

*A Microeconomic Test of the Theory of Exhaustible Resources**

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Abstract

This paper generalizes extant developments of the economic theory of exhaustible resource production, derives and extends a Halvorsen and Smith (1991) type test of the theory, and applies the test to a sample of natural gas resources. To facilitate our empirical test, we extend the model developed in Chermak and Patrick (1995) to explicitly account for the fact that the extracted resource (gross product) must be processed to obtain the final (saleable) product. Duality theory is used to derive econometric models with which the theory is statistically tested, using panel data from 29 natural gas wells. Shadow prices of the resource stock through time, which are generally unobservable but necessary for the test, are estimated via the indirect cost function. Contrary to the extant literature, we find, *inter alia*, (i) that, at any point in time, *ceteris paribus*, the *in situ* resource price (a) decreases with gross production and (b) increases with final production, and (ii) we cannot reject the theory of exhaustible resources, i.e., producer behavior is consistent with the theory. (*JEL* Q00, L71)

1.0 Introduction

In 1914, Gray argued that modification of Ricardo's 19th century classical notion of rent is required to accurately account for the exhaustibility of natural resources. Gray, focusing on the theory of the mine as the appropriate level of aggregation for the analysis, hypothesized that a price-taking profit maximizing firm, facing u-shaped costs,

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would adjust mine production so that the difference between price and marginal costs of production would increase at the rate of interest.¹ Seventeen years later, Hotelling (1931), *inter alia*, formalized this rule at the level of the mine, the firm, and the industry. This research received little attention until the second half of the twentieth century when concerns of resource scarcity resulted in a renewed interest in the economics of exhaustible resources. A substantive body of literature extending and empirically testing the theory has emerged.² However, lack of data has generally hampered empirical tests of the theory at all but highly aggregated levels. Little empirical indication of support for the theory of exhaustible resources has been found, leaving the question of the theory's usefulness in describing and predicting the behavior of exhaustible resource producers unresolved.

Existing tests of the theory have, for the most part, used annual data aggregated across industries (Halvorsen and Smith 1991, Slade 1982, Smith 1979), or monthly industry level data (Heal and Barrow 1980).³ However, Blackorby and Schworm (1982) demonstrate that aggregating heterogeneous deposits of a natural resource can result in an aggregate characterization of technology that doesn't represent the resource deposits or actual production technologies. Tests that do not allow for optimal production from the perspective of each individual deposit have little hope of yielding optimal aggregate production levels. As such, the focus of this paper is on whether the actual production of individual resource deposits is consistent with the theory of exhaustible resources. The ultimate question addressed here is the

¹ Cairns (1994) proposes this be called the "Gray r -per-cent rule."

² For a review of the literature, including extensions to the original model, see, for example, Sweeney (1993). Withagen (1998) discusses empirical tests of the theory.

³ Farrow (1985), Slade and Thille (1997), and Stollery (1983) are exceptions, they test the theory using data for a single mine or firm. However, no strong support for the theory is found, as is discussed in more detail below.

empirical relevance of a model that is widely used as a positive description of the behavior of exhaustible resource producers.

This paper develops a test of the theory of exhaustible resources at the level of individual natural resource deposits. Chermak and Patrick (1995) generalize the economic theory of exhaustible resource production to (a) allow decreasing marginal production costs, (b) consider physical bounds on periodic production, and (c) account for interdependency (feedback) between the stock of the resource, the periodic production bounds, and the chosen production path. In this paper we extend this model to explicitly recognize that the firm not only extracts the (gross) resource but also at least partially processes it to arrive at a final product to market. These generalizations are empirically relevant for the natural gas resources we analyze.⁴ Duality theory is used to provide the econometric model with which the theory is statistically tested. The data set, comprised of 29 wells from five natural gas producing firms, is unique and of sufficient detail to allow rigorous testing of the theory. Shadow prices of the resource stock through time are necessary for the test but are not directly observable. Duality theory is used to develop observable measures of these shadow prices and test the exhaustible resource theory under alternative specifications of the indirect final cost function, as well as the gross cost function. We find that the *in situ* price of the resource decreases with gross (unprocessed) production at any point in time, and increases with final production at any point in time. Our tests do not reject the economic theory of exhaustible resources. That is, our estimated final and gross cost functions indicate that the natural gas resources included in our data were produced in a manner that is consistent with exhaustible resource theory.

The format of the paper is as follows. The next section reviews extant empirical tests of the exhaustible resource theory, followed by a discussion of the appropriate level of aggregation at which to test. Our generalized model of exhaustible resource production is presented in Section 4. The econometric model, based on this theory, as well as the specific test of the theory we employ are presented in Section 5. Data and econometric results are provided in Section 6. A summary and our conclusions are in Section 7.

2.0 Previous Tests of Exhaustible Resource Theory

The implications of increased natural resource scarcity and its effects on economic growth have been discussed since at least the 19th century (Ricardo 1817, Gray 1914, Hotelling 1931). However, the theory was not empirically tested until the second half of the 20th century. Extant empirical tests of the theory have been categorized by Slade and Thille (1997) into three groups; 1) price behavior, 2) shadow price behavior, and 3) the Hotelling-Valuation Principle.

The earliest tests were price behavior tests and focused primarily on estimating price trends as indicators of scarcity. That is, increasing prices indicate increasing scarcity. Barnett and Morse (1963) plotted annual (1870-1957) industry-level unit prices of extractive products for five group aggregates (all extractive, minerals, agriculture, forestry, and fishing). With the exception of forestry and possibly fishing, they found horizontal price trends for the groups. Smith (1979), in an econometric study of four natural resource aggregates, utilized annual data

⁴ Contrary to conventional natural gas production, the common pool and joint-production (with oil) problems are not relevant here since we analyze data from "tight-sand" natural gas wells [see Hargoort

(1900-1973) and found little evidence of upward trends in prices.⁵ Smith (p. 426) concludes "... evaluations of resource scarcity without detailed analysis of the character of the markets for the specific commodities within each aggregate, as well as the institutional changes during the period, do not seem possible." Slade (1982) analyzed long run price movements using annual data (1870-1978) aggregated into major metals and fuels commodities. Estimating a quadratic functional form, she found support for three distinct price trends; declining, flat, and increasing. Slade hypothesized the trends come at different points in the life cycle of the exhaustible resource.

The above studies were based on the examination of product prices and did not test the coherence of market data with exhaustible resource theory. Heal and Barrow (1980) examined industry level monthly tin, lead, zinc, and copper data (July, 1965 through June, 1977), modeling metals prices as a function of interest rates and changes in interest rates. Their hypothesis is that if an exhaustible resource is an asset, whose price appreciation is a factor in the holding decision, there are potential arbitrage opportunities between resource markets and other capital asset markets. They found the change in interest rate, rather than the interest rate itself, was the significant independent variable. These results (based on commodity prices rather than *in situ* prices) are, as the authors conclude, "very tentative" and offer that the results "...suggest that the matter is considerably more complex than simple equilibrium theory would suggest" (p. 161).

Shadow prices are not directly observable and hence must be estimated. The shadow price behavior tests can be sub-divided into two types; estimates which include either explicit or

(1988, Chapter 7) for gas reservoir engineering details in this regard], as discussed in more detail below.

⁵ The groups included total extraction, minerals, agriculture, and forestry.

implicit price expectations.⁶ The former calculates shadow price as the difference between (expected) price and marginal cost, which requires specification of an explicit expected price path through time. This literature includes the following research.

