4.	$\mathtt{j} \in {}^{\{\}}[\![\lambda j(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)]\!]^{g[I/\mathtt{c}1]}$	D2.3.λ
5.	$[\![\lambda j(Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)]\!]^{g[I/\mathtt{c}1]}(\mathtt{j}) = 1$	A3.${}^{\{\}}\mathtt{f}$
6.	$[\![Ij \wedge sick\ \top\omega_1 j\ \top\delta_1 j)]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]} = 1$	D2.3.λ
7.	$[\![I]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}([\![j]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}) = 1$ $\&\ [\![sick]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}([\![\top\omega_1]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}([\![j]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}))([\![\top\delta_1]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}([\![j]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]})) = 1$	D2.3.$\wedge$, A
8.	$\mathtt{c}_1(\mathtt{j}) = 1$ $\&\ [\![sick]\!]([\![\top\omega_1]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}(\mathtt{j}))([\![\top\delta_1]\!]^{g[I/\mathtt{c}1][j/\mathtt{j}]}(\mathtt{j})) = 1$	D2.3.b, D2.2
9.	$\mathtt{j} \in {}^{\{\}}\mathtt{c}_1$ $\&\ [\![sick]\!](((\top\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1)) = 1$	A3 D2.3.a
10.	$\mathtt{j} \in \{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\}$ $\&\ [\![sick]\!](((\top\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1)) = 1$	F1^2.$\mathtt{c}_1$, A3
11.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\ \&\ [\![sick]\!](((\top\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1)) = 1$	df. {–|–}
12.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\ \&\ [\![sick]\!](((\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle)_\omega)_1)(((\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle)_\delta)_1) = 1$	A2.$\top\mathtt{i}$
13.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\ \&\ [\![sick]\!]((\langle\mathtt{w}\rangle)_1)((\langle[\![ole]\!]\rangle)_1) = 1$	A2.$(\mathtt{x})_a$, $\mathtt{w} \in D_\omega$ $[\![ole]\!] \in D_\delta$
14.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\ \&\ [\![sick]\!](\mathtt{w})([\![ole]\!]) = 1$	A2.$(\mathtt{x})_n$
15.	$\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\ \&\ \langle\mathtt{w}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$	A3.${}^{\{\}}$

FACT 2^3. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}$ and ${}^{\{\}}[\![sick]\!] = \{\langle\mathtt{w}_0, [\![ole]\!]\rangle\}$, then

$\mathtt{c}_1[\![[sick_{\top\omega}\ \top\delta]]\!]^g$

$= \ {}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\ \rangle\rangle\}$

3. From state c_2 to c_3

FACT 3[1]. $[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}]$

$\equiv\ \lambda I\lambda j\ \exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})$ A4.ii, v

FACT 3[2]. $c_2[\![[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}]$

$= {}^{\chi}\{\langle\langle p_1, [\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle|\ w \in {}^{\{\}}p_0\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$

$\&\ p_1 = {}^{\chi}\{v|\ v \in {}^{\{\}}p_0\ \&\ \langle v, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$ $=: c_3$

Proof: For any index $j \in D_s$, (1) iff (16):

1. $j \in {}^{\{\}}(c_2[\![[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])$
2. $j \in {}^{\{\}}(c_2[\![\lambda I\lambda j\ \exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})]\!]^g)$ F3[1]
3. $j \in {}^{\{\}}([\![\lambda I\lambda j\ \exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})]\!]^g(c_2))$ $af := f(a)$
4. $j \in {}^{\{\}}[\![\lambda j\ \exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})]\!]^{g[I/c2]}$ D2.3.λ
5. $[\![\lambda j\ \exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})]\!]^{g[I/c2]}(j) = 1$ A3.$^{\{\}}f$
6. $[\![\exists\mathbf{p}\exists i(j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\})]\!]^{g[I/c2][j/j]} = 1$ D2.3.λ
7. $\exists p_1 \in D_\Omega \exists i \in D_s$: $[\![j = (\mathbf{p}\cdot i) \wedge Ii \wedge \mathbf{p} = \top\omega_1\{I\}]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]} = 1$ D2.3.Q$\exists$
8. $\exists p_1 \in D_\Omega \exists i \in D_s$: D2.3.$\wedge$, D2.1
   $[\![j]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]} = [\![\mathbf{p}\cdot i]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}$ D2.3.=
   $\&\ [\![I]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}([\![i]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}) = 1$ D2.3.A
   $\&\ [\![\mathbf{p}]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]} = [\![\top\omega_1\{I\}]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}$ D2.3.=
9. $\exists p_1 \in D_\Omega \exists i \in D_s$:
   $j = \langle(g[I/c_2][j/j][\mathbf{p}/p_1][i/i](\mathbf{p})\cdot\top[\![i]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}), \bot[\![i]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}\rangle$ D2.3.$\cdot$, b, D2.2
   $\&\ c_2(i) = 1\ \&\ p_1 = [\![\top\omega_1\{I\}]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}$ D2.3.b, D2.2
10. $\exists p_1 \in D_\Omega \exists i \in D_s$:
   $j = \langle(p_1\cdot\top i), \bot i\rangle$ D2.3.b, D2.2
   $\&\ i \in {}^{\{\}}c_2\ \&\ p_1 = {}^{\chi}\{[\![\top\omega_1]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}(k)|\ k \in {}^{\{\}}[\![I]\!]^{g[I/c2][j/j][\mathbf{p}/p1][i/i]}\}$ A3.$^{\{\}}$, D2.3.{}
11. $\exists p_1 \in D_\Omega \exists i \in D_s$: D2.3.$a$, b, D2.2
   $j = \langle(p_1\cdot\top i), \bot i\rangle\ \&\ i \in {}^{\{\}}c_2\ \&\ p_1 = {}^{\chi}\{((\top k)_\omega)_1|\ k \in {}^{\{\}}c_2\}$
12. $\exists p_1 \in D_\Omega \exists i \in D_s$: F2[2].$c_2$, A3
   $j = \langle(p_1\cdot\top i), \bot i\rangle\ \&\ i \in \{\langle\langle[\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle|\ w \in {}^{\{\}}p_0\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
   $\&\ p_1 = {}^{\chi}\{((\top k)_\omega)_1|\ k \in \{\langle\langle[\![ole]\!], v, p_0\rangle, \langle\ \rangle\rangle|\ v \in {}^{\{\}}p_0\ \&\ \langle v, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$
13. $\exists p_1 \in D_\Omega \exists w \in {}^{\{\}}p_0$: elim. i, k
   $j = \langle(p_1\cdot\top\langle\langle[\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle), \bot\langle\langle[\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle\rangle\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
   $\&\ p_1 = {}^{\chi}\{v|\ \exists v' \in p_0(v = ((\top\langle\langle[\![ole]\!], v', p_0\rangle, \langle\ \rangle\rangle)_\omega)_1\ \&\ \langle v', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!])\}$
14. $\exists p_1 \in D_\Omega \exists w \in {}^{\{\}}p_0$: A2.$\top$, $\bot$
   $j = \langle(p_1\cdot\langle[\![ole]\!], w, p_0\rangle), \langle\ \rangle\rangle\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
   $\&\ p_1 = {}^{\chi}\{v|\ \exists v' \in p_0(v = ((\langle[\![ole]\!], v', p_0\rangle)_\omega)_1\ \&\ \langle v', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!])\}$
15. $\exists p_1 \in D_\Omega \exists w \in {}^{\{\}}p_0$:
   $j = \langle\langle p_1, [\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$ A2.$\cdot$
   $\&\ p_1 = {}^{\chi}\{v|\ \exists v' \in {}^{\{\}}p_0(v = v'\ \&\ \langle v', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!])\}$ A2.$(x)_a$, $(x)_n$
16. $\exists p_1 \in D_\Omega \exists w \in {}^{\{\}}p_0$:
   $j = \langle\langle p_1, [\![ole]\!], w, p_0\rangle, \langle\ \rangle\rangle\ \&\ \langle w, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
   $\&\ p_1 = {}^{\chi}\{v|\ v \in {}^{\{\}}p_0\ \&\ \langle v, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$ elim. $\exists v'$

