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Ronald L. Breiger

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# The Duality of Persons and Groups\*

RONALD L. BREIGER, *Harvard University*

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## ABSTRACT

A metaphor of classical social theory concerning the "intersection" of persons within groups and of groups within the individual is translated into a set of techniques to aid in empirical analysis of the interpenetration of networks of interpersonal ties and networks of intergroup ties. These techniques are useful in the study of director interlocks, clique structures, organizations within community and national power structures, and other collectivities which share members. The "membership network analysis" suggested in this paper is compared to and contrasted with sociometric approaches and is applied to the study by Davis *et al.* (1941) of the social participation of eighteen women.

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Consider a metaphor which has often appeared in sociological literature but has remained largely unexploited in empirical work. Individuals come together (or, metaphorically, "intersect" one another) within groups, which are collectivities based on the shared interests, personal affinities, or ascribed status of members who participate regularly in collective activities. At the same time, the particular patterning of an individual's affiliations (or the "intersection" of groups within the person) defines his points of reference and (at least partially) determines his individuality.<sup>1</sup>

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<sup>1</sup> Simmel (1955) entitled one of his essays "The Intersection of Social Circles," but Reinhard Bendix changed the title in translation because "a literal translation of this phrase . . . is almost meaningless . . . Simmel often plays with geometric analogies; it has seemed advisable to me to minimize this play with words . . ." (Simmel, 1955:125). For an assertion that Simmel's original title is not at all inappropriate, see Walter's essay (1959). For a more complete explication of the "dualism" inherent in Simmel's thought, see the essays by D. Levine, Lipman, and Tenbruck in Wolff (1959). A similar metaphor was put forward in America by Charles H. Cooley (1902:148), who wrote that "A man may be regarded as the point of intersection of an indefinite number of circles representing social groups, having as many arcs passing through him as there are groups." Much later, Sorokin (1947:345) observed that "the individual has as many social egos as there are different social groups and strata with which he is connected." On the "much neglected" development of the concept of "social circle" since Simmel's writings, see Kadushin (1966).

The following discussion consists of a translation of this metaphor into a set of techniques which aid in the empirical analysis of the interpenetration of networks of persons and networks of the groups that they comprise. My usage of the term "group" is restrictive in that I consider only those groups for which membership lists are available—through published sources, reconstruction from field observation or interviews, or by any other means. Such groups include corporation boards of directors (J. Levine, 1972), organizations within a community or national power structure (Lieberson, 1971; Perrucci and Pilisuk, 1970), cliques or organizations in a high school (Bonacich, 1972; Coleman, 1961), and political factions.

Donald Levine (1959:19–22) writes that "the concept of dualism" is a key principle "underlying Simmel's social thought." Levine explicates Simmel's dualism as "the assumption . . . that the subsistence of any aspect of human life depends on the coexistence of diametrically opposed elements." My own usage of the comparable term "duality" is specified with respect to Equations 3 and 4 below.<sup>2</sup>

## THE BASIC CONCEPTION

Consider a set of individuals and a set of groups such that the value of a *tie* between any two individuals is defined as the number of groups of which they both are members. The value of a tie between any two groups is de-

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<sup>2</sup> The "directional duality principle" enunciated by Harary *et al.* (1965) is to be distinguished from my conception. The former principle consists in reversing the directionality of lines in a graph; in the method of this paper, the lines in one graph are transformed into the points of its dual graph, and vice versa.

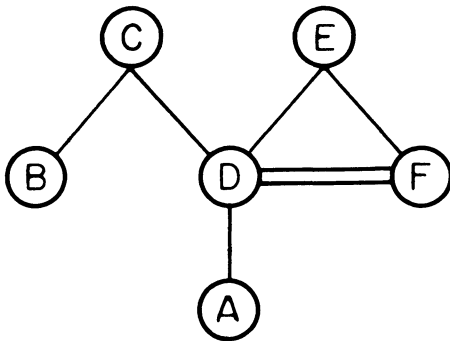
defined conversely as the number of persons who belong to both. A fictitious example is provided in Figure 1.A-1 and 1.A-2, where individuals are named by capital letters and their groups are named by integers. In concrete applications we might take U.S. Congressmen as the individuals and their committees as the groups, or schoolchildren as the individuals and their cliques as the groups, and so forth.

We may construct a matrix of interpersonal ties (denoted *P*) and a separate matrix of intergroup ties (*G*) in the usual way (Figure 1.B): let the (*i*, *j*)th entry of *P* indicate the number of groups to which both person *i* and person *j* belong, and let the (*i*, *j*)th entry of *G* indicate the number of persons who are members both of group *i* and group *j*. Each matrix is square; its row- and column-headings are identical strings of the names of all persons (in the *P*

matrix) or all groups (in the *G* matrix) under study. These matrices are mutually noncomparable in the following ways: they represent different levels of structure (persons and groups); they are not of the same dimension; and they differ in their cell-by-cell entries.