Stollery (1983) tests the theory using a time series of *in situ* prices. Based on an extension of the Hotelling pure depletion model that includes the effects of depletion, possible technical change, and exogenous mineral discoveries, he estimates the *in situ* price of nickel for the International Nickel Company (INCO) as the difference between marginal revenue and marginal cost of extraction. A price leadership model, with INCO as the price leader, is the assumed market structure. A log-linear demand function and a Cobb-Douglas production function were estimated with annual data from INCO for 1952-1973. Stollery finds the resource rent for nickel rose over the time period at a rate compatible with a 15% discount rate. Employing a capital asset pricing model Stollery argues, given market conditions over this time period, a 15% discount rate is consistent for INCO.

Farrow (1985) tests the exhaustible resource theory using monthly price and cost data (January, 1975 through December, 1981) on an underground hard rock mine that produced several metals commodities as joint products. User cost is calculated as the difference between product price (estimated as a weighted average of the individual metals commodity prices) and the marginal extraction cost from an estimated translog cost function. Three model variations are tested; the basic model with a dynamic discount rate, the basic model extended to include price expectations, and an extension that include an output constraint. At the level of a single mine, Farrow does not find support for the exhaustible resource theory.

⁶ Chermak and Patrick (1999) present a unified analysis of extant tests of the theory.

Slade and Thille (1997) integrate the Hotelling theory and the capital asset pricing model (with extensions) and test this model using annual data (from Young 1992) for fourteen Canadian copper mines, which operated between 1956 and 1982. They estimate user cost as the difference between price and marginal extraction costs under several alternative restrictions. They do not reject the basic theory, although the authors point out the parameter estimates on the Hotelling variables are “vacuous.” Although the traditional Hotelling model coefficients have the expected signs and the test results fail to reject this modified Hotelling theory, as the authors note, only the depletion effect is marginally significant. When risk is incorporated via CAPM and extensions the empirical performance of the theory improves.

The second type of the behavior of shadow price tests employs duality theory, which results in the shadow price being estimated directly from the indirect cost function. The advantage of this test type is that price expectations are included implicitly through the actions of the firm, whereas the first group requires the price path to be explicitly included, which also does not allow for different expectations across producers.

Halvorsen and Smith (1984, 1991) utilize duality theory to demonstrate that the *in situ* price (user cost) of the resource at any time can be observed by differentiating the indirect cost function with respect to quantity produced. In their 1991 paper, an econometric model is derived which provides a statistical test of the theory utilizing annual data from the Canadian metal mining industry (1954-1974) to test the hypothesis that the time path of *in situ* prices satisfies the dynamic optimality condition. The optimality condition is tested under various discount rates, both constant and varying over the time horizon. The null hypothesis is rejected under all discount rates tested (2-20%). However, as the authors point out (p. 137) "...the data

used to estimate the econometric model are at a high level of aggregation, the empirical results obtained here should be considered as only tentative."

The third class of test, formulated by Miller and Upton (1985a, b), is a variation of estimating the transition equation, although it does not hold when there are stock effects in the cost function. The test is based on an alternative implication derived from Hotelling's analysis. It is essentially (1985a, p. 24), "... the value of a unit of reserves in the ground is the same as its current value above ground less the marginal cost of extracting it, regardless of when the reserves are extracted." This formulation was, in part, a response to lack of adequate time series data with which to test the more traditional implications of the theory. They test this valuation principle by regressing the market value of oil reserves on their estimated Hotelling values. The market values are based on a firm's stock prices, while the Hotelling values are based on well-head prices, extraction costs, and estimated reserves (extracted from Securities and Exchange Commission data).⁷ The (1985a) results are robust under a variety of specifications, and the empirical test finds support for the principle under the restriction of no stock effects on costs. The (1985b) results do not explain as much variation as the (1985a) data. Miller and Upton conjecture (1985b, p. 1015) the difference in results is due to the different pricing histories from the two periods, coupled with measurement error.

Withagen (1998) reviews the extant empirical literature and finds the empirical tests have been hampered by data difficulties (both lack and aggregation of), as well as lack of information concerning price expectations. The appropriate level of aggregation in exhaustible resource analyses is examined by Blackorby and Schworm, who theoretically explore two

aggregation scenarios with firms extracting from (i) separate resource deposits and (ii) a single resource deposit. They find that an aggregated model may not be able to replicate actual behavior of industries or economies extracting from heterogeneous deposits with heterogeneous technologies. Furthermore, if the restrictions implied by such aggregation assumptions are not checked, the resulting models may be internally inconsistent with the individual technologies.

If aggregation is appropriate, at what level should we aggregate; the firm level or the industry level? The following section gives a brief overview of natural gas reservoirs and their affect on the appropriate level of analysis.

3.0 Level of Analysis

As stated above, Blackorby and Schworm find aggregation may not replicate actual behavior for an industry when firms are producing from heterogeneous deposits with heterogeneous technologies. Our research focuses on natural gas resources. In order to determine the level of analysis at which to conduct this research, a brief description of natural gas reservoirs and how they are produced is important to determine if aggregation is appropriate, and if so, at what level.

The majority of natural gas reservoir rocks were formed either by (1) deposition of rock fragments of older rocks, or (2) chemical or organic precipitation. The amount of gas potentially available for production is a function of the reservoir characteristics, specifically; porosity, permeability, and water saturation, which measure the total amount of void space in the rock, the ability of fluids to move through the rock, and the amount of the void space that is

⁷ The 1985a data coverage is December 1979-August 1981, while the 1985b data coverage is August 1981-December 1983. The authors (1985b, p. 1015) point out that the first data set covers a time period of large

water-filled, respectively. These reservoir characteristics are affected not only by the depositional environment, but also by the post-depositional environment. The spatial environment can rapidly change and, consequently, not only can porosity, permeability, and water saturation vary between reservoirs, they can vary within reservoirs. It follows that the production potential of one well may not be the same as the production potential of another well in the same field. Aggregating wells within the same field, which have heterogeneous reservoir characteristics would be the equivalent of aggregating heterogeneous deposits.⁸

The second consideration in the level of analysis is technology. For natural gas production, this focuses on (i) how the well is completed, and (ii) how the well is produced. By completion, we refer to procedures that enhance recovery. The most common being fracturing which induces manmade conduits (fractures) from the wellbore into the surrounding reservoir to allow easier gas flow to the wellbore. The technology of hydraulic fracturing has progressed over the years, as has the understanding of the effect of fracture geometry on production. Fracturing techniques vary across the wells comprising our data, which implies heterogeneous technologies across these wells.

Production technology includes not only the physical diameter of the wellbore pipe, but also the production choice. Natural gas production is normally pressure driven. That is, the natural pressure within the reservoir is used as the mechanism for pushing the gas to the surface.

variation in oil prices, while the second data set has relatively little price variation.

⁸ An important consideration in the analysis is that of the common pool. High permeability indicates easy fluid flow, which can result in the common pool problem, that is, production in one sector of the field can drain gas from another sector, adversely affecting reserves. Tight gas reservoirs are defined (legally) as reservoirs with an average reservoir permeability of less than .01 millidarcy, which is very low. As the name might suggest, tight gas reservoirs do not exhibit easy flow of fluids through the formation and interaction between wells is not common. The data used in this research is from tight sand gas reservoirs, and so the common pool problem is not considered in the analysis.

