Intervals in the Semantics of Gradable Adjectives[*]

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April 24, 2001

PART I. INTRODUCTION

It is natural to think of comparisons in terms of points on a scale. Jack is taller than Jill if the point associated with Jack on the height scale is higher than Jill's point. Jack is much taller than Jill is if Jack's point is separated from Jill's by a sizable amount. It is also natural to think of temporal discourse in terms of points on a time line. The analogy between the two is worth taking seriously.

In both domains, it has been claimed that points are useful but need not be taken as primitive. It is possible that events are ordered and that moments of times are equivalence classes of events relative to this ordering. Similarly, Cresswell(1976) defines points on a scale as equivalence classes of individuals ordered along some dimension.

A series of ordered points can also be interdefined with ordered intervals. In the semantics of tense, it has been argued that intervals are indispensable. Some events happen at a moment, but others, accomplishments, happen over an interval which is not a moment. I will make a parallel argument for the semantics of gradable predicates. Part II consists of a number of puzzles, many of which have been discussed elsewhere, but all of which, I claim, arise because the semantics is point-based. Part IV is a sketch of how a semantics based on intervals resolves the puzzles of part II. One might suspect that the puzzles of part II are due to our assumption of a semantics based on scales. Part III briefly addresses this worry. Part V discusses how measure phrases are analyzed in an interval semantics.

PART II. PUZZLES

P1. Reverse Russell.

Russell is sometimes credited with having been the first to propose a degree analysis of comparatives. In "On Denoting" he used such an analysis to analyze the ambiguity of A's remark in (1) below in terms of the relative scope of a degree quantifier and the propositional attitude verb:

[*] This paper borrows heavily from Schwarzschild and Wilkinson(to appear). I thank my co-author for bringing the initial problem to my attention and for the work we did together on it. Some of the ideas were further fleshed out in a Seminar at Rutgers in the Spring of 2001. The class was wonderful and I am very grateful for that feedback. Barbara Partee is to be thanked for urging me to advertise my 'Reverse Russell' puzzle. That was the impetus for this paper. Finally, the paper benefits from the careful scrutiny it received from Kathrin Koslicki who commented on it at a presentation at Rutgers.

(1) A: I thought your yacht was larger than it is.
B: No, my yacht is not larger than it is

(2) [The size that I thought your yacht was] is greater than [the size that your yacht is]
(3) I thought [[the size that your yacht] was greater than [the size of your yacht]]

The puzzle I wish to describe is perhaps easier to see if we reverse the surface order of the comparative and the propositional attitude verb, as in the following examples:

(4) Bertrand's yacht is larger than I thought it was.
(5) Sam's monkey is heavier than Smith expected it to be.
(6) John took more books than he agreed to take.

A degree analysis of the second example runs roughly as follows:

(7) $\exists d_1 \exists d_2$ Sam's monkey is d_1-heavy and Smith expected it to be d_2-heavy and $d_1 > d_2$

Suppose now that the monkey weighs 20 lbs. For (7) to be true, there has to be some weight, below 20lbs, such that Smith expected the monkey to weigh that much. This is not a good analysis of (5) because (5) could very well be true without there being any such weight. Smith may have expected the monkey to weigh somewhere between 5 and 10 lbs, like most of the pet monkeys he's seen. For the monkey to weigh say 5 lbs would be consistent with Smith's expectations, likewise for any other weight up to 10 lbs. But we wouldn't want to say that Smith expected the monkey to weigh 5 lbs and he expected it to weigh 6 lbs etc, for that would be to attribute to Smith inconsistent expectations, which we have no reason to do.

Our first puzzle is then: How is it possible that Sam's monkey is heavier than Smith expected it to be, even when there is no specific weight that Smith expected the monkey to be?

P2 Incommensurability

Imagine you have a scarf which varies in color from grayish white up through pure white. Take a patch of pure white cloth, and a patch of the grayish white.

(8) The first patch is whiter than the second.

It is also true that

(9) That fresh snow over there is whiter than the whole scarf is.

The statements in (8) and (9) are true, but the ones in (10) and (11) are **not**:

(10) The first patch is whiter than the whole scarf.

(11) The whole scarf is whiter than the first patch is.