Although these differences between the interpersonal network and the intergroup network are quite evident, the *P* and *G* matrices nonetheless stand in intimate relation to one another. Following Simmel (1955:125-8, 147), think of each tie between two groups as a set of persons who form the "intersection" of the groups' memberships. In the dual case, think of each membership tie between two persons as the set of groups in the "intersection" of their individual affiliations.

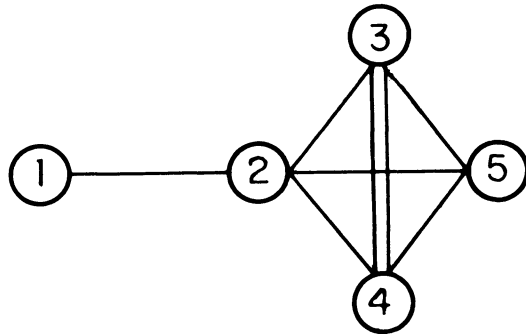
Define a binary adjacency matrix *A* (Figure 1.C) whose (*i*, *j*)th entry is "1" if person *i* is



A-1. Interpersonal network

	A	B	C	D	E	F
A	0	0	0	1	0	0
B	0	0	1	0	0	0
C	0	1	0	1	0	0
D	1	0	1	0	1	2
E	0	0	0	1	0	1
F	0	0	0	2	1	0

B-1. Matrix representation (*P*) of Figure 1. A-1



A-2. Intergroup network

	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	1	1
3	0	1	0	2	1
4	0	1	2	0	1
5	0	1	1	1	0

B-2. Matrix representation (*g*) of Figure 1. A-2

	1	2	3	4	5
A	0	0	0	0	1
B	1	0	0	0	0
C	1	1	0	0	0
D	0	1	1	1	1
E	0	0	1	0	0
F	0	0	1	1	0

C. The binary adjacency matrix (*A*) of person-to-group affiliations

Figure 1. Fictitious Data

affiliated with group  $j$ ; "0" otherwise. Where there are  $p$  persons and  $g$  groups under consideration,  $A$  has dimension  $p \times g$ , while the  $P$  and  $G$  matrices have dimension  $p \times p$  and  $g \times g$  respectively.

Notice that if we intersect any rows  $i$  and  $j$  of the  $A$  matrix (that is, lay one row atop the other, according to the value "1" only to those entries which are "1" in the same column of each row) and count the number of ones in the intersection, we discover the  $(i, j)$ th entry of the  $P$  matrix of Figure 1.B-1 (and dually for the intersection of pairs of columns of  $A$  with respect to the  $G$  matrix of Figure 1.B-2). This result is purely definitional. As will be seen below, it will be useful to formulate the definition in matrix notation.

$$P_{ij} = \sum_{k=1}^g A_{ik}A_{jk} \quad (1)$$

and similarly for ties between groups:

$$G_{ij} = \sum_{k=1}^p A_{ki}A_{kj} \quad (2)$$

The matrix  $A^T$  of group-to-person ties is equivalent to  $A$  except that its rows are interchanged or "transposed" with its columns; that is,  $A^T$  is of dimension  $g \times p$  and  $A^T_{ij} = A_{ji}$  for any  $i$  and  $j$ . Hence we may rewrite the above equations using the person-to-group "translation" matrix  $A$  and its transpose to obtain the fundamental equalities:

$$P = A(A^T) \quad (3)$$

$$G = (A^T)A \quad (4)$$

where the multiplication is ordinary (inner product) matrix multiplication. Thus: two distinct matrices, one of person-to-person relations ( $P$ ) and one of group-to-group relations ( $G$ ), are uniquely defined by and derivable from a single "translation" matrix ( $A$ ) of person-to-group affiliations.<sup>3</sup>

#### COMPARISON OF MEMBERSHIP NETS AND SOCIOMETRIC NETS

There are crucial sociological and mathematical differences between the approach of this paper and that of conventional sociometry.<sup>4</sup> An

<sup>3</sup> Notice that the products in Equations 3 and 4 differ from the  $P$  and  $G$  matrices of Figure 1 in that the former have non-zero main diagonal entries. (The main diagonal of a square matrix consists of cells [1, 1], [2, 2], and so on to [ $p$ ,  $p$ ] or [ $g$ ,  $g$ ]). Implications of this difference are discussed in the following section.

<sup>4</sup> For a review of sociometric and related methods, see Glanzer and Glaser (1959).

elaboration of both types of differences will help to clarify the nature and potential utility of each approach.

Erving Goffman (1971:188), in his discussion of "tie signs," writes that "the individual is linked to society through two principal social bonds: to collectivities through membership and to other individuals through social relationships. He in turn helps make a network of society by linking through himself the social units linked to him."

