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Fiber Lifetime Predictions

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ABSTRACT

A computer program is described that can analyze optical fiber fatigue data from a variety of fatigue experiments and for a variety of crack growth kinetics models and can then predict long term static fatigue behavior. The key feature of the program is its ability to analyze the statistical uncertainty in the predictions due to scatter in the data to which the models are fitted. It is shown that uncertainty in the lifetime predictions is often dominated by the uncertainty in the choice of the appropriate crack growth model. It is therefore concluded that a range of models should be considered when making lifetime predictions.

1. INTRODUCTION

The design life for optical fiber cables is often in excess of 20 years so it is not possible to conduct experiments to directly assess reliability on such time scales. In order to ensure reliability of the system it is therefore necessary to perform accelerated experiments in the laboratory and to extrapolate these results to less severe in-service conditions. In the case of mechanical reliability, accelerated testing usually involves applying large stresses to the fibers and measuring the time to failure. The maximum allowed stress that ensures survival for the design life is then estimated by extrapolating these data to lower applied stresses using an appropriate model for the mechanism that leads to failure.

The model for fatigue, or delayed failure, that is exclusively used is the sub-critical crack growth model which assumes that the fiber surface contains microcracks which grow in size under the combined influence of applied stress and environmental moisture. The applicability of this model may be questioned since firstly, it is unlikely that fiber contains sharp, stress-free cracks, whether in the pristine state or in a weaker condition, and secondly because the model does not account for observations such as strength reduction during zero stress aging and the static fatigue "knee". However, since there is no other quantitative model currently available and since reliability estimates must be made, the subcritical crack growth model must be used, though its predictions should be treated with considerable conservatism.

Having assumed the subcritical crack growth model, it is then necessary to assume a model for the crack growth kinetics. Power law crack growth is often assumed which relates the crack growth rate, \( \dot{c} \), and the applied stress intensity factor, \( K_1 \)

\[
\dot{c} = A \left( \frac{K_1}{K_{1c}} \right)^n ,
\]  

(1)

where \( n \) is the stress corrosion susceptibility parameter and \( K_{1c} \) is the critical stress intensity factor. The stress intensity factor is related to the applied stress, \( \sigma \), and the crack length, \( c \), by the Griffith relation,

\[
K_1 = \sigma Y c^{1/2} .
\]  

(2)

Once a particular loading scheme is specified, Eqs. (1) and (2) may be integrated for the crack growing from its initial length, \( c_i \),

\[
K_{1c} = \sigma_i Y c_i^{1/2} ,
\]  

(3)

where \( \sigma_i \) is the inert or initial strength, to its final length, \( c_f \),
at which point the crack grows catastrophically causing failure.

For static fatigue, the applied stress is a constant, \( \sigma = \sigma_0 \), and integration of Eqs. (1) and (2) gives the relationship between the time to failure, \( t_f \), and the applied stress

\[
t_f = \frac{2K_{IC}^2 \sigma_0^{n-2}}{AY^2(n-2) \sigma_0^n},
\]

(assuming \( c_f \neq c_t \)) which may be expressed in the form

\[
t_f = B \sigma_0^{-n} \sigma_1^{n-2}.
\]

Eq. (6) may be fitted to accelerated fatigue data to determine values for the fatigue parameters \( A \) and \( n \) and can then be used to extrapolate to in-service conditions. This might involve:

- extrapolating to lower applied stress or longer times to failure. \( t_f \) then represents the service lifetime and \( \sigma_0 \) the maximum service stress to ensure that lifetime.
- extrapolating to lower initial fiber strength, \( \sigma_0 \). The strength of short fiber lengths used in accelerated tests is higher than the practical strength of fiber which might be lower due to handling damage or manufacturing defects. If the fiber strength is proof tested, \( \sigma_1 \) is replaced by the proof stress since proof testing assures the initial or inert strength rather than the fatigue strength. Extrapolating to lower initial strength is also equivalent to extrapolating to longer lengths as the expected strength decreases with increasing specimen length due to the statistical distribution of flaw size and position.
- extrapolating to lower failure probability. Eq. (6) is often fitted to the mean or median values of the variables. The predictions therefore give the median or mean allowed service stress. However, one is usually interested in the value of the allowed stress that gives a low failure probability. Such statistical effects can be accounted for (e.g. Ref. 8).

