RELIABILITY ESTIMATION AND PREVENTIVE MAINTENANCE FOR COMPLEX MULTI-COMPONENT SYSTEMS SUBJECT TO MULTIPLE DEPENDENT COMPETING FAILURE PROCESSES

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Some complex systems experience multiple dependent competing failure processes, and many traditional approaches to reliability are inadequate or inappropriate because there are two or more dependent/correlated failure processes and multiple components subject to failure. A typical design example representative of these systems is Micro-Electro-Mechanical Systems. The dependency among the failure processes and the dependency among the component failure times present challenging issues in reliability modeling. In reality, for many actual engineering problems, there are interactions between soft failure and catastrophic failure that should not be ignored. Also, interaction among components should be considered. This paper develops a new reliability model and optimal preventive maintenance policies for complex multi-component systems experiencing multiple failure processes due to simultaneous exposure to degradation and shock loads. The developed reliability and maintenance models are demonstrated for a multi-component systems example. These models can also be applied directly or customized for other complex systems that experience multiple dependent competing failure processes.

Keywords: Multiple dependent competing failure processes, Multi-component systems, Degradation, Random shocks, Preventive maintenance

1. Introduction

Common failure mechanisms and causes of complex system failure include wear

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degradation, corrosion, shock loads, fatigue, etc. The issue of multiple failure processes is of particular interest to researchers because it is a critical problem that systems have experienced in the field, and one that could limit further developments and advancements of advanced and evolving technologies. For many engineering design problems, traditional failure-based reliability methods are not applicable due to the lack of failure data. Multiple competing failure processes may be independent or dependent. When they are dependent, it creates a unique and challenging problem to analyze and predict the system reliability performance. In this article, we develop models to analyze the reliability of complex multi-component systems that experience Multiple Dependent Competing Failure Processes (MDCFP) [1], particularly due to degradation and/or shock loads. The model can be applied when there is a single failure process or multiple competing failure processes, which offers greater flexibility and capability compared to most other prior work.

2. Background

Previous studies on degradation focus on developing degradation models and estimating time-to-failure distributions [2-8], using experimental design to improve reliability [9-12], and exploring maintenance policies for continuously monitored degrading systems [13-15].

For a complex system involving both degradation and shocks, independent multiple catastrophic and degradation failure processes [16] have been investigated. Simultaneous quality and reliability optimization model for systems composed of degrading components has also been studied [17].

However, relatively little research has been devoted to the reliability analysis of multi-component systems with MDCFP. The dependency among the failure processes and interaction among components present challenging issues in reliability modeling. In this paper, we develop reliability models and preventive maintenance policies for series systems of multiple components with dependent failure times, and each component experiences two dependent/correlated failure processes: (i) a soft failure process caused by continuous smooth degradation and additional abrupt degradation damages due to a shock process, and (ii) catastrophic failures caused by excessive shock magnitudes. These two failure processes are competing, since either failure process can cause the component or system to fail. The failure is caused by whichever failure process reaches the critical threshold first. In addition, these two failure processes are dependent or correlated because the effects from the same shock process contribute to both failure processes [18]. Some key
theoretical arguments and reliability models are presented [19]. Furthermore, there are multiple components that have failure times correlated due to shocks affecting the assembled systems.

Different maintenance strategies for degrading systems have been extensively examined in the literature. We consider a fixed replacement interval maintenance policy for a series system with multiple components each exposed to two competing dependent failure processes. We demonstrate the developed reliability model and maintenance optimization for multi-component systems subject to MDCFP using a representative example.

3. Reliability Analysis for MDCFP with Degradation and Shocks

3.1. System Description

As shown in Fig. 1 for a single component of a multi-component system, each individual component may fail due to two competing dependent failure modes that involve the same shock process: soft failures and catastrophic/hard failures. Soft failure occurs when a critical threshold \( H_i \) for the \( i \)th component is exceeded by the total wear volume. Random shock processes can cause catastrophic/hard failures when the load magnitude from a single shock exceeds a critical strength level \( D_i \). The component fails when either of the two competing failure modes occurs. For multiple-component series systems, each component is behaving somewhat similarly, and the system fails when the first component fails.

![Figure 1. Two dependent competing failure processes for component i](image)
The notation used in formulating the reliability and maintenance models in Sections 3 and 4 is now listed.

