On Optimal Bidding in Concurrent Auctions

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Abstract: Online auctions have become a popular and effective tool for Internet-based E-markets. We investigate problems and models of optimal adaptive automated bidding in an environment of concurrent online auctions, where multiple auctions for identical items are running simultaneously. We develop new models for a firm where its item valuation derives from the sale of the acquired items via their demand distribution, sale price, acquisition cost, salvage value and lost sales. We discuss possible monotonicity properties for the value function and the optimal dynamic bid strategies that can be employed in developing efficient computations.

Key–Words: Auctions, Dynamic Bidding, Simultaneous, Multi-period

1 Introduction

Online auctions have become a popular and effective tool for Internet-based E-markets. We investigate problems and models of optimal adaptive automated bidding in an environment of concurrent online auctions, where multiple auctions for identical items are running simultaneously. We develop new models for a firm where its item valuation derives from the sale of the acquired items via their demand distribution, sale price, acquisition cost, salvage value and lost sales. We discuss possible monotonicity properties for the value function and the optimal dynamic bid strategies that can be employed in developing efficient computations.

In the current paper we present two models which extends our work in [13] and [17]. In the first model the buyer has to acquire a fixed number of items. These items can be acquired either at a fixed buy-it-now price or by participating in sequence of concurrent auctions. In the second model, which is similar to [17], there are two phases. Each cycle starts with Phase 1 when items are bought in a sequence of concurrent auctions. This is followed by Phase 2 when items are sold and any remaining items are salvaged for a fixed salvage cost. In addition in the present paper we the allow the demand distribution to be continuous and the set of allowable bids to be a compact interval.

A key notion in both the models is that the probability of the buyer winning in an auction is a known function of the bid and the number of opponents present in an auction. These probabilities are in general functions of the market’s collective valuation of the items, cf. [10], [13].

Procurement auctions have become a primary way with which firms acquire goods and services. The main drivers of this phenomenon are factors such as the belief that auctions are an impartial way of setting the price for an item, and the proliferation of internet which has made conducting auctions and participating in them efficient. Few examples can be found in the following papers. (cf. [22], [24], [18], [12]). Other recent related work includes [11], [15], [13], [5], [25], [26], [6] and [21]. For a recent survey we refer to [1]. Further literature is provided in [17]. To our knowledge this is the first study of a multi-period stochastic inventory problem where replenishment is done via auctions.

The paper is organized as follows. In section 2 we present the fixed demand model. In section 3 we present the variable demand model. In section 4, we present concluding remarks.

2 Fixed Demand Model

2.1 Problem Formulation

In this model we assume that the buyer has to acquire a fixed number \(L\) of items during the current cycle. To that end, the buyer participates in a sequence of concurrent sequential auctions of identical items. There are a fixed number \(N\) of sequential bidding periods each cycle. During each bidding period, the bidder can bid in a random number of concurrent auctions. The number of concurrent auctions during each bid-
dging period is a Markov Chain. The transition probability \( c_{yy'} \) is the probability that number of auctions in the next bidding period is \( m' \) given that the number of auctions in the current bidding period was \( m \). The initial distribution of auctions at the beginning of each period is represented using \( c_y \).

Every bidder submits a sealed bid, which is the bid in all the concurrent auctions. One of the highest bidders wins the auction. If there is a tie we assume that the bidder is chosen at random from the set of highest bidders. The set of bids available for each bidder is the set \( \mathcal{A} = [a_0, a_p] \). We assume that \( a_0 = 0 \) represents the action of not bidding and that \( a_p \) i the buy-it-now price. The probability of the buyer winning in an auction is a function of the bid \( a \) and the number of concurrent auctions \( y \) and is represented by the function \( p_y(a) \).

We next present the Markov Decision Process formulation of the problem described above.

1. The state space \( \mathcal{X} \) is the set of triplets \((n, x, y)\) where \( n (1 \leq n \leq N) \) represents the number of remaining bidding periods, \( x (0 \leq x \leq L) \) represents the number of items already acquired by the buyer, and \( y \) represents the number of concurrent auctions during the current bidding period.

2. In any state \((n, x, y)\) the bidder can takes one of the actions from the set \( \mathcal{A}(n, x, y) = \{a_0, a_p\} \).

3. When an action \( a \in A(n, x, y) \) is taken in state \((n, x, y)\) the following transitions are possible.
   
   i) If \( x = L \) the only possible transition is back to the state \((n - 1, L, y')\) with probability \( c_{yy'} \).
   
   ii) If \( x < L \) depending the next state is \((n - 1, x + y, y')\) with probability \( p_y(a) c_{yy'} \) or \((n - 1, x, y')\) with probability \((1 - p_y(a)) c_{yy'} \).

4. The following costs are incurred.
   
   i) In states \((n, L, y)\) there is no cost.
   
   ii) In all other states \((n, x, y)\) a cost is incurred only if the item is won in the auction. The expected cost when action \( a \) is taken is \( a y p_y(a) \).