By (8), there is a point on the whiteness scale that corresponds to the first patch and which is higher than the one for the second patch. By (9), there is a point on the scale associated with the whole scarf which is below the point associated with the fresh snow. If the first patch and the scarf are both associated with a point on the scale and if, in virtue of being a scale, we take these points to be connected, then either the patch's point is above the scarf's or the other way around, in which case either (10) or (11) ought to be true.

Here's another example which shows the same thing but with a different scale. The temperature in the living room varies across the parts of the room from 60º to 70º. The living room is warmer than inside the refrigerator and the cup of lukewarm water (65º) is warmer than inside the refrigerator, but the living room is not warmer than the cup of water nor is the reverse true.

P3 Quantifier Scope

Suppose that it is 90° Fahrenheit in New York but between 70° and 85° in most other East Coast cities. It is then true that:

(12) It is hotter in New York than it is in most other East Coast cities.

if (12) is analyzed as:

(13) $\exists d_1\ \exists d_2$ it is d_1-hot in New York and it is d_2-hot in most other E.C. cities and $d_1 > d_2$

then we confront a problem similar to the one confronted in our reverse-Russell puzzle. (13) requires most East Coast cities to have the same temperature, but (12) carries no such requirement.

In order to be sure that this is a genuine puzzle for the degree analysis, we need to first consider the possibility that the nominal quantifier actually lies in the scope of the comparative at logical form. For if it didn't, we would have an analysis like in (14), in which the problem just mentioned evaporates:

(14) For most cities c: $\exists d_1\ \exists d_2$ it is d_1-hot in New York and it is d_2-hot in c and $d_1 > d_2$

To posit a logical form like the one in (14) is to make the dubious assumption that a quantifier can take scope outside the clause that contains it (on the surface). It would furthermore lead one to expect scopal interactions between quantifiers inside and outside the comparative, interactions that we do not find. The following example illustrates one such case:

(15) At least one of the students is exactly 1 in taller than is every member of the faculty.

The only possible reading of this sentence requires all the faculty to be of the same height and for there to be one student an inch taller. However, if the quantifier under *than* were allowed to take scope outside the comparative, we should expect a more plausible interpretation according to which for each faculty member there is one student who is 1 inch taller.

One might respond to this observation (Lerner and Pinkal (1992)?) by claiming that quantifiers can somehow scope outside the comparative but not very far. But pushing the higher quantifier closer in doesn't seem to help. (16) is hard to interpret but in any case it cannot mean (17).

(16) It's an inch closer to one of the trees than it is to most of the buildings.

(17) [For most of the buildings: b] [there is one tree: t] it's an inch closer to t then it is to b.

And even if this constellation can be explained away on syntactic grounds, we would still be left with cases in which a scope bearing element intervenes between the comparative morpheme and the quantifier in question, as in the scheme below:

(18) [more….[X….. [Quantifier….

In such a case, if the quantifier has scope over the comparative, it will have to have scope over the intervener, X. The following are representative examples of (18) in which X is a propositional attitude verb or a modal:

(19) John gave me more than he must give each of the others.
(20) Simian is heavier than Smith predicted that each of the others would be.

Suppose that Smith predicted that the others would range in weight from 5 to 10 lbs and he further predicted that each monkey would have a different weight. If Simian weighs 20 lbs then (20) is true. The degree-analysis has us choose between the following two possibilities:

(21) $\exists d_1\ \exists d_2$ Simian is d_1-heavy and Smith predicted that each of the others would be d_2-heavy and $d_1 > d_2$

(22) For each of the others m: $\exists d_1\ \exists d_2$ Simian is d_1-heavy and Smith predicted that m would be d_2-heavy and $d_1 > d_2$

(21) is wrong, because Smith did not predict a specific weight for each of the others. (22) is wrong because Smith did not make any prediction about particular monkeys. Although these examples bear some resemblance to those in P1, the reverse-Russell puzzle, they are not the same problem. The problem with (21) is no different from the problem in (13) where no propositional attitude report is involved. The problem in (22) has to do with the fact that (20) is true in sensu composito, but (22) assigns it a reading in sensu diviso.

Quantifiers in the scope of the comparative remain a puzzle that will not be solved by shifting scope.