This paper is primarily concerned with the first item; extrapolating to long times to failure since this extrapolation is fundamental to all reliability predictions. Extrapolating to lower inert strength will also be considered briefly.

When calculating lifetime estimates it is important to have some idea of the uncertainty in such estimates. For example, if it is calculated that the expected lifetime under a given stress is 25 years, what exactly does this mean? Could it mean failure is expected in 25 years but could occur between 24 and 26 years, or between 5 and 50 years or between 10 seconds and 10 centuries? Clearly, an understanding of the uncertainty is useful, if not essential, and is probably more important than the expected lifetime itself. In general, the literature on lifetime estimates ignores estimating confidence intervals, most likely because such calculations are significantly more involved than estimating the expected lifetime.

Uncertainty in lifetime estimates comes from at least two important sources. Firstly, from uncertainty in the values of the fatigue parameters found by fitting to accelerated data due to the statistical scatter in that data. Secondly, there is uncertainty in the nature of the fatigue model and, in particular, uncertainty in the model for the crack growth kinetics. For example, the power law, Eq. (1), is an empirical relation that fits data for the growth of macroscopic cracks quite well. It is widely used principally because it is readily integrable for a variety of loading conditions but it is not based on any physical model for the crack growth mechanism. Further, the power law growth equation does not incorporate the temperature dependence in a consistent way. The prefactor, \( A \), exhibits Arrhenius type temperature dependence but the activation energy calculated from that dependence has been found to be stress dependent. Therefore the stress appears in Eq. (1) both explicitly in \( K_{IC} \) but also implicitly in \( A \), yet the stress dependence of \( A \) is ignored in the analysis.

Various other models for crack growth kinetics have been proposed in the literature and four will be considered here:
\[ \dot{c} = A_1 \left[ \frac{K_1}{K_{1c}} \right]^{n_1} \]  
Model 1

\[ \dot{c} = A_2 \exp \left[ n_2 \left( \frac{K_1}{K_{1c}} \right) \right] \]  
Model 2

\[ \dot{c} = A_3 \exp \left[ n_3 \left( \frac{K_1}{K_{1c}} \right)^2 \right] \]  
Model 3

\[ \dot{c} = A_4 \left( \frac{K_1}{K_{1c}} \right) \exp \left[ n_4 \left( \frac{K_1}{K_{1c}} \right) \right] \]  
Model 4

Model 1 is the power law as in Eq. (1). Model 2 is based on simple chemical kinetics in which the stress at the crack tip modifies the effective activation energy for the bond rupture that leads to crack growth.\textsuperscript{10} Model 3 is derived from an atomistic model for crack growth,\textsuperscript{11} and model 4 has been found to closely fit some fatigue data.\textsuperscript{12} All four crack growth equations are analytically integrable under conditions of static fatigue (constant applied stress).\textsuperscript{13} Under dynamic fatigue conditions of constant stressing rate \( \sigma = \dot{\sigma}t \), only model 1 is analytically integrable:

\[ \sigma^{n_1+1} = 2 \dot{\sigma} \frac{(n_1+1)}{(n_1-2)} \left( \frac{K_{1c}}{K_1} \right)^{n_1-2} \frac{1}{A_1^{n_1^2}} \]  

Jakus et al.\textsuperscript{13} used a numerical integration technique\textsuperscript{14} to fit models 2, 3 and 4 to dynamic fatigue data. Their work showed that the different models can give quite different lifetime predictions although uncertainties in the predictions were not examined and so it could not be proved that the differences were significant. Bubel and Mathewson\textsuperscript{15} used linear regression to fit the four models to static fatigue data and showed that the differences between the crack growth models could indeed be significant and that uncertainty due to statistical scatter could be insignificant in comparison.

