

PLURALITIES

1. INTRODUCTION

• *Collectivity and distributivity*

- (1) a. John and Mary | *they are a nice couple.* collective VP
 b. * John *is a nice couple.*
- (2) a. John and Mary *are students.* distributive VP
 b. John *is a student.* (2a) |= (2b)
- (3) John and Mary *bought a house* ambiguous VP
 C. John and Mary *bought a house together | jointly.* C: collective rdg
 |≠ John *bought a house.*
 *John *bought a house together | jointly.*
 D. John and Mary *each bought a house.* D: distributive rdg
 |= John *bought a house.*
- (4) John and Mary invited *their_{dis} parents to their_{coll} apartment.* mixed VP

Basic ideas:

- *e*-domain structured by *part-of* relation; *pluralities* are plural entities (with proper parts).
- *collective predicates* are predicated of pluralities
- *distributive predicates* are distributed over pluralities
- *mixed predicates* involve both collective predication and distribution

• *Definiteness and number*

- *the np_{PL} ~ np₁ and np₂ | they*

- (5) In the seminar I taught last semester...
 ... *the students worked in pairs.* collective VP
 ... *the students each wrote a paper.* distributive VP

- *the np_{SG} ~ John | s/he*

- (6) Last semester...
 ... * *the professor worked in pairs.* collective VP
 ... * *the professor each wrote a paper.* distributive VP

Basic ideas:¹

- PL *np*'s are predicates of *pluralities*; SG *np*'s, predicates of *atoms* (no proper parts)
- [*the np*] denotes the *maximum* of the set of entities denoted by the *np*

¹ Let \leq be a partial order on a non-empty set A (i.e. transitive, reflexive, and anti-symmetric relation in A), $B \subseteq A$, and $x \in A$. The element x is the \leq -maximum of B , written $x = \max_{\leq} B$, iff $x \in B$ & $\forall y \in B: y \leq x$

2. TYPED LOGIC OF PLURALS (LP) (a la Sharvy 1980, Link 1987)

DEFINITION 1.1 (LP-types)

- i. $t, e \in \mathbf{Typ}$
- ii. $(ab) \in \mathbf{Typ}$ if $a, b \in \mathbf{Typ}$

DEFINITION 1.2 (Basic LP-terms)

- | | |
|---|---|
| <ul style="list-style-type: none"> i. $\mathbf{Con}_e = \{j, m, \dots\}$ $\mathbf{Con}_{(ee)} = \{fa, ma, \dots\}$ $\mathbf{Con}_{(et)} = \{house, couple, pair, \dots\}$ $\mathbf{Con}_{(e(et))} = \{buy, wrt, prnt^{of}, \dots\}$ $\mathbf{Con}_{(e(e(et)))} = \{giv^{to}, inv^{to}, \dots\}$ $\mathbf{Con}_{((et)(et))} = \{^D, *, \dots\}$ (Link 1983, 1987) \vdots | <ul style="list-style-type: none"> ii. $\mathbf{Var}_e = \{x, y, z, \dots\}$ $\mathbf{Var}_{(ee)} = \{f, \dots\}$ $\mathbf{Var}_{(et)} = \{P, \dots\}$ $\mathbf{Var}_{(e(et))} = \{R, \dots\}$ $\mathbf{Var}_{(e(e(et)))} = \{S, \dots\}$ $\mathbf{Var}_{((et)(et))} = \{M, \dots\}$ \vdots |
|---|---|

DEFINITION 1.3 (LP-syntax)

- $b. A \in \mathbf{Term}_a$ if $A \in \mathbf{Con}_a$ or $A \in \mathbf{Var}_a$
- $\lambda. \lambda u[B] \in \mathbf{Term}_{(ab)}$ if $u \in \mathbf{Var}_a$ and $B \in \mathbf{Term}_b$
- $f. B(A) \in \mathbf{Term}_a$ if $B \in \mathbf{Term}_{(ab)}$ and $A \in \mathbf{Term}_a$
- $=. (A = B) \in \mathbf{Term}_t$ if $A \in \mathbf{Term}_a$ and $B \in \mathbf{Term}_a$
- $\neg. \neg A \in \mathbf{Term}_t$ if $A \in \mathbf{Term}_t$
- $\wedge. (A \wedge B) \in \mathbf{Term}_e$ if $A \in \mathbf{Term}_t$ and $B \in \mathbf{Term}_t$
- $\oplus. (A \oplus B) \in \mathbf{Term}_e$ if $A \in \mathbf{Term}_e$ and $B \in \mathbf{Term}_e$ (Link 1987:152)
- $\sigma. \sigma u.B \in \mathbf{Term}_e$ if $u \in \mathbf{Var}_e$ and $B \in \mathbf{Term}_t$ (Link 1987:152)
- $\iota. \iota u.B \in \mathbf{Term}_e$ if $u \in \mathbf{Var}_e$ and $B \in \mathbf{Term}_t$ (Sharvy 1980)

DEFINITION 2.1 (LP-frames). An LP-frame is a family of sets $\{D_a: a \in \mathbf{Typ}\}$ such that:

- i. $D_t = \{1, 0\}$, where $1 := \{\emptyset\}$ and $0 := \emptyset$
 $D_e = \{X \mid \emptyset \subset X \subseteq \mathbf{A}\}$, for some a non-empty set \mathbf{A} (set of *proto-atoms*) disjoint from D_t
- ii. $D_{(ab)} = \{f \mid \emptyset \subset \text{Dom } f \subseteq D_a \ \& \ \text{Ran } f \subseteq D_b\}$

DEFINITION 2.2 (LP-models and assignments)

M. An LP-model is a structure $M = \langle D, \llbracket \cdot \rrbracket \rangle$ such that:

- (i) $D = \{D_a: a \in \mathbf{Typ}\}$ is an LP-frame
- (ii) $\llbracket \cdot \rrbracket$ is an interpretation function that assigns to any $A \in \mathbf{Con}_a$ a denotation $\llbracket A \rrbracket \in D_a$.
- (iii) for any $f \in D_{(et)}$, $d \in D_e$:¹
 - $\llbracket ^D \rrbracket(f)(d) = 1$ iff $\{a \in D_e: |a| = 1 \ \& \ a \subseteq d\} \subseteq \text{set } f$
 - $\llbracket * \rrbracket(f)(d) = 1$ iff $\exists X(\emptyset \subset X \subseteq \text{set } f \ \& \ d = \cup X)$
 - ($\text{set } f$ is the set characterized by f)
- g.** An *M-assignment* is a function g that assigns to any variable $u \in \mathbf{Var}_a$ a value $g(u) \in D_a$.
 If $d \in D_a$, then $g[u/d] := (g \setminus \{\langle u, g(u) \rangle\}) \cup \{\langle u, d \rangle\}$