The deeper the reservoir, in general, the higher the initial reservoir pressure.⁹ Low production quantities, in a given time period, are achieved by using only a small portion of the reservoir pressure, i.e., restricting production by allowing only a portion of the physically producible gas to come to surface. All wells are not produced in a similar manner. This brings us to our final aggregation consideration; the effect of past production on future potential. As a well is produced, the reservoir pressure declines. Different production paths lead to different pressure decline profiles, which result in different future potentials. This further complicates the ability to aggregate.

Given the heterogeneity of wells from the physical, technological, and past production history aspects, the appropriate level of analysis is at the level of the deposit, which is the individual well in the case of natural gas from tight sands.

4.0 The Theory of Exhaustible Resources

This section presents the profit-maximizing problem for the price-taking firm producing a non-renewable resource subject to a capacity constraint on periodic production as well as the traditional resource stock constraint. Furthermore, we consider the effect of extraction on the production capacity and stock constraints, common to natural gas production, as well as the fact that some processing of gross production may be necessary.

Formally, we consider maximum production at t , $Q(t) = Q(\mathbf{q})$, (where $\mathbf{q} = \{q(t) : t \in [0, t]\}$), as a functional of the function $q(t)$, the gross production at time t . Each

⁹ Initial reservoir pressure is a function of, among other things, the depth from surface of the reservoir. The U.S. Energy Information Administration (1990) reports natural gas production depths from less than 3,000 to in excess of 15,000 feet. Average production depths for active wells range from 5,500 feet for pre-1957 vintage wells to 6,600 feet for 1985-1988 vintage wells. Again, heterogeneity among wells is indicated.

$Q(t)$, $t \in (t, T]$, may be affected by previous gross production levels, $q(t)$ for all $t \in [0, t]$.¹⁰

$R(t)$ is the associated continuous and piecewise differentiable state variable, also defined on the time interval $t \in [0, T]$. Let $\nabla_q Q$ be the first partial

derivative of the functional Q with respect to the function $q(t)$ at the point t .¹¹ The stock of the resource at $t = 0$ is then¹²

$$R(0) = \int_0^T Q(\mathbf{q}, t) dt. \quad (1)$$

The initial stock of the resource will vary with the chosen production path unless

$q(t) = Q(\mathbf{q})$ for all $t \in [0, T]$. (1) implies that the remaining stock at any t is

$$R(t) = \int_0^T Q(\mathbf{q}, t) dt - \int_0^t q(x) dx. \quad (2)$$

The transition equation on the resource stock is

$$\dot{R}(t) = S(\mathbf{q}) - q(t), \quad R(T) \geq 0, \quad (3)$$

where $S(\mathbf{q})$, the effect of past production on the recoverable stock, is

$$S(\mathbf{q}) = \int_0^T (\nabla_q Q \nabla_t \mathbf{q} + Q_t) dt \leq 0. \quad (4)$$

This effect is specific to pressure produced resources, such as natural gas and oil. The signs of the first and second derivatives of $S(\mathbf{q})$, which follow from reservoir engineering theory, are

¹⁰ $q(t)$ is a piecewise continuous control function with, at most, a finite number of discontinuities, with finite jumps (i.e., one-sided limits) at each point of discontinuity.

¹¹ In general, if F maps a Banach space Y into a Banach space Z and there is a bounded linear operator L mapping Y into Z such that $\|F(y+h) - F(y) - Lh\| = o(\|h\|)$, $h \in Y$, then Lh is the Fréchet differential of F at y , and the operator L is the Fréchet derivative of $F(y)$ at y , which we will denote $L = \nabla_y F$. There is a complete calculus for Fréchet derivatives, see, e.g., Luenberger (1969). For further detail in this regard, and for the optimal control of functional equations and Fréchet derivatives see Warga (1972) and Li and Yong (1995).

¹² $R(0)$ is a function not only of the physical reservoir characteristics but also of the production choice through, among other things, capital inputs or contract constraints. In the case of natural gas production from tight sands, the capital expenditure for completion is a determinant of the resultant fracture, which, in turn, affects $R(0)$. We do not directly consider such issues here as they would serve only to complicate the

nonpositive. $S(\mathbf{q}) = 0$ for non-pressure produced resources, in which case (3) becomes

$\dot{R}(t) = -q(t)$, i.e., the transition equation in traditional exhaustible resource models.

Gross production from the well is not necessarily the amount of gas that can be sold by the firm. The removal of non-hydrocarbon gases from the natural gas and other shrinkage generally prohibit this.¹³ Let $q(t) = g(\mathbf{X}^s, R, t)$ represent the gross production function where $\mathbf{X}^s = \{x_j(t) : j = 1, \dots, J\}$ are inputs into the gross production process. The final production of natural gas (i.e., the amount that is input into the pipeline to be transported to purchasers) is given by $z(t) = f(\mathbf{X}^f, q, t)$, where $\mathbf{X}^f = \{x_k(t) : k = 1, \dots, K\}$ are inputs used to process gross production to arrive at the final product at each t , $z(t)$. Naturally, final production can be no greater than gross production, i.e., $z \leq q$.

The firm's problem is to choose the input production paths, the gross production path, and the production time horizon to maximize profits, π , of the quasi-fixed resource.¹⁴ The firm's objective is then to choose \mathbf{X}^f , $q(t)$ for $t \in [0, T]$, and T to maximize

$$\mathbf{p} = \int_0^T e^{-rt} [P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^s, q(t), R(t), t)] dt, \quad (5)$$

where $P(t)$ is the output price per unit of final production, $z(t)$, $\mathbf{W}^f = \{w_k(t) : k = 1, \dots, K\}$ are processing input prices, $\mathbf{W}^s = \{w_g(t) : j = 1, \dots, J\}$ are prices of inputs into gross production, and the gross cost function (the dual to the gross production function) is given by

economics without adding any compensating insight. See Kuller and Cummings (1974) in the case of oil production and Patrick and Chermak (1992) and Chermak and Patrick (1995) in natural gas production.

¹³ The US Energy Information Administration (*Natural Gas Annuals*, various years) distinguishes gross withdrawals, defined as full well-stream volume excluding condensate separated at the lease from dry natural gas production defined as gross withdrawals less gas diverted for repressuring, quantities vented or flared, non-hydrocarbon gas removed in the treatment or processing, and extraction losses.

¹⁴ Generally, the firm would have a number of wells under its control and, with costs estimated at the well level, would choose the number of wells to produce and simply sum revenues and costs of all wells.

$$C(\mathbf{W}^g, q(t), R(t), t) \equiv \min_{\mathbf{X}^g} \mathbf{W}^g \mathbf{X}^g \text{ s. t. } q = g(\mathbf{X}^g, R, t). \quad (5')$$

Profits are maximized subject to the resource stock constraint, (3), and bounds on gross production at any time

$$0 \leq q(t) \leq Q(\mathbf{q}), \quad (6)$$

where $\nabla_q Q \leq 0$, which implies, e.g., that increasing q at any t may reduce Q for all future t , and $\nabla_{qq} Q \geq 0$. The upper bound on periodic production, Q , is a natural physical constraint, based on current reservoir pressure. Initial reservoir pressure is a function of depth and formation characteristics. Once extraction begins, the upper bound declines over time due to declining reservoir pressure. The rate of decline in pressure is affected by past production. In practice Q and R are incorporated into the economic model through reservoir engineering theory.¹⁵

The Hamiltonian for this problem is

$$H(t) = e^{-rt} [P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t)] + I [S(\mathbf{q}) - q(t)], \quad (7)$$

where I is the multiplier (*in situ* resource price) on (3), the transition equation of the resource stock. The generalized Hamiltonian is

$$L(t) = H(t) + \mathbf{h}[Q(\mathbf{q}) - q(t)], \quad (7')$$

where \mathbf{h} is the multiplier on (6), the upper bound production constraint at each t . Necessary conditions include

$$-L_R = \dot{I} = e^{-rt} C_R, \quad (8)$$

However, considering the number of wells to produce adds to the notational complexity of the analysis without providing compensating insights or any qualitative differences in the results presented below.

