Irreconcilable Differences in Tests of Hotelling's Theory of Exhaustible Resources

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Abstract

Empirical tests of the theory of exhaustible resources have provided mixed results regarding empirical validity of the theory. This paper considers differences in the primary testing methodologies and alternative measures of the *in situ* resource price. We demonstrate that *in situ* price measures common in the literature are theoretically equivalent for the vertically integrated firm. We also show that empirical shadow price test methodologies imply the same restricted econometric model, while the unrestricted models diverge. We find that the Halvorsen and Smith (1991) test methodology is relatively efficient, and, using data on natural gas wells, reject the unrestricted transition equation of traditional methodology. We conclude that discrepant empirical test results may be an artifact of the econometric testing methodologies rather than a reflection on the validity of the theory.

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Extensions and New Applications of the Hotelling Model **(Q3)**

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Robert H. Patrick, Rutgers University, and Janie M. Chermak, University of New Mexico--*Irreconcilable Differences: Tests of the Theory of Exhaustible Resources*

Michael R. Caputo, University of Central Florida--*A Nearly Complete Test of a Vertically Integrated, Capital Accumulating, Nonrenewable Resource Extracting Model of the Competitive Firm*

Ujjayant Chakravorty, University of Central Florida and University of Toulouse, Michel Moreaux, University of Toulouse, and Mabel Tidball, University of Montpellier--*Ordering the Extraction of Polluting Nonrenewable Resources* Peter Hartley and Kenneth

B. Medlock III, Rice University--*A Model of the Operation and Development of a National Oil Company*

Discussants: Peter Berck, University of California, Berkeley
Scott Farrow, University of Maryland, Baltimore County
Stephen Holland, University of North Carolina at Greensboro
Denise Young, University of Alberta

1.0 Introduction

> *My approach has always been based on a twofold conviction:*
> *- the conviction that, without theory, knowledge inevitably remains confused and that an accumulation of facts only constitutes a chaotic and unavoidably incomprehensible aggregate;*
> *- and the even stronger conviction that a theory which cannot be confronted with the facts or which has not been verified quantitatively by observed data, is, in fact, devoid of any scientific value.*
>
> Maurice Allais (1990, p. 5)

The implication of increased resource scarcity on economic growth was discussed by Ricardo in the early as the 1800s in the context of land quality. Gray (1914) modified Ricardo's discussion of rent to include a scarcity rent of an exhaustible resource, which is referred to today in a variety of terms, for example, *in situ* resource price, user cost, scarcity value, shadow price, or option value. It wasn't until 1931 that a formal theory of optimal exploitation was presented in the literature when Hotelling published his seminal paper on the theory of exhaustible resources. While Hotelling developed his theoretical model under a number of assumptions, the key elements are the unit price of the resource, cost function characteristics, the quantity of the resource exploited, and the *in situ* resource price.[1]

Hotelling's theory of exhaustible resources has been tested using data on various exhaustible resources; across different levels of aggregation, time periods, market structure, etc.; by means of a variety of theoretical approaches and statistical/econometric techniques; with varying results as to the empirical validity of the theory.[2] The first empirical examinations of the theory appeared in the literature in the 1960s when there was international speculation of the potential of exhaustion of some resources. This genre of

[1] See Solow (1974) for an excellent description of the main points of the original paper.
[2] See Berck (1995), Withagen (1998) and Chermak and Patrick (2002) for more thorough reviews of the literature.

research focuses on exhaustible resource price behavior, either through explicit tests of the theory or inquiry into price paths. Price behavior tests (e.g., Barnett and Morse 1963, Smith 1979, Slade 1982), based on variations of the Hotelling's r-percent rule (i.e., net resource price will increase at the rate of interest).[3] These tests have not resulted in support for the theory. Shadow price tests have dominated the relatively recent empirical literature (Stollery 1983, Farrow 1985, Miller and Upton 1985a, 1985b, Halvorsen and Smith 1991, Slade and Thille 1997, Berck and Bentley 1997, Cairns and Davis 1998, and Chermak and Patrick 2001). The shadow price tests have disparate results and the adequacy of the theory of exhaustible resources in describing firm behavior continues to be argued. Across these tests, findings are that Barnett and Morse (1963), Smith (1979), Slade (1982), Farrow (1985), Halvorsen and Smith (1991), Young (1992), and Young and Ryan (1996) reject the theory; Miller and Upton (1985a, 1985b) and Cairns and Davis (1998) find mixed results; while Stollery (1983), Slade and Thille (1997), Berck and Bentley (1997), and Chermak and Patrick (2001) do not reject the theory.

Testing the Hotelling model requires a measure of user cost, which is generally not directly observable. The traditional user cost measure, for the price-taking firm, has been estimated using the necessary condition on optimal extraction, i.e., the difference between price and the marginal cost of gross product. Thus, price is explicitly included. While this is the measure of user cost primarily applied in the empirical literature, it is not the only theoretically developed measure. Recognizing that exhaustible resource production requires processing after extraction, in 1984 Halvorsen and Smith (HS) developed an alternative user cost measure for the vertically integrated firm. This duality-based formulation measures user

[3] Berck and Roberts (1996), although not testing the Hotelling theory, use time-series techniques to estimate the likelihood that real market prices of resources will increase and find that trend-stationary modeling predicts

cost as the negative of the marginal indirect cost function with respect to the quantity of unprocessed resource extracted, thus price is implicit in this formulation. For simplicity's sake we will refer to the two measures of user cost as the traditional measure and as the HS measure.[4]

There are two shadow price econometric test methodologies in the literature. The traditional methodology estimates the transition equation and then tests for theoretically consistent parameter restrictions by restricting the transition equation. The second methodology, a duality-based approach developed by Halvorsen and Smith (1991), estimates the indirect final cost function and the tests the theoretical restrictions implied by the dynamic optimality condition, i.e., the transition equation. Again, for simplicity the two test methodologies are referred to as the traditional test and the HS test.