I disagree with Goffman in that I see no reason why individuals cannot be linked to other individuals by bonds of common membership (as in interlocking directorates) or to collectivities through social relationships (as in "love" of one's country or "fear" of a bureaucracy). Moreover, Goffman's focus on the individual as his unit of analysis is a one-sided departure from Simmel's insight into duality. This demurrer notwithstanding, I fundamentally agree that there are two types of social ties: membership and social-relations. Following Goffman's terminology, I will refer to my approach as "membership network analysis" in contrast to the conventional "social-relations network analysis" typified by sociometry. A similar vocabulary is hinted by the anthropologist S. F. Nadel (1957:91, 95) in his discussion of "relational roles" and "membership roles":

. . . [B]elonging to a subgroup, being involved in its regular activities and rules of behaviour, has all the characteristics of role performance. Which means that the names describing persons in terms of the subgroups they belong to are true role names. And this means, further, that these membership roles, whether explicitly named or not, correspond to *relational* roles, since the very nature of groups depends on the relationships between the people comprising them. . . . The two networks [membership and relational], in other words, can exist side by side and interpenetrate.

All sociometric approaches specify that the points or "nodes" of a graph are actors (persons or—much more rarely—collectivities) and that the lines or "ties" of the graphs are social relationships (affect, avoidance, "helping," influence, etc.). Actors and relationships are conceived as irreducible phenomena. When the relationships are those of membership, however, this conception is radically at odds with Simmel's image (Wolff, 1959:350) in which "the fact of sociation puts the individual into the dual position . . . that he is both a link in the organism of sociation and an autonomous organic whole." With respect to the membership

network, on the other hand, persons who are actors in one picture (the  $P$  matrix) are with equal legitimacy viewed as connections in the dual picture (the  $G$  matrix), and conversely for groups. Formally, we have two classes (one for all people and one for all groups under consideration) of finite sets (each person is associated with the set of groups to which he belongs, and conversely for groups) and an axiom that the intersection of any two sets belonging to either class is contained in the power set of the other class.<sup>5</sup>

A second axiom of the membership network is symmetry: if person  $a$  is connected to person  $b$  by virtue of a shared membership, then  $b$  is connected to  $a$  as well. If two groups share at least one member, they are mutually related. This implies reflexivity: a person who belongs to any group relates to himself by that fact, and similarly for any group with members.

The main diagonal of a sociomatrix consists solely of zeroes if irreflexivity has been imposed (as is usual). This represents a crucial contrast with the membership network. As Harrison White (1971:31) has stated, "whether to assign self-choices ('loops') in a generator graph . . . is a fundamental theoretical issue, not a technicality of computation, as it has often been regarded." One advantage of the membership net is the intuitively clear meaning of reflexivity: the number of ties between a person and himself is the number of groups to which he belongs (and conversely in the dual matrix: the number of ties between a group and itself is the number of members), whereas to state in a sociometric analysis that a person "esteems" or "avoids" himself (say) three times has no meaning that has been developed. Moreover, the sum of the main-diagonal entries in  $P$  always equals the corresponding sum in  $G$ , as the affiliations not only create the differences between the two networks but unify them as well. (More formally: the sum of any row in the "translation" matrix  $A$  gives the number of groups to which a particular individual belongs; hence, by Equation 3, the vector of row-marginals of  $A$  is equivalent to the main diagonal of  $P$ ; similarly, the vector of column-marginals of  $A$  is equivalent to the main diagonal of  $G$  by Equation 4; moreover, the sum of row-marginals must equal the sum of column-marginals.)

<sup>5</sup> "Power set" denotes the set of all possible sets of the given elements; e.g., the power set of a set containing three objects consists of eight sets, including the empty and universal sets.

As a further theoretical implication of reflexivity, consider the group-to-group matrix  $G$ . A lower bound on the total number of persons who belong to all groups (i.e., a lower bound on the dimension of the  $P$  matrix) is given by the largest-valued cell on the main diagonal of  $G$ . An upper bound is given by the sum of main-diagonal cells in  $G$ . (That is: if all the persons belonging to all groups are found to belong to any single group, then the lower bound is the actual number of persons; at the opposite extreme, if no groups overlap, the upper bound is the actual number of persons.) And conversely in consideration of the dual ( $P$ ) matrix.

#### AN APPLICATION OF DUAL ANALYSIS: SOCIAL PARTICIPATION IN "OLD CITY"

In empirical work we might define some minimal level of connectivity among (say) groups, excluding any group connected to at least one other by at least  $k$  links, and then examine the dual person-to-person matrix resulting from this selection. The goal is to look for patterned relations among persons; the strategy is to perform operations on the (group-to-group) matrix dual to our interest. The value of  $k$  is set according to the "graininess" or connectivity ratio (defined below) desired in the resulting matrix.

Consider the study by Davis *et al.* (1941) of the social participation of eighteen women in "Old City." The method employed in their investigation is discussed in somewhat greater detail by Homans (1950). The researchers compiled a table with eighteen rows—one for each woman—and fourteen columns, one for each "event" (such as a club meeting, a church supper, a card party, and so on), held during the course of a year, for which it could be determined that various of the women were present. The goal of the study was to determine the clique structure among the women.