¹ Link 1987:171: $^D\text{VP} := \lambda x \forall y [y \cdot \Pi x \rightarrow \text{VP}(y)]$, where ' $x \cdot \Pi y$ ' stands for ' x is an atomic part of y '
 Link 1983:315: $\llbracket *P \rrbracket := \{x \in E \mid \exists X \subseteq \llbracket P \rrbracket \ \& \ X \neq \emptyset \text{ such that } x = \text{sup}_i X\}$

DEFINITION 2.3 (LP-semantics)¹

In the following rules ‘ $x \doteq y$ ’ abbreviates ‘ x is y , if y is defined, otherwise x is undefined’:

- $b.$ $\llbracket A \rrbracket^g = \llbracket A \rrbracket$ for any $A \in \mathbf{Con}_a$
- $\quad = g(A)$ for any $A \in \mathbf{Var}_a$
- $\lambda.$ $\llbracket \lambda u[A] \rrbracket^g(d) \doteq \llbracket A \rrbracket^{g[u/d]}$ for any $d \in D_a$
- $f.$ $\llbracket B(A) \rrbracket^g \doteq \llbracket B \rrbracket^g(\llbracket A \rrbracket^g)$
- $=.$ $\llbracket (A = B) \rrbracket^g = |\{ \langle d, d' \rangle \in D_a^2 : d = \llbracket A \rrbracket^g \ \& \ d' = \llbracket B \rrbracket^g \ \& \ d = d' \}|$
- $\neg.$ $\llbracket \neg A \rrbracket^g \doteq (1 \setminus \llbracket A \rrbracket^g)$
- $\wedge.$ $\llbracket (A \wedge B) \rrbracket^g \doteq (\llbracket A \rrbracket^g \cap \llbracket B \rrbracket^g)$
- $\oplus.$ $\llbracket (A \oplus B) \rrbracket^g \doteq (\llbracket A \rrbracket^g \cup \llbracket B \rrbracket^g)$
- $\sigma.$ $\llbracket \sigma u.A \rrbracket^g \doteq \cup \{d \in D_e : \llbracket A \rrbracket^{g[u/d]} = 1\}$
- $\iota.$ $\llbracket \iota u.A \rrbracket^g \doteq \max_{\leq} \{d \in D_e : \llbracket A \rrbracket^{g[u/d]} = 1\}$

DEFINITION 3 (Truth)

A is *true* in $M = \langle D, [\cdot] \rangle$ under g , written $\models_{M,g} A$, iff $\llbracket A \rrbracket^g = 1$

ABBREVIATIONS 1 (connectives & quantifiers).

- \bullet $(A \vee B) := \neg(\neg A \wedge \neg B)$
- \bullet $\forall u_b A := (\lambda u_b[A] = \lambda u_b[(u = u)])$
- \bullet $(A \rightarrow B) := \neg(A \wedge \neg B)$
- \bullet $\exists u_b A := \neg \forall u_b \neg A$

FACT 1:

- \bullet $\llbracket (A \vee B) \rrbracket^g \doteq (\llbracket A \rrbracket^g \cup \llbracket B \rrbracket^g)$
- \bullet $\llbracket \forall u_b A \rrbracket^g = 1$ iff $\forall d \in D_b : \llbracket A \rrbracket^{g[u/d]} = 1$
- \bullet $\llbracket (A \rightarrow B) \rrbracket^g \doteq (1 \setminus (\llbracket A \rrbracket^g \setminus \llbracket B \rrbracket^g))$
- \bullet $\llbracket \exists u_b A \rrbracket^g = 1$ iff $\exists d \in D_b : \llbracket A \rrbracket^{g[u/d]} = 1$

ABBREVIATIONS 2 (application, plural relations & properties)

- \bullet $BA_1 \dots A_n := B(A_1, \dots, A_n) := B(A_1) \dots (A_n)$
- \bullet $(A \leq B) := \exists x(A \oplus x) = B$ *A is part of B*
- \bullet $(A \circ B) := \exists x(x \leq A \wedge x \leq B)$ *A overlaps B*
- \bullet $(A \oslash B) := \neg \exists x(x \leq A \wedge x \leq B)$ *A does not overlap B*
- \bullet $\mathbf{1} := \lambda x[\forall y(y \leq x \rightarrow y = x)]$ *is an atom*
- \bullet $\mathbf{2} := \lambda x[\exists y \exists z(x = (y \oplus z) \wedge \mathbf{1}y \wedge \mathbf{1}z \wedge (y \oslash z))]$ *consists of two atoms*
- \bullet $\mathbf{3} := \lambda x[\exists y \exists z(x = (y \oplus z) \wedge \mathbf{1}y \wedge \mathbf{2}z \wedge (y \oslash z))]$ *consists of three atoms*
- \vdots \vdots \vdots

FACT 2: For any $A, B \in \mathbf{Term}_e, d \in D_e$:

- \bullet $\llbracket (A \leq B) \rrbracket^g = |\{ \langle d, d' \rangle \in D_e^2 : d = \llbracket A \rrbracket^g \ \& \ d = \llbracket B \rrbracket^g \ \& \ d \subseteq d' \}|$
- \bullet $\llbracket (A \circ B) \rrbracket^g = |\{ \langle d, d' \rangle \in D_e^2 : d = \llbracket A \rrbracket^g \ \& \ d = \llbracket B \rrbracket^g \ \& \ d \cap d' \neq \emptyset \}|$
- \bullet $\llbracket (A \oslash B) \rrbracket^g = |\{ \langle d, d' \rangle \in D_e^2 : d = \llbracket A \rrbracket^g \ \& \ d = \llbracket B \rrbracket^g \ \& \ d \cap d' = \emptyset \}|$
- \bullet $\llbracket \mathbf{1} \rrbracket^g = \{^{\lambda} d \in D_e : |d| = 1\}$ *(λA is the characteristic function of set A)*
- \bullet $\llbracket \mathbf{2} \rrbracket^g = \{^{\lambda} d \in D_e : |d| = 2\}$
- \vdots \vdots \vdots

¹ Let $\langle A, \leq \rangle$ be a partially ordered set (i.e. $A \neq \emptyset$ & \leq is a partial order on A), $B \subseteq A$, and $x \in A$. Then:

- \bullet x is the \leq -maximum of B , written $x = \max_{\leq} B$ iff $x \in B$ & $\forall y \in B : y \leq x$
- \bullet x is the \leq -minimum of B , written $x = \min_{\leq} B$ iff $x \in B$ & $\forall y \in B : x \leq y$
- \bullet x is the \leq -supremum of B , written $x = \sup_{\leq} B$, iff $x = \min_{\leq} \{z : z \in A \ \& \ \forall y \in B : y \leq z\}$

LP-PREDICATES OF PLURALITIES

1. COLLECTIVE (P) VS. DISTRIBUTIVE (${}^D P$)

English:

- | | |
|---|-----------------|
| (1) <i>John and Mary are a nice couple.</i> | collective VP |
| (2) <i>John and Mary are students.</i> | distributive VP |
| (3) <i>John and Mary bought a house</i> | ambiguous VP |

LP-translations, truth conditions & models:

- | | | |
|----------------------------|--|-------------------------|
| • <i>John and Mary</i> | - s/iv: $\lambda P[P(j \oplus m)]$ | Link 87:157 |
| • <i>are a nice couple</i> | - iv: <i>nice.couple</i> | collective VP |
| (1) | - s: <i>nice.couple</i> ($j \oplus m$) | |
| • <i>are students</i> | - iv: ${}^D(\textit{student})$ | distributive VP |
| (2) | - s: ${}^D(\textit{student})(j \oplus m)$ | |
| • <i>bought a house</i> | - iv: $\lambda x[\exists y(\textit{house}(y) \wedge \textit{buy}(x, y))]$ | coll. rdg. (...jointly) |
| (3 [⊕]) | - s: $\exists y(\textit{house}(y) \wedge \textit{buy}(j \oplus m, y))$ | |
| <i>bought a house</i> | - iv: ${}^D(\lambda x[\exists y(\textit{house}(y) \wedge \textit{buy}(x, y))])$ | dis. rdg. (...each) |
| (3 ^D) | - s: ${}^D(\lambda x[\exists y(\textit{house}(y) \wedge \textit{buy}(x, y))])(j \oplus m)$ | |

(1) *John and Mary are a nice couple.*

$$\models_{M,g} \textit{nice.couple}(j \oplus m)$$

$$\text{iff } \llbracket \textit{nice.couple} \rrbracket (\llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1$$

e.g. true if:

$$\llbracket j \rrbracket = \{J\} \quad \llbracket \textit{nice.couple} \rrbracket (\{J, M\}) = 1$$

$$\llbracket m \rrbracket = \{M\}$$

(2) *John and Mary are students.*

$$\models_{M,g} {}^D(\textit{student})(j \oplus m)$$

$$\text{iff } \forall a \in D_e (|a| = 1 \ \& \ a \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket \rightarrow \llbracket \textit{student} \rrbracket (a) = 1)$$

e.g. true if:

$$\llbracket j \rrbracket = \{J\} \quad \llbracket \textit{student} \rrbracket (\{J\}) = 1$$

$$\llbracket m \rrbracket = \{M\} \quad \llbracket \textit{student} \rrbracket (\{M\}) = 1$$

(3) *John and Mary bought a house.*

$$\models_{M,g} \exists y(\textit{house}(y) \wedge \textit{buy}(j \oplus m, y))$$

rdg. (3[⊕])

$$\text{iff } \exists b \in D_e: \llbracket \textit{house} \rrbracket (b) = 1 \ \& \ \llbracket \textit{buy} \rrbracket (\llbracket j \rrbracket \cup \llbracket m \rrbracket, b) = 1$$

e.g. true if:

$$\llbracket j \rrbracket = \{J\} \quad \llbracket \textit{house} \rrbracket (\{H\}) = 1 \quad \llbracket \textit{buy} \rrbracket (\{J, M\}, \{H\}) = 1$$

$$\llbracket m \rrbracket = \{M\}$$

$$\models_{M,g} {}^D(\lambda x[\exists y(\textit{house}(y) \wedge \textit{buy}(x, y))])(j \oplus m)$$

rdg. (3^D)

$$\text{iff } \forall a \in D_e (|a| = 1 \ \& \ a \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket \rightarrow \exists b \in D_e: \llbracket \textit{house} \rrbracket (b) = 1 \ \& \ \llbracket \textit{buy} \rrbracket (a, b) = 1)$$

e.g. true if:

$$\llbracket j \rrbracket = \{J\} \quad \llbracket \textit{house} \rrbracket (\{H_1\}) = 1 \quad \llbracket \textit{buy} \rrbracket (\{J\}, \{H_1\}) = 1$$

$$\llbracket m \rrbracket = \{M\} \quad \llbracket \textit{house} \rrbracket (\{H_2\}) = 1 \quad \llbracket \textit{buy} \rrbracket (\{M\}, \{H_2\}) = 1$$

2. DISTRIBUTIVE (${}^D P$) VS. PLURAL ($*P$)D2.2.M.iii For any $f \in D_{(et)}$, $d \in D_e$:¹

- $\llbracket {}^D \rrbracket(f)(d) = 1$ iff $\{a \in D_e: |a| = 1 \ \& \ a \subseteq d\} \subseteq \text{f} f$
iff $\forall a \in D_e(|a| = 1 \ \& \ a \subseteq d \rightarrow f(a) = 1)$
[every atomic part of d satisfies f]
- $\llbracket * \rrbracket(f)(d) = 1$ iff $\exists X(\emptyset \subset X \subseteq \text{f} f \ \& \ d = \cup X)$
[d is a sum of parts that satisfy f]

English:

(5) *John, Mary and Sue bought a house.*

LP-translation candidates:

 $(5^D) \quad {}^D(\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m \oplus s)$

(distributive VP)

 $(5^*) \quad *(\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m \oplus s)$

(plural VP)

Truth condition of (5^D) : $\models_{M, g} {}^D(\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m \oplus s)$ iff $\forall a \in D_e(|a| = 1 \ \& \ a \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket \cup \llbracket s \rrbracket$ $\rightarrow \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(a, b) = 1)$ *true if:*