While not a requirement, the empirical tests that employ the traditional measure of user cost have employed the traditional test, while the HS tests have employed the HS user cost measure.

Regardless of the user cost measure employed or the test methodology used, these tests have required an estimated cost function. The extant empirical tests have been performed across a variety of resources, over a variety of time frames, under varying market structures, at different levels of aggregations (across deposits, resources, and/or production periods), and other various assumptions using either the traditional or duality testing framework, with a variety of econometric estimation techniques. A difficulty with these results is that given the variety of resources across the tests and the implied restrictions

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increasing prices.

[4]The user cost measure has been used to further categorize the shadow price tests as explicit or implicit price tests. Farrow, Slade and Thille, Young, Young and Ryan, Stollery, Miller and Upton and Cairns and Davis tests include price explicitly, while the Halvorsen and Smith and Chermak and Patrick include it implicitly.

associated with each test, it is not easy to ascertain consistency across the results. Are the differences due to differences across resources, markets, and producers or are they due to the implicit restrictions in each of the methodologies?

Chermak and Patrick (2002) relax the implicit data restrictions due to different resources, levels of aggregations (across deposits, resources, and/or production periods), and time horizons of the original tests by employing a single resource, micro-level data set to perform four of the extant tests. Two are variations of the traditional test with the traditional measure of user cost; Farrow (1985) and Slade and Thille (1992), while the other two are variations of the HS test with an HS measure of user cost; Halvorsen and Smith (1991) and Chermak and Patrick (2001). Of the original tests the Farrow and Halvorsen and Smith results do not find support for the theory, while the Slade and Thille and Chermak and Patrick tests do. The new tests of the single resource data set *still* end in conflicting results. The two HS tests find support for the theory, while the two traditional tests do not. Thus, the 2002 Chermak and Patrick tests are consistent with the original Farrow and Chermak and Patrick results, but inconsistent with the original Slade and Thille and Halvorsen and Smith results.

Obviously, data restrictions are not responsible for these mixed results. Given that the two HS tests found support, while the two traditional tests did not, leads us to focus on the interpretation of the theory and the empirical tests methodologies. In this paper we consider differences in the primary testing methodologies and alternative measures of the *in situ* resource price.

We first use duality theory to develop alternative measures of the *in situ* resource price and the relationship between gross (unprocessed) and final (commodity) costs. We demonstrate the traditional and the HS measures of user cost applied in the literature are theoretically equivalent. That is, regardless of the test methodology (traditional or HS), the negative of the marginal gross cost of final production is, theoretically, an appropriate measure of user cost for the vertically integrated exhaustible resource producing firm. This development negates the need for the explicit price path expected or faced by the producer, reducing the required data.[5] Using the Chermak and Patrick data, we employ the HS user cost measure in a traditional test and still have discrepant results in that this new test does not find support for the theory, while the parallel 2002 HS test find support.

Finally, we evaluate the consistency of the traditional test, which directly restricts the estimated transition equation and the HS test, which restricts the indirect cost function. We find the two methodologies imply identical restricted econometric models, but the implied specifications of the unrestricted models diverge. The traditional test has a relatively complex unrestricted model and requires estimating more parameters than the HS test. The HS test is preferred to the traditional test in terms of (statistical) efficiency. Empirically, we test the statistical consistency of the transition equations associated with these two tests methods and find that they are statistically different. This demonstrates, for this single resource, micro-level dataset, that the conflicting conclusions are due to differences in the test methodologies.

The remainder of the paper is organized as follows. Section 2 presents the theory of the price-taking, vertically integrated, exhaustible resource producing firm. Section 3 uses

[5] The potential impact of this can be seen by comparing forecast and realized prices. For example, Wong-Parodi et. al., (2005) compare the Annual Energy Outlook average annual price forecasts for natural gas to the actual, realized, annual average price. Between 1996 and 2003, the average difference between the forecast and actual price is over 19%. In 2003, the forecast price was \$2.85. The actual average price was \$4.98. The forecast underestimated the price of natural gas by 42%. Employing the realized price to test the theory (if the producer made his production plan based on the forecast price) would result in estimated user costs being larger than the use cost associated with the production plan.

duality theory to develop the alternative user cost measures and demonstrate their equivalence. Section 4 discusses information and data considerations in testing the theory. Section 5

2.0 The Vertically Integrated Exhaustible Resource Producing Firm

This section presents the profit-maximizing problem for the vertically integrated, price-taking firm producing a non-renewable resource, subject to the traditional resource stock constraint.