At the start of the analysis the rows were arranged arbitrarily and the columns chronologically, as in the  $A$  matrix of Figure 2a which I have adapted from Homans' presentation and in which the  $(i, j)$ th entry represents the presence or absence of woman  $i$  at event  $j$ . The reader will observe that the  $A$  matrix fits precisely my conception of a "translation matrix." The researchers were aware that they could derive the woman-to-woman relations from  $A$ , but they chose not to do so. A glance at the  $P$  matrix of Figure 2b will, I believe, indicate why. The researchers were attempting to dis-

cover the clique cleavages among the women; however, connectivity in the  $P$  matrix is 91 percent.<sup>6</sup> Since everyone was connected to virtually everyone else, identification of sub-groupings became problematic. As Homans (1950:82-3) describes it:

The chart in its rough form will not reveal very much. (If you do not believe this, try making such a chart for yourself.) For one thing, the columns are probably arranged in the chronological order of events, and the women are probably in no particular order at all. But then we begin to reshuffle lines and columns. As far as columns are concerned, we put in the center the columns representing the events . . . at which a large number of the women were present, and we put toward the edges the columns representing the events . . . at which only a few of the women were present. As far as lines are concerned, we put toward the top or bottom the lines representing those women that participated most often together in social events. A great deal of reshuffling may have to be done before any pattern appears.

There can be no doubt that the researchers were operating with an implicit conception of duality, although they were uninterested in event-to-event relations. A more explicit conception might have led them to a much less time-consuming approach (particularly as no computer was available) as follows. Begin with the unpermuted  $A$  matrix of Figure 2a, even though it is in "rough form." By Equation 4, create the matrix—call it  $G$ —of membership overlaps among events (see Figure 3a). Impose the assumption that only those events which have zero overlap with at least one other event are likely to separate the women into socially meaningful subgroupings. Therefore, by inspection of the  $G$  matrix (which is dual to the matrix of our interest), note each column which contains no "zero" entry (i.e., columns 4, 6, 8, and 12) and eliminate the corresponding column in the  $A$  matrix, creating the modified translation matrix  $A2$  of woman-to-event relations. By Equation 3, create  $P2$ , the new matrix of woman-to-woman relations (Figure 3b), which may be thought of as the "skeleton structure" of the original  $P$  matrix. Inspection of  $P2$  will show that connectivity has been significantly reduced to 30 percent—but is this reduction *meaningful*? The answer is affirmative with one minor qualification: although the two cliques (of sizes seven and five, respec-

tively) that Homans describes<sup>7</sup> are contained person for person in the graph of  $P2$  (see Figure 4), each clique in the latter graph also contains one additional woman (Ruth and Verne, respectively), whom Homans (1950:84) describes as "marginal" to both cliques. ("The pattern is frayed at the edges, but there is a pattern.")<sup>8</sup>

#### DUALITY AND TRANSITIVITY

While the analysis of the previous section was predicated on knowledge of the "translation" matrix  $A$ , this section indicates that information about "reachability" in either the person-to-person or the group-to-group matrix may be derived from knowledge of its dual matrix. In the graph of person-to-person ties, two persons are mutually "reachable" along a path of length  $n$  if there exists a sequence of  $n$  contiguous ties between them (that is, if there exist  $n-1$  intermediate persons on a connected path from one person to the other). The number of person-to-person ties of length  $n$  between every two persons is given by entries of the binarized  $P$  matrix raised to the  $n$ th power (Harary *et al.*, 1965). With reference to the fictitious data of Figure 1.A-1, for example, persons  $B$  and  $D$  are connected by one 2-path ( $B-C-D$ ; this is the shortest path), but also by all  $(2 + 3k)$ -paths ( $k$  any positive integer), including two 5-paths ( $B-C-D-E-F-D$  and  $B-C-D-F-E-D$ ; these

<sup>7</sup> The clique membership reported by Homans (1950) is as follows. Clique 1: Evelyn, Laura, Theresa, Brenda, Charlotte, Frances, Eleanor. Clique 2: Myrna, Katherine, Sylvia, Nora, Helen. Women not clearly belonging to either clique: Pearl, Ruth, Verne, Dorothy, Olivia, Flora.

<sup>8</sup> As Homans (1950) notes, the analysis of Davis *et al.* follows the logic of Forsyth and Katz, which—as several authors (see Glanzer and Glaser, 1959 for a review) have observed—involves much awkward and tedious manipulation. More recent methods of clique detection (methods of Festinger and of Luce and Perry, reviewed by Glanzer and Glaser, 1959:326-28; see also Alba, 1973) are applicable only to square sociomatrices: e.g., to the  $P$  matrix of Figure 2b rather than to the rectangular  $A$  matrix of Figure 2a. Since connectivity in the  $P$  matrix approaches unity (91 percent), the problem for clique detection is the reduction of connectivity—hence the concern for operations on the (group-to-group) matrix *dual* to the sociomatrix, rather than with powers of the latter. A new algorithm (Breiger *et al.*, 1974) for detecting structure in multiple relational matrices combines this duality approach with the blocking and structural equivalence concepts of Harrison White (1974; White and Breiger, 1974) and has yielded highly interpretable results on this data and on various other social network data.