2. NUMERICAL ANALYSIS TECHNIQUES

This paper describes results of continuation of the work of Bubel and Mathewson.\textsuperscript{15} A computer program has been developed that, in principal, can fit any type of crack growth model to data from any type of fatigue experiment and can then predict the behavior under any other type of fatigue conditions. In particular, the program can fit the four crack growth models of Eqs. (7) to dynamic fatigue data and can then either extrapolate the dynamic fatigue data or predict and extrapolate under conditions of static fatigue. A key feature of this program is that, by careful statistical analysis, it is able to determine the statistical uncertainty in any of the predictions. The program performs calculations in several steps:

1. At the heart of the program is a procedure that can integrate the crack growth from the initial to the final crack length \( (\sigma_1 \text{ to } \sigma_f) \) as defined in Eqs. (3) and (4)) under the specified loading conditions (e.g. constant stress rate) for given fatigue parameters, \( A \) and \( n \) (the \( A_1 \) and \( n_1 \) of Eqs. (7)). The initial value problem involved in this calculation is solved using a Runge-Kutta-Fehlberg method\textsuperscript{16} which adapts the integration step length to achieve a specified level of accuracy. This method is very efficient for this problem since the crack growth equation is rapidly changing - small steps are used only when needed. Simpler methods with a constant step length are inefficient; the step length is too small in regions where the function to be integrated is shallow leading to long execution times and accumulation of rounding errors, but is too coarse in regions where the function is steep leading to significant truncation errors.

2. The values of \( A \) and \( n \) are calculated from variables \( x_1 \) and \( x_2 \) which are constructed to roughly correspond to the slope and position of the fatigue data on log-log axes. In this way the problem is cast in terms of variables that have sampling distributions that are not skewed and may be taken as the same as those for the slope and position of a straight line regression fit. Note that this does not assume that the data are straight on a log-log plot, only that they are roughly straight; this is a weak (i.e. not restrictive) assumption.

3. Best fit values for \( x_1 \) and \( x_2 \) are calculated by minimizing the sum-of-squares of the residual differences between the calculated behavior and the fatigue data. A variable metric steepest descent algorithm is used for this minimization. Convergence on secondary minima can be a problem in a general minimization procedure. However, secondary minima are unusual in sum-of-squares functions. In addition, by fitting an approximately straight line function (step
2) secondary minima are almost impossible since the sum-of-squares is nearly quadratic in the fit parameters, $x_1$ and $x_2$, and so can only have one minimum. In addition, the starting values for $x_1$ and $x_2$ are calculated from the power law model for which analytic values are available, ensuring that the starting values are very close to the best fit values. Overall these techniques provide a highly robust and accurate fitting process.

4. At this stage the variables, $x_1$ and $x_2$, are correlated in much the same way as the slope and intercept of a linear regression fit are correlated; changes in the slope produce changes in the intercept. The variable $x_2$ is replaced by a linear transformation, $ax_1 + x_2$. The value of $a$ for which the covariance of $x_1$ and $x_2$ is negligible is then found iteratively using a Newton-Raphson method. $x_1$ and $x_2$ are now orthogonal variables for which the variances can be calculated from the shape of the sum-of-squares minimum.

5. The best fit values of $A$ and $n$ are then calculated from the best fit values of $x_1$ and $x_2$. A pair of bounding values of $A$ and $n$ are also calculated by examining the effects of the uncertainties in $x_1$ and $x_2$ independently.

6. The predicted value of any function of $A$ and $n$ is then calculated from the best fit values of $A$ and $n$. In addition the variance of the predicted function value is calculated from the pairs of bounding values of $A$ and $n$; in effect the variance is calculated by summing the function changes due to the variances of the variables $x_1$ and $x_2$. This is only valid because $x_1$ and $x_2$ have been made orthogonal in step 4. The function that is predicted may be, for example, static fatigue behavior and may involve integration of the crack growth, as in step 1, for different loading conditions.

This program has been used to analyze dynamic fatigue data in order to illustrate the principles behind the analysis of prediction uncertainties.