¹⁵ See Patrick and Chermak (1992) and Chermak, et al., (1999).

$$\begin{aligned}
& L_q^*(q - q^*) + \sum_{k=1}^K L_{x_k}^*(x_k - x_k^*) = e^{-rt} \sum_{k=1}^K (Pz_{x_k} - w_k)(x_k - x_k^*) \\
& + [e^{-rt}(Pz_q - C_q) + \mathbf{I}(\nabla_q S(\mathbf{q}^*) - 1) + \mathbf{h}(\nabla_q Q(\mathbf{q}^*) - 1)](q - q^*) \leq 0 \quad (9) \\
& \forall q \in \{q : [\nabla_q Q(\mathbf{q}^*) - 1](q - q^*) \geq 0, Q(\mathbf{q}^*) - q^* \geq 0\},
\end{aligned}$$

$$\mathbf{h} \geq 0, (\mathbf{h} = 0 \text{ if } Q(\mathbf{q}^*) > q^*), \quad (10)$$

$$\begin{aligned}
H(T^*) &= e^{-rT^*} [P(T^*)z(T^*) - \mathbf{W}^f(T^*) \cdot \mathbf{X}^f(T^*) - C(\mathbf{W}^g(T^*), q^*(T^*), R^*(T^*), T^*)] \\
& + \mathbf{I}(T^*)[S(\mathbf{q}^*(T^*)) - q^*(T^*)] = 0, \quad (11)
\end{aligned}$$

and the transversality condition is

$$\mathbf{I}(T^*) \geq 0 \quad (= 0 \text{ if } R^*(T^*) > 0), \quad (12)$$

where the “*” superscripts represent the optimal level of the respective variable. If (7) is concave in (q, \mathbf{X}, R) , where $\mathbf{X} = \mathbf{X}^f \cup \mathbf{X}^g$, for each $t \in [0, T]$ then (8)-(12) are necessary and sufficient for (q^*, \mathbf{X}^*, R^*) to solve the problem. If (7) is strictly concave in (q, \mathbf{X}, R) then (q^*, \mathbf{X}^*, R^*) are unique. Eswaren, Lewis, and Heaps (1983) and Fisher and Karp (1993) consider, *inter alia*, the non-existence of a competitive equilibrium when extraction costs are non-convex, as we find in our empirical consideration below. However, in our model, in addition to the physical restriction on periodic production, we consider, among other things, the naturally occurring production effects on the stock of the resource. This implies an interior solution to our problem may exist even if marginal cost is decreasing with production at any time. Although more complex in the present model due to the additional choice variables from the explicit consideration of processing gross production into final product, the demonstration of the existence of this potential interior solution follows directly from the proof developed in Chermak and Patrick (1995).

(12) is the present value of the terminal time *in situ* price and (11) is the necessary condition on choosing the optimal terminal time, T^* . (10) is the shadow price condition for the

physical capacity constraint. (9) is the optimality condition on gross production and other inputs into final production, q^* and \mathbf{X}^{f*} , for each $t \in [0, T^*]$. $I(t) \geq 0$ for all $t \in [0, T^*]$ is implied by (8) and (12), i.e., $I(T^*) \geq 0$ by (12) and (8) implies the present value user cost is declining over t , $\dot{I} = e^{-rt} C_R < 0$, since $C_R < 0$ (cost increases as the stock of the resource is depleted). (8) is the dynamic optimality condition for the exhaustible resource, which is used in the econometric tests developed in the next section.

5.0 The Econometric Model

This section develops alternative empirical measures of the *in situ* resource price and our econometric test of the theory of exhaustible resources.

The current value form of (8), the dynamic optimality condition for the exhaustible resource, is

$$\dot{m} = rm - L_R = rm + C_R, \quad (13)$$

where $m \equiv e^{rt} I$ is the current value *in situ* resource price,

$$L \equiv e^{rt} L(t) = P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t) + m[S(\mathbf{q}) - q(t)] + n[Q(\mathbf{q}) - q(t)],$$

and $n \equiv e^{rt} h$. $m(t) \geq 0$ for all $t \in [0, T^*]$ because $m(t) \equiv e^{rt} I(t)$ and $I(t) \geq 0$ for all

$t \in [0, T^*]$. (13) implies that the current value *in situ* resource price increases at the rate of

interest less the increase in future costs from extracting the marginal unit. (13) indicates that

empirical testing of the implications of exhaustible resource theory for the dynamic behavior of

producing firms requires estimates of m , r , and C_R . We now turn our attention to relationships

which allow us to measure m and C_R .

Consider the cost minimization problem of producing given levels of gross and final product. Let $q = \bar{q} = q^*$ and $z = \bar{z} = z^*$ be the profit maximizing levels of gross and final output, respectively, from the profit maximization problem of the previous section. The physical constraint on production, $\bar{q} \leq Q$, also applies. The Lagrangian for the problem is

$$\bar{L} = \mathbf{W} \cdot \mathbf{X} + \mathbf{g}[\bar{z} - f(\mathbf{X}^f, \bar{q}, t)] + \mathbf{d}[\bar{q} - g(\mathbf{X}^g, R, t)] + \mathbf{r}(Q - \bar{q}), \quad (14)$$

where $\mathbf{W} = \mathbf{W}^f \cup \mathbf{W}^g$, \mathbf{g} is the multiplier on the final production constraint, \mathbf{d} is the multiplier on the gross production constraint, and \mathbf{r} is the multiplier on the physical capacity constraint.

Necessary conditions include

$$\bar{L}_{x_j} = w_j - \mathbf{d}g_{x_j} = 0, \quad j = 1, \dots, J, \quad (15)$$

for inputs into gross production, and

$$\bar{L}_{x_k} = w_k - \mathbf{g}f_{x_k} = 0, \quad k = 1, \dots, K, \quad (16)$$

for the choice of inputs into final production. The condition on the multiplier of the well's periodic capacity constraint is

$$\mathbf{r} \geq 0 \quad (= 0 \text{ if } q < Q). \quad (17)$$

The solution to this cost minimization problem yields the (indirect) final cost function

$$\bar{C}(\mathbf{W}, z, q, R, t, \dots). \quad (18)$$

The following three envelope theorem results are useful in developing observable measures of the *in situ* resource price.