Appendix: 2-sorted Update with Centering (UC2)

DEFINITION 1.1 The set of UC2 types is the smallest set **Typ** such that:
• $t, \delta, \omega, s \in \mathbf{Typ}$ • $(ab) \in \mathbf{Typ}$, if $a, b \in \mathbf{Typ}$
The set of discourse object types is the subset $\mathbf{DTyp} = \{\delta, \omega, \omega t\} \cup \{(at): a \in \{\delta, \omega, \omega t\}\}$

ABBREVIATIONS 1
$a_1 \ldots a_n b \;:=\; (a_1 \ldots (a_n b))$ $[\,] \;:=\; (st)st$
$\Omega \;:=\; \omega t$ $[a] \;:=\; a[\,]$

DEFINITION 1.2 (Basic UC2 terms). For each type $a \in \mathbf{Typ}$, there is a set of a-constants, $\mathbf{Con}_a$, a set of plain a-variables, $^{\perp}\mathbf{Var}_a$, and a set of topic-setting a-variables, $^{\top}\mathbf{Var}_a$, including:

$a \in \mathbf{Typ}$	$^{\perp}\mathbf{Var}_a$	$^{\top}\mathbf{Var}_a$	$\mathbf{Con}_a$	Name of objects
δ	x, y	$\mathbf{x}, \mathbf{y}$	*ole*	(ordinary) individuals
ω	w, v	$\mathbf{w}, \mathbf{v}$		worlds
Ω	p, q	$\mathbf{p}, \boldsymbol{q}$		propositions
δt	X, Y	$\mathbf{X}, \mathbf{Y}$		sets of individuals
Ωt	Q	$\mathbf{Q}$		sets of propositions
$\omega\delta t$			*sbd*, *dog*, *sick*, …	world-dependent sets of individuals
s	i, j, k, h			indices (*aka* topic-background lists)
st	I, J			infotention states
$(st)st$	K			updates

DEFINITION 1.3 (UC2 syntax). For each type $a \in \mathbf{Typ}$ the set of a-terms is defined as follows:

b	$A \in \mathbf{Term}_a$	if $A \in \mathbf{Con}_a \cup {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$
a	$\top a_n, \perp a_n \in \mathbf{Term}_{sa}$	if $a \in \mathbf{DTyp}$ & $n \in \{1, 2, \ldots\}$
{}	$B\{A\} \in \mathbf{Term}_{at}$	if $a \in \mathbf{DTyp}$ & $B \in \mathbf{Term}_{sa}$ & $A \in \mathbf{Term}_{st}$
·	$(u_a \cdot B) \in \mathbf{Term}_s$	if $a \in \mathbf{DTyp}$, $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_s$
;	$(A; B) \in \mathbf{Term}_{(st)st}$	if $A, B \in \mathbf{Term}_{(st)st}$
λ	$\lambda u_a(B) \in \mathbf{Term}_{ab}$	if $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_b$
A	$BA \in \mathbf{Term}_b$	if $B \in \mathbf{Term}_{ab}$ & $A \in \mathbf{Term}_a$
C	$\neg A, (A \rightarrow B), (A \wedge B), (A \vee B) \in \mathbf{Term}_t$	if $A, B \in \mathbf{Term}_t$
Q	$\forall u_a B, \exists u_a B \in \mathbf{Term}_t$	if $u_a \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$ & $B \in \mathbf{Term}_t$
=	$(A = B) \in \mathbf{Term}_t$	if $A, B \in \mathbf{Term}_a$

DEFINITION 2.1 (UC2 frames). A UC2 frame is a set of sets $\{D_a\}_{a \in \mathbf{Typ}}$ such that:
i. $D_t = \{1, 0\}$, D_δ, and D_ω are non-empty pairwise disjoint sets
ii. $D_s = \cup_{n,\, m \geq 0} (D^n \times D^m)$, where $D = \cup_{a \in \mathbf{DTyp}} D_a$
iii. $D_{ab} = \{\mathtt{f} \mid \varnothing \subset \mathrm{Dom}\ \mathtt{f} \subseteq D_a \;\&\; \mathrm{Ran}\ \mathtt{f} \subseteq D_b\}$

ABBREVIATIONS 2 (lists, projections & extensions). Let $D = \cup_{a \in \mathbf{DTyp}} D_a$.
- For $\mathtt{i} = \langle \mathtt{i}_1, \mathtt{i}_2 \rangle \in D^n \times D^m$, $\top\mathtt{i} = \mathtt{i}_1$ is the *topic list* & $\perp\mathtt{i} = \mathtt{i}_2$ is the *background list* of $\mathtt{i}$
- For $\mathtt{x} \in D^{n+m}$, $(\mathtt{x})_n$ is $\mathtt{x}_n$ (nth coordinate) & $(\mathtt{x})_a$ is the subsequence of type a coordinates
- For $\mathtt{d}_0, \ldots, \mathtt{d}_n \in D$: $(\mathtt{d}_0 \cdot \langle\,\rangle) = \langle \mathtt{d}_0 \rangle$ & $(\mathtt{d}_0 \cdot \langle \mathtt{d}_1, \ldots, \mathtt{d}_n \rangle) := \langle \mathtt{d}_0, \mathtt{d}_1, \ldots, \mathtt{d}_n \rangle$ ($\mathtt{d}_0$-*extensions*)

DEFINITION 2.2 (UC2-models and assignments)
- A UC2-*model* is a pair, $M = \langle \{D_a\}_{a \in \mathbf{Typ}}, [\![\cdot]\!] \rangle$, s.t. (i) $\{D_a\}_{a \in \mathbf{Typ}}$ is a standard UC2-frame, (ii) $[\![\cdot]\!]$ is an *interpretation function* that assigns to each $\alpha \in \mathbf{Con}_a$ a denotation $[\![\alpha]\!] \in D_a$.
- An *M-assignment* is a function g that assigns $g(u) \in D_a$ to each $u \in {}^{\top}\mathbf{Var}_a \cup {}^{\perp}\mathbf{Var}_a$. If $\mathtt{d} \in D_a$ then $g[u/\mathtt{d}]$ is the M-assignment s.t. (i) $g[u/\mathtt{d}](u') = g(u')$ for all $u' \neq u$, (ii) $g[u/d](u) = \mathtt{d}$.

ABBREVIATIONS 3 (Functions & sets). For $\mathtt{f} \in D_{a1\ldots an\,t}$ and $\langle \mathtt{a}_1, \ldots, \mathtt{a}_n\rangle \in D_{a1} \times \ldots \times D_{an}$:

- $\mathtt{f}(\mathtt{a}_1, \ldots, \mathtt{a}_n) := \mathtt{f}(\mathtt{a}_1)\ldots(\mathtt{a}_n)$
- ${}^{\{\}}(\mathtt{f}) = \{\langle \mathtt{a}_1,\ldots \mathtt{a}_n\rangle: \mathtt{f}(\mathtt{a}_1,\ldots \mathtt{a}_n) = 1\}$ (*set characterized* by function $\mathtt{f}$)
- ${}^{\chi}(\mathtt{A})$ (*characteristic function* of set $\mathtt{A}$) is the function $\mathtt{f}$ such that ${}^{\{\}}(\mathtt{f}) = \mathtt{A}$