3. EXPERIMENTAL

For the experimental study, 125μm fused silica fiber coated in a UV-curable polyurethane acrylate was used. The strength of the fiber was measured in ambient laboratory air at $22\pm 2^\circ\text{C}$ and $50\pm 10\%\text{RH}$. A two-point bending technique was used to break the fibers in which the fibers are held between two face plates that are brought together by a stepper motor until the fibers break. Multi-grooved face plates were used that could simultaneously test nine fibers, the individual fractures being detected acoustically. The strength was measured as a function of loading rate and 18 specimens were broken at each rate. The constant loading rate was achieved by continuously varying the stepper motor step rate. The stepping profile was calculated accurately so that the face plate separation was always within $\frac{1}{2}$μm of its correct position. While constant stress rate results can be directly compared to results from other techniques, such as bending, it has been shown that two-point bending under conditions of constant stressing rate and constant face-plate velocity do, however, give essentially the same results.

Specimens were pre-loaded to approximately half of their final failure stress before closing the face plates at the appropriate loading rate. This significantly reduces the overall experiment time but does not influence the results. This was confirmed by the software program described above which gives essentially the same results whether the fiber is loaded from zero load or from half the failure load. In fact, if the preload did affect the results, it can be accounted for in the numerical analysis.

The two point bending technique used in this work essentially applies a strain to the fiber by constraining its shape. The strength of the fiber, and also the constant stressing rate loading profile, both require knowledge of the elastic modulus of the fiber in order to calculate stress from strain or vice versa. Throughout this work the strain dependence of the elastic modulus of silica has been accounted for. It is assumed that the modulus, $E(\varepsilon)$, at a strain, $\varepsilon$, is given by

$$E(\varepsilon) = E_0 \left(1 + \alpha \varepsilon + \beta \varepsilon^2 \right)$$

where $E_0$ is the modulus at zero strain. Values of $E_0=72.2\text{GPa}$, $\alpha=3.2$ and $\beta=8.48$ were used which may be calculated from acoustic measurements of modulus in the range of 0 to 8% strain.
4. RESULTS

Fig. 1 shows the results of strength measurement as a function of the loading rate where the loading rate spans five decades. The fastest loading rate (1000MPa·s⁻¹) corresponds to a time to failure of a few seconds while the slowest loading rate corresponds to a time to failure of a few days. This is the largest span of loading rate that can be conveniently achieved. Faster rates require a very high speed stepper motor and detection of the fiber fracture would need very high bandwidth electronics. Slower loading rates would monopolize the equipment for inconveniently long times.

The error bars in Fig. 1 represent a 95% confidence interval on the mean strength and are not drawn when comparable to the size of the plotted point. The points showing error bars correspond to results where the two sets of nine measurements gave somewhat different values due to slight fluctuations in temperature and humidity which were not controlled. The analysis program described above weights the calculation of residuals according to the uncertainty in the mean.

The solid line shown in Fig. 1 represents the best fit of the power law crack growth model (model 1) and the dashed lines represent a 95% confidence interval on the fit. These results were obtained using the program described above, but essentially identical results are found by linear regression since the crack growth equations are readily integrable for model 1. This provides a useful check of the data analysis program. The fit to the data is quite good. Fig. 2 shows the data of Fig. 1 but now the simple exponential crack growth kinetics model (model 2) is fitted to the data. Again the fit is good although in this case the residual sum-of-squares is approximately half that of model 1. The improved fit is a result of some slight curvature apparent in the data. Model 4, for these data in particular, but also generally, gives very similar results to model 2. Fig. 3 shows the data of Fig. 1 but now with model 3 fitted. This model gives the worst fit of the four models under consideration with a residual sum-of-squares approximately 50% larger than that for model 1. However, the fit is still quite reasonable.
When several different models are fitted to experimental data one of them necessarily will give the best fit. However, this does not necessarily mean that it is the most appropriate model to use since the goodness of fit depends to some extent on the statistical scatter in the results. Therefore, any model that gives a good fit is admissible for consideration. Since the four models considered here all give good fits to the data, none can be excluded on the grounds of goodness of fit and so all should be considered when making lifetime predictions.
Fig. 4. Predictions for the static fatigue behavior deduced from the dynamic fatigue data of Fig. 1.