$$\bar{L}_q = \bar{C}_q = -\mathbf{g}^f + \mathbf{d} - \mathbf{r}, \quad (19)$$

$$\bar{L}_R = \bar{C}_R = -\mathbf{d}g_R, \quad (20)$$

and

$$\bar{L}_z = \bar{C}_z = \mathbf{g}. \quad (21)$$

Now consider the firm's problem of minimizing the cost of producing z , treating q as an endogenous input into the production of z . The inputs, \mathbf{X} , are now chosen to minimize costs, defined as $\mathbf{W} \cdot \mathbf{X} + mg(\mathbf{X}^g, R, t)$, where m is the (unobserved) shadow price of the resource *in situ*, subject to the constraints on final and periodic production, $z = \bar{z} = z^*$ and $0 \leq q \leq Q$. The Lagrangian for the cost minimization problem when gross production, q , is endogenous is then

$$L = \mathbf{W} \cdot \mathbf{X} + mg(\mathbf{X}^g, R, t) + \tilde{\mathbf{g}}[\bar{z} - f(\mathbf{X}^f, g(\mathbf{X}^g, R, t), t)] + \tilde{\mathbf{r}}[Q - g(\mathbf{X}^g, R, t)], \quad (22)$$

where $\tilde{\mathbf{g}}$ is the multiplier on the final production constraint and $\tilde{\mathbf{r}}$ is the multiplier on the physical capacity constraint. Necessary conditions include

$$L_{x_j} = w_j + [m - \tilde{\mathbf{g}}_g^f - \tilde{\mathbf{r}}]g_{x_j} = 0, \quad j = 1, \dots, J, \quad (23)$$

for the optimal choice of inputs into gross production;

$$L_{x_k} = w_k - \tilde{\mathbf{g}}f_{x_k} = 0, \quad k = 1, \dots, K, \quad (24)$$

for the optimal choice of inputs (other than q) into final production and

$$\tilde{\mathbf{r}} \geq 0 \quad (= 0 \text{ if } q < Q). \quad (25)$$

Envelope theorem results include

$$L_R = C_R = (m - \tilde{\mathbf{g}}_q^f - \tilde{\mathbf{r}})g_R, \quad (26)$$

The optimal \mathbf{X} for this problem is identical to that from the previous problem since $q = \bar{q} = q^*$ and $z = \bar{z} = z^*$. (16) and (24) then imply that $\mathbf{g} = \tilde{\mathbf{g}}$. (15), (17), (23), and (25) then lead to $\mathbf{r} = \tilde{\mathbf{r}}$ and $\mathbf{d} = -m + \tilde{\mathbf{g}}_q^f + \tilde{\mathbf{r}}$. Substituting these expressions into (19) leads to the result stated in Lemma 1.

Lemma 1. The current value *in situ* resource price is equal to the negative of the marginal final cost of gross production, i.e., $m = -\bar{C}_q$.

Halvorsen and Smith (1984, 1991), as discussed in Section 2, derived this result in their exhaustible resource model. Lemma 1 and $m \geq 0$ imply that $\bar{C}_q \leq 0$, i.e., the marginal final cost of gross production is nonpositive. The intuition here is that additional q , all else held constant, reduces the marginal cost of producing a given level of z . Lemma 1 also implies that the current value *in situ* resource price can be observed by differentiating the estimated indirect final cost function with respect to gross production, q . Analogous to the demonstration of Lemma 1, substituting $\mathbf{d} = -m + \tilde{\mathbf{g}}_q + \tilde{\mathbf{r}}$ into (26) and using (20), it follows that $\bar{L}_R = L_R$ and hence $\bar{C}_R = C_R$. Thus, by estimating the indirect final cost function, (18), and differentiating with respect to q and R we can observe m and C_R , which are required in testing (13), the dynamic optimality condition.

To obtain another observable measure of the *in situ* resource price, as well as gain further insight into Lemma 1, consider the profit maximization model in Section 4 and recall that the above cost minimization problems are evaluated at the profit maximizing levels of z and q . For simplicity in allowing us to focus on measuring the *in situ* price, we write the model in current value form, take the feedback effect as insignificant, and assume an interior solution.¹⁶ The current value Hamiltonian for the profit maximization problem is then

$$H(t) = P(t)z(t) - \mathbf{W}^f \cdot \mathbf{X}^f - C(\mathbf{W}^g, q(t), R(t), t) - mq(t). \quad (27)$$

Necessary conditions include

$$-H_R = \dot{m} = rm + C_R, \quad (28)$$

¹⁶ The relationship between gross and final production is more complicated than modeled here, and requires the incorporation of reservoir engineering theory, as well as the gas properties to discern it to a greater degree than in this consideration. Although we have made significant progress in such modeling (see Chermak, *et al.*, 1999), this is beyond the scope of this paper and is left for future research. Nonetheless, our relatively simple exhaustible resource models tested here are consistent with producer behavior.

$$H_q = Pf_q - C_q - m = 0, \quad (29)$$

$$H_{x_k} = Pf_{x_k} - w_k = 0, \quad (30)$$

$$\begin{aligned} H(T^*) &= P(T^*)z^*(T^*) - \mathbf{W}^f(T^*) \cdot \mathbf{X}^f(T^*) \\ &\quad - C(\mathbf{W}^g(T^*), q^*(T^*), R^*(T^*), T^*) - m(T^*)q^*(T^*) = 0, \end{aligned} \quad (31)$$

and the transversality condition is

$$m(T^*) \geq 0 \quad (= 0 \text{ if } R^*(T^*) > 0), \quad (32)$$

As above, since $\mathbf{I}(t) \geq 0$ and $m(t) \equiv e^{rt}\mathbf{I}(t)$, $m \geq 0$. Substituting (29) into (30) we have

$$(w_k f_q / f_{x_k}) = C_q + m. \quad (30')$$

(16) and (21) into (30') lead to $m = \mathbf{g}_q - C_q = \bar{C}_z f_q - C_q$. We summarize this result in the following Lemma.

Lemma 2. The current value *in situ* resource price is equal to the value (in terms of the marginal final cost of z) of the marginal product of q less the marginal gross cost, i.e.,

$$m = \bar{C}_z f_q - C_q \geq 0.$$

The gross cost minimization problem, (5'), in Section 4 implies $C_q \geq 0$. Given production occurs, the marginal product of q , f_q , is positive. We expect the marginal final cost (the marginal cost of producing the final product) to be positive, $\bar{C}_z > 0$. The value of the marginal product of q must then be greater than the marginal gross cost of q , $\bar{C}_z f_q > C_q$, for the inequality in Lemma 2 to hold.

Further development of the implications of Lemma 2, and other results which follow, are developed in Chermak and Patrick (1999).

The differential equation (13) can be solved to provide a more convenient form for testing the dynamic optimality condition since our estimated indirect gross and final cost function is (statistically) autonomous. (13), multiplied by e^{-rt} , can be rewritten as

$$d(e^{-rt}m) / dt = e^{-rt}C_R,$$

which, after integrating and rearranging, leads to¹⁷

$$m(t) - e^{rt}m(0) = e^{rt} \int_0^t e^{-rx} C_R dx,$$

or, in discrete time,

$$m(t) - e^{rt}m(0) = \sum_{x=0}^t (1+r)^{t-x} C_R.$$

Substituting $m = -\bar{C}_q$ and $C_R = \bar{C}_R$, from our results above, yields

$$e^{rt} [\bar{C}_q |_{t=0}] - \bar{C}_q = \sum_{x=0}^t (1+r)^{t-x} \bar{C}_R, \quad (13')$$

which is the form of the dynamic optimality condition we use to restrict the parameters of the indirect final cost function, (18), in econometrically testing the exhaustible resource theory.