$\llbracket j \rrbracket = \{J\}$	$\llbracket \text{house} \rrbracket(\{H_1\}) = 1$	$\llbracket \text{buy} \rrbracket(\{J\}, \{H_1\}) = 1$
$\llbracket m \rrbracket = \{M\}$	$\llbracket \text{house} \rrbracket(\{H_2\}) = 1$	$\llbracket \text{buy} \rrbracket(\{M\}, \{H_2\}) = 1$
$\llbracket s \rrbracket = \{S\}$	$\llbracket \text{house} \rrbracket(\{H_3\}) = 1$	$\llbracket \text{buy} \rrbracket(\{S\}, \{H_3\}) = 1$

false if: $\llbracket j \rrbracket = \{J\} \quad \neg \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\{J\}, b) = 1$ Truth condition of (5^*) : $\models_{M, g} *(\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m \oplus s)$ iff $\exists X(\emptyset \subset X \subseteq \{a \in D_e \mid \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(a, b) = 1\}$
& $\llbracket j \rrbracket \cup \llbracket m \rrbracket \cup \llbracket s \rrbracket = \cup X)$ *true if:*

$\llbracket j \rrbracket = \{J\}$	$\neg \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\{J\}, b) = 1$
$\llbracket m \rrbracket = \{M\}$	$\llbracket \text{house} \rrbracket(\{H_1\}) = 1 \quad \llbracket \text{buy} \rrbracket(\{J, M\}, \{H_1\}) = 1$
$\llbracket s \rrbracket = \{S\}$	$\llbracket \text{house} \rrbracket(\{H_2\}) = 1 \quad \llbracket \text{buy} \rrbracket(\{J, S\}, \{H_2\}) = 1$

since $X = \{\{J, M\}, \{J, S\}\}$ satisfies the scope of $\exists X$ also *true if:*

$\llbracket j \rrbracket = \{J\}$	$\neg \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\{J\}, b) = 1$
$\llbracket m \rrbracket = \{M\}$	$\neg \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\{M\}, b) = 1$
$\llbracket s \rrbracket = \{S\}$	$\neg \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\{S\}, b) = 1$
	$\llbracket \text{house} \rrbracket(\{H\}) = 1 \quad \llbracket \text{buy} \rrbracket(\{J, M, S\}, \{H\}) = 1$

since $X = \{\{J, M, S\}\}$ satisfies the scope of $\exists X$ ¹ $\cup X := \{z \mid \exists Y: Y \in X \ \& \ z \in Y\}$

APPENDIX: DERIVATIONS

(1) *John and Mary are a nice couple.*

1. $\models_{M,g} \text{nice.couple}(j \oplus m)$

2. $\llbracket \text{nice.couple}(j \oplus m) \rrbracket^g = 1$

D3

3. $\llbracket \text{nice.couple} \rrbracket(\llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1$

D2.3: f, \oplus, b (2) *John and Mary are students.*

1. $\models_{M,g}^D (\text{student})(j \oplus m)$

2. $\llbracket^D \text{student}(j \oplus m) \rrbracket^g = 1$

D3

3. $\llbracket^D \rrbracket(\llbracket \text{student} \rrbracket)(\llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1$

D2.3: f, \oplus, b

4. $\{a \in D_e: |a| = 1 \ \& \ a \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket\} \subseteq \llbracket^D \text{student} \rrbracket$

D2.2:^D

5. $\forall a \in D_e: |a| = 1 \ \& \ a \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket \rightarrow \llbracket \text{student} \rrbracket(a) = 1$

df. $\subseteq, \{-:-\}, \llbracket^D f \rrbracket$ (3[⊕]) *John and Mary bought a house (jointly).*

1. $\models_{M,g} \exists y(\text{house}(y) \wedge \text{buy}(j \oplus m, y))$

2. $\llbracket \exists y(\text{house}(y) \wedge \text{buy}(j \oplus m, y)) \rrbracket^g = 1$

D3

3. $\exists d \in D_e: \llbracket \text{house}(y) \wedge \text{buy}(j \oplus m, y) \rrbracket^{g[y/d]} = 1$

F1: \exists

4. $\exists d \in D_e: \llbracket \text{house} \rrbracket(g[y/d](d)) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\llbracket j \rrbracket \cup \llbracket m \rrbracket), g[y/d](d)) = 1$

D2.3: \wedge, f, \oplus, b

5. $\exists d \in D_e: \llbracket \text{house} \rrbracket(d) = 1 \ \& \ \llbracket \text{buy} \rrbracket(\llbracket j \rrbracket \cup \llbracket m \rrbracket), d) = 1$

D2.2: $g[u/d]$ (3^D) *John and Mary bought a house (each).*

1. $\models_{M,g}^D (\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m)$

2. $\llbracket^D (\lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y))])(j \oplus m) \rrbracket^g = 1$

D3

3. $\llbracket^D \rrbracket(\llbracket \lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y)) \rrbracket^g \rrbracket)(\llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1$

D2.2: f, \oplus, b

4. $\{a \in D_e: |a| = 1 \ \& \ a \subseteq (\llbracket j \rrbracket \cup \llbracket m \rrbracket)\}$

$\subseteq \{a \in D_e: \llbracket \lambda x[\exists y(\text{house}(y) \wedge \text{buy}(x, y)) \rrbracket^g(a) = 1\}$

D2.2:^D

5. $\{a \in D_e: |a| = 1 \ \& \ a \subseteq (\llbracket j \rrbracket \cup \llbracket m \rrbracket)\}$

$\subseteq \{a \in D_e: \exists b \in D_e(\llbracket \text{house}(y) \wedge \text{buy}(x, y) \rrbracket^{g[x/a][y/b]} = 1)\}$

D2.3: $\lambda, F1: \exists$

6. $\{a \in D_e: |a| = 1 \ \& \ a \subseteq (\llbracket j \rrbracket \cup \llbracket m \rrbracket)\}$

$\subseteq \{a \in D_e: \exists b \in D_e(\llbracket \text{house}(y) \wedge \text{buy}(x)(y) \rrbracket^{g[x/a][y/b]} = 1)\}$

df. $f(a_1, \dots, a_n)$

7. $\{a \in D_e: |a| = 1 \ \& \ a \subseteq (\llbracket j \rrbracket \cup \llbracket m \rrbracket)\}$

$\subseteq \{a \in D_e: \exists b \in D_e(\llbracket \text{house} \rrbracket(b) = 1 \ \& \ (y) \ \& \ \llbracket \text{buy} \rrbracket(a)(b) = 1)\}$

D2.3: $\wedge, f, b, D2.2: [u/d]$

8. $\forall a \in D_e(|a| = 1 \ \& \ a \subseteq (\llbracket j \rrbracket \cup \llbracket m \rrbracket))$