Fig. 4 shows the predictions for the static fatigue behavior of the fiber used in these experiments, as predicted from the dynamic fatigue results of Fig. 1. For short times to failure, high applied stress, the four models give similar predictions since this time scale corresponds to the time scale of the dynamic fatigue experiments. However, as the results are extrapolated to longer times to failure, lower applied stresses, the predictions of the models diverge. For a design life of 25 years the magnitudes of the allowed stress predicted by the different models are quite different. The order of the predictions (model 1 giving the highest, model 3 the lowest allowed stress) is not data specific; Bubel and Matthewson\textsuperscript{15} have shown that for fitting to static fatigue data this order is always preserved and have provided simple approximate relationships between the allowed stresses for the four models.

The 95% confidence intervals on the predictions of the allowed stress for 25 year survival are represented by the dashed lines in Fig. 4. For these particular data the uncertainty due to the scatter in the dynamic fatigue data is small compared to the differences between the models. Therefore, for accurate lifetime predictions it is more important to employ the correct crack growth model than to have “better” (i.e. more) data. Similar results were obtained by Bubel and Matthewson\textsuperscript{15} for fitting to static fatigue data. Interestingly, the uncertainty due to scatter is less significant for the dynamic fatigue data presented here than for typical static fatigue data. Fig. 5 (from Ref. 15) shows the extrapolations of the fits to static fatigue data of Wang and Zupko.\textsuperscript{21} While the differences between the models are more important than the uncertainty due to statistical scatter, the confidence intervals are larger than those shown in Fig. 4. This is at first sight surprising because the static fatigue data extend out to over 1 month time to failure, which approaches two orders of magnitude longer times than those used in the dynamic fatigue experiments. Therefore, the extrapolation distance is much shorter for the static fatigue data and statistical uncertainty is expected to be correspondingly smaller, but this is not the case. Examination of the data indicates that the scatter in the static fatigue data of Wang and Zupko is larger than that in the dynamic fatigue data and may be of a systematic nature. This might result if the test environment fluctuated during...
Some tests. Such fluctuations are more likely in long term tests than in short term tests. It is therefore tempting to conclude that well controlled short term tests can be a better predictor of long term behavior than longer term tests conducted under somewhat less well controlled conditions. This conclusion would be correct if the appropriate fatigue model is known, but it is clearly shown here that a range of models are possible and that this uncertainty dominates.

This latter point may be illustrated by analyzing a subset of the data of Fig. 1. Fig. 6 compares the power law (model 1) fit to all the data of Fig. 1 to the fit to only four of the points (arrowed). These four points, spanning three decades in loading rate, roughly correspond to the loading rates and number of specimens defined in a standard test procedure (although the experimental conditions do not). The fits are similar and the uncertainty in the fits are of comparable size although somewhat larger for the fit to the data subset. Fig. 7 shows the predicted static fatigue behavior for fits to both the full and subset data. The predicted allowed stresses for 25 year survival are similar and still very different from the allowed stress predicted by the other models. Therefore the choice of fatigue model is more important than obtaining more extensive data. This conclusion could be viewed in two ways. Either the data may be considered inadequate since they are unable to distinguish between the different models, or, a relatively small amount of data is required to make predictions with little statistical scatter; obtaining more extensive data (11 points spanning 5 decades instead of 4 points spanning 3 decades, for example) does not improve the situation since we are already in the region of diminishing (√N) returns.

Fig. 5. Extrapolation of the static fatigue data of Wang and Zupko. After Ref. 15.
Fig. 6. Comparison of fits of the power law crack growth model (Eq. 7a) to all the data of Fig. 1 and to a subset of the data (points arrowed).