The MDP equations: We use the following notation. Let \( v(n, x, y) \) denote the value function in state \((n, x, y)\) and \( a_{n, x, y} \) the optimal action in the state \((n, x, y)\). Let \( w(n, x, y; a) \) be the expected future reward when action \( a \) is taken in state \((n, x, y)\) and an optimal policy is followed thereafter. Note that \( v(n, x, y) = w(n, x, y; a_{n, x, y}) \). Finally, let

\[
u(n, x, y) = \sum_{y' = 1}^{\infty} c_{yy'} v(n, y', x)
\]

and

\[
u_1(n, x, y) = \sum_{m = 1}^{\infty} c_m v(n, x, y).
\]

The MDP equations are:

\[
v(n, x, y) = \max_{a \in \mathcal{A}} \{w(n, x, y; a)\}
\]

where \( w(n, x, y; a) \)

\[
\begin{cases}
- a y p_y(a) + p_y(a) u(n - 1, x + y, y) & \text{if } x \neq L, \\
(1 - p_y(a)) u(n - 1, x, y) & \text{if } x = L,
\end{cases}
\]

2.2 Structure of the Optimal Policy

In this section we derive structural properties of the optimal bidding policy under the following assumptions.

Assumption 1. The function \( p_y(a) \) is twice differentiable in \( a \) with \( \frac{\partial p_y(a)}{\partial a} > 0 \) and \( \frac{\partial^2 p_y(a)}{\partial a^2} \leq 0 \).

Assumption 2. The function \( p_y(a) \) is a decreasing function of \( m \), i.e.

\[p_y + 1(a) \leq p_y(a)\]

Assumption 3. There exists a function \( G \) with \( \sum_{i = -\infty}^{\infty} G(i) = 1 \) such that:

\[
c_{yy'} = \begin{cases} G(y' - y) & \text{if } y' > 1, \\ \sum_{k = y - 1}^{\infty} G(k) & \text{if } y' = 1. \end{cases}
\]

The rationale behind assumption 1 is that when \( m \) is constant, the probability of winning is greater when \( a \) is larger and there are diminishing returns for increasing values of \( a \). Assumption 2 implies that for the same bid \( a \) the probability of winning is lower when there are more concurrent auctions. Assumption 3 implies that the probability of there being \( y' \) concurrent auctions in the next bidding period given that there are \( y \) auctions in the current bidding period depends only on the difference \( y' - y \).
Theorem 1. Under assumptions 1, 2, and 3 the following relations hold for all \((n, x, y), (n, x, y + 1) \in \mathcal{X}\):

\[
\frac{\partial w(n, x, y; a)}{\partial x} \geq 0, \quad (4)
\]

\[
\frac{\partial v(n, x, y)}{\partial x} \geq 0, \quad (5)
\]

\[
\frac{\partial a_{n,x,y}}{\partial x} \leq 0. \quad (6)
\]

\[
v(n, x, y) \leq v(n + 1, x, y) \quad (7)
\]

\[
a_{n,x,y} \geq a_{n+1,x,y} \quad (8)
\]

\[
v(n, x, y) \geq v(n, x, y + 1) \quad (9)
\]

\[
a_{n,x,y} \leq a_{n,x,y+1} \quad (10)
\]

In the next theorem we establish monotonicity of the optimal value function and the optimal bid with respect to the number of opposing bidders \(m\).

3 Random Demand Model

3.1 Problem Formulation

We assume there are two phases in each cycle. Phase 1 is identical to what was described in the previous model. During Phase 2, the items bought in the above auctions are sold in the firm’s market where the demand distribution \(D\) is assumed to be i.i.d. per period with a continuous probability distribution function \(f(\cdot)\) and a cumulative distribution function \(F(\cdot)\). The sales price is \(r\). Excess demand is lost with a penalty and unsold items at the end of the period have same salvage value. Let \(\delta(x)\) denote the penalty associated with \(x\) units of excess demand and let \(s\) be the unit salvage value. We assume that \(s < r\).

We next present the Markov Decision Process formulation of the problem described above.

Theorem 2. Under assumptions 1, 2, and 3 the following relations hold for all \((n, x, y), (n, x, y + 1) \in \mathcal{X}\):

\[
\frac{\partial w(n, x, y; a)}{\partial x} \geq 0, \quad (11)
\]

\[
\frac{\partial v(n, x, y)}{\partial x} \geq 0, \quad (12)
\]

\[
\frac{\partial a_{n,x,y}}{\partial x} \leq 0. \quad (13)
\]
\[ v(n, x, y) \leq v(n+1, x, y) \] (14)

\[ a_{n,x,y} \geq a_{n+1,x,y} \] (15)

\[ v(n, x, y) \geq v(n, x, y+1) \] (16)

\[ a_{n,x,y} \leq a_{n,x,y+1} \] (17)

### 4 Conclusion

In this paper we presented two models of a firm that participates in sequential bidding periods with concurrent auctions. In the first model the objective was to acquire a fixed number of items. In the second model the objective was to acquire items that were to be sold in a secondary market with a known demand distribution. In each auction the buyer may acquire a discrete amount of new inventory.

We formulated both problems as a Markov Decision Processes and established, under pertinent assumptions, that the optimal value function is a decreasing function of the number of remaining auctions, increasing function of the number of opponents and decreasing function of the inventory on hand. We also prove that the optimal bid is an increasing function of the number of remaining auctions, a decreasing function of the number of opponents and an increasing function of the inventory on hand. These properties can be used to obtain efficient computational methods. This model can be extended in several directions. Some are discussed in [17]. We note that the current model does not capture aspects of “collusion” between bidders such as the bidders placing coordinated bids in order to acquire the items at lower cost.

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**References:**