Specifically, we want to test the null hypothesis, that (13') is satisfied by our parameter estimates for (18), against the alternative hypothesis, that (13') is not satisfied by our parameter estimates for (18). This hypothesis is tested with a Hausman (1978) specification test by estimating (18) with and without the restriction implied by (13'). We perform an analogous, although more complicated, test on the gross cost function, (5'), using Lemma 2.

¹⁷Rather than using the initial time we could use the traditional terminal time development of the constant of integration. Using the terminal time condition, $H(T)=0$, it is also easy to see that $m(t) \geq 0$. That is, since $P(T) - [\mathbf{W}^f(T) \mathbf{X}^f(T) + C(T)]/q(T) \geq 0$ and $C_R < 0$, we have

$$m(t) = e^{rt} \int_0^t e^{-rx} C_R dx + e^{rt} m(0) = e^{r(t-T)} \left[P(T) - [\mathbf{W}^f(T) \mathbf{X}^f(T) + C(T)]/q(T) \right] - e^{rt} \int_t^T e^{-rx} C_R dx \geq 0.$$

See Chermak and Patrick (1995) for further detail.

Hausman's test requires that we have (i) an asymptotically efficient estimator under the null hypothesis which is not consistent under the alternative hypothesis and (ii) another estimator that is asymptotically less efficient than the first under the null hypothesis and consistent under the alternative. Estimation of equation (18) and (5') provide consistent estimates of the parameters of the indirect final and gross cost functions, respectively, regardless of whether the time path of *in situ* prices conforms to (13'). Estimation of the constrained equations, (18) with (13') and (5') with (13'), provide consistent and asymptotically efficient estimates of the parameters of the indirect final and gross cost functions under the null hypothesis in each case, but inconsistent estimates under the alternative. Denote the first estimator by the vector $\hat{\mathbf{b}}$ and the second by the vector $\tilde{\mathbf{b}}$. Let $\hat{\mathbf{V}}$ be a consistent estimator of asymptotic variance-covariance matrix of $(\hat{\mathbf{b}} - \tilde{\mathbf{b}})$. The Hausman statistic is then

$$(\hat{\mathbf{b}} - \tilde{\mathbf{b}})' \hat{\mathbf{V}}^{-1} (\hat{\mathbf{b}} - \tilde{\mathbf{b}}), \quad (22)$$

which, under the null hypothesis, is asymptotically χ^2 distributed with degrees of freedom equal to the dimension of the vector \mathbf{b} .

6.0 Data, Empirical Specification and Results

The test of the theory of exhaustible resources is performed on our econometrically estimated indirect cost function developed above. The data set is comprised of 443 observations from 29 tight sand gas wells. Seventeen of the wells are located in Wyoming and twelve in Texas. The data is monthly, over the time period of 1987 through 1991.¹⁸

¹⁸ Monthly observations are used since they are an accepted norm in the industry. A possible concern would be if there is a detectable change on a monthly basis. Certainly from the production perspective, monthly changes are discernible. The Energy Information Administration (1990) shows monthly production declines between 4% and 8% a month during the first year of production, resulting in 50% drop in

The wells in our data set vary in age from the initial month of production through the 29th year of production. The wells also vary in, among other things, physical characteristics such as (i) gross pay thickness (21 to 133 feet), (ii) production depths (5,400 to 10,400 feet), and (iii) reservoir characteristics of porosity, permeability, and water saturation. The data, provided by five firms, include (1) recoverable reserves, (2) gross monthly production, (3) monthly extraction and production costs, (4) the producing formation, (5) location, and (6) the beginning month of production for each well, as well as the physical characteristics for each well. In addition to this data, described in detail in Patrick and Chermak (1992) and Chermak and Patrick (1995), we also use final production data from ten of these wells.

We specify the indirect final cost function in the form of a Generalized Cobb-Douglas, where monthly production costs, \bar{C} , are hypothesized to be a function of gross production, q , final production, z , remaining reserves, R , and production month, pm . We use a fixed effects model which stratifies the panel data by firm and year. Our general (unrestricted) specification of the indirect final cost function is

$$(FC_U) \quad \ln \bar{C} = \sum_{i=1}^N \mathbf{a}_i D_i + \sum_{y=2}^{Y_i} \mathbf{a}_y D_y + \mathbf{b}_q \ln q + \mathbf{b}_z \ln z + \mathbf{b}_R \ln R + \mathbf{b}_{pm} \ln pm + e,$$

$i=1, \dots, N$ firms, $y=1, \dots, Y$ years, and e_{wt} = the customary error term, a random number with zero mean. Since there is a strong relationship between z and q , this specification of the final cost function may be problematic in terms of separating the effects of z and q on final costs.

Therefore, in addition to the unrestricted form above (FC_U), we estimate two restricted forms of

production quantities from the beginning to the ending of the first year of production. During later production periods, the declines are lower. This results in a steeply downward sloping production path early in a well's life, followed by a fairly flat production path during later stages. Of course, the actual path and percentage changes during a given month depend on the specific well and the production plan followed by the producer.

the final cost function. Given the theoretical developments above, we estimate a function (FC_R) that restricts the parameter estimates on final and gross production to be of opposite sign but equal in absolute value (i.e., $\mathbf{b}_z = -\mathbf{b}_q$). We also estimate an average final cost function

$$(FC_A) \quad \ln \bar{C} - \ln z = \sum_{i=1}^N \mathbf{a}_i D_i + \sum_{y=2}^{Y_i} \mathbf{a}_y D_y + \mathbf{b}_q \ln q + \mathbf{b}_R \ln R + \mathbf{b}_{pm} \ln pm + e$$

which imposes a parameter estimate of positive one on z , i.e., constant marginal cost of final production. Halvorsen and Smith (1991) estimate an analogous average cost function in their test of the theory.¹⁹ Finally, we estimate an indirect gross cost function (GC), which is specified analogous to the general form of the final cost function with the restriction $z=0$, i.e.,

$$(GC) \quad \ln C = \sum_{i=1}^N \mathbf{a}_i D_i + \sum_{y=2}^{Y_i} \mathbf{a}_y D_y + \mathbf{b}_q \ln q + \mathbf{b}_R \ln R + \mathbf{b}_{pm} \ln pm + e.$$

The three final cost models (FC_U, FC_R, and FC_A) are employed as our empirical specifications of (18), while the gross cost function, GC, is employed as our empirical specification of (5'). Differentiating the resulting final cost specification (FC_U, FC_R, or zFC_A) with respect to q and R , applying Lemma 1, and substituting into (13') provides the parametric form of the dynamic optimality condition for the final cost models. Testing GC requires an analogous process, using Lemma 2, to derive the implied constraints on the parameters of the gross cost function.

Unfortunately, final production data are not available for all our observations on gross production and the other available data. However, as discussed in the theoretical development above, final production is a function of, and can be no greater than, gross production. To avoid discarding important information contained in the incomplete data and improve efficiency of our

estimates, missing z observations were estimated from the available data (Griliches 1986, Greene 1997) according to $\hat{z} = \hat{a}_z q^{\hat{b}_{zq}} pm^{\hat{b}_{zp}}$, subject to $z \leq q$, where parameters are defined analogous to the above. Production month is included to allow for changes in the composition of the natural gas over the life of the well. Changes in gas composition may, for instance, result in the increased production of non-hydrocarbon gases over time. Our econometrically estimated final production function is then

$$\ln z = -.3116 + 1.0349 \ln q - .0228 \ln pm,$$

$$(.0365) \quad (.0039) \quad (.004)$$

where standard errors are in parentheses.²⁰ For comparison sake, the average value on gross production is 19,057 mcf per month, relative to a mean of 18,557 for final production, with a standard deviations of 19,308 and 19,164, respectively.

