

*Measuring Transfer Prices with Unknown Subcosts*

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**Abstract**

This paper develops a model of transfer pricing by a vertically integrated firm where divisional costs and distributions are unobservable, and uses duality theory to derive measures of optimal transfer prices.

Keywords: Transfer pricing; vertical integration; asymmetric information; duality; capacity.

JEL classification: D2, D8, D82, M1, L51, H26

**1. Introduction**

Transfer prices are the internal prices at which goods and services are traded across divisions of an integrated firm. A difficulty in measuring these prices is that they are generally unobservable as they are not determined in markets, but rather internally within the integrated firm. Intrafirm determination of these prices and informational asymmetries leave opportunities for strategic manipulation. Clausing (2003), for example, finds “substantial evidence of tax-motivated transfer pricing” to increase

corporate profits by reducing tax liabilities across the approximately 40% of US international trade that is intrafirm. For (partially) regulated integrated firms, transfer prices may be used to facilitate cross-subsidization to increase firm profitability (e.g., Berg and Tschirhart 1988; Bradley, Colvin, and Panzar 1999). They are also of concern to divisional and corporate management and shareholders since they affect divisional and corporate profitability (e.g., Milgrom and Roberts 1992).

With symmetric information, the optimal transfer price equals the marginal cost of the transferred product (Hirshleifer 1956). Besanko and Sibley (1991) consider transfer pricing and compensation under asymmetric information and find optimal transfer prices generally depart from marginal cost. Holmstrom and Tirole (2001), in their analysis of organizational form and transfer pricing, also argue that it may be difficult to determine marginal cost, the opportunity cost of capacity is rarely known, and even if this information were available it may not be truthfully revealed.<sup>1</sup> Laffont and Tirole (1993), *inter alia*, present Bayesian models of multiproduct firms where subcosts are unobservable and address the usefulness of subcost observation in reducing informational rents in incentive mechanisms.

This paper develops a simple transfer pricing model for a vertically integrated firm where subcosts and their distributions are unknown to the principal (corporate management, tax authority, or regulator). Organizational structure is taken as given: the firm is decentralized in terms of input and output decisions; and division managers' compensation depend on divisional profits; which are common corporate characteristics. The realized total cost, divisional outputs, and market prices are verifiable by the principal, which are also common assumptions in the multiproduct Bayesian principal-agent literature (Laffont and Tirole 1993). Given the firm's technology is convex and monotonic, duality theory can

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<sup>1</sup> There is a very large "transfer pricing" literature, an EconLit search (6/25/07) listed 382 cites.

be used with the observable (aggregate) information to develop the integrated firm's restricted cost function.<sup>2</sup> Through this approach, even though the various divisional components of costs (operating expenditures and investment) are unobservable and cannot be allocated among the integrated firm's divisions, it is demonstrated that this restricted cost function can be used to measure efficient transfer prices. If, in addition, the firm is capacity constrained and capacity is observable, the optimal capacity constrained transfer price is likewise measurable.

## 2.0 Analysis

In developing the analysis, the problem is discussed from the perspective of corporate management setting the transfer price at which the firm's divisions trade. However, the development of the optimal transfer price here is equally relevant to the regulator or tax authority. Corporate and divisional management have coalescing incentives in dealing with regulators and tax authorities but divergent incentives internally. Given divisional managers are compensated according to divisional profit and seek to maximize divisional profits, the downstream division's manager naturally prefers a relatively lower transfer price and will have an incentive to understate the value of the upstream input, while the upstream division prefers a relatively higher transfer price and will have the incentive to overstate costs of its output.