P4. Positives

As the paraphrases used up to now indicate, degree accounts take gradable adjectives, as well as other gradable predicates, to have degree arguments. What happens then in a sentence like *the monkey is fat* where the degree argument of the gradable adjective *fat* is left free? The standard answer is that in such a case the degree argument is supplied by context. Such uses are indeed context dependent and the dependence does intuitively have to do with the degree of obesity required at the moment. But the account can't be that simple. It cannot be that the context is called upon simply to supply a degree. For it is possible in one and the same context to say that *monkey A is fat* and *monkey B is fat* and yet *monkey A is fatter than monkey B.* Spelling these assertions out in degree-terms makes the difficulty clear:

(23) Monkey A is d_1-fat.
(24) Monkey B is d_2-fat.
(25) $\exists d \exists d'$ monkey A is d-fat and monkey B is d'-fat and $d > d'$.

Assuming that the context supplies a salient degree of obesity, (23)-(25) should be impossible. By (25) d_1 and d_2 are different, but if they are both the contextually supplied degree, they must be identical.

The solution that is normally proposed is that not only does the context supply a value for the free degree-variable, but the meaning of the adjective in *the monkey is fat* is more than what you find in the comparative. In these cases, the adjective undergoes a meaning change by which it takes on a comparative sense, something along the lines of "more Adj than X" where X is supplied by context. As Klein(1980) has pointed out, this is somewhat surprising, given that it is rare or impossible to find a language where the relevant operation is morphologically realized.

PART III. OTHER ACCOUNTS

The basic assumption of this paper is that the semantics of comparatives should make reference to scales. There are however alternatives in the literature. While I will not discuss these in any detail, I will briefly illustrate how the puzzles enumerated here crop up on those accounts. A number of writers view the comparative as a quantifier over comparison classes (Wheeler 1972:321, see also McConnell-Ginet 1973, Klein 1980, Ludlow 1989). (26) is analyzed as (27) on these accounts:

(26) Jack is heavier than Jill is.

(27) There is a comparison class **c** such that:
Jack is heavy relative to **c**
and
it is not the case that Jill is heavy relative to **c**.

Now consider P1, the reverse-Russell puzzle:

(28) Simian is heavier than Smith expected him to be.

Remember that Simian weighs 20 lbs, while Smith expected him to weigh somewhere between 5 and 10 lbs. The claim was that (28) is true but the degree-analysis predicts it to be false, since there is no specific weight that Smith expected Simian to be. A comparison class analysis gives us:

(29) There is a comparison class **c** such that:
Simian is heavy relative to **c**
and
it is not the case that Smith expected Simian to be heavy relative to **c**.

In fact, (29) correctly comes out true. Let **c** be a class of monkeys relative to which a 20lb monkey is heavy but a monkey weighing 15 lbs or less is not. In that case, Smith did not expect Simian to be heavy relative to **c**. So far, so good, but now suppose that Simian weighs 9 lbs. In that case, (28) is false, but (29) remains true. This time let **c** be a class of monkeys relative to which a 9 lb monkey is heavy but a monkey weighing 7 lbs or less is not. In that case, Smith did not expect Simian to be heavy relative to **c**, for that matter Smith didn't expect Simian not to be heavy, but that is irrelevant to the truth of (29).

P3, the quantificational puzzle, crops up in alternative accounts as well. Larson(1988) was one of the first to identify the puzzle and he was working with the semantics of Klein(1980).

It is possible that **P2,** the incommensurability puzzle, is less troubling on some alternative accounts, but it is interesting to note that Williamson(1994 p125 in 1998 edition) very briefly raises such cases as an objection to fuzzy-semantics analyses of the comparative.

PART IV. SOLUTIONS

P2, the incommensurability puzzle, had to do with the scarf which varied in color from grayish white to pure white. If one tries to order a white patch from the scarf along a scale of whiteness, one finds that while the scarf and the patch are both located on the scale, they are not ordered with respect to one another. This type of situation has an analogue in the temporal domain. The second battle of WWI is located on the time line, so is WWI itself. But they are not temporally ordered with respect to one another:

(30) The second battle of WWI happened before WWII.
(31) WWI happened before WWII.

(32) NOT: The second battle of WWI happened before WWI.
(33) NOT: WWI happened before the second battle of WWI.