<sup>6</sup> That is, of the  $153 = (1/2)(18)(17)$  possible binary ties among the eighteen women, we find by inspection of  $P$  that 139 ties actually existed. The ratio  $139/153$  is 0.91.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Eleanor	0	1	0	1	0	0	0	1	0	0	0	1	0	0
Brenda	0	1	0	1	0	0	1	1	0	1	0	1	1	0
Dorothy	0	0	0	0	0	1	0	0	0	0	0	1	0	0
Verne	0	0	0	1	1	1	0	0	0	0	0	1	0	0
Flora	1	0	0	0	0	1	0	0	0	0	0	0	0	0
Olivia	1	0	0	0	0	1	0	0	0	0	0	0	0	0
Laura	0	1	1	1	0	0	1	1	0	1	0	1	0	0
Evelyn	0	1	1	0	0	1	1	1	0	1	0	1	1	0
Pearl	0	0	0	0	0	1	0	1	0	0	0	1	0	0
Ruth	0	1	0	1	0	1	0	0	0	0	0	1	0	0
Sylvia	0	0	0	1	1	1	0	0	1	0	1	1	0	1
Katherine	0	0	0	0	1	1	0	0	1	0	1	1	0	1
Myrna	0	0	0	0	1	1	0	0	1	0	0	1	0	0
Theresa	0	1	1	1	0	1	1	1	0	0	0	1	1	0
Charlotte	0	1	0	1	0	0	1	0	0	0	0	0	1	0
Frances	0	1	0	0	0	0	1	1	0	0	0	1	0	0
Helen	1	0	0	1	1	0	0	0	1	0	0	1	0	0
Nora	1	0	0	1	1	1	0	1	1	0	1	0	0	1

Figure 2a. The A Matrix Indicating Presence ("1") or Absence ("0") of Each of Eighteen Women at Each of Fourteen Social Events. Adapted from Homans (1950:83). (Row headings name the women. Column headings name each event in chronological order.)

	E1	Br	Do	Ve	Fl	Ol	La	Ev	Pe	Ru	Sy	Ka	My	Th	Ch	Fr	He	No
Eleanor	4	4	1	2	0	0	4	3	2	3	2	1	1	4	2	3	2	2
Brenda	4	7	1	2	0	0	6	6	2	3	2	1	1	6	4	4	2	2
Dorothy	1	1	2	2	1	1	1	2	2	2	2	2	2	2	0	1	1	1
Verne	2	2	2	4	1	1	2	2	2	3	4	3	3	3	1	1	3	3
Flora	0	0	1	1	2	2	0	1	1	1	1	1	1	1	0	0	1	2
Olivia	0	0	1	1	2	2	0	1	1	1	1	1	1	1	0	0	1	2
Laura	4	6	1	2	0	0	7	6	2	3	2	1	1	6	3	4	2	2
Evelyn	3	6	2	2	1	1	6	8	3	3	2	2	2	7	3	4	1	2
Pearl	2	2	2	2	1	1	2	3	3	2	2	2	2	3	0	2	1	2
Ruth	3	3	2	3	1	1	3	3	2	4	3	2	2	4	2	2	2	2
Sylvia	2	2	2	4	1	1	2	2	2	3	7	6	4	3	1	1	4	6
Katherine	1	1	2	3	1	1	1	2	2	2	6	6	4	2	0	1	3	5
Myrna	1	1	2	3	1	1	1	2	2	2	4	4	4	2	0	1	3	3
Theresa	4	6	2	3	1	1	6	7	3	4	3	2	2	8	4	4	2	3
Charlotte	2	4	0	1	0	0	3	3	0	2	1	0	0	4	4	2	1	1
Frances	3	4	1	1	0	0	4	4	2	2	1	1	1	4	2	4	1	1
Helen	2	2	1	3	1	1	2	1	1	2	4	3	3	2	1	1	5	4
Nora	2	2	1	3	2	2	2	2	2	2	6	5	3	3	1	1	4	8

Figure 2b. The P Matrix of Woman-to-Woman Relations, Derived from Matrix A by Equation 3. (Each off-diagonal entry is the number of events at which two given women were jointly present. Each main-diagonal entry is the total number of events attended by a single woman.)

are termed degenerate paths). Similarly for the group-to-group ties: the number of *n*-paths between every two groups is contained in the matrix  $G^n$ .