Formally, let $q(t)$, $t \in [0,T]$, represent the gross production at time *t*.[6] $R(t)$ is the associated continuous and piecewise differentiable state variable, also defined on the time interval $t \in [0,T]$. The stock of the resource at $t = 0$ is $R(0) = R^0$. The stock of the resource is reduced by extraction at each *t*, implying that the transition equation on the resource stock is given by

$$\dot{R}(t) = -q(t),\ R(0) = R^0,\ R(T) \geq 0\ . \qquad (1)$$

Gross production is not necessarily the amount of the resource that can be sold by the firm.[7] The gross production function is given by $q(t) = g(\mathbf{X}^g, R(t), t)$, where $\mathbf{X}^g = \left\{ x_j(t) : j = 1,..., J \right\}$ are inputs into the gross production process. The processed resource (final production) is given by $z(t) = f(\mathbf{X}^f, q(t), t)$, where $\mathbf{X}^f = \left\{ x_k(t) : k = 1,..., K \right\}$ are inputs used to process gross production to arrive at the final product, *z*(*t*), at each *t*. Naturally, final

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[6] $q(t)$ is a piecewise continuous control function with, at most, a finite number of discontinuities, with finite jumps (i.e., one-sided limits) at each point of discontinuity.

[7] For example, with natural gas, the removal of non-hydrocarbon gases and other shrinkage generally prohibit this. The US Energy Information Administration (*Natural Gas Annuals*, various years) distinguishes gross withdrawals, defined as full well-stream volume excluding condensate separated at the lease from dry natural gas production defined as gross withdrawals less gas diverted for re-pressuring, quantities vented or flared, non-

production can be no greater than gross production, i.e., $z(t) \le q(t)$, where a strict equality would imply that gross and final products are equivalent.

The firm's problem is to choose the input production paths, the gross production path, and the production time horizon to maximize profits, π, of the fixed resource. The firm's objective is then to choose $\{\mathbf{X}^f, q(t)\}$ for $t \in [0,T]$, and T to maximize

$$\pi = \int_0^T e^{-rt}[P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t)]dt, \qquad (2)$$

where $P(t)$ is the output price per unit of final production, $z(t)$, $\mathbf{W}^f = \{w_k(t) : k = 1,...,K\}$ are processing input prices, $\mathbf{W}^g = \{w_g(t) : j = 1,...,J\}$ are prices of inputs into gross production, and the gross cost function (the dual to the gross production function) is given by

$$C(\mathbf{W}^g, q(t), R(t), t) \equiv \min_{X^g} \ \mathbf{W}^g \mathbf{X}^g \text{ s.t. } q = g(\mathbf{X}^g, R(t), t). \qquad (3)$$

Profits are maximized subject to the resource stock constraint, (1). The Hamiltonian for this problem is

$$H(t) = e^{-rt}[P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t)] - \lambda(t)q(t), \qquad (4)$$

where $\lambda(t)$ is the multiplier (*in situ* resource price) on (1), the transition equation of the resource stock. Necessary conditions include[8]

$$-H_R = \dot{\lambda} = e^{-rt}C_R, \qquad (5)$$

$$H_q = e^{-rt}(Pf_q - C_q) - \lambda = 0, \qquad (6)$$

$$H_{x_k} = e^{-rt}(Pf_{x_k} - w_k) = 0, \qquad (7)$$

$$\begin{aligned} H(T^*) = e^{-rt}[&P(T^*)z^*(T^*) - \mathbf{W}^f(T^*) \cdot \mathbf{X}^f(T^*) \\ &- C(\mathbf{W}^g(T^*), q^*(T^*), R^*(T^*), T^*)] - \lambda(T^*)q^*(T^*) = 0, \end{aligned} \qquad (8)$$

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hydrocarbon gas removed in the treatment or processing, and extraction losses.

and the transversality condition is

$$\lambda(T^*) \geq 0 \;\; (= 0 \text{ if } R^*(T^*) > 0), \tag{9}$$

where the "*" superscripts represent the optimal level of the respective variable (naturally, conditions (5)-(9) hold for the optimal value candidates of the choice variables). If (4) is concave in (q, $\mathbf{X}$, R), where $\mathbf{X} = \mathbf{X}^f \cup \mathbf{X}^g$, for each $t \in [0,T]$ then (5)-(9) are necessary and sufficient for $(q^*, \mathbf{X}^*, R^*)$ to solve the problem. If (4) is strictly concave in (q, $\mathbf{X}$, R) then $(q^*, \mathbf{X}^*, R^*)$ are unique.

(9) is the present value of the terminal time *in situ* price and (8) is the necessary condition on choosing the optimal terminal time, T^*. (7) is the condition for optimal choice of reproducible inputs into final production, $\mathbf{X}^{f*}$, for each $t \in [0,T^*]$. (6) is the optimality condition on gross production, q^*, for each $t \in [0,T^*]$. This necessary condition is a more general form of the measure of the *in situ* resource price in the traditional test of the theory. The difference being that this measure explicitly considers processing, which implies that the necessary condition for optimal gross production in our model is that the discounted value of the marginal product of q is equated to the sum of the discounted marginal gross cost and the *in situ* resource price. If $z = q$, i.e., there is no processing, we have the traditional result that $\lambda = e^{-rt}(P - C_q)$. $\lambda(t) \geq 0$ for all $t \in [0,T^*]$ is implied by (5) and (9). That is, $\lambda(T^*) \geq 0$ by (9) and (5) implies the present value user cost is declining over t, $\dot{\lambda} = e^{-rt} C_R < 0$, since $C_R < 0$ (cost increases as the stock of the resource is depleted). (5) is the dynamic optimality condition for the exhaustible resource that is used in the econometric tests. However, to carry out these tests we must first develop a measure of *in situ* resource prices.

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8 We drop the time argument for ease of exposition, only using it from here on when necessary for clarity.