DEFINITION 2.3 (UC2 semantics). The value $[\![A]\!]^g$ of a term A on a model $M = \langle\{D_a\}_{a\,\in\,\mathbf{Typ}}, [\![\cdot]\!]\rangle$ under an assignment g is defined as follows (Note: (i) we use von Neuman's definitions $0 := \{\}$ & $1 := \{\varnothing\}$, (ii) '$X \doteq Y$' means 'X is Y if Y is defined, else X is undefined', and (iii) $\mathtt{a}\mathtt{f} \doteq \mathtt{f}(\mathtt{a})$)

b	$[\![A]\!]^g$	$=$	$[\![A]\!]$	if $A \in \mathbf{Con}_a$
		$=$	$g(A)$	if $A \in {}^{\top}\mathbf{Var}_a \cup {}^{\bot}\mathbf{Var}_a$
a	$[\![\top a_n]\!]^g(\mathtt{i})$	$\doteq$	$((\top\mathtt{i})_a)_n$	for any $\mathtt{i} \in D_s$
	$[\![\bot a_n]\!]^g(\mathtt{i})$	$\doteq$	$((\bot\mathtt{i})_a)_n$	for any $\mathtt{i} \in D_s$
{}	$[\![B\{A\}]\!]^g$	$\doteq$	${}^{\chi}\{[\![B]\!]^g(\mathtt{i}) \mid \mathtt{i} \in {}^{\{\}}[\![A]\!]^g\}$	
·	$[\![(u_a \cdot B)]\!]^g$	$\doteq$	$\langle (g(u_a) \cdot \top[\![B]\!]^g), \bot[\![B]\!]^g\rangle$	for any $u_a \in {}^{\top}\mathbf{Var}_a$
		$\doteq$	$\langle \top[\![B]\!]^g, (g(u_a) \cdot \bot[\![B]\!]^g)\rangle$	for any $u_a \in {}^{\bot}\mathbf{Var}_a$
;	$\mathtt{c}[\![(A; B)]\!]^g$	$\doteq$	$\mathtt{c}[\![A]\!]^g[\![B]\!]^g$	for any $\mathtt{c} \in D_{st}$
$\boldsymbol{\lambda}$	$[\![\lambda u_a(B)]\!]^g(\mathtt{d})$	$\doteq$	$[\![B]\!]^{g[u/\mathtt{d}]}$	for any $\mathtt{d} \in D_a$
A	$[\![BA]\!]^g$	$\doteq$	$[\![B]\!]^g([\![A]\!]^g)$	
C	$[\![\neg A]\!]^g$	$\doteq$	$(1 - [\![A]\!]^g)$	
	$[\![(A \to B)]\!]^g$	$\doteq$	$(1 - ([\![A]\!]^g - [\![B]\!]^g))$	
	$[\![(A \wedge B)]\!]^g$	$\doteq$	$([\![A]\!]^g \cap [\![B]\!]^g)$	
	$[\![(A \vee B)]\!]^g$	$\doteq$	$([\![A]\!]^g \cup [\![B]\!]^g)$	
Q	$[\![\forall u_a A]\!]^g$	$=$	1	if $\forall \mathtt{d} \in D_a$: $[\![A]\!]^{g[u/\mathtt{d}]} = 1$
		$=$	0	otherwise
	$[\![\exists u_a A]\!]^g$	$=$	1	if $\exists \mathtt{d} \in D_a$: $[\![A]\!]^{g[u/\mathtt{d}]} = 1$
		$=$	0	otherwise
=	$[\![(A = B)]\!]^g$	$=$	1	if $[\![A]\!]^g, [\![B]\!]^g \in D_a$ & $[\![A]\!]^g = [\![B]\!]^g$
		$=$	0	if $[\![A]\!]^g, [\![B]\!]^g \in D_a$ & $[\![A]\!]^g \neq [\![B]\!]^g$

DEFINITION 3.1 (Initial context & default state). An *initial context* is a proposition $\mathtt{p}_0 \in D_\Omega$ s.t. ${}^{\{\}}\mathtt{p}_0 \neq \varnothing$. This induces the *default infotention state* $*\mathtt{p}_0 = {}^{\chi}\{\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle \mid \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\}$.

DEFINITION 3.2 (Topic, assertion, truth value). For a model M, initial context $\mathtt{p}_0$, and $(st)st$-term K:

i. $\top_{M,\mathtt{p}0}\mathsf{K} := \{(\top\mathtt{i})_1 \mid \forall g: \mathtt{i} \in {}^{\{\}}(*\mathtt{p}_0[\![\mathsf{K}]\!]^g)\}$ (set of *primary topics* introduced by K wrt M and $\mathtt{p}_0$)

ii. Given M and $\mathtt{p}_0$, K can be used to *assert* a proposition $\mathtt{q} \in D_\Omega$, iff $\top_{M,\mathtt{p}0}\mathsf{K} = \{\mathtt{q}\}$

iii. K is *true* in world $\mathtt{w}$ wrt M and $\mathtt{p}_0$, iff $\exists \mathtt{q} \in D_\Omega$: $\top_{M,\mathtt{p}0}\mathsf{K} = \{\mathtt{q}\}$ & $\mathtt{w} \in {}^{\{\}}\mathtt{q}$
K is *false* in world $\mathtt{w}$ wrt M and $\mathtt{p}_0$, iff $\exists \mathtt{q} \in D_\Omega$: $\top_{M,\mathtt{p}0}\mathsf{K} = \{\mathtt{q}\}$ & $\mathtt{w} \notin {}^{\{\}}\mathtt{q}$

ABBREVIATIONS 4 (Syntactic sugar)

i. $A_a \in B_{at}$	:=	BA		(set theory)
ii. $\top a, \bot a$	:=	$\top a_1, \bot a_1$	if $a \in \mathbf{DTyp}$	(dynamic terms)
$A_a{}^{\circ}$	:=	$\lambda i(A)$	if $a \in \mathbf{DTyp}$	
	:=	A	if $a \in \{sb \mid b \in \mathbf{DTyp}\}$	
iii. $(B_W\, A)$	:=	$\lambda i(B\, W^{\circ}i\, A^{\circ}i)$		(conditions)
$(A = B)^{\circ}$	:=	$\lambda i(A^{\circ}i = B^{\circ}i)$		
(C_1, C_2)	:=	$\lambda i(C_1 i \wedge C_2 i)$		
iv. $[C]$	:=	$\lambda I\lambda j(Ij \wedge Cj)$		(local drt-boxes)
$[u_1\ldots u_n]$	:=	$\lambda I\lambda j\, \exists u_1\ldots u_n\exists i(j = (u_1 \cdot \ldots(u_n \cdot i)) \wedge Ii)$		
$[u_1\ldots u_n \mid C]$	:=	$\lambda I\lambda j\, \exists u_1\ldots u_n\exists i(j = (u_1 \cdot \ldots(u_n \cdot i)) \wedge Ii \wedge Ci)$		
v. $[u_{at} \mid u = A_{sa}\{\mid\}]$	:=	$\lambda I\lambda j\, \exists u\exists i(j = (u \cdot i) \wedge Ii \wedge u = A\{I\})$		(global drt-boxes)
$[u_a \mid u \in A_{sa}\{\mid\}]$	:=	$\lambda I\lambda j\, \exists u\exists i(j = (u \cdot i) \wedge Ii \wedge u \in A\{I\})$		

Lecture 15
QUESTIONS AS ATTENTION UPDATE

1. Overview

• Speaking up focuses attention on the input CG, e.g.