Suppose we know the *G* matrix but do not know the *P* matrix (for example, suppose we are given information on director-interlocks between corporations but we have no knowledge of director-to-director ties). Suppose further that we have person-to-group information for only two (or several, say  $p^* \ll p$ ) of

the *p* persons. We can find the *l*-paths among these (two or several) persons by  $A(A^T)$  where *A* has  $p^*$  rows and one column for each group in *G*. But it appears that we cannot find paths of length two or more among our  $p^*$  people because we don't know who the intermediate persons are, or how these intermediaries are connected to others. In this case we are aided by

Lemma 1.  $P^n = A(G^{n-1})A^T ; G^n = A^T(P^{n-1})A$ .

The proof follows from associativity and substitution of Equations 3 and 4; e.g.,

$$P^n = (AA^T)^n = A(A^T A)^{n-1} A^T = A(G^{n-1})A^T.$$

In this manner, we can determine the number of paths of any length among our  $p^*$  people by examining the dual paths in  $G$ . What of the number of *groups* that a person can reach (and conversely the number of persons that a group can reach)?

Lemma 2.  $P^n A = (G^n A^T)^T$

Proof:  $P^n A = (AA^T)^n A = AG^n = ((AG^n)^T)^T$   
 $= ((G^n)^T (A)^T)^T = (G^n A^T)^T.$

The assertion of Lemma 2 is that if we play out all chains of person-to-person ties as far as we like and then observe the groups that the last persons reach, we come out with the same endpoints as if we had played out all group-to-group chains to the same length and then looked at persons reached by the last groups.

The extension of lengths of paths in any graph has a natural limit; there exists some minimal  $m$  such that each node reaches all other nodes it will ever reach by paths of length  $m$  (at most): that is, converting the values of ties to their binary form and conceiving matrix multiplication as Boolean (Harary

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4	0	0	2	2	3	0	1	2	0	1	1	0	1
2	0	8	3	6	0	3	6	6	0	3	0	7	4	0
3	0	3	3	2	0	2	3	3	0	2	0	3	2	0
4	2	6	2	10	4	5	4	5	3	2	2	8	3	2
5	2	0	0	4	6	5	0	1	5	0	3	5	0	3
6	3	3	2	5	5	12	2	4	4	1	3	9	2	3
7	0	6	3	4	0	2	6	5	0	3	0	5	4	0
8	1	6	3	5	1	4	5	8	1	3	1	7	3	1
9	2	0	0	3	5	4	0	1	5	0	3	4	0	3
10	0	3	2	2	0	1	3	3	0	3	0	3	2	0
11	1	0	0	2	3	3	0	1	3	0	3	2	0	3
12	1	7	3	8	5	9	5	7	4	3	2	14	3	2
13	0	4	2	3	0	2	4	3	0	2	0	3	4	0
14	1	0	0	2	3	3	0	1	3	0	3	2	0	3

Figure 3a. The G Matrix of Event-to-Event Relations, Derived from Matrix A by Equation 4. (Each off-diagonal entry is the number of women who participated in both of two given events. Each main-diagonal entry is the total number of women who attended a given event. Observe that only columns 4, 6, 8, and 12 have no zero entry.)

	E1	Br	Do	Ve	F1	O1	La	Ev	Pe	Ru	Sy	Ka	My	Th	Ch	Fr	He	No
Eleanor	1	1	0	0	0	0	1	1	0	1	0	0	0	1	1	1	0	0
Brenda	1	4	0	0	0	0	3	4	0	1	0	0	0	3	3	2	0	0
Dorothy	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Verne	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1	1
Flora	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1
Olivia	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1
Laura	1	3	0	0	0	0	4	4	0	1	0	0	0	3	2	2	0	0
Evelyn	1	4	0	0	0	0	4	5	0	1	0	0	0	4	3	2	0	0
Pearl	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ruth	1	1	0	0	0	0	1	1	0	1	0	0	0	1	1	1	0	0
Sylvia	0	0	0	1	0	0	0	0	0	0	4	4	2	0	0	0	2	4
Katherine	0	0	0	1	0	0	0	0	0	0	4	4	2	0	0	0	2	4
Myrna	0	0	0	1	0	0	0	0	0	0	2	2	2	0	0	0	2	2
Theresa	1	3	0	0	0	0	3	4	0	1	0	0	0	4	3	2	0	0
Charlotte	1	3	0	0	0	0	2	3	0	1	0	0	0	3	3	2	0	0
Frances	1	2	0	0	0	0	2	2	0	1	0	0	0	2	2	2	0	0
Helen	0	0	0	1	1	1	0	0	0	0	2	2	2	0	0	0	3	3
Nora	0	0	0	1	1	1	0	0	0	0	4	4	2	0	0	0	3	5

Figure 3b. The New Matrix P2 of Woman-to-Woman Relations, Formed by Eliminating Columns 4, 6, 8, and 12 of Matrix A and Then by Applying Equation 3. For the Graph of P2, See Figure 3.