The parameter estimates for the three specifications of the final cost function and the gross cost specification, are presented in Table 1.²¹ The first four parameter estimates are associated with gross production, final production, remaining reserves, and the production month, respectively. The remaining nine parameter estimates are fixed effects terms for firms one through five and years 1988 through 1991, respectively.

¹⁹ Contrary to this average cost specification, Halvorsen and Smith's (1991) specified average cost per unit *value* of output rather than the average cost per unit of output.

²⁰ The regression was estimated using full feasible generalized least squares (FGLS, see, e.g., Greene 1997). First order autocorrelation was indicated by the Durbin Watson statistic; an iterative Prais-Winsten algorithm was used in the correction. Results from both Ramsey RESET and White tests indicate the null hypothesis of homoskedasticity cannot be rejected. 127 observations were available to estimate this equation, which is also characterized by a $r = .41235$ (with a standard error of .08116) and a log-likelihood of 281.24.

²¹ As above, FGLS was employed for all regressions. Durbin Watson statistics indicated first order autocorrelation in each of the four models. An iterative Prais-Winsten algorithm was used in the corrections. Ramsey RESET and White tests indicate the null hypothesis of homoskedasticity cannot be rejected.

As expected, parameter estimates in all four models indicate costs increase as the resource is exhausted. The parameter estimate on R , the stock of the resource remaining at time t , is significant at the 5% level in FC_U and the 10% level in FC_R , FC_A , and GC . Production month is not significant in FC_U , but is significant at the 5% level in FC_R ; the 1% level in FC_A , and the 10% level in GC . In all specifications, the individual firm intercepts are positive and significant at the 5% level. The parameter estimates on the years are significant in 1989 and 1990 (at 10% and 5%, respectively) in FC_R and FC_A , and in 1990 (at 10%) in both FC_U and GC .

The estimated parameters on z and q are significant at least at the 5% level in all four regression equations. According to the theory developed above, the final cost function should be characterized by $\bar{C}_z \geq 0$ and $\bar{C}_q \leq 0$, while for the gross cost function, $C_q \geq 0$. This implies the estimated parameters should be $\mathbf{b}_z \geq 0$ and $\mathbf{b}_q \leq 0$ for final costs, and, for gross costs, $\mathbf{b}_q \geq 0$. With the exception of FC_U , the signs on the estimated parameters are as we would expect from the theory. Contrary to the theory, FC_U indicates the parameter on z is significantly negative, i.e., final costs are inversely related to final production.²² However, final production is naturally restricted to be no greater than gross production and there exists a strong relationship between gross and final production, although this does not prohibit obtaining significant parameter estimates on these variables. The parameter estimates \mathbf{b}_z and \mathbf{b}_q in FC_U have a relatively large and negative covariance. We therefore examine final cost functional forms which

²² Although there is some intuitive appeal to these results. Holding all else constant, increased final production means an increased ratio of final to gross product. Higher ratios of final to gross product translate into less non-hydrocarbon gases and water, which must be separated and disposed of during processing, which results in lower processing costs.

do not explicitly include separate independent variables for z and q , i.e., FC_R and FC_A , as specified above.

Next we statistically compare the three forms of the final cost function estimated. If we restrict $\mathbf{b}_z = 1$ and divide FC_U by z , we have FC_A , which is analogous the functional specification assumed in Halvorsen and Smith (1991). Restricting FC_U by $\mathbf{b}_z = -\mathbf{b}_q$, leads to FC_R . To compare the three forms of the final cost function we use a Wald statistic to test the following restrictions on the parameters of FC_U ;

$$FC_A: H_0 : \mathbf{b}_z = 1 \text{ versus } H_1 : \mathbf{b}_z \neq 1$$

and

$$FC_R: H_0: \mathbf{b}_z = -\mathbf{b}_q \text{ versus } H_1: \mathbf{b}_z \neq -\mathbf{b}_q.$$

The resulting Wald values are 9.383 and .000049, respectively. The Wald value for FC_A is greater than the critical χ^2 value, indicating we reject the null hypotheses (.05 level of significance) of the average final cost model. The Wald value for FC_R is less than the critical χ^2 value, indicating we do not reject the null hypotheses (.01 level of significance) that \mathbf{b}_q and \mathbf{b}_z are of opposite signs and equal in absolute value. Thus we have statistical support for the FC_R specification of the final cost function.

Contrary to the findings of others, our gross and final cost function estimates imply that marginal costs of gross and final production are decreasing with increases in q and z , respectively. That is, for GC, $C_q > 0$ and $C_{qq} < 0$, and, for FC_R , $\bar{C}_z > 0$ and $\bar{C}_{zz} < 0$. How the *in situ* resource price varies with changes in q and z is also of interest. Again, our results are contrary to extant results. To see this, differentiate FC_R with respect to q , which leads to $\bar{C}_q < 0$ and $\bar{C}_{qq} > 0$. Since $m = -\bar{C}_q$ (Lemma 1), $m_q = -\bar{C}_{qq} < 0$, which implies, *ceteris*

paribus, that the *in situ* resource price decreases with gross production at any point in time.

Considering the relationship between final production and the *in situ* resource price in an analogous fashion, we have $\overline{C}_{qz} < 0$ which leads to $m_z = -\overline{C}_{qz} > 0$. This implies, *ceteris paribus*, that the *in situ* resource price increases with final production at any point in time.

The dynamic optimality condition is tested under all three specifications of the final cost function using Lemma 1, the specified gross cost function using Lemma 2, and a range of discount rates for all models. If natural gas wells are produced in a manner consistent with the theory then the production decision is effectively made taking into account the opportunity cost of current production (the *in situ* resource price or user cost) and the path is economically optimal according to our theory. Table 2 presents the Hausman statistics for the four models at various discount rates. As indicated, the theory is not rejected at the 0.05 significance level in any of our estimated models or for any of the discount rates tested.²³ That is, our estimated final and gross cost functions indicate that the natural gas resources included in our data were produced in a manner that is consistent with Hotelling's theory.

Finally, we estimate the user cost (*in situ* resource price) from FC_R using Lemma 1. User cost across all wells ranges from \$0.000004 (per mcf) to \$3.18, with a mean value of \$0.0766, a median of \$0.0233, and a standard deviation of \$0.21. These values, however, are aggregated across the entire data set, which has a tendency to mask variations across wells. The importance of using the appropriate level of analysis is highlighted by the variation of user cost across wells. As can be seen from Table 3, there are significant differences across wells.

²³ As is common in the literature (e.g., Halvorsen and Smith 1991), we present the tests for discount rates of no greater than 20%. However, we performed the test for larger discount rates and reached exactly the same conclusions.

These differences across the wells may be attributed to differences in reservoir properties, age of the well, ownership, and location, among other things.²⁴

While Table 3 presents the summary statistics, it does not adequately represent the user cost path through time for any of the wells. Figure 1 presents the daily production and user cost paths for Well 12b.²⁵ All else equal, we expect user cost values to monotonically decline over time, as theoretically demonstrated in Section 4. However, all else is not held constant in Figure 1. Overlaying the daily production path helps to explain the deviations: when gross production is relatively low, user cost is high; conversely, when gross production is relatively high, user cost is relatively low. This is the expected relationship, as shown above, given our result of declining marginal costs of gross production.