$\rightarrow \exists b \in D_e: \llbracket \text{house} \rrbracket(b) = 1 \ \& \ (y) \ \& \ \llbracket \text{buy} \rrbracket(a, b) = 1)$

df. $\subseteq, \{-:-\},$
 $f(a_1, \dots, a_n)$

DEFINITENESS AND NUMBER

1. DASH OF LATTICE THEORY

Definitions

Let $\langle A, \leq \rangle$ be a (partially) ordered set (i.e. A is a non-empty set, and \leq is a transitive, reflexive, anti-symmetric relation on A). Consider any $B \subseteq A$. Then:

- i. x is a \leq -maximal element of B ,
iff $x \in B \ \& \ \neg \exists y (y \in B \ \& \ x < y)$
- ii. x is the \leq -maximum (aka the \leq -greatest element) of B , written $x = \max_{\leq} B$,
iff $x \in B \ \& \ \forall y (y \in B \rightarrow y \leq x)$
- iii. x is the \leq -minimum (aka the \leq -least element) of B , written $x = \min_{\leq} B$,
iff $x \in B \ \& \ \forall y (y \in B \rightarrow x \leq y)$
- iv. the set of \leq -upper bounds of B , written B'' (read as 'B upper'),
is $\{x \mid x \in A \ \& \ \forall y (y \in B \rightarrow y \leq x)\}$
- v. x is the \leq -supremum (aka least upper bound) of B , written $x = \sup_{\leq} B$,
iff $x = \min_{\leq} B''$

Example 1

$$A = \mathcal{P}(\{a, b, c\}) \setminus \{\emptyset\} = \underline{\hspace{15em}}$$

$$\leq = \subseteq$$

$$B_1 = \{\{b\}\}$$

$$\max_{\leq} B_1 \doteq \text{the } x \text{ s.t. } x \in B_1 \ \& \ \forall y (y \in B_1 \rightarrow y \subseteq x)$$

$$\doteq \underline{\hspace{15em}}$$

$$B_1'' = \{x \mid x \in A \ \& \ \forall y (y \in B_1 \rightarrow y \subseteq x)\}$$

$$= \underline{\hspace{15em}}$$

$$\sup_{\leq} B_1 \doteq \min_{\leq} B_1''$$

$$\doteq \underline{\hspace{15em}}$$

Example 2

$$A = \mathcal{P}(\{a, b, c\}) \setminus \{\emptyset\} = \underline{\hspace{15em}}$$

$$\leq = \subseteq$$

$$B_2 = \{\{a\}, \{b\}\}$$

$$\max_{\leq} B_2 \doteq \text{the } x \text{ s.t. } x \in B_2 \ \& \ \forall y (y \in B_2 \rightarrow y \subseteq x)$$

$$\doteq \underline{\hspace{15em}}$$

$$B_2'' = \{x \mid x \in A \ \& \ \forall y (y \in B_2 \rightarrow y \subseteq x)\}$$

$$= \underline{\hspace{15em}}$$

$$\sup_{\leq} B_2 \doteq \min_{\leq} B_2''$$

$$\doteq \underline{\hspace{15em}}$$

Example 3

$$A = \mathcal{P}(\{a, b, c\}) \setminus \{\emptyset\} = \underline{\hspace{15em}}$$

$$\leq = \subseteq$$

$$B_3 = \{\{a, b\}\}$$

$$\max_{\subseteq} B_3 \doteq \text{the } x \text{ s.t. } x \in B_3 \ \& \ \forall y (y \in B_3 \rightarrow y \subseteq x)$$

$$\doteq \underline{\hspace{15em}}$$

$$B_3'' = \{x \mid x \in A \ \& \ \forall y (y \in B_3 \rightarrow y \subseteq x)\}$$

$$= \underline{\hspace{15em}}$$

$$\sup_{\subseteq} B_3 \doteq \min_{\subseteq} B_3''$$

$$\doteq \underline{\hspace{15em}}$$

Example 4

$$A = \mathcal{P}(\{a, b, c\}) \setminus \{\emptyset\} = \underline{\hspace{15em}}$$

$$\leq = \subseteq$$

$$B_4 = \{\{a, b\}, \{b, c\}, \{a, c\}\}$$

$$\max_{\subseteq} B_4 \doteq \text{the } x \text{ s.t. } x \in B_4 \ \& \ \forall y (y \in B_4 \rightarrow y \subseteq x)$$

$$\doteq \underline{\hspace{15em}}$$

$$B_4'' = \{x \mid x \in A \ \& \ \forall y (y \in B_4 \rightarrow y \subseteq x)\}$$

$$= \underline{\hspace{15em}}$$

$$\sup_{\subseteq} B_4 \doteq \min_{\subseteq} B_4''$$

$$\doteq \underline{\hspace{15em}}$$

2. 'THE *P*' AS THE GREATEST *P* (Sharvy 1980)Singular descriptions**the boy** |– np: $\iota x. \text{boy}(x)$ $\llbracket \iota x. \text{boy}(x) \rrbracket^g$ $\doteq \max_{\subseteq}^{\exists} \llbracket \text{boy} \rrbracket$

e.g.

- if $\exists \llbracket \text{boy} \rrbracket = \{\{b\}\}$,
then $\llbracket \iota x. \text{boy}(x) \rrbracket^g$
 $\doteq \max_{\subseteq} \{\{b\}\}$
 $= \{b\}$
cf. intuition: ✓
- if $\exists \llbracket \text{boy} \rrbracket = \{\{a\}, \{b\}\}$,
then $\llbracket \iota x. \text{boy}(x) \rrbracket^g$
 $\doteq \max_{\subseteq} \{\{a\}, \{b\}\}$
 $= \text{undefined}$
cf. intuition: ✓

Plural descriptions**the boys** |– np: $\iota x. * \text{boy}(x)$ $\llbracket \iota x. * \text{boy}(x) \rrbracket^g$ $\doteq \max_{\subseteq} \{y \mid \exists X: \emptyset \subset X \subseteq \exists \llbracket \text{boy} \rrbracket \ \& \ y = \cup X\}$ $\doteq \cup^{\exists} \llbracket \text{boy} \rrbracket$

e.g.