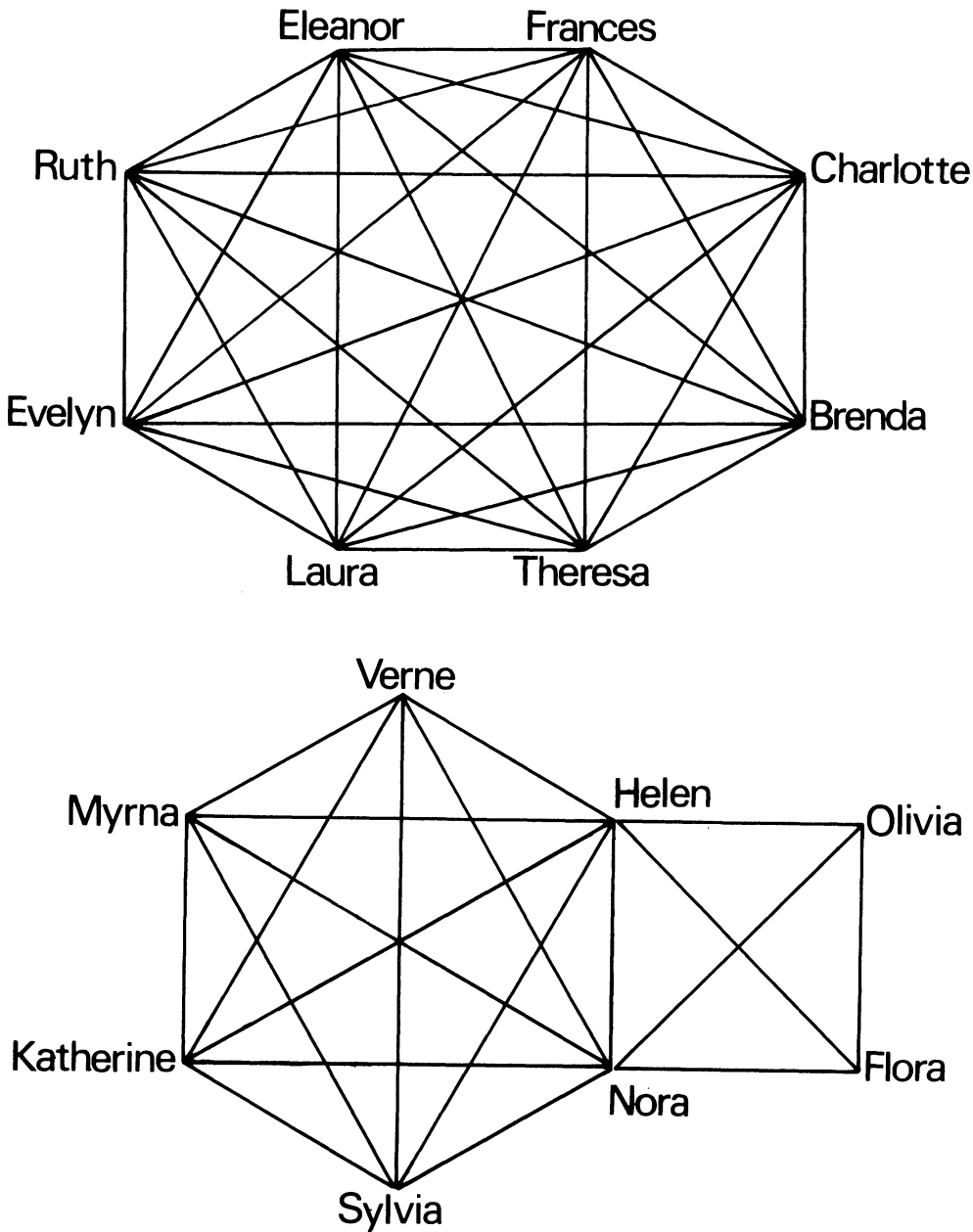


Figure 4. The Graph of Matrix P2. (For simplicity, the ties are shown in their binary form.)

*et al.*, 1965), there is some minimal  $m$  for which the matrix  $P^{m+1}$  is contained in the union of the first  $m$  powers of  $P$ , and some minimal  $n$  for which  $G^{n+1}$  is contained in the union of the first  $n$  powers of  $G$ . The matrices  $P$  and  $G$  are then said to have reached transitive closure.

*Theorem.* If  $P$  reaches transitive closure at the  $m$ th power and  $G$  reaches transitive closure at the  $n$ th power, then the absolute difference of  $m$  and  $n$  is at most 1.

Here is a sketch of the proof. It follows from Lemmas 1 and 2 that the matrix which is the union of the first  $k$  powers of  $P^i A$  ( $i = 1, \dots, k$ ) specifies (for minimal  $k$ ) all groups ever reached by each person if and only if the union of the first  $k$  powers of  $G^i A^T$  specifies all persons ever reached by each group ( $P^{k+1} A \subseteq \bigcup_{i=1}^k P^i A$  if and only if  $G^{k+1} A^T \subseteq \bigcup_{i=1}^k G^i A^T$ ). Since the nodes on a path from a person to a group may be conceived as an alternating sequence of persons and groups, all persons reach all

groups they will ever reach (by paths not exceeding length  $k$ , at most) *only* if they have just reached all *persons* they will ever reach ( $P$  reaches transitive closure at the  $k$ th power) or if they are about to do so at the next remove ( $P$  reaches transitive closure at the  $k+1$ st power). And similarly for  $G$  ( $G$  reaches transitive closure only at the  $k$ th or the  $k+1$ st power).