3.0 Measuring *In Situ* Resource Prices

Below, we restate the Chermak and Patrick (2001) derivation of Lemma 1 to facilitate further developments. The relationship between the gross and final indirect cost functions is derived in Proposition 1. Next, the vertically integrated firm counterpart to the traditional measure of *in situ* resource price is developed in Lemma 2. Finally, in Proposition 2, we show regardless of the test type (traditional or HS test), the appropriate measure of *in situ* resource price is theoretically equivalent.

We begin with the development of the current value of the dynamic optimality condition for exhaustible resources. Let

$$\mathrm{L} \equiv e^{rt}L(t) = P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t) - mq(t) + n[Q - q(t)].$$

Where $n \equiv e^{rt}\eta$ and where $m \equiv e^{rt}\lambda$ is the current value *in situ* resource price. Furthermore, $m(t) \geq 0$ for all $t \in [0, T^*]$ because $m(t) \equiv e^{rt}\lambda(t)$ and $\lambda(t) \geq 0$ for all $t \in [0, T^*]$. The current value form of (5), the dynamic optimality condition, i.e., the transition equation, for the exhaustible resource, is

$$\dot{m} = rm - \mathrm{L}_R = rm + C_R, \qquad (10)$$

which implies that the current value *in situ* resource price increases at the rate of interest less the increase in future costs from extracting the marginal unit. (10) indicates that empirical testing of the implications of exhaustible resource theory for the dynamic behavior of producing firms requires estimates of *m*, *r*, and C_R. We now turn to estimating *m* and C_R.

Consider the cost minimization problem of producing given levels of gross and final product. Let $q = \overline{q} = q^*$ and $z = \overline{z} = z^*$ be the profit maximizing levels of gross and final output from the profit maximization problem of the previous section. The physical constraint on production, $\overline{q} \leq Q$, (where Q is the upper bound on production) also applies. The Lagrangian for the problem is

$$\overline{L} = \mathbf{W} \cdot \mathbf{X} + \gamma[\overline{z} - f(\mathbf{X}^f, \overline{q}, t)] + \delta[\overline{q} - g(\mathbf{X}^g, R, t)] + \rho(Q - \overline{q}), \qquad (11)$$

where $\mathbf{W} = \mathbf{W}^f \cup \mathbf{W}^g$, γ is the multiplier on the final production constraint, δ is the multiplier on the gross production constraint, and ρ is the multiplier on the physical capacity constraint. Necessary conditions include

$$\overline{L}_{x_j} = w_j - \delta g_{x_j} = 0,\ j\text{=}1,\ldots,J\ , \qquad (12)$$

for inputs into gross production, and

$$\overline{L}_{x_k} = w_k - \gamma f_{x_k} = 0,\ k\text{=}1,\ldots,K\ , \qquad (13)$$

for the choice of inputs into final production. The condition on the multiplier of the capacity constraint is

$$\rho \geq 0\ (= 0 \text{ if } q < Q). \qquad (14)$$

The solution to this cost minimization problem yields the (indirect) final cost function

$$\overline{C}(\mathbf{W}, z, q, R, t, \ldots). \qquad (15)$$

From this, we have the following envelope theorem results.

$$\overline{L}_q = \overline{C}_q = -\gamma f_q + \delta - \rho, \qquad (16)$$

$$\overline{L}_R = \overline{C}_R = -\delta g_R, \qquad (17)$$

and

$$\overline{L}_z = \overline{C}_z = \gamma. \qquad (18)$$

These results are used below in developing observable measures of the *in situ* resource price.

Now consider the firm's problem of minimizing the cost of producing *z*, treating *q* as an endogenous input into the production of *z*. The inputs, **X**, are now chosen to minimize costs, defined as $\mathbf{W}\cdot\mathbf{X}+mg(\mathbf{X}^g,R,t)$, where *m* is the (unobserved) shadow price of the resource *in situ*, subject to the constraints on final and periodic production, $z=\overline{z}=z^*$ and $0\le q\le Q$. The Lagrangian for the cost minimization problem when gross production, *q*, is endogenous is then

$$L=\mathbf{W}\cdot\mathbf{X}+mg(\mathbf{X}^g,R,t)+\tilde{\gamma}[\overline{z}-f(\mathbf{X}^f,g(\mathbf{X}^g,R,t),t)]+\tilde{\rho}[Q-g(\mathbf{X}^g,R,t)], \qquad (19)$$

where $\tilde{\gamma}$ is the multiplier on the final production constraint and $\tilde{\rho}$ is the multiplier on the physical capacity constraint. Necessary conditions include

$$L_{x_j}=w_j+[m-\tilde{\gamma}f_g-\tilde{\rho}]g_{x_j}=0,\ j\text{=}1,\ldots,J, \qquad (20)$$

for the optimal choice of inputs into gross production;

$$L_{x_k}=w_k-\tilde{\gamma}f_{x_k}=0,\ k\text{=}1,\ldots,K, \qquad (21)$$

for the optimal choice of inputs (other than *q*) into final production and

$$\tilde{\rho}\ge 0\ (=0 \text{ if } q<Q). \qquad (22)$$

Envelope theorem results include

$$L_R=C_R=(m-\tilde{\gamma}f_q-\tilde{\rho})g_R, \qquad (23)$$

The optimal **X** for this problem is identical to that from the previous problem since $q=\overline{q}=q^*$ and $z=\overline{z}=z^*$. (13) and (21) then imply that $\gamma=\tilde{\gamma}$. (12), (14), (20), and (22) then lead to $\rho=\tilde{\rho}$ and $\delta=-m+\tilde{\gamma}f_q+\tilde{\rho}$. Substituting these expressions into (16) leads to

Lemma 1. The current value *in situ* resource price is equal to the negative of the marginal final cost of gross production, i.e., $m=-\overline{C}_q$.