These facts would be puzzling if we were to assume that *before* orders a connected series of points. If instead we take it to order points as well as intervals a pattern like in (30)-(33) is expected. The intervals during which WWI and the second battle of WWI occurred are both ordered prior to WWII, but since they overlap neither is prior to the other.

I propose that we view the semantics of the comparative in the same way. The scale of whiteness consists of points but it consists of intervals as well. It doesn't matter for the moment if the intervals are defined in terms of the points or vice-versa, the important idea is that the semantics is given in terms of intervals. A comparative like (34) should be understood as in (35):

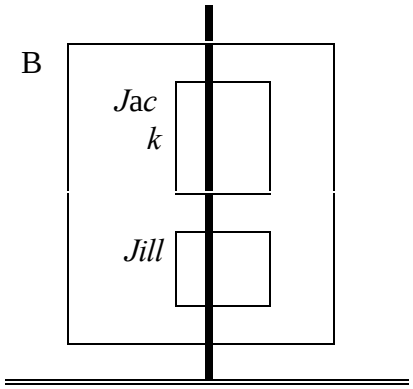
(34) That fresh snow over there is whiter than the whole scarf is.
(35) $\exists I\ \exists J$ that snow is I-white and the scarf is J-white and $I > J$.

And while the scarf and the white patch are both associated with intervals in the whiteness scale, these intervals overlap and so neither of the following is true:

(36) The patch is whiter than the whole scarf.
(37) The whole scarf is whiter than the patch is.

Within the semantics of tense, a distinction is sometimes made between being true **at** an interval versus being true **in** an interval. If my car began to roll at 7:01:02 AM and stopped at 9 AM, then it's rolling happened **at** the 7:01:02-9 AM interval. It also happened **in** that interval, but it happened in the month containing that interval as well as in the year containing that interval. This **in** relation is <u>persistent</u>. And it is this persistent relation that we ought to take as the basis for an interval semantics for comparatives. Given persistence in the temporal domain, for any two events that have occurred, there is some interval in which they both occurred. Even though Jack's birth and Jack's pubescence occurred at different times, in different intervals of time, Jack's lifespan is an interval that contains both of these events. Similarly, although there is an interval of the height scale that is associated with Jack and a different one associated with Jill, there is a larger interval that is associated with both of them. There are in fact many such intervals and the scale itself is the largest interval that is associated with both of them.

(38)



Persistence will now allow us to solve P3, the puzzle of quantifier scope. Given our new way of looking at things, an example like:

(39) It is hotter in New York than it is in most other East Coast cities.

is now analyzed as:

(40) ∃I ∃J it is I-hot in New York and it is J-hot in most other E.C. cities and I > J.

Suppose that it is still 90° Fahrenheit in New York but between 70° and 85° in most other East Coast cities. Suppose Boston is one of the other East Coast cities and it is 75° in Boston. If we let 75° name the relevant point/interval on the scale, then we could write that Boston is 75°-hot. Using similar notation, by persistence, we can write that Boston is [70°-85°]- hot. Likewise for any city whose temperature lies in that interval.

I hope that it is clear how P1, the reverse-Russell is solved. Smith expects Simian to weigh between 5 and 10 lbs. If Simian weighs say 6 lbs, that is consistent with Smith's expectations, but if Simian weighs 13 lbs, that would not be consistent. What Smith expects is that Simian will be [5-10]-heavy. A 6 lb Simian, by persistence, is [5-10]-heavy, a 13 lb Simian is not. The point on the scale for 13lbs is not contained in the [5-10] interval. Remembering now that Simian in fact weighs 20lbs we have the following:

(41)

Simian is 20lb-heavy.
Smith expected Simian to be [5-10]- lbs heavy.
[20lbs] > [5-10lbs]

∃I ∃J Simian is I-heavy
Smith expected Simian to be J-heavy.
I > J

and so:

(42) Simian is heavier than Smith expected him to be.

On the interval view, the role of context in interpreting positives reduces to the choice of an interval of the scale as the argument for the positive. Since two individuals can be of different weights but associated with some of the same intervals of the weight scale, it is entirely possible that *monkey A is fat* and *monkey B is fat* and yet *monkey A is fatter than monkey B.* No change of meaning is necessary in the formation of the positive, though there is a constraint on the choice of the contextually supplied interval: there must be no points on the scale that lie above it. This accounts for the intuition that if *B* counts as fat and *A* is fatter than *B*, then *A* counts as fat.