$p_0 = {}^{\chi}\{w_0, w_1, w_2\}$ $\quad \langle w_0, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\langle w_1, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\langle w_2, [\![ole]\!]\rangle \notin {}^{\{\}}[\![sick]\!]$

• Specifically, speaking up focuses attention on p_0 (default Ω-topic) and the various p_0-worlds (default topic worlds). That is, it induces the following *default infotention state*:

$$*p_0 = {}^{\chi}\{\langle\langle w, p_0\rangle, \langle\,\rangle\rangle \mid w \in {}^{\{\}}p_0\} \qquad \text{D3.1}$$

$$= {}^{\chi}\{\langle\langle w_0, p_0\rangle, \langle\,\rangle\rangle \quad \langle\langle w_1, p_0\rangle, \langle\,\rangle\rangle \quad \langle\langle w_2, p_0\rangle, \langle\,\rangle\rangle\} \qquad p_0 = {}^{\chi}\{w_0, w_1, w_2\}$$

• $*p_0$ is then updated based on what is said in the speech act, e.g.:

YN be.TNS Ole sick **?** (prosody)

$*p_0[\![[w|\ w \in \top\omega\{|\}];\ [\mathbf{x}|\ (\mathbf{x} = ole)°];\ [sick_{\perp\omega}\ \top\delta];\ [p|\ p = \perp\omega\{|\}];\ [\mathbf{Q}|\ \mathbf{Q} = \perp\Omega\{|\}]]\!]^g$

$= *p_0[\![[w|\ w \in \top\omega\{|\}]]\!]^g[\![[\mathbf{x}|\ (\mathbf{x} = ole)°]]\!]^g[\![[sick_{\perp\omega}\ \top\delta]]\!]^g[\![[p|\ p = \perp\omega\{|\}]]\!]^g[\![[\mathbf{Q}|\ \mathbf{Q} = \perp\Omega\{|\}]]\!]^g$

c'_1

c'_2

c'_3

c'_4

c'_5

1. <u>From default infotention state</u> $*\mathtt{p}_0$ <u>to</u> $\mathtt{c}'_1$

FACT Q1[1]. $[w|\ w \in \top\omega\{|\}]$

$\equiv\ \lambda I \lambda j\ \exists w \exists i (j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w)$	A4.ii, v, i

FACT Q1[2]. $*\mathtt{p}_0[\![[w|\ w \in \top\omega\{|\}]]\!]^g$

$=\ {}^{\chi}\{\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$	$=: \mathtt{c}'_1$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (16):

1. $\mathtt{j} \in {}^{\{\}}(*\mathtt{p}_0[\![[w|\ w \in \top\omega\{|\}]]\!])$	
2. $\mathtt{j} \in {}^{\{\}}(*\mathtt{p}_0[\![\lambda I \lambda j\ \exists w \exists i (j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w)]\!]^g)$	FQ1[1]
3. $\mathtt{j} \in {}^{\{\}}([\![\lambda I \lambda j\ \exists w \exists i (j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w)]\!]^g(*\mathtt{p}_0))$	$\mathtt{a}\mathfrak{f} := \mathfrak{f}(\mathtt{a})$
4. $\mathtt{j} \in {}^{\{\}}[\![\lambda j\ \exists w \exists i (j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w)]\!]^{g[I/*\mathtt{p}0]}$	D2.3.λ
5. $[\![\exists w \exists i (j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w)]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}]} = 1$	A2, D2.3.λ
6. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$: $[\![j = (w \cdot i) \wedge Ii \wedge \top\omega_1\{I\}w]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]} = 1$	D2.3.Q$\exists$
7. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	D2.3.$\wedge$, D2.1
$[\![j]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]} = [\![w \cdot i]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}$	D2.3.=
$\&\ [\![I]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}([\![i]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}) = 1$	D2.3.A
$\&\ [\![\top\omega_1\{I\}]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}([\![w]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}) = 1$	D2.3.A
8. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	
$\mathtt{j} = \langle \top[\![i]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}, (g[I/*\mathtt{p}_0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}](w) \cdot \bot[\![i]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]})\rangle$	D2.3.$\cdot$, b, D2.2
$\&\ *\mathtt{p}_0(\mathtt{i}) = 1\ \&\ [\![\top\omega_1\{I\}]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}(\mathtt{v}) = 1$	D2.3.b, D2.2
9. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	
$\mathtt{j} = \langle \top\mathtt{i}, (\mathtt{v} \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} \in {}^{\{\}}(*\mathtt{p}_0)\ \&\ \mathtt{v} \in {}^{\{\}}[\![\top\omega_1\{I\}]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}$	D2.3.b, D2.2, A3
10. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	
$\mathtt{j} = \langle \top\mathtt{i}, (\mathtt{v} \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} \in {}^{\{\}}(*\mathtt{p}_0)$	
$\&\ \mathtt{v} \in \{[\![\top\omega_1]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}(\mathtt{k})|\ \mathtt{k} \in {}^{\{\}}[\![I]\!]^{g[I/*\mathtt{p}0][j/\mathtt{j}][w/\mathtt{v}][i/\mathtt{i}]}\}$	D2.3.{}, A3
11. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	
$\mathtt{j} = \langle \top\mathtt{i}, (\mathtt{v} \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} \in {}^{\{\}}(*\mathtt{p}_0)\ \&\ \mathtt{v} \in \{((\top\mathtt{k})_\omega)_1|\ \mathtt{k} \in {}^{\{\}}(*\mathtt{p}_0)\}$	D2.3.a, b, D2.2
12. $\exists \mathtt{v} \in D_\omega \exists \mathtt{i} \in D_s$:	
$\mathtt{j} = \langle \top\mathtt{i}, (\mathtt{v} \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} \in \{\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\}$	D3.1.$*\mathtt{p}_0$, A3
$\&\ \mathtt{v} \in \{((\top\mathtt{k})_\omega)_1|\ \mathtt{k} \in \{\langle\langle \mathtt{w}', \mathtt{p}_0\rangle, \langle\ \rangle\rangle|\ \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\}\}$	D3.1.$*\mathtt{p}_0$, A3
13. $\exists \mathtt{v} \in D_\omega \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0$:	elim. $\mathtt{i}$, $\mathtt{k}$
$\mathtt{j} = \langle \top\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle, (\mathtt{v} \cdot \bot\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle)\rangle\ \&\ \mathtt{v} \in \{((\top\langle\langle \mathtt{w}', \mathtt{p}_0\rangle, \langle\ \rangle\rangle)_\omega)_1|\ \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\}$	
14. $\exists \mathtt{v} \in D_\omega \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0$:	A2.$\top$, $\bot$
$\mathtt{j} = \langle\langle \mathtt{w}, \mathtt{p}_0\rangle, (\mathtt{v} \cdot \langle\ \rangle)\rangle\ \&\ \mathtt{v} \in \{((\langle \mathtt{w}', \mathtt{p}_0\rangle)_\omega)_1|\ \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\}$	
15. $\exists \mathtt{v} \in D_\omega \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle\ \&\ \mathtt{v} \in \{\mathtt{w}'|\ \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\}$	A2.$\cdot$, $(\mathtt{x})_a$, $(\mathtt{x})_n$
16. $\exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle$	$\{-\|-\}$, ${}^{\{\}}\mathtt{p}_0 \subseteq D_\omega$

FACT Q1[3]. If $p_0 = {}^{\chi}\{w_0, w_1, w_2\}$ then

$$
\begin{aligned}
&{}^{*}p_0[\![[w|\ w \in \top\omega\{|\}]]\!]^g \\
= \ & {}^{\chi}\{\langle\langle w_0, p_0\rangle, \langle w_0\rangle\rangle, \\
&\langle\langle w_0, p_0\rangle, \langle w_1\rangle\rangle, \\
&\langle\langle w_0, p_0\rangle, \langle w_2\rangle\rangle, \\
&\langle\langle w_1, p_0\rangle, \langle w_0\rangle\rangle, \\
&\langle\langle w_1, p_0\rangle, \langle w_1\rangle\rangle \\
&\langle\langle w_1, p_0\rangle, \langle w_2\rangle\rangle \\
&\langle\langle w_2, p_0\rangle, \langle w_0\rangle\rangle, \\
&\langle\langle w_2, p_0\rangle, \langle w_1\rangle\rangle, \\
&\langle\langle w_2, p_0\rangle, \langle w_2\rangle\rangle\}
\end{aligned}
$$

Homework 6

Given

FACT Q1[2]. $*\mathtt{p}_0\llbracket[w|\ w \in \top\omega\{|\}]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$ $\quad =: \mathtt{c}'_1$

FACT 1[1]. $[\mathbf{x}|\ (\mathbf{x} = ole)°]$

$\equiv \ \lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))$

FACT Q3[1]. $[sick_{\bot\omega}\ \top\delta]$

$\equiv \ \lambda I\lambda j(Ij \wedge sick\ \bot\omega_1 j\ \top\delta_1 j)$

FACT Q4[1]. $[p|\ p = \bot\omega\{|\}]$

$\equiv \ \lambda I\lambda j\ \exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})$

prove

FACT Q2[2]. $\mathtt{c}'_1\llbracket[\mathbf{x}|\ (\mathbf{x} = ole)]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle \llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$ $\quad =: \mathtt{c}'_2$