First, consider a simple transfer pricing model where divisions cooperate to maximize firm profits. The upstream division produces  $q^u$  and transfers it to the downstream division, which uses it as an input into the downstream product,  $q^d$ , which is sold in an external market at price  $P$  per unit. For simplicity, all market prices are taken

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<sup>2</sup> The duality between cost and production functions was originally demonstrated by Shephard (1953,

as given. Let  $q^u = g(\mathbf{X}^u)$  represent the subproduction function for the upstream product, where  $\mathbf{X}^u = \{x_1, \dots, x_{J-1}, Q\}$  are  $J$  inputs into production of  $q^u$ ,  $Q$  is the upstream production capacity, and  $q^u = g(0, \dots, 0, Q) = 0$ . The firm's production function is

$$q^d = f(\mathbf{X}^d, q^u), \quad (1)$$

where  $\mathbf{X}^d = \{x_i : i = 1, \dots, N\}$  are market inputs used to produce  $q^d$ . The integrated firm's cooperative profit maximization problem is then

$$\max_{\mathbf{X}^d, q^u, Q} \pi = Pf(\mathbf{X}^d, q^u) - \mathbf{W}^d \cdot \mathbf{X}^d - C(\mathbf{W}^u, q^u, Q) \text{ subject to } Q \geq q^u, \quad (2)$$

where  $\mathbf{W}^d = \{w_i : i = 1, \dots, N\}$  are downstream input prices,  $\mathbf{W}^u = \{w_j : j = 1, \dots, J\}$  are prices of inputs into upstream production, and

$$C(\mathbf{W}^u, q^u, Q) \equiv \min_{\mathbf{X}^u} \mathbf{W}^u \cdot \mathbf{X}^u \text{ subject to } q^u = g(\mathbf{X}^u) \leq Q, \quad (3)$$

the minimal cost function dual to the upstream subproduction function. The Lagrangian for the firm's capacity constrained profit maximization problem is

$$L = Pf(\mathbf{X}^d, q^u) - \mathbf{W}^d \cdot \mathbf{X}^d - C(\mathbf{W}^u, q^u, Q) + \lambda(Q - q^u). \quad (4)$$

Necessary conditions for interior solutions in  $q^u$ ,  $\mathbf{X}^d$ , and  $Q$  include<sup>3</sup>

$$L_{q^u} = Pf_{q^u} - C_{q^u} - \lambda = 0, \quad (5)$$

$$L_{x_i} = Pf_{x_i} - w_i = 0, \quad i = 1, \dots, N, \quad (6)$$

$$L_Q = -C_Q + \lambda = 0, \quad (7)$$

and

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1970). Chambers (1988), Cornes (1992), and Diewert (1993) provide monographs on duality.

<sup>3</sup> Subscripts represent derivatives, i.e.,  $L_{q^u} \equiv \partial L / \partial q^u$ , etc.

$$\lambda \geq 0, \lambda(Q - q^u) = 0. \quad (8)$$

$\lambda \geq 0$  follows from sufficient conditions for a constrained maximum. (5) implies that  $q^u$  is chosen so that the value of the marginal product of  $q^u$  is equal to the sum of the marginal cost of  $q^u$  plus the capacity scarcity rent (shadow price),  $\lambda$ . According to (6), the value of the marginal product of each downstream input equals the price of the input. If the firm chooses capacity optimally, (7) will hold,  $\lambda = C_Q$ , the capacity scarcity rent equals the marginal cost of capacity.

In decentralized firms, input and output decisions are made by divisional managers. The transfer price,  $\rho$ , the price at which  $q^u$  is transferred from the upstream to the downstream division of the firm, is set by corporate management in our model.

The upstream division, given the transfer price, chooses  $q^u$  and  $Q$  to maximize

$$\pi^u = \rho q^u - C(W^u, q^u, Q) \text{ subject to } Q \geq q^u. \quad (9)$$

The Lagrangian for the upstream division's profit maximization is

$$L^u = \rho q^u - C(W^u, q^u, Q) + \lambda^u (Q - q^u). \quad (10)$$

Necessary conditions for upstream production include

$$L^u = \rho - C_{q^u} - \lambda^u = 0, \quad (11)$$

$$L^u_Q = -C_Q + \lambda^u = 0, \quad (12)$$

and

$$\lambda^u \geq 0, \lambda^u (Q - q^u) = 0. \quad (13)$$

Given the transfer price  $\rho$ , the downstream division chooses  $\mathbf{X}^d$  and  $q^u$  to maximize

$$\pi^d = Pf(\mathbf{X}^d, q^u) - \mathbf{W}^d \cdot \mathbf{X}^d - \rho q^u. \quad (14)$$