Fig. 7. Comparison of the predicted static fatigue data for the fits to all the data and to a subset of the data as shown in Fig. 6.
5. EXTRAPOLATION TO LOWER INERT STRENGTH

The practical strength of long lengths of fiber is lower than the strength encountered in the short lengths used in accelerated testing. The defects leading to this weakness are usually associated with extrinsic defects such as handling damage or surface contamination. It is therefore important to be able to account for lower inert fiber strength, \( \sigma_i \), when making reliability estimates of practical fiber systems. The subcritical crack growth model may be used to make predictions for the behavior at lower inert strength, though strictly this is not extrapolation. Accelerated data are usually gathered for the single inert strength of pristine fiber and the predictions for lower inert strengths are entirely dependent on the subcritical crack growth model and are not based on empirically determined trends. Weak fiber has been modelled by abrading fiber and there is some indication that the fatigue parameters are somewhat different from the values for pristine material. However, abrasion is difficult to control and can not give a specified residual strength. Indentation of the fiber by a Vickers diamond pyramid has been proposed as a suitable model for weak fiber. This technique gives controlled residual strengths by applying an appropriateindentation load. Lin and Matthewson extended the existing data to lower indentation loads and hence higher residual strengths to cover the entire strength region of interest, ranging from typical proof stress levels to nearly pristine strength. Their results do not indicate any dramatic changes in the fatigue processes but none of the cited work can be considered to give, as yet, sufficient data to make empirical extrapolations on inert strength.

Fig. 8 shows how the allowed stress for 25 year survival varies with the inert strength as predicted by the subcritical crack growth model for the four crack growth kinetics models of Eqs. (7). As expected, the allowed stress falls as the inert strength (expressed as a ratio to the inert strength of the fiber used for the accelerated data) decreases. The predictions for the four models converge at low inert strength and similar results were seen by Bubel and Matthewson for the analysis of static fatigue data. Scaling considerations show that a reduction in the inert strength has a similar effect on the time to failure as increasing the applied stress. Therefore, decreasing the inert strength effectively moves the predictions nearer the conditions of the accelerated data where the models converge since they are all fitted to the data at this position. However, the implication of Fig. 8 is that the uncertainty in the allowed stress due to uncertainty in the crack growth model decreases for decreasing inert strength. This is not reasonable since the nature of the defects in the pristine fiber are intrinsic while the defects in weak fiber are extrinsic in nature and may behave quite differently and so
the predictions for weak fiber behavior must contain considerable uncertainty. Predictions for weak fiber behavior must therefore be only considered as rough indicators of behavior in the absence of supporting experimental data.

6. DISCUSSION AND CONCLUSIONS

A computer program has been developed that can analyze data from a range of fatigue experiments and can then predict the behavior under other fatigue conditions. A range of crack growth kinetics models can be incorporated. The key feature of this program is that it performs a rigorous statistical analysis of the data so that confidence intervals can be placed on the predictions. An important use for the program is fitting to dynamic fatigue data, which is experimentally convenient to obtain but requires numerical techniques to analyze, and then, using the fitted parameters, predict the lifetime under static fatigue conditions.

Analysis of both static and dynamic fatigue data indicates that the uncertainty in the nature of the appropriate crack growth kinetics model is often far more important than the statistical uncertainty due to scatter in the fitted data. Therefore it is more important to know the correct growth kinetics model than to have an extensive data base. Goodness of fit alone does not distinguish between different candidate models; while a simple exponential model fits the data presented here the best, the power law model gives the best fit to some data. Therefore, since not all models can be excluded on physical or poorness of fit grounds, a range of models must be considered when making lifetime estimates.

The simple exponential model is a prime candidate for being the appropriate crack growth kinetics model since it is based on a physical model and does give the best fit to many sets of data. In addition, Michalske et al. have shown that the exponential model predicts the correct slope for fiber fatigue from measurements on crack velocities. However, while this is evidence supporting an exponential form for the crack growth kinetics, it does not necessarily give conclusive evidence for the underlying mechanism assumed for the fatigue process; namely, the subcritical growth of sharp cracks. Whatever the underlying fatigue mechanism, it does involve chemical attack of strained silicon-oxygen bonds and so is expected to have Arrhenius type kinetics giving an exponential dependence on stress. Also, while the nature of the strength limiting defects in pristine silica is a matter of some debate, there is good evidence that the defects are not sharp cracks and so may not behave as such.

Finally, the results presented here indicate that, given the uncertainty in the crack growth kinetics, short-term dynamic fatigue experiments can sometimes be as good a predictor of long term behavior as medium-term static fatigue experiments. In general, the short term experiments are more convenient to perform.

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8. REFERENCES


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