- if $\exists \llbracket \text{boy} \rrbracket = \{\{b\}\}$,
then $\llbracket \iota x. * \text{boy}(x) \rrbracket^g$
 $\doteq \cup \{\{b\}\}$
 $= \{b\}$
cf. intuition: ✓ (see Sec. 4)
- if $\exists \llbracket \text{boy} \rrbracket = \{\{a\}, \{b\}\}$,
then $\llbracket \iota x. * \text{boy}(x) \rrbracket^g$
 $\doteq \cup \{\{a\}, \{b\}\}$
 $= \{a, b\}$
cf. intuition: ✓

the pair |– np: $\iota x. \text{pair}(x)$ $\llbracket \iota x. \text{pair}(x) \rrbracket^g$ $\doteq \max_{\subseteq}^{\exists} \llbracket \text{pair} \rrbracket$

e.g.

- if $\exists \llbracket \text{pair} \rrbracket = \{\{a, b\}\}$
then $\llbracket \iota x. \text{pair}(x) \rrbracket^g$
 $\doteq \max_{\subseteq} \{\{a, b\}\}$
 $= \{a, b\}$
cf. intuition: ✓
- if $\exists \llbracket \text{pair} \rrbracket = \{\{a, b\}, \{b, c\}, \{a, c\}\}$
then $\llbracket \iota x. \text{pair}(x) \rrbracket^g$
 $\doteq \max_{\subseteq} \{\{a, b\}, \{b, c\}, \{a, c\}\}$
 $= \text{undefined}$
cf. intuition: ✓

the pairs |– np: $\iota x. * \text{pair}(x)$ $\llbracket \iota x. * \text{pair}(x) \rrbracket^g$ $\doteq \max_{\subseteq} \{y \mid \exists X: \emptyset \subset X \subseteq \exists \llbracket \text{pair} \rrbracket \ \& \ y = \cup X\}$ $\doteq \cup^{\exists} \llbracket \text{pair} \rrbracket$

e.g.

- if $\exists \llbracket \text{pair} \rrbracket = \{\{a, b\}\}$
then $\llbracket \iota x. * \text{pair}(x) \rrbracket^g$
 $\doteq \cup \{\{a, b\}\}$
 $= \{a, b\}$
cf. intuition: ✓ (see Sec. 4)
- if $\exists \llbracket \text{pair} \rrbracket = \{\{a, b\}, \{b, c\}, \{a, c\}\}$
then $\llbracket \iota x. * \text{pair}(x) \rrbracket^g$
 $\doteq \cup \{\{a, b\}, \{b, c\}, \{a, c\}\}$
 $= \{a, b, c\}$
cf. intuition: ?/✓

3. 'THE P ' AS THE SUPREMUM OF P (Link 1983, 1987)Singular descriptions**the boy** |– np: $\alpha x. \text{boy}(x)$ $\llbracket \alpha x. \text{boy}(x) \rrbracket^g$ $\doteq \cup \{y \in D_e \mid \llbracket \text{boy} \rrbracket^{g[x/y]} = 1\}$ $\doteq \cup^{\exists} \llbracket \text{boy} \rrbracket$ $\doteq \text{sup}_{\subseteq}^{\exists} \llbracket \text{boy} \rrbracket$

e.g.

- if $\cup^{\exists} \llbracket \text{boy} \rrbracket = \{\{b\}\}$,
then $\llbracket \alpha x. \text{boy}(x) \rrbracket^g$
 $\doteq \text{sup}_{\subseteq} \{\{b\}\}$
 $= \{b\}$
cf. intuition: \checkmark
- if $\cup^{\exists} \llbracket \text{boy} \rrbracket = \{\{a\}, \{b\}\}$,
then $\llbracket \alpha x. \text{boy}(x) \rrbracket^g$
 $\doteq \text{sup}_{\subseteq} \{\{a\}, \{b\}\}$
 $= \{a, b\}$
cf. intuition: $*$

Plural descriptions**the boys** |– np: $\alpha x. * \text{boy}(x)$ $\llbracket \alpha x. * \text{boy}(x) \rrbracket^g$ $\doteq \cup \{y \mid \exists X: \emptyset \subset X \subseteq \cup^{\exists} \llbracket \text{boy} \rrbracket \ \& \ y = \cup X\}$ $\doteq \cup^{\exists} \llbracket \text{boy} \rrbracket$ $\doteq \llbracket \alpha x. \text{boy}(x) \rrbracket^g$

e.g.

- if $\cup^{\exists} \llbracket \text{boy} \rrbracket = \{\{b\}\}$,
then $\llbracket \alpha x. * \text{boy}(x) \rrbracket^g$
 $\doteq \cup \{\{b\}\}$
 $= \{b\}$
cf. intuition: \checkmark (see Sec. 4)
- if $\cup^{\exists} \llbracket \text{boy} \rrbracket = \{\{a\}, \{b\}\}$,
then $\llbracket \alpha x. * \text{boy}(x) \rrbracket^g$
 $\doteq \cup \{\{a\}, \{b\}\}$
 $= \{a, b\}$
cf. intuition: \checkmark

the pair |– np: $\alpha x. \text{pair}(x)$ $\llbracket \alpha x. \text{pair}(x) \rrbracket^g$ $\doteq \cup \{y \in D_e \mid \llbracket \text{pair} \rrbracket^{g[x/y]} = 1\}$ $\doteq \cup^{\exists} \llbracket \text{pair} \rrbracket$ $\doteq \text{sup}_{\subseteq}^{\exists} \llbracket \text{pair} \rrbracket$

e.g.

- if $\cup^{\exists} \llbracket \text{pair} \rrbracket = \{\{a, b\}\}$
then $\llbracket \alpha x. \text{pair}(x) \rrbracket^g$
 $\doteq \text{sup}_{\subseteq} \{\{a, b\}\}$
 $= \{a, b\}$
cf. intuition: \checkmark
- if $\cup^{\exists} \llbracket \text{pair} \rrbracket = \{\{a, b\}, \{b, c\}, \{a, c\}\}$
then $\llbracket \alpha x. \text{pair}(x) \rrbracket^g$
 $\doteq \text{sup}_{\subseteq} \{\{a, b\}, \{b, c\}, \{a, c\}\}$
 $= \{a, b, c\}$
cf. intuition: $*$

the pairs |– np: $\alpha x. * \text{pair}(x)$ $\llbracket \alpha x. * \text{pair}(x) \rrbracket^g$ $\doteq \cup \{y \mid \exists X: \emptyset \subset X \subseteq \cup^{\exists} \llbracket \text{pair} \rrbracket \ \& \ y = \cup X\}$ $\doteq \cup^{\exists} \llbracket \text{pair} \rrbracket$ $\doteq \llbracket \alpha x. \text{pair}(x) \rrbracket^g$

e.g.