Necessary conditions for downstream profit maximization include

$$\pi_{q^u}^d = Pf_{q^u} - \rho = 0 \quad (15)$$

and

$$\pi_{x_i}^d = Pf_{x_i} - w_i = 0, \quad i = 1, \dots, N. \quad (16)$$

If divisional costs were available, it would be straightforward for corporate management to set the transfer price so that the solutions to the divisional profit maximization problems are equivalent to the firm profit maximization problem. That is, if  $\rho = Pf_{q^u} = C_{q^u} + \lambda$  then (11), (15), and (5) are equivalent; (16) is equivalent to (6); (12) to (7); (13) to (8); and  $\lambda = \lambda^u$  so that an equilibrium exists across the divisional problems and the optimal  $\{\mathbf{X}^d, q^u\}$  are equivalent across the cooperative versus divisional profit maximization problems.

**Lemma 1.** If  $\rho = Pf_{q^u} = C_{q^u} + \lambda$  then the cooperative and divisional profit maximization outcomes are equivalent.

The difficulty is that corporate management does not generally observe the marginal product or marginal cost of  $q^u$ , or the capacity shadow price, and hence the optimal transfer price cannot be determined at this point.

However, regardless of the informational asymmetry, duality theory can be used with the available information to measure of the optimal transfer price.<sup>4</sup> To see this, first consider the problem of choosing market inputs to minimize costs subject to given levels of

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<sup>4</sup> Lau (1976) demonstrates conditions under which the solution to the more general profit maximization problem is identical to the restricted cost minimization problem.

upstream and downstream production and capacity. Let  $q^u = \bar{q}^u = q^{u*}$ ,  $q^d = \bar{q}^d = q^{d*}$ , and  $Q = \bar{Q} = Q^*$  be the profit maximizing levels of upstream output, downstream output, and capacity from the firm's constrained profit maximization problem above. The Lagrangian for the cost minimization problem is then

$$\bar{L} = \mathbf{W} \cdot \mathbf{X}' + \gamma[\bar{q}^d - f(\mathbf{X}^d, \bar{q}^u)] + \delta[\bar{q}^u - g(\mathbf{X}^u, \bar{Q})] + \mu(\bar{Q} - \bar{q}^u), \quad (17)$$

where the prices of inputs purchased outside the integrated firm are  $\mathbf{W} = \mathbf{W}^u \cup \mathbf{W}^d$ , and  $\gamma$ ,  $\delta$ , and  $\mu$  are multipliers on the respective downstream and upstream production, and capacity constraints. Necessary conditions include

$$\bar{L}_{x_i} = w_i - \gamma f_{x_i} = 0, \quad i=1, \dots, N, \quad (18)$$

for the choice of downstream inputs,

$$\bar{L}_{x_j} = w_j - \delta g_{x_j} = 0, \quad j=1, \dots, J-1, \quad (19)$$

for upstream inputs, and, for the capacity constraint,

$$\mu \geq 0, \quad \mu(\bar{Q} - \bar{q}^u) = 0. \quad (20)$$

The solution to this restricted cost minimization problem yields the restricted cost function

$$\bar{C}(\mathbf{W}, q^d, q^u, Q), \quad (21)$$

where  $\bar{C}$  is the minimal total expenditure given  $\{\mathbf{W}, q^d, q^u, Q\}$ . Given the assumptions above,  $\bar{C}$ ,  $\mathbf{W}, q^d, q^u$ , and  $Q$  in (21) are observable, providing sufficient information for estimation of this restricted cost function dual to the firm's production function (1). An essential envelope theorem result follows,

$$\bar{C}_{q^u} = \bar{L}_{q^u} = -\gamma f_{q^u} + \delta - \mu \quad (22)$$

which can be interpreted by relaxing the  $q^u = \bar{q}^u = q^{u*}$  assumption in the cost minimization problem above.