7.0 Summary and Conclusions

This paper presents a test of the theory of exhaustible resources at the level of individual natural resource deposits. We generalize extant developments of the economic theory of exhaustible resource production, derive and extend a Halvorsen and Smith (1991) type test of this theory, and apply the test to a sample of natural gas resources. To facilitate our empirical test, the model developed in Chermak and Patrick (1995) is extended to explicitly account for the fact that the resource is not only extracted but also processed. Duality theory is then used to derive the econometric model with which the theory is statistically tested, employing panel data from 29 natural gas wells. Shadow prices of the resource stock through time that are generally

²⁴ See Patrick and Chermak (1992) and Chermak and Patrick (1995) for additional detail on differences across the wells comprising this data set.

²⁵ Well 12 was chosen for illustration purposes since it has one of the longest production histories of any of the wells in the data set. The other wells comprising the data set present analogous results.

unobservable but necessary for the test, are estimated via the indirect final cost function.

Contrary to the extant literature, our test, carried out at the level of the individual resource, indicates that the theory can, indeed, be useful in describing the behavior of an individual producer.

The theory and the test as derived above assume, among other things, that all decisions are made under complete certainty.²⁷ A natural extension to this research is to explicitly incorporate risk into the analysis. However, given the results we obtain, it is likely that generalizing the model to explicitly consider uncertainty would only serve to reinforce our results concerning the predictive ability of the theory for the natural gas resources considered in this paper.

²⁷ Gilbert (1979), Pindyck (1980), Dasgupta and Stiglitz (1981), and Deshmukh and Pliska (1985) generalize the traditional Hotelling model to formally consider various aspects of uncertainty.

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Table 1: Regression Results

Parameter	FC _U		FC _R		FC _A		GC	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
\$ _q	3.117 ^b	1.340	-0.863 ^a	0.032	-0.996 ^a	0.011	0.038 ^a	0.011
\$ _z	-2.976 ^b	1.296	0.863 ^a	0.032	-	-	-	-
\$ _R	-0.057 ^b	0.028	-0.048 ^c	0.029	-0.052 ^c	0.028	-0.053 ^c	0.029
\$ _p	-0.014	0.032	0.056 ^b	0.023	0.059 ^a	0.022	0.041 ^c	0.022
" _{Firm 1}	6.084 ^a	0.642	7.351 ^a	0.508	7.383 ^a	0.497	7.057 ^a	0.494
" _{Firm 2}	5.719 ^a	0.615	6.926 ^a	0.488	6.957 ^a	0.478	6.647 ^a	0.474
" _{Firm 3}	7.539 ^a	0.634	8.823 ^a	0.493	8.842 ^a	0.485	8.525 ^a	0.482
" _{Firm 4}	7.923 ^a	0.613	9.222 ^a	0.465	9.241 ^a	0.455	8.91 ^a	0.452
" _{Firm 5}	7.113 ^a	0.632	8.849 ^a	0.480	8.449 ^a	0.472	8.114 ^a	0.468
" ₁₉₈₈	-0.106	0.221	-0.157	0.224	-0.161	0.223	-0.147	0.222
" ₁₉₈₉	-0.318	0.228	-0.383 ^c	0.231	-0.382 ^c	0.230	-0.366	0.229
" ₁₉₉₀	-0.377 ^c	0.226	-0.449 ^b	0.229	-0.451 ^b	0.228	-0.432 ^c	0.227
" ₁₉₉₁	-0.234	0.236	-0.326	0.237	-0.329	0.237	-0.305	0.236
ρ	0.72 ^a	0.033	0.731 ^a	0.032	0.728 ^a	0.033	0.726 ^a	0.033
Log-L	-57.88		-62.61		-62.64		-60.55	

s.e.≡standard error; level of significance: a=0.01, b=0.05, and c=0.10

Table 2: Dynamic Optimality Test Results*

Discount Rate	Hausman Statistics			
	FC _U	FC _R	FC _A	GC
0.02	8.028	3.828	6.622	0.52098
0.05	7.678	1.799	6.365	0.52097
0.10	7.279	0.993	6.051	0.52094
0.12	7.184	1.781	5.961	0.52093
0.14	7.093	2.359	5.889	0.52093
0.15	7.047	2.571	5.859	0.52099
0.16	6.994	2.734	5.832	0.52092
0.17	6.993	2.849	5.809	0.52091
0.18	6.970	2.922	5.789	0.52099
0.19	6.916	2.957	5.771	0.52090
0.20	6.920	2.958	5.757	0.52099

*Distributed χ^2 with 4 degrees of freedom for the FC_U function and 3 degrees of freedom for the FC_R, FC_A, and GC models. The critical value, at the 0.05 level of significance, of the test statistic is 9.488 for 4 degrees of freedom and 7.815 for three degrees of freedom.

TABLE 3: *In Situ* Resource Price Summary Statistics (\$/mcf)

WELL	MEAN	MEDIAN	S.D.	MIN	MAX
1	0.045403	0.016853	0.109975	0.011564	0.539374
2	0.143274	0.135364	0.028912	0.107679	0.232192
3	0.210166	0.198573	0.053081	0.162773	0.452002
4	0.523268	0.431558	0.320705	0.289788	1.406445
5	0.049406	0.048963	0.005694	0.040987	0.063665
6	0.030388	0.013397	0.055413	0.010515	0.231854
7	0.015759	0.016946	0.005264	0.005758	0.024646
8	0.209025	0.019042	0.767903	0.010064	3.188560
9	0.021090	0.020658	0.002332	0.018651	0.024872
10	0.244624	0.041013	0.572365	0.037445	1.661108
11	0.015812	0.015415	0.001772	0.012801	0.018782
12(a) *	0.000017	0.000014	0.000009	0.000010	0.000041
12(b) *	0.102597	0.092587	0.028073	0.082101	0.202578
12(all) *	0.061565	0.087874	0.055272	0.000010	0.202578
13	0.000031	0.000028	0.000006	0.000022	0.000042
14	0.070902	0.056291	0.044138	0.036664	0.189397
15	0.058237	0.044176	0.034011	0.027809	0.145935
16	0.038588	0.029117	0.022569	0.020923	0.090375
17	0.041794	0.027793	0.041223	0.015691	0.165709
18	0.064496	0.053762	0.031250	0.032286	0.142233
19	0.022708	0.019412	0.008851	0.014364	0.044792
20	0.027950	0.024072	0.012030	0.013910	0.052832
21	0.018774	0.014783	0.012510	0.007138	0.047392
22	0.069305	0.063948	0.035380	0.024208	0.158955
23	0.056153	0.054375	0.028449	0.022671	0.119454
24	0.000004	0.000004	0.000001	0.000004	0.000006
25	0.000018	0.000018	0.000006	0.000011	0.000034
26	0.000024	0.000024	0.000003	0.000021	0.000029
27	0.000033	0.000033	0.000010	0.000018	0.000049
28	0.000004	0.000004	0.000001	0.000004	0.000005
29	0.000164	0.000168	0.000019	0.000138	0.000203
ALL	0.076574	0.023262	0.210104	0.000004	3.188560

*Well 12 changed ownership at the end of the 16th month of production.

Figure 1: *In Situ* Resource Price and Gross Production Paths for Well 12b