- \( D_i \) = threshold for catastrophic failures of \( i \)th component;
- \( N(t) \) = number of shock loads that have arrived by time \( t \);
- \( n \) = number of components in a series system;
- \( \lambda \) = arrival rate of random shocks;
- \( W_{ij} \) = size/magnitude of the \( j \)th shock load on the \( i \)th component;
- \( F_{i,W}(w) \) = cumulative distribution function (cdf) of \( W_i \);
- \( H_i \) = critical wear degradation failure threshold of the \( i \)th component;
- \( X_i(t) \) = wear volume of the \( i \)th component due to continuous degradation at \( t \);
- \( X_{Si}(t) \) = total wear volume of the \( i \)th component at \( t \) due to both continual wear and instantaneous damage;
- \( Y_{ij} \) = damage size of the \( i \)th component caused by the \( j \)th shock load;
- \( S(t) \) = cumulative shock damage size of the \( i \)th component at \( t \);
- \( G(x_i, t) \) = cdf of \( X_i(t) \) at \( t \);
- \( F_{X_S}(x_i, t) \) = cdf of \( X_{Si}(t) \) at \( t \);
- \( f_{Y_i}(y) \) = probability density function (pdf) of \( Y_i \);
- \( f_{Y_i < k}(y) \) = pdf of the sum of \( k \) independent and identically distributed (i.i.d.) \( Y_i \) variables;
- \( f_{T}(t) \) = pdf of the failure time, \( T \);
- \( F_{T}(t) \) = cdf of the failure time, \( T \);
- \( V \) = periodic replacement interval;
- \( C(t) \) = cumulative maintenance cost by time \( t \);
- \( CR(V) \) = average long-run maintenance cost rate;
- \( E[U] \) = expected value of the first renewal cycle length, \( U \);
- \( E[TC] \) = expected value of the total maintenance cost of the first renewal cycle, \( TC \);
- \( C_R \) = replacement cost per unit;
- \( C_F \) = cost of replacement caused by failure.

There are some specific assumptions used for the reliability and maintenance modeling in this article.

1. Soft failure occurs when the overall degradation of the any component is beyond its threshold value \( H_i \) and hard/catastrophic failure occurs when the shock load itself exceeds the maximum strength of the materials \( D_i \).
2. Random shocks arrive according to a Poisson process.
3. The system fails when an individual component reaches either of the two failure conditions.
4. The model is for systems that are packaged and sealed together, making it impossible or impractical to repair or replace individual components within
3. The system is replaced at periodic intervals. However, if the system fails before the specified replacement interval, it will be replaced immediately. Replacements are assumed to be instantaneous and perfect.

6. For this model formulation, no continuous monitoring is performed on the system due to cost concerns and practicality issues.

3.2. Modeling for Catastrophic Failures Due to Shocks

In this paper, component catastrophic/hard failures occur according to extreme shock model. Figure 1(b) shows that a component fails due to fracture when the $j^{th}$ load/stress of the $i^{th}$ component exceeds the maximal fracture strength $D_i$. The probability that the $i^{th}$ component survives the applied stress from the $j^{th}$ shock is [20]:

$$P(W_{ij} < D_i) = F_{i,w}(D_i) \text{ for } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots$$ (1)

If the $W_{ij}$ are assumed to be i.i.d. random variables distributed as a normal distribution, $W_{ij} \sim N(\mu_w, \sigma_w^2)$, then the probability that each component survival of a shock becomes [1]:

$$P_{Li} = F_{i,w}(D_i) = \Phi\left(\frac{D_i - \mu_w}{\sigma_w}\right) \text{ for } i = 1, 2, \ldots, n,$$ (2)

where $\Phi(\cdot)$ is the cdf of a standard normally-distributed random variable.

3.3. Modeling for Soft Failures Due to Degradation and Shocks

Soft failures of the $i^{th}$ component can occur when the overall degradation of the component is beyond a threshold level $H_i$. As shown in Fig. 1(a), the total degradation accrued, $X_i(t)$, is the sum of the degradation due to continual wear and the instantaneous damages due to shocks. A linear degradation path is shown in Fig. 1(a), $X_i(t) = \varphi_i + \beta t$, where the initial value $\varphi_i$ and the degradation rate $\beta$ can be constant or random variables.

Degradation changes or shifts can accumulate instantaneously when a shock arrives. Each shock impacts all components and affects both failure processes of each component. The cumulative damage size due to random shocks until time $t$, $S_i(t)$, is given as

$$S_i(t) = \begin{cases} \sum_{j=1}^{N(t)} Y_{ij}, & \text{if } N(t) > 0, \\ 0, & \text{if } N(t) = 0, \end{cases}$$ (3)
where \( N(t) \) is the total number of shocks to the system that have arrived by time \( t \). The overall degradation of the \( i \)th component is expressed as \( X_S(t) = X_i(t) + S_i(t) \).