The firm's minimizing total cost while determining  $q^u$  endogenously problem follows. The inputs,  $\mathbf{X}$ , are now chosen to minimize total costs,  $\mathbf{W} \cdot \mathbf{X}' + \rho g(\mathbf{X}^u)$ , where  $\rho$  is the (unobserved) transfer price, subject to the constraints,  $q^d = \bar{q}^d = q^{d*}$  and  $Q = \bar{Q} = Q^*$ .

The Lagrangian is

$$\bar{L} = \mathbf{W} \cdot \mathbf{X}' + \rho g(\mathbf{X}^u) + \bar{\gamma}[\bar{q}^d - f(\mathbf{X}^d, g(\mathbf{X}^u))] + \bar{\mu}(\bar{Q} - \bar{q}^u), \quad (23)$$

where  $\bar{\gamma}$  and  $\bar{\mu}$  are multipliers on the downstream production and capacity constraints.

Necessary conditions include

$$\bar{L}_{x_j} = w_j + [\rho - \bar{\gamma} f_{q^u}] g_{x_j} = 0, \quad j=1, \dots, J-1, \quad (24)$$

for the optimal choice of upstream inputs, which determine  $q^u$ ,

$$\bar{L}_{x_i} = w_i - \bar{\gamma} f_{x_i} = 0, \quad i=1, \dots, N, \quad (25)$$

for the optimal choice of other downstream inputs, and

$$\bar{\mu} \geq 0, \quad \bar{\mu}(\bar{Q} - \bar{q}^u) = 0. \quad (26)$$

The optimal  $\mathbf{X}$  for this problem is identical to that from the previous problem since

$q^u = \bar{q}^u = q^{u*}$  and  $q^d = \bar{q}^d = q^{d*}$ . (18) and (25) imply that  $\gamma = \bar{\gamma}$ . (20) and (26) imply

$\mu = \bar{\mu}$ . (19) and (24) then imply  $\delta = -\rho + \bar{\gamma} f_{q^u}$ . Given the observable information,

substituting these expressions into (22) leads to the optimal transfer price measure.

**Proposition 1.** The optimal transfer price is equal to the negative of the derivative of the firm's restricted cost function with respect to upstream production, i.e.,

$$\rho = -\bar{C}_{q^u}(\mathbf{W}, q^d, q^u, Q).$$

Given  $\rho > 0$  ( $C_{q^u} > 0$ ,  $\lambda \geq 0$ ,  $P > 0$ , and  $f_{q^u} > 0$ ),  $\bar{C}_{q^u} < 0$ , i.e., the marginal restricted cost of upstream production is nonpositive. The intuition is that additional  $q^u$ , all else constant, reduces the marginal cost of producing a given level of  $q^d$ . The proposition implies that the optimal transfer price can be observed by differentiating the restricted cost function with respect to upstream production,  $q^u$ , and evaluating at the levels of the observable determinants.

The transfer pricing literature has not generally explicitly considered capacity constraints, as above.<sup>5</sup> An analogous transfer pricing result follows if the firm is not capacity constrained. Dropping the capacity constraint from the above implies an indirect cost function of the form  $\bar{C}(\mathbf{W}, q^d, q^u)$ , and straightforwardly following the proof of the proposition above leads directly to

**Corollary 1.**  $\rho = -\bar{C}_{q^u}(\mathbf{W}, q^d, q^u)$ .

### 3.0 Conclusions

Even though the various divisional components of costs and their distributions are unobservable and thus cannot be allocated among the integrated firm's divisions, duality theory results are derived that enable the principal (tax authority, regulator, corporate management) to measure optimal transfer prices. Interesting extensions include the consideration of quality of the transferred product in the context of Lewis and Sappington (1991) and implications of subcost observability in reducing information rents in

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<sup>5</sup> Harris, Kriebel, and Raviv (1982) is an exception. An EconLit search (6/25/07) on the intersection of "transfer pricing" and "capacity" yielded 11 cites.

multiproduct principal-agent models (as discussed in Laffont and Tirole 1993) in relation to the results here.

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