FACT Q3[2]. $\mathtt{c}'_2\llbracket[sick_{\bot\omega}\ \top\delta]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle \llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, \llbracket ole\rrbracket\rangle \in {}^{\{\}}\llbracket sick\rrbracket\}$ $\quad =: \mathtt{c}'_3$

FACT Q4[2]. $\mathtt{c}'_3\llbracket[p|\ p = \bot\omega\{|\}]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle \llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{p}_1, \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, \llbracket ole\rrbracket\rangle \in {}^{\{\}}\llbracket sick\rrbracket$

$\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}'|\ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}', \llbracket ole\rrbracket\rangle \in {}^{\{\}}\llbracket sick\rrbracket\}\}$ $\quad =: \mathtt{c}'_4$

and spell out the following infotention states as matrices (i.e. one index per row, all aligned):

FACT Q2[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$, then

$\mathtt{c}'_1\llbracket[\mathbf{x}|\ (\mathbf{x} = ole)°]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle,$

⋮

$\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle\}$

FACT Q3[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$ and ${}^{\{\}}\llbracket sick\rrbracket = \{\langle \mathtt{w}_0, \llbracket ole\rrbracket\rangle, \langle \mathtt{w}_1, \llbracket ole\rrbracket\rangle\}$, then

$\mathtt{c}'_2\llbracket[sick_{\bot\omega}\ \top\delta]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle,$

⋮

$\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle\}$

FACT Q4[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$ and ${}^{\{\}}\llbracket sick\rrbracket = \{\langle \mathtt{w}_0, \llbracket ole\rrbracket\rangle, \langle \mathtt{w}_1, \llbracket ole\rrbracket\rangle\}$, then

$\mathtt{c}'_3\llbracket[p|\ p = \bot\omega\{|\}]\rrbracket^g$

$= \ {}^{\chi}\{\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle,$

⋮

$\langle\langle$_____________$\rangle, \langle$______________$\rangle\rangle\}$

Solution to homework 6 + Update 5

• **Update 2**: from state $\mathtt{c}'_1$ to $\mathtt{c}'_2$

FACT Q2[2]. $\mathtt{c}'_1[\![[\mathbf{x}|\ (\mathbf{x} = ole)]]\!]^g$
$= {}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$ =: $\mathtt{c}'_2$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (18):

1. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_1[\![[\mathbf{x}|\ (\mathbf{x} = ole)^\circ]]\!]^g)$
2. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_1[\![\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^g)$ F1[1]
3. $\mathtt{j} \in {}^{\{\}}([\![\lambda I\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^g(\mathtt{c}'_1))$ $\mathtt{af} := \mathtt{f(a)}$
4. $\mathtt{j} \in {}^{\{\}}[\![\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^{g[I/\mathtt{c}'1]}$ D2.3.λ
5. $[\![\lambda j\ \exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^{g[I/\mathtt{c}'1]}(\mathtt{j}) = 1$ A3.${}^{\{\}}\mathtt{f}$
6. $[\![\exists\mathbf{x}\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}]} = 1$ D2.3.λ
7. $\exists\mathtt{d} \in D_\delta$: $[\![\exists i(j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole))]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}]} = 1$ D2.3.Q$\exists$
8. $\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$: $[\![j = (\mathbf{x} \cdot i) \wedge Ii \wedge (\mathbf{x} = ole)]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = 1$ D2.3.Q$\exists$
9. $\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$: D2.3.$\wedge$, D2.1
   $[\![j]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = [\![\mathbf{x} \cdot i]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}$ D2.3.=
   $\&\ [\![I]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}([\![i]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}) = 1$ D2.3.A
   $\&\ [\![\mathbf{x}]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]} = [\![ole]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}$ D2.3.=
10. $\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$:
    $\mathtt{j} = \langle(g[I/\mathtt{c}'_1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}](\mathbf{x}) \cdot \top[\![i]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}), \bot[\![i]\!]^{g[I/\mathtt{c}'1][j/\mathtt{j}][\mathbf{x}/\mathtt{d}][i/\mathtt{i}]}\rangle$ D2.3.$\cdot$, b, D2.2
    $\&\ \mathtt{c}'_1(\mathtt{i}) = 1\ \&\ \mathtt{d} = [\![ole]\!]$ D2.3.b, D2.2
11. $\exists\mathtt{d} \in D_\delta\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle(\mathtt{d} \cdot \top\mathtt{i}, \bot\mathtt{i}\rangle\ \&\ \mathtt{c}'_1(\mathtt{i}) = 1\ \&\ \mathtt{d} = [\![ole]\!]$ D2.3.b, D2.2
12. $\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![ole]\!] \cdot \top\mathtt{i}, \bot\mathtt{i}\rangle\ \&\ \mathtt{c}'_1(\mathtt{i}) = 1$ elim. $\exists\mathtt{d}$
13. $\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![ole]\!] \cdot \top\mathtt{i}, \bot\mathtt{i}\rangle\ \&\ \mathtt{i} \in {}^{\{\}}(\mathtt{c}'_1)$ A3.${}^{\chi}\mathtt{f}$
14. $\exists\mathtt{i} \in D_s$: $\mathtt{j} = \langle([\![ole]\!] \cdot \top\mathtt{i}, \bot\mathtt{i}\rangle\ \&\ \mathtt{i} \in \{\langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$ FQ1[2].$\mathtt{c}'_1$, A3
15. $\exists\mathtt{i} \in D_s\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![ole]\!] \cdot \top\mathtt{i}), \bot\mathtt{i}\rangle\ \&\ \mathtt{i} = \langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle$ df. {–|–}
16. $\exists\mathtt{i} \in D_s\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![ole]\!] \cdot \langle\mathtt{w}, \mathtt{p}_0\rangle), \langle\mathtt{v}\rangle\rangle\ \&\ \mathtt{i} = \langle\langle\mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle$ A2.$\top$, $\bot$
17. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle([\![ole]\!] \cdot \langle\mathtt{w}, \mathtt{p}_0\rangle), \langle\mathtt{v}\rangle\rangle$ elim. $\exists\mathtt{i}$
18. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle$ A2.$\cdot$

FACT Q2[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$,
$\mathtt{c}'_1[\![[\mathbf{x}|\ (\mathbf{x} = ole)^\circ]]\!]^g$
$= {}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\mathtt{w}_2\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle$
$\langle\langle[\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\mathtt{w}_2\rangle\rangle$
$\langle\langle[\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle,$
$\langle\langle[\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle\mathtt{w}_2\rangle\rangle\}$

• **Update 3**: from state $\mathtt{c}'_2$ to $\mathtt{c}'_3$

FACT Q3[2]. $\mathtt{c}'_2\llbracket[sick_{\perp\omega}\ \top\delta]\rrbracket^g$

$$= {}^{\chi}\{\langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle | \ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \ \&\ \langle\mathtt{v}, \llbracket ole\rrbracket\rangle \in {}^{\{\}}\llbracket sick\rrbracket\} \qquad =: \mathtt{c}'_3$$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (15):

1. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_2\llbracket[sick_{\perp\omega}\ \top\delta]\rrbracket^g)$
2. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_2\llbracket\lambda I\lambda j(Ij \wedge sick\ \perp\omega_1 j\ \top\delta_1 j)\rrbracket^g)$ — FQ3[1]
3. $\mathtt{j} \in {}^{\{\}}(\llbracket\lambda I\lambda j(Ij \wedge sick\ \perp\omega_1 j\ \top\delta_1 j)\rrbracket^g(\mathtt{c}'_2))$ — $\mathtt{af} := \mathtt{f}(\mathtt{a})$
4. $\mathtt{j} \in {}^{\{\}}\llbracket\lambda j(Ij \wedge sick\ \perp\omega_1 j\ \top\delta_1 j)\rrbracket^{g[I/\mathtt{c}'2]}$ — D2.3.λ
5. $\llbracket\lambda j(Ij \wedge sick\ \perp\omega_1 j\ \top\delta_1 j)\rrbracket^{g[I/\mathtt{c}'2]}(\mathtt{j}) = 1$ — A3.${}^{\{\}}\mathtt{f}$
6. $\llbracket Ij \wedge sick\ \perp\omega_1 j\ \top\delta_1 j)\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]} = 1$ — D2.3.λ
7. $\llbracket I\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\llbracket j\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}) = 1$ — D2.3.$\wedge$, A
   $\&\ \llbracket sick\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\llbracket\perp\omega_1\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\llbracket j\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}))(\llbracket\top\delta_1\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\llbracket j\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]})) = 1$
8. $\mathtt{c}'_2(\mathtt{j}) = 1$ — D2.3.*b*, D2.2
   $\&\ \llbracket sick\rrbracket(\llbracket\perp\omega_1\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\mathtt{j}))(\llbracket\top\delta_1\rrbracket^{g[I/\mathtt{c}'2][j/\mathtt{j}]}(\mathtt{j})) = 1$
9. $\mathtt{j} \in {}^{\{\}}\mathtt{c}'_2$ — A3
   $\&\ \llbracket sick\rrbracket(((\perp\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1)) = 1$ — D2.3.*a*
10. $\mathtt{j} \in \{\langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle | \ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\}$ — FQ2[2].$\mathtt{c}'_2$, A3
    $\&\ \llbracket sick\rrbracket(((\perp\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1)) = 1$
11. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — df. $\{-|-\}$
    $\mathtt{j} = \langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle \ \&\ \llbracket sick\rrbracket(((\perp\mathtt{j})_\omega)_1)(((\top\mathtt{j})_\delta)_1) = 1$
12. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — A2.$\perp\mathtt{i}$, $\top\mathtt{i}$
    $\mathtt{j} = \langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle \ \&\ \llbracket sick\rrbracket(((\langle\mathtt{v}\rangle)_\omega)_1)(((\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle)_\delta)_1) = 1$
13. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — A2.$(\mathtt{x})_a$
    $\mathtt{j} = \langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle \ \&\ \llbracket sick\rrbracket((\langle\mathtt{v}\rangle)_1)((\langle\llbracket ole\rrbracket\rangle)_1) = 1$
14. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — A2.$(\mathtt{x})_n$
    $\mathtt{j} = \langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle \ \&\ \llbracket sick\rrbracket(\mathtt{v})(\llbracket ole\rrbracket) = 1$
15. $\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — A3.${}^{\{\}}$
    $\mathtt{j} = \langle\langle\llbracket ole\rrbracket, \mathtt{w}, \mathtt{p}_0\rangle, \langle\ \rangle\rangle \ \&\ \langle\mathtt{v}, \llbracket ole\rrbracket\rangle \in {}^{\{\}}\llbracket sick\rrbracket$

FACT Q3[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$ and ${}^{\{\}}\llbracket sick\rrbracket = \{\langle\mathtt{w}_0, \llbracket ole\rrbracket\rangle, \langle\mathtt{w}_1, \llbracket ole\rrbracket\rangle\}$, then

$$\begin{aligned}
&\mathtt{c}'_2\llbracket[sick_{\perp\omega}\ \top\delta]\rrbracket^g \\
= {}^{\chi}\{&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle, \\
&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_0, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle, \\
&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle, \\
&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_1, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle \\
&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_2, \mathtt{p}_0\rangle, \langle\mathtt{w}_0\rangle\rangle, \\
&\langle\langle\llbracket ole\rrbracket, \mathtt{w}_2, \mathtt{p}_0\rangle, \langle\mathtt{w}_1\rangle\rangle\}
\end{aligned}$$

• **Update 4**: from state $\mathtt{c}'_3$ to $\mathtt{c}'_4$

FACT Q4[2]. $\mathtt{c}'_3[\![[p|\ p = \bot\omega\{|\}]]\!]^g$

$$= {}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$$
$$\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}'|\ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\} \qquad =: \mathtt{c}'_4$$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (15):

1. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_3[\![[p|\ p = \bot\omega\{|\}]\!])$
2. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_3[\![\lambda I\lambda j\ \exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})]\!]^g)$ — FQ4[1]
3. $\mathtt{j} \in {}^{\{\}}([\![\lambda I\lambda j\ \exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})]\!]^g(\mathtt{c}'_3))$ — $\mathtt{af} := \mathtt{f}(\mathtt{a})$
4. $\mathtt{j} \in {}^{\{\}}[\![\lambda j\ \exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})]\!]^{g[I/\mathtt{c}'3]}$ — D2.3.λ
5. $[\![\lambda j\ \exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})]\!]^{g[I/\mathtt{c}'3]}(\mathtt{j}) = 1$ — A3.${}^{\{\}}\mathtt{f}$
6. $[\![\exists p\exists i(j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\})]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}]} = 1$ — D2.3.λ
7. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$: $[\![j = (p \cdot i) \wedge Ii \wedge p = \bot\omega_1\{I\}]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]} = 1$ — D2.3.Q$\exists$
8. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$: — D2.3.$\wedge$, D2.1
   $[\![j]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]} = [\![p \cdot i]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}$ — D2.3.=
   $\&\ [\![I]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}([\![i]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}) = 1$ — D2.3.A
   $\&\ [\![p]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]} = [\![\bot\omega_1\{I\}]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}$ — D2.3.=
9. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$:
   $\mathtt{j} = \langle\top[\![i]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}, (g[I/\mathtt{c}'_3][j/\mathtt{j}][p/\mathtt{p}_1][i/\mathtt{i}](p) \cdot \bot[\![i]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]})\rangle$ — D2.3.$\cdot$, b, D2.2
   $\&\ \mathtt{c}'_3(\mathtt{i}) = 1\ \&\ \mathtt{p}_1 = [\![\bot\omega_1\{I\}]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}$ — D2.3.b, D2.2
10. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$:
   $\mathtt{j} = \langle\top\mathtt{i}, (\mathtt{p}_1 \cdot \bot\mathtt{i}\rangle$ — D2.3.b, D2.2
   $\&\ \mathtt{i} \in {}^{\{\}}\mathtt{c}'_3\ \&\ \mathtt{p}_1 = {}^{\chi}\{[\![\bot\omega_1]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}(\mathtt{k})|\ \mathtt{k} \in {}^{\{\}}[\![I]\!]^{g[I/\mathtt{c}'3][j/\mathtt{j}][p/\mathtt{p}1][i/\mathtt{i}]}\}$ — A3.${}^{\{\}}$, D2.3.$\{\}$
11. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$: — D2.3.$a$, b, D2.2
   $\mathtt{j} = \langle\top\mathtt{i}, (\mathtt{p}_1 \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} \in {}^{\{\}}\mathtt{c}'_3\ \&\ \mathtt{p}_1 = {}^{\chi}\{((\bot\mathtt{k})_\omega)_1|\ \mathtt{k} \in {}^{\{\}}\mathtt{c}'_3\}$
12. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s$: — FQ3[2].$\mathtt{c}'_3$, A3
   $\mathtt{j} = \langle\top\mathtt{i}, (\mathtt{p}_1 \cdot \bot\mathtt{i})\rangle$
   $\&\ \mathtt{i} \in \{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{((\bot\mathtt{k})_\omega)_1|\ \mathtt{k} \in \{\langle\langle[\![ole]\!], \mathtt{w}', \mathtt{p}_0\rangle, \langle\mathtt{v}'\rangle\rangle|$
   $\mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$
13. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{i} \in D_s\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — df. $\{-|-\}$
   $\mathtt{j} = \langle\top\mathtt{i}, (\mathtt{p}_1 \cdot \bot\mathtt{i})\rangle\ \&\ \mathtt{i} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{v}\rangle\rangle\ \&\ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{((\bot\mathtt{k})_\omega)_1|\ \exists\mathtt{w}' \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v}' \in {}^{\{\}}\mathtt{p}_0$:
   $\mathtt{k} = \langle\langle[\![ole]\!], \mathtt{w}', \mathtt{p}_0\rangle, \langle\mathtt{v}'\rangle\rangle\ \&\ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
14. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$: — A2.$\top$, $\bot$,
   $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, (\mathtt{p}_1 \cdot \langle\mathtt{v}\rangle)\rangle\ \&\ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$ — elim. $\mathtt{i}$, $\mathtt{k}$, $\mathtt{w}'$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{((\langle\mathtt{v}'\rangle)_\omega)_1|\ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
15. $\exists\mathtt{p}_1 \in D_\Omega\exists\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\exists\mathtt{v} \in {}^{\{\}}\mathtt{p}_0$:
   $\mathtt{j} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle\ \&\ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$ — A2.$\cdot$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}'|\ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$ — A2.$(\mathtt{x})_a$, $(\mathtt{x})_n$