This is the HS measure of user cost. The original insight for this result is from Halvorsen and Smith (1984) and it is used in the HS test. Lemma 1 and $m\ge 0$ imply that $\overline{C}_q\le 0$, i.e., the marginal final cost of gross production is non-positive. The intuition is that additional *q*, all else held constant, reduces the marginal cost of producing a given level of z. Lemma 1 also implies that the current value *in situ* resource price can be observed by differentiating the estimated indirect final cost function with respect to gross production, q. Analogously, substituting $\delta = -m + \tilde{\gamma} f_q + \tilde{\rho}$ into (23) and using (17), it follows that $\overline{L}_R = L_R$ and hence we have $\overline{C}_R = C_R$. Thus, by estimating the indirect final cost function, (15), and differentiating with respect to q and R, we have observable measures of m and C_R, which are required in testing (10), the dynamic optimality condition.

Now consider the profit maximization model developed in the previous section. For simplicity in allowing us to focus on measuring the *in situ* price, we write the model in current value form.[9] The current value Hamiltonian for the profit maximization problem is then

$$H(t) = P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t) - mq(t) . \tag{24}$$

Necessary conditions include

$$-H_R = \dot{m} = rm + C_R , \tag{25}$$

$$H_q = Pf_q - C_q - m = 0 , \tag{26}$$

$$H_{x_k} = Pf_{x_k} - w_k = 0, \tag{27}$$

$$\begin{aligned} H(T^*) = {} & P(T^*)z^*(T^*) - \mathbf{W}^f(T^*) \cdot \mathbf{X}^f(T^*) \\ & - C(\mathbf{W}^g(T^*), q^*(T^*), R^*(T^*), T^*) - m(T^*)q^*(T^*) = 0. \end{aligned} \tag{28}$$

The transversality condition is

$$m(T^*) \geq 0 \ \ (= 0 \text{ if } R^*(T^*) > 0) . \tag{29}$$

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[9]The relationship between gross and final production is likely to be more complicated than modeled here. For example, considering natural gas resources requires the incorporation of reservoir engineering theory, as well as the gas properties to discern it to a greater degree than in this abstract consideration. See Chermak, *et al*., (1999), for a detailed model of such considerations.

As above, since $\lambda(t) \geq 0$ and $m(t) \equiv e^{rt}\lambda(t)$, $m \geq 0$. Substituting (26) into (27) we have

$$(w_k f_q / f_{x_k}) = C_q + m. \tag{30}$$

Substituting (13) and (18) into (30) leads to

$$m = \bar{C}_z f_q - C_q \geq 0. \tag{31}$$

That is, the current value *in situ* resource price is equal to the value (in terms of the marginal final cost of *z*) of the marginal product of *q* less the marginal gross cost of *q*. The cost minimization problem in the previous section implies $C_q \geq 0$. Given production occurs, the marginal product of *q* is positive, $f_q > 0$. Naturally, we expect the marginal final cost (the marginal cost of producing the final product) to be positive, $\bar{C}_z > 0$. The value of the marginal product of *q* must then be greater than the marginal gross cost of *q*, $\bar{C}_z f_q > C_q$, for the inequality in (31) to hold. Equating (31) and the expression in Lemma 1, and rearranging, leads to

Proposition 1. Marginal gross cost equals the sum of the value (in terms of the marginal final cost of *z*) of marginal product of *q* and the marginal final cost of *q*, i.e., $C_q = \bar{C}_z f_q + \bar{C}_q$.

This proposition provides the implied theoretical relationship between the final and gross indirect cost functions. Lemma 1 and $m \geq 0$ imply $\bar{C}_q \leq 0$ and, since $C_q \geq 0$, $\bar{C}_z f_q \geq \left|\bar{C}_q\right|$.

Next, consider the value (in terms of the marginal final cost of *z*) of marginal product of *q*. Since (13) and (21) imply that $\gamma = \tilde{\gamma}$, we can substitute (18) and (21) into (27), which leads to $P = \bar{C}_z$. That is, the unit price of the final product is equal to the value of the marginal final cost of *z*. This is the expected result for the price-taking firm; produce your final product to the point where price equals marginal cost. Substituting this into Proposition

1, we then have $C_q = \bar{C}_z f_q + \bar{C}_q = Pf_q + \bar{C}_q$. Or, more importantly in measuring the *in situ* resource price, $\bar{C}_q = C_q - Pf_q$ which, with Lemma 1, leads to

Lemma 2. The current value *in situ* resource price is equal to the value (in terms of the price of *z*) of the marginal product of *q* less the marginal gross cost of *q*, i.e., $m = Pf_q - C_q$.

This is equivalent to (26) and to the current value specification of (6). Contrary to the measure of the *in situ* resource price in Lemma 1, which does not require the price of the final product, Lemma 2 requires final product price to measure user cost (as in traditional test methodology).

Finally, consider that the current value *in situ* resource price developed in Lemma 2 is the vertically integrated firm analog to the traditional measure of user cost. Proposition 1 and Lemma 2 yields $m = (P - \bar{C}_z) f_q - \bar{C}_q$. Substituting $P = \bar{C}_z$, as developed above, into this expression completes the link between the traditional measurement of user cost and the HS measure. This result is summarized in Proposition 2.

Proposition 2. The HS measure of the *in situ* resource price (Lemma 1) is equal to the vertically integrated firm analog to the traditional measure of the *in situ* resource price (Lemma 2), i.e., $m = Pf_q - C_q = -\bar{C}_q$.