Then the probability that the total degradation at time \( t \) is less than \( x_i \), \( F_X(x_i, t) \), can be derived as

\[
F_X(x_i, t) = P(X_S(t) < x_i) = \sum_{j=0}^{\infty} P(X_i(t) + S_i(t) < x_i \mid N(t) = j) P(N(t) = j) \quad (4)
\]

Furthermore, if we consider \( G(x_i, t) \) to be the cdf of \( X_i(t) \) at \( t \), \( f_{Y_i}(y) \) to be the pdf of \( Y_i \), and \( f^{\sum_{k}}(y) \) to be the pdf of the sum of \( k \) i.i.d. \( Y_i \) variables, then the cdf of \( X_S(t) \) in Eq. (4) can be derived using a convolution integral:

\[
F_X(x_i, t) = \sum_{j=0}^{\infty} \left( \int_0^{x_i} G(x_i - u, t) f_{Y_i}(u) du \right) P(N(t) = j) \quad (5)
\]

If the shock damage sizes of the \( i \)th component are i.i.d. normal random variables, \( Y_i \sim N(\mu_{Y_i}, \sigma_{Y_i}^2) \), and the degradation path is linear with a constant initial value \( \phi_i \) and a normal-distributed degradation rate \( \beta_i \) with \( \beta_i \sim N(\mu_{\beta_i}, \sigma_{\beta_i}^2) \), then a more specific model can be determined based on Equation (4) [17]:

\[
F_X(x_i, t) = \sum_{j=0}^{\infty} \Phi \left( \frac{x_i - (\mu_{\beta_i} t + \phi_i + j \mu_{Y_i})}{\sqrt{\sigma_{\beta_i}^2 t^2 + j \sigma_{Y_i}^2}} \right) \exp\left(-\frac{\lambda t}{\sqrt{\sigma_{\beta_i}^2 t^2 + j \sigma_{Y_i}^2}}\right) \frac{1}{j!} \quad (6)
\]

The probability that component \( i \) does not experience soft failure before time \( t \) is expressed as

\[
P(X_S(t) < H_i) = F_X(H_i, t) \quad (7)
\]

### 3.4 System Reliability Analysis

Figure 2 shows a series system made up of \( n \) components. The reliability of this system at time \( t \) is the probability that the system survives each of the \( N(t) \) shock loads (\( W_{ij} < D_i \) for \( j=1, 2, \ldots \)) and the total degradation is less than the threshold level (\( X_S(t) < H_i \)) for each component:

![Figure 2. Series system example](image-url)
The number of shocks $N(t)$ has an effect on each component. When $N(t)$ is large enough, the sum of damage size of each component caused by shocks is large, and a failure is more frequent for all components. Alternatively, when there are relatively few shocks, times to failure are relatively longer for all components. Thus, this makes those component failure times probabilistically dependent. Using condition probability, we have:

$$R(t) = P\left[ W_{i_1} < D_{i_1}, W_{i_2} < D_{i_2}, \ldots, W_{i_k} < D_{i_k}, X_{i_1}(t) < H_{i_1} \right] \cap \left[ W_{i_2} < D_{i_2}, W_{i_3} < D_{i_3}, \ldots, W_{i_k} < D_{i_k}, X_{i_2}(t) < H_{i_2} \right] \cap \ldots \cap \left[ W_{i_k} < D_{i_k}, W_{i_{k+1}} < D_{i_{k+1}}, \ldots, W_{i_k} < D_{i_k}, X_{i_k}(t) < H_{i_k} \right]$$

(8)

Shocks arriving at random time intervals are modeled as a renewal process with Poisson distribution. When the system is shocked (at rate $\lambda$), all components experience a shock. Then, the system reliability function can be derived for the general case as:

$$R(t) = \sum_{m=0}^{\infty} \prod_{i=1}^{n} P(W_i < D_i)^m P\left[ X_i(t) + \sum_{j=1}^{m} Y_{ij} < H_i \right] \left[ N(t) = m \right] \frac{\exp(-\lambda t)(\lambda t)^m}{m!}$$

(9)

Based on Equations (2) and (5), the reliability function can be expressed as:

$$R(t) = \sum_{m=0}^{\infty} \prod_{i=1}^{n} F_{i}(H_{i} - u) f_{j_{i}(m)}(u) \int_{0}^{H_{i}} G_{j}(H_{i} - u) f_{j_{i}(m)}(u) du \frac{\exp(-\lambda t)(\lambda t)^m}{m!}$$

(10)

The reliability function for the more specific case with normally distributed $W_{ij}, Y_{ij},$ and $\beta$ can be expressed as

$$R(t) = \sum_{m=0}^{\infty} \prod_{i=1}^{n} P_{Li} \Phi \left( \frac{H_{i} - \left( \mu_{Li} t + \phi_i + m\mu_{Yi} \right)}{\sqrt{\sigma^2_{Li} t^2 + m\sigma^2_{Yi}}} \right) \exp(-\lambda t)(\lambda t)^m$$

(11)

where $P_{Li}$ is given by Eq. (2) and $\Phi(\cdot)$ is the pdf of a standard normally distributed variable.