- if $\cup^{\exists} \llbracket \text{pair} \rrbracket = \{\{a, b\}\}$
then $\llbracket \alpha x. * \text{pair}(x) \rrbracket^g$
 $\doteq \cup \{\{a, b\}\}$
 $= \{a, b\}$
cf. intuition: \checkmark (see Sec. 4)
- if $\cup^{\exists} \llbracket \text{pair} \rrbracket = \{\{a, b\}, \{b, c\}, \{a, c\}\}$
then $\llbracket \alpha x. * \text{pair}(x) \rrbracket^g$
 $\doteq \cup \{\{a, b\}, \{b, c\}, \{a, c\}\}$
 $= \{a, b, c\}$
cf. intuition: $?/\checkmark$

4. POSSESSED DEFINITES

(1) *John and Mary invited their_{dis} parents to their_{coll} apartment.*

$$1. \models_{M,g} \lambda x \uparrow^D (\lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x')]) (x')](j \oplus m)$$

∴

$$n. \forall x (x \in D_e \ \& \ |x| = 1 \ \& \ x \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket \\ \rightarrow \llbracket \text{inv}^{to} \rrbracket (x, \cup \{y \mid \llbracket \text{prnt}^{of} \rrbracket (y, x) = 1\}, \max_{z \subseteq \{z \mid \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket\}} = 1\}) = 1)$$

e.g. true if:

$$\begin{array}{lll} \llbracket j \rrbracket = \{j\} & \{y \mid \llbracket \text{prnt}^{of} \rrbracket (y, \{j\}) = 1\} = \{\{J\}, \{J'\}\} & \llbracket \text{inv}^{to} \rrbracket (\{j\}, \{J, J'\}, \{a\}) = 1 \\ \llbracket m \rrbracket = \{m\} & \{y \mid \llbracket \text{prnt}^{of} \rrbracket (y, \{m\}) = 1\} = \{\{M\}\} & \llbracket \text{inv}^{to} \rrbracket (\{m\}, \{M\}, \{a\}) = 1 \\ & \{z \mid \llbracket \text{apt}^{of} \rrbracket (z, \{j, m\}) = 1\} = \{\{a\}\} & \end{array}$$

APPENDIX: DETAILS OF (1)

(1) *John and Mary invited their_{dis} parents to their_{coll} apartment.*

1. iff 14.:

1. $\models_{M,g} \lambda x \uparrow^D (\lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x')]) (x')) (j \oplus m)$
2. $\llbracket \lambda x \uparrow^D (\lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x')]) (x')) (j \oplus m) \rrbracket^g = 1$ |=
3. $\llbracket^D (\lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x')]) (x')) \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket]} = 1$ $\lambda, \oplus, \mathbf{b}$
4. $\llbracket^D (\llbracket \lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x') \rrbracket \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket]}) (\llbracket j \rrbracket \cup \llbracket m \rrbracket)) = 1$ $\mathbf{f}, \mathbf{b}, [u/d]$
5. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \lambda x [\text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x') \rrbracket \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket]}(y) = 1)$ $^D, \subseteq, \{f$
6. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to}(x, \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y), \iota z. \text{apt}^{of}(z, x') \rrbracket \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y]} = 1)$ λ
7. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \llbracket \iota y. *(\lambda y [\text{prnt}^{of}(y', x)])(y) \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y]}, \llbracket \iota z. \text{apt}^{of}(z, x') \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y]} = 1)$ \mathbf{f}
8. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \max_{\subseteq} \{d \in D_e: \llbracket *(\lambda y [\text{prnt}^{of}(y', x)])(y) \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][y/d]} = 1\},$
 $\max_{\subseteq} \{d \in D_e: \llbracket \text{apt}^{of}(z, x') \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][z/d]} = 1\}) = 1)$ ι
9. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \max_{\subseteq} \{d \in D_e: \llbracket *(\llbracket \lambda y [\text{prnt}^{of}(y', x)] \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][y/d]}) (d) = 1\},$
 $\max_{\subseteq} \{d \in D_e: \llbracket \text{apt}^{of} \rrbracket (d, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ $\mathbf{f}, \mathbf{b}, [u/d]$
10. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \max_{\subseteq} \{d \in D_e: \exists X (\emptyset \subset X \subseteq \{ \llbracket \lambda y [\text{prnt}^{of}(y', x)] \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][y/d]} \ \& \ d = \cup X \},$
 $\max_{\subseteq} \{z: \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ $*$, rename d
11. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \cup \{d \in D_e: \llbracket \lambda y [\text{prnt}^{of}(y', x)] \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][y/d]}(d) = 1\},$
 $\max_{\subseteq} \{z: \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ \max_{\subseteq}, \cup
12. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \cup \{d: \llbracket \text{prnt}^{of}(y', x) \rrbracket^{g[x'/\llbracket j \rrbracket \cup \llbracket m \rrbracket][x/y][y/d][y'/d]} = 1\},$
 $\max_{\subseteq} \{z: \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ λ , simplify
13. $\forall y (y \in D_e \ \& \ |y| = 1 \ \& \ y \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (y, \cup \{d: \llbracket \text{prnt}^{of} \rrbracket (d, y) = 1\}, \max_{\subseteq} \{z: \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ $\mathbf{f}, \mathbf{b}, [u/d]$
14. $\forall x (x \in D_e \ \& \ |x| = 1 \ \& \ x \subseteq \llbracket j \rrbracket \cup \llbracket m \rrbracket$
 $\rightarrow \llbracket \text{inv}^{to} \rrbracket (x, \cup \{y \mid \llbracket \text{prnt}^{of} \rrbracket (y, x) = 1\}, \max_{\subseteq} \{z \mid \llbracket \text{apt}^{of} \rrbracket (z, \llbracket j \rrbracket \cup \llbracket m \rrbracket) = 1\}) = 1)$ rename y, d

NUMERALS AND QUANTIFIERS

1. NUMERALS AND PARTITIVITY

Examples:

- (1) *Two cars are heavier than one.*
(cf. *A truck is heavier than a car.*)
- (2) *Two students presented a paper.*
(cf. *A first-year student presented a paper.*)
- (3) *The two students presented a paper.*
- (4) *Two of the students presented a paper.*

Basic ideas:

- *Numerals* translate into cardinality predicates and combine with the head noun like intersective adjectives (e.g. *two tables ~ four-legged table*).
- A bare numeral+noun is associated with a contextually determined *quantificational force* (e.g. universal in (1), existential in (2)), like an indefinite (see Lewis 1975, Heim 1982)
- Partitive *of* translates into the part-of relation (\leq).