Regardless of whether we use the traditional or the HS approach to measure user cost, the derived measures are theoretically equivalent for the vertically integrated firm.

The theoretical importance of this result is that user cost can be appropriately measured as the negative of the marginal indirect final cost of gross production for the vertically integrated firm, irrespective of the approach used in testing the theory. The empirical significance of this is that an explicit price path is not required for the test. This alleviates potential problems of employing an *ex post* price path to an *ex ante* production decision.[10]

Given the results from Proposition 2, we retest the theory. We use the HS measure of user cost, directly estimate the transition equation and restrict the parameter estimates to test for the consistency.

4.0 The Traditional Test Using Proposition 2

In this section, we use $m = -\bar{C}_q$ as the measure of user cost in the context of the transition equation test. We use the empirical final cost function from Chermak and Patrick (2001) presented in Table 1.[11] The function is specified in the form of a Generalized Cobb-Douglas, where monthly final production costs, $\bar{C}$, are hypothesized to be a function of gross production, *q*, final production, *z*, and remaining reserves, *R*. Production month, *t,* is also included to account for differences in the wells across their respective production horizons. A fixed effects model, which stratifies the panel data by firm and by year, is employed. The specification for the indirect final cost function is;

$$\ln \bar{C} = \sum_{i=1}^{N} \alpha_i D_i + \sum_{y=2}^{Y} \alpha_y D_y + \beta_q \ln q + \beta_z \ln z + \beta_R \ln R + \beta_t \ln t + e \qquad (32)$$

where there are *N*=5 firms, *Y*=5, and *e* = the customary error term, a random number with zero mean. As discussed above, Chermak and Patrick (2001) apply the HS test with the HA user cost measure and do not reject the theory of exhaustible resources.

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[10] For a more detailed discussion of data issues see Chermak and Patrick (2005).

[11] The data are 443 monthly observations from May 1987 to June 1991 for 29 tight gas sand wells. Seventeen of the natural gas wells are located in Wyoming, ten in west Texas, and the remaining two in east Texas. The wells vary in, among other things physical characteristics, age, and completion technology. Four companies provided individual well information. For a complete description of the data, see Patrick and Chermak (1992).

TABLE 1: Final Cost Function Estimation

Parameter	*Estimate*	*Standard Error*
β_q	-0.863[a]	0.032
β_z	0.863[a]	0.032
β_R	-0.048[c]	0.029
β_t	0.056[a]	0.023
$\beta_{\text{Firm 1 User Cost (t-1)}}$	7.351[a]	0.508
$\beta_{\text{Firm 2 User Cost (t-1)}}$	6.926[a]	0.488
$\beta_{\text{Firm 3 User Cost (t-1)}}$	8.823[a]	0.493
$\beta_{\text{Firm 4 User Cost (t-1)}}$	9.222[a]	0.465
$\beta_{\text{Firm 5 User Cost (t-1)}}$	8.849[a]	0.480
β_{1988}	-0.157	0.224
β_{1989}	-0.383[c]	0.231
β_{1990}	-0.449[b]	0.229
β_{1991}	-0.326	0.237
ρ	0.731[a]	0.032
Log-L	-62.61	

Level of significance: a=0.01, b=0.05, and c=0.10

Combining Proposition 2 with (10), the discrete form of the dynamic optimality condition yields

$$m(t) - m(t-1) = rm(t-1) + \overline{C}_R. \tag{33}$$

Employing the final indirect cost function presented in the above table we estimate $\Delta m(t),\ rm(t\text{-}1),$ and $\overline{C}_R,$ for discount rates of 2% and 15%. Table 3 provides the descriptive statistics for $\Delta m(t),\ m(t\text{-}1),$ and $\overline{C}_R$.

TABLE 2: Descriptive Statistics for Traditional Test Data

Variable	*Mean*	*Standard Deviation*	*Minimum*	*Maximum*	*N*
$\Delta m(t)$	0.0034	0.6440	-8.4977	8.4220	414
$m(t-1)$	0.1614	0.4999	0.0055	8.6601	414
$\overline{C}_R$	- 0.0001	0.0005	-0.0044	-0.0000006	414

We specifically allow for parameter variation across firms, i.e.,

$$\Delta m_i(t) = \sum_{i=1}^{N} \left[\beta_{0i} D_i + \beta_{1i} r m_i (t-1) + \beta_{2i} \overline{C}_{Ri}(t) \right].$$

The tests are of the null hypothesis H_{0i} : $\beta_{0i} = 0$, and $\beta_{1i} = \beta_{2i} = 1$, $i = 1,...,5$. That is, if the data are consistent with this test of the theory: the constant term for each firm is not statistically different from zero, $\beta_{0i} = 0$; the parameter estimate on the discount take, lagged user cost is equal to one, $\beta_{1i} = 1$, and the parameter estimate on the firm's marginal cost with respect to remaining reserves is equal to one, $\beta_{2i} = 1$. The parameter estimates are presented in Table 3.