PART V. MEASURE PHRASES

The italicized phrases in the following examples are sometimes called **measure phrases**:

(43) Jack is *much* happier than Jill is.
(44) Jack read *many* more books than Jill did.
(45) Jack is *10 years* older than Jill is.
(46) Jack is *10 years* old.
(47) The pole is *10 ft* high and the building is *2in* higher than the pole.

In the first example, (43), *much* tells us something about the gap between Jack's degree of happiness and Jill's. As a first guess then, we might analyze (43) as follows[2]:

(48) ∃I ∃J Jack is I-happy and Jill is J-happy and much([J-I])

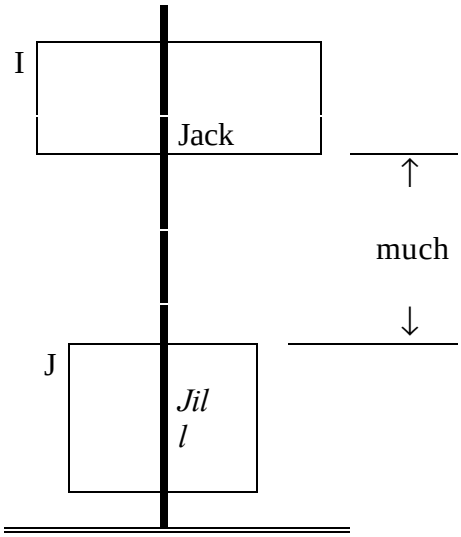
much is a predicate operating on intervals with the following semantics:

(49) much(K) = 1 iff K is large.

In (48), [I-J] names the interval that fills the gap that runs from J up to I. *much* is a context dependent adjective, like it's count counterpart *many*, and so what counts as 'large' will be a function of a variety of factors affecting expectations about size (cf. Fernando and Kamp(1996), Lappin 2000).

[2] For a more detailed account of the relationship between the 'upper' and 'lower' intervals and the measure phrase, see Schwarzschild & Wilkinson (to appear).

(50)



Numerical measure phrases work in a similar fashion. The phrases *6 inches* and *30° Celsius* can be thought of as predicates of intervals (contrary to what we said above about the metalanguage '75°' in "Boston is 75°-hot"). The stretch from 70° to 100° is a 30° interval. It is this predicative use of *30°* that we find in *it was 30° hotter in Rome than it was in Venice.* On this view, comparatives are a kind of expression that may take a predicate of intervals argument. That argument specifies the size of the gap. Positive adjectives do not take such arguments, which explains the ungrammaticality of:

(51) *Jack is much happy.
(52) *It is 30° hot in here.

Contrasting with (52), we do find:

(53) It wasn't <u>that</u> hot in here yesterday.

Positive adjectives denote interval-taking functions. In (53), *that* refers to an interval, hence the combination is possible. This difference between interval denoting *that* and predicative *much* shows up as well in the contrast between (51), (53)and (54):

(54) *It was <u>that</u> hotter in Texas than it was in New Jersey.

Given what we have said so far, we predict that with the exception of *that,* we should not find measure phrases combining with positive adjectives. Indeed some languages pattern that way, possibly most languages. Nevertheless, we do find lexical variation both within and across languages. While it is true that English does not allow **2ft low, *10 years young* we do find *2ft high, 10 years old*. And while English doesn't allow **20° hot* or **2lbs heavy* or **20mph fast*, German does. So there is a story to be told about how these exceptions come about, about what accounts for subregularities notably the

difference between antonym pairs such as *high* and *low* and finally about the distribution of measure phrases in noun phrases: *10lbs of water, 10 years of work, *10° of water*.

PART V. CONCLUSION

The key idea here is that gradable predicates denote relations between individuals and elements in an ordered set of intervals. Two intervals can be non-overlapping in which case one is higher than the other. This is required in the case of comparatives. Because of persistence, two individuals can be associated with distinct intervals and at the same time be associated with a third common interval. This possibility is unavailable when scales are viewed simply as a set of linearly ordered points and this was the source of many of the difficulties raised in Part II.

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