PRIMARY AFFILIATIONS AND ASYMMETRIC TIES

We have, until now, imposed symmetry on a network of membership ties; indeed, most writers (e.g., Bonacich, 1972; Perrucci and Pilisuk, 1970) conceive such ties as symmetric only. There are, however, cases (such as corporate interlocks or coalitions among parties or factions) in which it is more interesting to conceive of an asymmetric tie from one person or group to another. This creation of asymmetric orientations out of the symmetry of group membership was formulated by Simmel (1955: 138, 155) in terms of primary and secondary affiliations.

[O]ne group appears as the original focus of an individual's affiliation, from which he then turns toward affiliation with other, quite different groups on the basis of his special qualities, which distinguish him from other members of his primary group. His bond with his primary group may well continue to exist . . .

An infinite range of individualizing combinations is made possible by the fact that the individual belongs to a multiplicity of groups. . . . The instinctive needs of man prompt him to act in these mutually conflicting ways: he feels and acts *with* others but also *against* others.

The generalization of this asymmetry occurs in Simmel's discussion (1955:62) of competition. "Modern competition is described as the fight of all against all, but at the same time it is the fight of all *for* all"—and thus results not in the chaos of Hobbes (which necessitates external control) but in an intrinsically ordered interweaving of relations based on "the possibilities of gaining favor and connection."

Here is a method for building asymmetry into the basic approach of this paper. Begin with  $p$  people,  $g$  groups, a  $p \times g$  matrix  $F$  whose  $(i, j)$ th entry is "1" if person  $i$  has group  $j$  as his *primary* affiliation and is "0" otherwise, and a  $p \times g$  matrix  $A$  (as above) showing all affiliations of each person. Partition the memberships in  $A$  among primary and secondary (i.e., all other) affiliations. Define the  $p \times g$  matrix  $S$  of secondary affiliations by  $S = A \cap \sim F$ .

Let us say that two people mutually influence each other if they share a common primary affiliation. Our substantive conceptualization of a particular problem (for example, influence among directors of corporations) might suggest specifying that an asymmetric tie exists from person  $i$  to person  $j$  (" $i$  is influenced by  $j$ ") if a group which is *is* primary affiliation is a secondary affiliation for  $j$  (the assumption here being that directors of higher-status corporations are more sought after to lend their prestige to the boards of other corporations).<sup>9</sup>

Following from this conception is a matrix  $P'$  of asymmetric ties among persons:

$$P' = FF^T \cup FST = F(F^T \cup S^T) = FA^T$$

By reasoning analogous to that of Equations 3 and 4, we find the dual matrix  $G'$  of asymmetric ties among groups:  $G' = A^T F$ . Moreover, the above reasoning on duality and transitivity is easily extended to the asymmetric case; as the reader may verify, for example,

$$(P')^n = (FA^T)^n = F(A^T F)^{n-1} A^T = F(G')^{n-1} A^T$$

is the analogue to Lemma 1 for the asymmetric case.

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<sup>9</sup> Mace (1971:90) quotes this observation of a company official: "You want to communicate to the various publics that if any company is good enough to attract the president of a large New York bank as a director, it just *has* to be a great company."

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## Testing Theoretical Hypotheses: A PRE Statistic

KENNETH J. BERRY, *Colorado State University*

THOMAS W. MARTIN, *Colorado State University*

KEITH F. OLSON, *Colorado State University*

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### ABSTRACT

Departures from statistical independence are conjoined with an assessment of predictive accuracy in a coefficient of association for nominal-level 2 x 2 contingency tables which is both interpretable as a *PRE* measure and consistent with research hypotheses of manifold forms. Measurement assumptions and operating characteristics of the measure are delineated; definitional and computational formulae are derived from classical probability theory; comparisons with other relevant statistics are made; and the test of significance is shown to be the traditional chi-square test.

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Social scientists have long recognized that the assessment of association at the nominal level presents an especially difficult problem in both experimental and survey research. In order to arrive at a useful and meaningful measure of association for nominal-level variables, two considerations must be kept in mind: (a) the form of the empirical test must be identical with the form of the research hypothesis (see

Costner, 1965; Duggan and Dean, 1968; Francis, 1961; Kang, 1972, 1973; Leik and Gove, 1969) and (b) the measure of association should be "operationally interpretable" in terms of the proportional reduction in error of estimation made possible by the relationship" (Costner, 1965:342). The purpose of this paper is to introduce *K*, a measure of association for dichotomized qualitative variables