TABLE 3: Traditional Test with HS User Cost Measure: $m = -\overline{C}_q$

Parameter	***Estimate***	***Standard Error***
β Firm 1	0.0785[c]	0.0452
β Firm 2	0.0142	0.2009
β Firm 3	0.1744	1.1014
β Firm 4	- 0.1993	0.6833
β Firm 5	0.0189	0.0783
β Firm 1 User Cost (t-1)	- 0.9776[a]	0.0505
β Firm 2 User Cost (t-1)	- 0.7971	1.3459
β Firm 3 User Cost (t-1)	- 0.9099	3.7031
β Firm 4 User Cost (t-1)	- 0.7222	1.2791
β Firm 5 User Cost (t-1)	- 0.1827	0.6814
β Firm 1 $\overline{C}_R$	-4701.38[a]	1409.65
β Firm 2 $\overline{C}_R$	-2410.18	22864.93
β Firm 3 $\overline{C}_R$	781.51	8525.03
β Firm 4 $\overline{C}_R$	-2498.21	5716.76
β Firm 5 $\overline{C}_R$	-64.35	230.31

R^2(adjusted) = 46.7% Level of significance: a=0.01, c=0.15

The calculated *F*-statistics for the restricted transition equation for each firm (at *r* = 2% and *r* = 15%) are presented in Table 4. F_{CRIT} = 2.63, for $\alpha = .05$. The restrictions on β_{0i}, β_{1i} and β_{2i} are tested individually for each firm using an asymptotic *F*-test. Note in Table 3 that the parameter estimates for Firm 1 are significantly different from zero, while those for the other firms are not. However, for completeness we present the results for all five firms. For Firm 1 the $F_{CALC} > F_{CRIT}$ at both discount rates, resulting in a rejection of firm behavior consistent with the theory. In the case of the other four firms $F_{CALC} < F_{CRIT}$ for both discount rates, however, the unrestricted parameter estimates are not significantly different from zero, thus we do not find valid support for the theory.

TABLE 4: *F*-Test

Firm	***F_{CALC} at Discount Rate: F_{CRIT}=2.63 (3,411)***		
	2%	***15%***	***Result***
1	130.3	166.6	Reject at both levels
2	0.13	0.19	Do not reject*
3	0.02	0.04	Do not reject*
4	0.11	0.20	Do not reject*
5	0.03	0.46	Do not reject*

* Parameter estimates not statistically significantly different from zero

Coupled with the results in Chermak and Patrick (2002), these results indicate that even when we alleviate any differences in the measurement of the user cost between the traditional test and the HS test are contradictory in terms of rejecting the theory or not.

5.0 The HS versus the Traditional Test

Alternative measures of the *in situ* resource price developed in Section 3.0 can be used to evaluate the alternative testing methodologies found in the extant literature. The parametric test methodologies found in the literature depend on what function is estimated with which to test the theory, i.e., the traditional test or the HS test. We consider these results in terms of structure of the econometric tests and the restrictions implied on the models.

Let $\bar{C} = g\left(\boldsymbol{\beta}, z, q, R, \mathbf{W}, \varepsilon\right)$ be the nonlinear parameterized model of the indirect cost function, where $\boldsymbol{\beta}$ is a vector of *K* parameters to be estimated and ε represents error. The unrestricted model in the HS test is then

$$\bar{C} = g\left(\boldsymbol{\beta}, z, q, R, \mathbf{W}, \varepsilon\right). \tag{34}$$

Using Proposition 2, Lemma 1, and (34), the dynamic optimality condition is

$$-g_q\left(\boldsymbol{\beta}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t = (1+r)\left[-g_q\left(\boldsymbol{\beta}, z, q, R, \mathbf{W}, \varepsilon\right)\right]\Big|_{t-1} + g_R\left(\boldsymbol{\beta}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t. \tag{35}$$

For the restricted model, (34) is estimated subject to (35), which is the form of the transition equation associated with the HS test.

For the traditional test, define the indirect cost function analogous to (34) as

$$\bar{C} = g\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right), \tag{36}$$

where the parameters $\boldsymbol{\gamma}$ are analogous to $\boldsymbol{\beta}$ in the cost function test, both are *K*-dimensional vectors. The transition equation, using Proposition 2 and Lemma 1 with (36), is then parameterized as

$$\begin{aligned} &-g_q\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t \\ &\quad = \theta_0 + \theta_1 (1+r)\left[-g_q\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\right]\Big|_{t-1} + \theta_2 g_R\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t, \end{aligned} \tag{37}$$

where $\theta_i,\ i = 0,1,2,$ are additional parameters required to parameterize the traditional test. (37) is the unrestricted transition equation for the traditional test.

The restrictions on (37) implied by exhaustible resource theory are $\theta_0 = 0$ and $\theta_1 = \theta_2 = 1$. Applying these restrictions to (37) implies the restricted transition equation

$$-g_q\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t = (1+r)\left[-g_q\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\right]\Big|_{t-1} + g_R\left(\boldsymbol{\gamma}, z, q, R, \mathbf{W}, \varepsilon\right)\Big|_t . \quad (38)$$

Note that the only differences between the restricted model for the HS test (equations (34) and (35)) and the restricted model for the traditional test (equations (36) and (38)) are the $\boldsymbol{\gamma}$ and the $\boldsymbol{\beta}$. If $\boldsymbol{\gamma} = \boldsymbol{\beta}$ then (35) and (38) are equivalent and (34) and (36) are equivalent. If the same data and estimation techniques are used in the HS and traditional tests, then $\boldsymbol{\gamma} = \boldsymbol{\beta}$ across the models. The restricted models are equivalent.

Now consider the respective unrestricted models. For the traditional test the unrestricted model is given by (37), the transition equation with K γ parameters and three θ parameters to estimate. The unrestricted model for the HS test is simply equation (34), which has K parameters to estimate. The traditional test requires more degrees of freedom and so, on efficiency grounds, the HS test is preferred. However, this is of little, if any, consequence in large samples.