FACT Q4[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$ and ${}^{\{\}}[\![sick]\!] = \{\langle \mathtt{w}_0, [\![ole]\!]\rangle, \langle \mathtt{w}_1, [\![ole]\!]\rangle\}$, then

$$\begin{aligned}
&\mathtt{c}'_3[\![[p|\ p = \bot\omega\{|\}]]\!]^g\\
=\ &{}^{\chi}\{\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,\\
&\langle\langle[\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle,\\
&\langle\langle[\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,\\
&\langle\langle[\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle\\
&\langle\langle[\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,\\
&\langle\langle[\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle\}
\end{aligned}$$

• **Update 5**: from state $\mathtt{c}'_4$ to $\mathtt{c}'_5$

FACT Q5[1]. $[\mathbf{Q}|\ \mathbf{Q} = \bot\Omega\{|\}]$

$\equiv\ \lambda I\lambda j\ \exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\})$ A4.v, ii

FACT Q5[2]. Given $\{\mathtt{u}|\ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\} \neq \varnothing$,

$$\begin{aligned}
&\mathtt{c}'_4[\![[\mathbf{Q}|\ \mathbf{Q} = \bot\Omega\{|\}]]\!]^g\\
=\ &{}^{\chi}\{\langle\langle{}^{\chi}\{\mathtt{p}_1\}, [\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle \mathtt{p}_1, \mathtt{v}\rangle\rangle|\\
&\quad \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\\
&\quad \&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u}|\ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\} \quad =: \mathtt{c}'_5
\end{aligned}$$

Proof: For any index $\mathtt{j} \in D_s$, (1) iff (18):

1. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_4[\![[\mathbf{Q}|\ \mathbf{Q} = \bot\Omega\{|\}])$
2. $\mathtt{j} \in {}^{\{\}}(\mathtt{c}'_4[\![\lambda I\lambda j\ \exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\})]\!]^g)$ FQ5[1]
3. $\mathtt{j} \in {}^{\{\}}([\![\lambda I\lambda j\ \exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge p = \bot\Omega_1\{I\})]\!]^g(\mathtt{c}'_4))$ $\mathtt{af} := \mathtt{f}(\mathtt{a})$
4. $\mathtt{j} \in {}^{\{\}}[\![\lambda j\ \exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\})]\!]^{g[I/\mathtt{c}'4]}$ D2.3.λ
5. $[\![\lambda j\ \exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\})]\!]^{g[I/\mathtt{c}'4]}(\mathtt{j}) = 1$ A3.${}^{\{\}}\mathtt{f}$
6. $[\![\exists\mathbf{Q}\exists i(j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\})]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}]} = 1$ D2.3.λ
7. $\exists\mathtt{Q}_1 \in D_{\Omega t}\exists\mathtt{i} \in D_s$: $[\![j = (\mathbf{Q}\cdot i) \wedge Ii \wedge \mathbf{Q} = \bot\Omega_1\{I\}]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]} = 1$ D2.3.Q$\exists$
8. $\exists\mathtt{Q}_1 \in D_{\Omega t}\exists\mathtt{i} \in D_s$: D2.3.$\wedge$, D2.1
   $[\![j]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]} = [\![\mathbf{Q}\cdot i]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}$ D2.3.=
   $\&\ [\![I]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}([\![i]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}) = 1$ D2.3.A
   $\&\ [\![\mathbf{Q}]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]} = [\![\bot\Omega_1\{I\}]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}$ D2.3.=
9. $\exists\mathtt{Q}_1 \in D_{\Omega t}\exists\mathtt{i} \in D_s$: D2.3.$\cdot$, *b*, D2.2
   $\mathtt{j} = \langle(g[I/\mathtt{c}'_4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}_1][i/\mathtt{i}](\mathbf{Q})\cdot\top[\![i]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}), \bot[\![i]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}\rangle$
   $\&\ \mathtt{c}'_4(\mathtt{i}) = 1\ \&\ \mathtt{Q}_1 = [\![\bot\Omega_1\{I\}]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}$ D2.3.*b*, D2.2
10. $\exists\mathtt{Q}_1 \in D_{\Omega t}\exists\mathtt{i} \in D_s$:
    $\mathtt{j} = \langle(\mathtt{Q}_1\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle$ D2.3.*b*, D2.2
    $\&\ \mathtt{i} \in {}^{\{\}}\mathtt{c}'_4\ \&\ \mathtt{Q}_1 = {}^{\chi}\{[\![\bot\Omega_1]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}(\mathtt{k})|\ \mathtt{k} \in {}^{\{\}}[\![I]\!]^{g[I/\mathtt{c}'4][j/\mathtt{j}][\mathbf{Q}/\mathtt{Q}1][i/\mathtt{i}]}\}$ A3.${}^{\{\}}$, D2.3.{}
11. $\exists\mathtt{Q}_1 \in D_{\Omega t}\exists\mathtt{i} \in D_s$: D2.3.*a*, *b*, D2.2
    $\mathtt{j} = \langle(\mathtt{Q}_1\cdot\top\mathtt{i}), \bot\mathtt{i}\rangle\ \&\ \mathtt{i} \in {}^{\{\}}\mathtt{c}'_4\ \&\ \mathtt{Q}_1 = {}^{\chi}\{((\bot\mathtt{k})_{\Omega})_1|\ \mathtt{k} \in {}^{\{\}}\mathtt{c}'_4\}$

12. $\exists \mathtt{Q}_1 \in D_{\Omega t} \exists \mathtt{i} \in D_s$: FQ3[2].$\mathtt{c}'_3$, A3
$\mathtt{j} = \langle(\mathtt{Q}_1 \cdot \top\mathtt{i}), \bot\mathtt{i}\rangle$
$\& \ \mathtt{i} \in \{\langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle | \ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!] \ \& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$
$\& \ \mathtt{Q}_1 = {}^{\chi}\{((\bot\mathtt{k})_\Omega)_1 | \ \mathtt{k} \in \{\langle\langle[\![ole]\!], \mathtt{w}', \mathtt{p}_0\rangle, \langle\mathtt{q}_1, \mathtt{v}'\rangle\rangle | \ \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!] \ \& \ \mathtt{q}_1 = {}^{\chi}\{\mathtt{u}' | \ \mathtt{u}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}\}$

13. $\exists \mathtt{Q}_1 \in D_{\Omega t} \exists \mathtt{i} \in D_s \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{p}_1$: df. $\{-|-\}$
$\mathtt{j} = \langle(\mathtt{Q}_1 \cdot \top\mathtt{i}), \bot\mathtt{i}\rangle \ \& \ \mathtt{i} = \langle\langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
$\& \ \mathtt{Q}_1 = {}^{\chi}\{((\bot\mathtt{k})_\Omega)_1 | \ \exists \mathtt{w}' \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{q}_1: \ \mathtt{k} = \langle\langle[\![ole]\!], \mathtt{w}', \mathtt{p}_0\rangle, \langle\mathtt{q}_1, \mathtt{v}'\rangle\rangle \ \& \ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!] \ \& \ \mathtt{q}_1 = {}^{\chi}\{\mathtt{u}' | \ \mathtt{u}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$