LP translations:

- *students* \vdash $*(std)$
- *two* \vdash $\lambda P \lambda x [2(x) \wedge P(x)]$
- $[\forall]$ \vdash $\lambda P \lambda P' [\forall x (P'(x) \rightarrow P(x))]$
- $[\exists]$ \vdash $\lambda P \lambda P' [\exists x (P'(x) \wedge P(x))]$
- *the* \vdash $\lambda P [\iota x. Px]$
- *of* \vdash $\lambda z \lambda x [x \leq z]$

- $[\forall]$ *two cars* \vdash $\lambda P [\forall x (2(x) \wedge *(car)(x) \rightarrow P(x))]$ $[\forall] + \text{numeral} + \text{noun}$
- *heavier than one* \vdash $\lambda x [\forall y (1(y) \wedge car(y) \rightarrow hvr(x, y))]$
- (1) \vdash $\forall x (2(x) \wedge *(car)(x) \rightarrow \forall y (1(y) \wedge car(y) \rightarrow hvr(x, y)))$

- $[\exists]$ *two std.* \vdash $\lambda P [\exists x (2(x) \wedge *(std)(x) \wedge P(x))]$ $[\exists] + \text{numeral} + \text{noun}$
- *present a paper* \vdash $*(\lambda x [\exists y (ppr(y) \wedge prs(x, y))])$ coll. or dist., $* > \exists$
- (2) \vdash $\exists x (2(x) \wedge *(std)(x) \wedge *(\lambda z [\exists y (ppr(y) \wedge prs(z, y))])(x))$

- *the two std.* \vdash $\iota x (2(x) \wedge *(std)(x))$ *the* + numeral + noun
- *present a paper* \vdash $*(\lambda x [\exists y (ppr(y) \wedge prs(x, y))])$ coll. or dist., $* > \exists$
- (3) \vdash $*(\lambda z [\exists y (ppr(y) \wedge prs(z, y))]) (\iota x. (2(x) \wedge *(std)(x)))$

- *of the students* \vdash $\lambda x [x \leq \iota z. *(std)(z)]$ partitive *of*-NP
- $[\exists]$ *two of ...* \vdash $\lambda P [\exists x (2(x) \wedge x \leq \iota z. *(std)(z) \wedge P(x))]$ $[\exists] + \text{num} + \text{of-NP}$
- *present a paper* \vdash $*(\lambda x [\exists y (ppr(y) \wedge prs(x, y))])$ coll. or dist., $* > \exists$
- (4) \vdash $\exists x (2(x) \wedge x \leq \iota z. *(std)(z) \wedge *(\lambda z [\exists y (ppr(y) \wedge prs(z, y))])(x))$

3. SOME TRUTH CONDITIONS AND MODELS

(1) $[[\forall]$ two cars] are heavier than one.

$$\models_{M,g} \forall x(\mathbf{2}(x) \wedge *(car)(x) \rightarrow \forall y(\mathbf{1}(y) \wedge car(y) \rightarrow hvr(x, y)))$$

$$\text{iff } \forall x \forall y (x, y \in D_e \ \& \ |x| = 2 \ \& \ x \in \{\cup Y: Y \subseteq {}^{\exists}[[car]]\}$$

$$\ \& \ |y| = 1 \ \& \ y \in {}^{\exists}[[car]] \rightarrow [[hvr]](x, y) = 1)$$

e.g. *true* if:

$${}^{\exists}[[car]] = \{\{a\}, \{b\}, \{c\}\}$$

$$[[hvr]](x, y) = 1 \text{ iff } \langle x, y \rangle \in \underline{\hspace{15cm}}$$

e.g. *false* if:

$${}^{\exists}[[car]] = \{\{a\}, \{b\}, \{c\}\}$$

$$[[hvr]](\underline{\hspace{2cm}}, \underline{\hspace{2cm}}) = 1$$

(2) $[[\exists]$ two students] presented a paper.

$$\models_{M,g} \exists x(\mathbf{2}(x) \wedge *(std)(x) \wedge *(\lambda z[\exists y(ppr(y) \wedge prs(z, y))])(x))$$

$$\text{iff } \exists x(x \in D_e \ \& \ |x| = 2 \ \& \ x \in \{\cup Y: Y \subseteq {}^{\exists}[[std]]\}$$

$$\ \& \ x \in \{\cup Y: Y \subseteq \{z: \exists y(y \in {}^{\exists}[[ppr]] \ \& \ [[prs]](z, y) = 1)\}\})$$

e.g. if:

$${}^{\exists}[[std]] = \{\{s_1\}, \{s_2\}, \{s_3\}\}$$

$${}^{\exists}[[ppr]] = \{\{p_1\}, \{p_2\}, \{p_3\}\}$$

$$[[prs]](x, y) = 1 \text{ iff } \langle x, y \rangle \in \{\langle \{s_1\}, \{p_1\} \rangle, \langle \{s_2\}, \{p_2\} \rangle\}$$

e.g. if:

$${}^{\exists}[[std]] = \{\{s_1\}, \{s_2\}, \{s_3\}\}$$

$${}^{\exists}[[ppr]] = \{\{p_1\}, \{p_2\}, \{p_3\}\}$$

$$[[prs]](x, y) = 1 \text{ iff } \langle x, y \rangle \in \{\langle \{s_1\}, \{p_1\} \rangle, \langle \{s_2\}, \{p_2\} \rangle, \langle \{s_3\}, \{p_3\} \rangle\}$$

e.g. if:

$${}^{\exists}[[std]] = \{\{s_1\}, \{s_2\}, \{s_3\}\}$$

$${}^{\exists}[[ppr]] = \{\{p_1\}, \{p_2\}, \{p_3\}\}$$

$$[[prs]](x, y) = 1 \text{ iff } \langle x, y \rangle \in \{\langle \{s_1, s_2\}, \{p_1\} \rangle, \langle \{s_1, s_2\}, \{p_2\} \rangle\}$$

e.g. if:

$${}^{\exists}[[std]] = \{\{s_1\}, \{s_2\}, \{s_3\}\}$$

$${}^{\exists}[[ppr]] = \{\{p_1\}, \{p_2\}, \{p_3\}\}$$

$$[[prs]](x, y) = 1 \text{ iff } \langle x, y \rangle \in \{\langle \{s_1, s_2, s_3\}, \{p_1\} \rangle, \langle \{s_1, s_2\}, \{p_1, p_3\} \rangle\}$$