4. Maintenance Modeling and Optimization

To evaluate the performance of the maintenance policy, we use an average long-
run maintenance cost rate model, in which the periodic replacement interval $V$ is the decision variable. In the model at time $V$, the system is replaced with a new one, with all new components. However, if the system fails before time $V$, it will be replaced immediately. The average long-run total maintenance cost per unit time can be evaluated by:

$$
\lim_{t \to \infty} \frac{E(C(t)/t)}{E(U)} = \frac{E(\text{expected maintenance cost between two replacements})}{E(\text{expected time between two replacements})}
$$

where $TC$ is the total maintenance cost of a renewal cycle, and $U$ is the length of a cycle that takes a value of $V$ or time to failure $T$ [21]. Then the expected total maintenance cost is given as

$$
E(U) = VR(V) + \int_0^V tf(t)dt = \int_0^V R(t)dt
$$

Based on Eqs. (13) to (15), the average long-run maintenance cost rate as a function of $V$, $CR(V)$, is given as

$$
CR(V) = \frac{C_F \left(1 - \sum_{n=0}^{\infty} \prod_{i=1}^{n} \left[p_{ij}^n \phi(H_i - (\mu_i V + \varphi_i + m \mu_i)) \frac{\exp(-\lambda V)(\lambda V)^n}{n!} \right] + C_R \right)}{\int_0^V \sum_{n=0}^{\infty} \prod_{i=1}^{n} \left[p_{ij}^n \phi(H_i - (\mu_i + \varphi_i + m \mu_i)) \frac{\exp(-\lambda t)(\lambda t)^n}{n!} \right] dt}
$$

To obtain an analytical result of the optimal solution, we calculate the first derivative of the objective function in Eq. (16), as given below:

$$
CR'(V) = -\left[C_F \left[ V - \int_0^V F(t)dt \right] - [C_F F(V) + C_R][1 - F(V)] \right] = 0
$$

5. Numerical Example

Consider a series system with three components. The parameters in Eq. (12) for reliability analysis are provided in Table 1. We assumed that $W_{ij}$ and $Y_{ij}$ follow a normal distribution. Without a lack of generality, we assume that those
parameters of all three components are the same. From Sandia’s micro-electromechanical systems (MEMS) experimental results, we estimate $H_i$, $D_i$, and $X_i(t) = \varphi_i + \beta_i t$. Based on Eq. (11), the reliability function $R(t)$ and the pdf of failure time $f_f(t)$ are plotted in Figure 3.

5.1. Optimal Maintenance Policy

Choosing $C_F=$$150 and $C_R=$$30, we can find the minimum average long-run maintenance cost rate of $1.175 \times 10^{-3}$/cycle, which is obtained at $V^*=0.8246 \times 10^5$, the optimal number of revolutions for periodic replacement. Figure 6 illustrates $C_F(V)$ as a function of $V$.

Table 1. Parameter values for multi-component system reliability analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_i$</td>
<td>0.00125 $\mu$m$^3$</td>
<td>Tanner and Dugger [18]</td>
</tr>
<tr>
<td>$D_i$</td>
<td>1.5 Gpa (for polysilicon Material used for springs)</td>
<td>Tanner and Dugger [18]</td>
</tr>
<tr>
<td>$\varphi_i$</td>
<td>0</td>
<td>Tanner and Dugger [18]</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>$\sim N(\mu_\beta, \sigma_\beta^2)$ for $i=1,2,...,\infty$, $\mu_\beta=8.4823 \times 10^{-9}$ and $\sigma_\beta=6.0016 \times 10^{-10}$</td>
<td>Tanner and Dugger [18] Peng et al. [17]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$2.5 \times 10^5$</td>
<td>Assumption</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>$\sim N(\mu_Y, \sigma_Y^2)$ for $i=1,2,...,\infty$, $\mu_Y=1 \times 10^{-4}$ and $\sigma_Y=2 \times 10^{-4}$</td>
<td>Assumption</td>
</tr>
<tr>
<td>$W_i$</td>
<td>$\sim N(\mu_W, \sigma_W^2)$ for $i=1,2,...,\infty$, $\mu_W=1.2$ GPa and $\sigma_W=0.2$ GPa</td>
<td>Assumption</td>
</tr>
</tbody>
</table>

Figure 3. Plots of reliability function $R(t)$ and failure time distribution $f_f(t)$.
6. Conclusions

In this article, we develop reliability and preventive maintenance models for multi-component systems that experience MDCFP. The reliability model is developed based on general degradation path and random shock models for individual components, where extreme and cumulative shock models are used in the modeling of the catastrophic and soft failure processes, respectively. The general reliability model is then extended to a specific model for a linear degradation path with a normal-distributed degradation rate and a random shock process with normal distributed shock load sizes and shock damage sizes. The average long-run maintenance cost rate is evaluated and optimized for the preventive maintenance policy where the periodic replacement interval is the decision variable.

Acknowledgments

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