14. $\exists \mathtt{Q}_1 \in D_{\Omega t} \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{p}_1$: A2.$\top$, $\bot$, elim. $\mathtt{i}$, $\mathtt{k}$, $\mathtt{w}'$
$\mathtt{j} = \langle(\mathtt{Q}_1 \cdot \langle[\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle), \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
$\& \ \mathtt{Q}_1 = {}^{\chi}\{((\langle\mathtt{q}_1, \mathtt{v}'\rangle)_\Omega)_1 | \ \mathtt{v}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!] \ \& \ \mathtt{q}_1 = {}^{\chi}\{\mathtt{u}' | \ \mathtt{u}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}\}$

15. $\exists \mathtt{Q}_1 \in D_{\Omega t} \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{p}_1$:
$\mathtt{j} = \langle\langle\mathtt{Q}_1, [\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$ A2.$\cdot$
$\& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$ df. $\{-|-\}$
$\& \ \mathtt{Q}_1 = {}^{\chi}\{\mathtt{q}_1 | \ \exists \mathtt{v}'(\mathtt{v}' \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{v}', [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]) \ \& \ \mathtt{q}_1 = \mathtt{p}_1\}$ A2.$(\mathtt{x})_a$, $(\mathtt{x})_n$

16. $\exists \mathtt{Q}_1 \in D_{\Omega t} \exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{p}_1$:
$\mathtt{j} = \langle\langle\mathtt{Q}_1, [\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$
$\& \ \mathtt{Q}_1 = {}^{\chi}\{\mathtt{p}_1\}$ **Given**: ${}^{\{\}}\mathtt{p}_1 \neq \emptyset$

17. $\exists \mathtt{w} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{v} \in {}^{\{\}}\mathtt{p}_0 \exists \mathtt{p}_1$: elim. $\mathtt{Q}$
$\mathtt{j} = \langle\langle{}^{\chi}\{\mathtt{p}_1\}, [\![ole]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\mathtt{p}_1, \mathtt{v}\rangle\rangle \ \& \ \langle\mathtt{v}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]$
$\& \ \mathtt{p}_1 = {}^{\chi}\{\mathtt{u} | \ \mathtt{u} \in {}^{\{\}}\mathtt{p}_0 \ \& \ \langle\mathtt{u}, [\![ole]\!]\rangle \in {}^{\{\}}[\![sick]\!]\}$

FACT Q5[3]. If $\mathtt{p}_0 = {}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1, \mathtt{w}_2\}$ and ${}^{\{\}}[\![sick]\!] = \{\langle\mathtt{w}_0, [\![ole]\!]\rangle, \langle\mathtt{w}_1, [\![ole]\!]\rangle\}$, then

$$\mathtt{c}'_4[\![[\mathbf{Q}| \ \mathbf{Q} = \bot\Omega\{|\}]]\!]^g$$
$$= {}^{\chi}\{\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,$$
$$\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_0, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle,$$
$$\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,$$
$$\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_1, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle$$
$$\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_0\rangle\rangle,$$
$$\langle\langle{}^{\chi}\{{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}\}, [\![ole]\!], \mathtt{w}_2, \mathtt{p}_0\rangle, \langle{}^{\chi}\{\mathtt{w}_0, \mathtt{w}_1\}, \mathtt{w}_1\rangle\rangle\}$$

Lecture 16
PRIMARY TOPIC, ASSERTION, AND TRUTH

FACT 4[1]. For any model $M = \langle\{D_a\}_{a \in \mathbf{Typ}}, [\![\cdot]\!]\rangle$ and initial context $\mathtt{p}_0$:
$\mathtt{q} \in \top_{M,\,\mathtt{p}0}\,([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])$
iff ${}^{\{\}}\mathtt{q} \neq \varnothing\ \&\ \mathtt{q} = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$

Proof: For any $\mathtt{q} \in D_\Omega$, (1) iff (10)

1. $\mathtt{q} \in \top_{M,\,\mathtt{p}0}\,([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])$
2. $\mathtt{q} \in \{(\top\mathtt{i})_1|\ \forall g\colon \mathtt{i} \in {}^{\{\}}(^*\mathtt{p}_0[\![([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])]\!]^g)\}$ D3.2.⊤
3. $\mathtt{q} \in \{(\top\mathtt{i})_1|\ \forall g\colon \mathtt{i} \in {}^{\{\}}(^*\mathtt{p}_0[\![([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ]]\!]^g[\![[\mathit{sick}_{\top\omega}\ \top\delta]]\!]^g[\![[\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])]\!]^g)$ D2.3.;
4. $\mathtt{q} \in \{(\top\mathtt{i})_1|\ \forall g\colon$ F1², 2², 3²
   $\mathtt{i} \in {}^{\{\}}({}^{\chi}\{\langle\langle \mathtt{p}_1, [\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}\}$
5. $\mathtt{q} \in \{(\top\mathtt{i})_1|$ A3, elim. g
   $\mathtt{i} \in \{\langle\langle \mathtt{p}_1, [\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}\}$
6. $\exists \mathtt{i}$: {–|–}
   $\mathtt{q} = (\top\mathtt{i})_1$
   $\&\ \mathtt{i} \in \{\langle\langle \mathtt{p}_1, [\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle|\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}\}$
7. $\exists \mathtt{i} \exists \mathtt{w} \exists \mathtt{p}_1$: {–|–}
   $\mathtt{q} = (\top\mathtt{i})_1\ \&\ \mathtt{i} = \langle\langle \mathtt{p}_1, [\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle\ \&\ \mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$
8. $\exists \mathtt{i} \exists \mathtt{p}_1$: A2.⊤i, $(x)_n$
   $\mathtt{q} = \mathtt{p}_1\ \&\ \mathtt{i} = \langle\langle \mathtt{p}_1, [\![\mathit{ole}]\!], \mathtt{w}, \mathtt{p}_0\rangle, \langle\,\rangle\rangle\ \&\ \exists \mathtt{w}(\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!])$ rearrange
   $\&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$
9. $\exists \mathtt{p}_1\colon\ \mathtt{q} = \mathtt{p}_1\ \&\ {}^{\{\}}\mathtt{p}_1 \neq \varnothing\ \&\ \mathtt{p}_1 = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$ elim. i
10. ${}^{\{\}}\mathtt{q} \neq \varnothing\ \&\ \mathtt{q} = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$ elim. $\mathtt{p}_1$

FACT 4[2]. Given any model $M = \langle\{D_a\}_{a \in \mathbf{Typ}}, [\![\cdot]\!]\rangle$, and initial context $\mathtt{p}_0 \in D_\Omega$:
$([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])$ is true in $\mathtt{w}$,
iff $\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$

Proof: (1) iff (6)

1. $([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}])$ is true in $\mathtt{w}$
2. $\exists \mathtt{q} \in D_\Omega\colon \top_{M,\,\mathtt{p}0}\,([\mathbf{x}|\,(\mathbf{x} = \mathit{ole})^\circ];\ [\mathit{sick}_{\top\omega}\ \top\delta];\ [\mathbf{p}|\ \mathbf{p} = \top\omega\{|\}]) = \{\mathtt{q}\}\ \&\ \mathtt{w} \in {}^{\{\}}\mathtt{q}$ D3.2.*true*
3. $\exists \mathtt{q} \in D_\Omega\colon {}^{\{\}}\mathtt{q} \neq \varnothing\ \&\ \mathtt{q} = {}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}\ \&\ \mathtt{w} \in {}^{\{\}}\mathtt{q}$ F4¹
4. $\mathtt{w} \in {}^{\{\}}({}^{\chi}\{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\})$ elim. q
5. $\mathtt{w} \in \{\mathtt{v}|\ \mathtt{v} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{v}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]\}$ A3.$^{\{\}}$, $^{\chi}$
6. $\mathtt{w} \in {}^{\{\}}\mathtt{p}_0\ \&\ \langle \mathtt{w}, [\![\mathit{ole}]\!]\rangle \in {}^{\{\}}[\![\mathit{sick}]\!]$ {–|–}