While the restricted models are equivalent under either test approach, the unrestricted models are markedly different. All else equal, the difference in results from these two tests (if performed on identical data, using an identical cost function and identical measures of *in situ* price, using identical statistical methods) is attributable to the impacts of the distinct unrestricted equations.

It is an empirical question as to whether the alternative structures and relative efficiency of the tests will necessarily lead to divergent conclusions. (35) and (37) provide the basis to test for statistical differences in the transition equations across the two tests for a specific functional form.

We derive the econometric estimate of the transition equations for the Cobb-Douglas cost function and then empirically test the statistical equivalence of the parameter estimates in the Chermak and Patrick (2001) dataset.

For the HS test, the indirect cost function is restricted by the transition equation;

$$m_t = (1+r)m_{t-1} + \bar{C}_R. \tag{39}$$

Substituting the Lemma 1 result into (39) and rearranging, leads to

$$\frac{q_t}{R_t} = \gamma_0 + \gamma_1 \left[(1+r) \left(\frac{\bar{C}_{t-1}}{\bar{C}_t} \frac{q_t}{q_{t-1}} \right) \right] + \varepsilon_t, \tag{40}$$

the unrestricted version of the transition equation for the HS test. The theoretical restriction implies $\gamma_0 = \gamma_1 = -\frac{\beta_1}{\beta_2}$, where β_1 and β_2 are the parameter estimates from (32).

The traditional test involves directly estimating the transition equation. That is,

$$m_t = \theta_0 + \theta_1 (1+r) m_{t-1} + \theta_2 \bar{C}_R, \tag{41}$$

subject to the unrestricted cost function, (32) , which is used in measuring the current value user cost, and the joint restrictions $\theta_0 = 0$ and $\theta_1 = \theta_2 = 1$. Econometrically;

$$\frac{q_t}{R_t} = \eta_0 + \eta_1 \left[(1+r) \left(\frac{\bar{C}_{t-1}}{\bar{C}_t} \frac{q_t}{q_{t-1}} \right) \right] + \eta_2 \left(\frac{q_t}{\bar{C}_t} \right) + \varepsilon_t, \tag{42}$$

a form of the unrestricted transition equation for the traditional test. (42) and (40) are the transition equations we compare. Restricting (42) by $\eta_0 = \eta_1 = \gamma_0 = \gamma_1$ and $\eta_2 = 0$ allows us to test for statistical differences between (42) and (40). Employing the previously discussed Chermak and Patrick data set, we test for differences in the transition equations. Allowing for differences across firms and across interest rates, an F-test is employed. Table 5 reports the results where F_{CRIT}=3.0 for α=.05. In all cases, $F_{CALC} > F_{CRIT}$, so the null is rejected.

Thus, for this data set, the parameter estimates on the transition equations associated with the two tests are statistically significantly different. The divergent test results from this dataset can be attributed to the differences in the structure of the tests.

Table 5: F-tests of the Transition Equations

Firm	***F_{CALC} at Interest Rate: F_{CRIT}=3.0 (2,396)***						
	2%	***5%***	***7%***	***10%***	***15%***	***20%***	***Result***
1	11.34	11.34	11.35	11.35	11.35	11.35	Reject at all levels
2	4.90	4.89	4.89	4.88	4.86	4.85	Reject at all levels
3	7.92	7.83	7.77	7.68	7.53	7.40	Reject at all levels
4	15.01	14.90	14.83	14.74	14.58	14.45	Reject at all levels
5	63.49	63.33	63.22	63.06	62.81	62.56	Reject at all levels

6.0 Summary and Conclusions

Conclusions in the empirical testing literature concerning the validity of the theory of exhaustible resources in explaining resource owners' production decisions are inconsistent.

This paper examines the differences in the primary testing methodologies and alternative measures of the *in situ* resource price. We begin by developing a theoretical framework for the vertically integrated firm for alternative measures of the unobservable *in situ* resource price and demonstrate that the HS and the traditional measures of user cost applied in the literature are theoretically equivalent. This result has both theoretical and empirical significance. In regards to the theoretical importance of this result is that user cost can be appropriately measured as the negative of the marginal indirect final cost of gross production for the vertically integrated firm, regardless of test methodology. Empirically, this means an explicit price path is not required to test the theory. Thus, reducing the required data and also reduces potential bias in results due to differences in the price path assumed by the producer and tested with by the researcher.

An evaluation of the consistency of the two most used empirical test methodologies; the traditional test, which directly restricts the estimated transition equation and the newer, duality-based Halvorsen and Smith test, finds that while the methodologies produce exactly the same implied restricted econometric model, the unrestricted models diverge. The traditional test requires more degrees of freedom than the Halvorsen and Smith test and, all else equal is the less efficient. Our empirical results show that the transition equations associated with the two tests are statistically different, leading to the conclusion that some empirical results from tests the theory of exhaustible resources may be an artifact of the econometric testing methodologies.

Much work remains to be done. First, a test to directly compare the implied unrestricted models across the tests of Hotelling's theory are of interest. Complicating this, as we've shown above, is that these unrestricted models are non-nested and their implied functional forms diverge. Second, Monte Carlo experiments could be informative in terms of properties of the tests. Third, a theoretical development and application of nonparametric tests of the theory would be useful to remove parametric assumptions on the vertically integrated exhaustible resource producing firms' underlying production technologies. Fourth, existing tests, including those in this paper, are conditional on additional assumptions that can also be tested rather than maintained (see Caputo 2006 for a theoretical development in this regard).

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