

## Semantics: Study of Meaning

Semantics studies the meaning of sentences. What does it mean to know the meaning of a sentence?

Consider the following situation.

### Situation A



Based on Situation A, which of the sentences are **True** and which are **False**?

- (1) John saw Susan.
- (2) A person saw another person.
- (3) A woman saw a man.
- (4) A man saw himself.

Given a situation, we can know whether a given sentence is **True** or **False**. Every declarative sentence can have a **Truth Value**.

- (5) **Truth value** = {True, False}

Now consider the following sentence:

- (6) John loves Mary.

Is (6) True or False?

We don't know if (6) is True or False but we can know it if we are given a situation. Because, we know the **conditions** under which (6) can be True or False. In other words, we know its **Truth Conditions**.

For some linguists/philosophers, to know the meaning of a sentence is to know its **Truth Conditions**. This is called **Truth-Conditional Semantics**, which was first used by Donald Davidson. We will try to derive the truth conditions of sentences *compositionally*.

## Semantic Relations

Sentences can have a number of relations among themselves. Given two sentences, we can detect some relations between the two sentences. The first type of relation that we will use is based on Truth. It is called Entailment.

### Entailment

Given two sentences **p** and **q**,  
Whenever p is true, if q is also true then p entails q.

- (7) John loves Mary.
- (8) A man loves a woman.

Given our world knowledge, whenever sentence (7) is true then sentence (8) must also be true. Hence, sentence (7) entails sentence (8).

We can represent entailment with a rightward arrow.

(9)  $p \rightarrow q$

The representation in (9) is read as *p entails q*. Thus for sentences (7) and (8), we can use  $(7) \rightarrow (8)$ .

Consider the following sentences.

(10) **p**: John loves Mary.  
**q**: A man loves Mary.

In (10), p entails q. ( $p \rightarrow q$ )

(11) **p**: John loves Mary.  
**q**: Mary loves John.

In (11), p does not entail q. ( $p \rightarrow q$ )

**Note** that entailment is not a bi-directional relation. In (11), p entails q but q does not entail p.

(12) **p**: John loves Mary.  
**q**: A man loves a woman.

In order to determine whether a sentence entails another sentence, we need to consider every possible situation. In other words, we should try to find a situation where p is True but q is False.

### Practice

- (13) **p**: Patrick is a smart boy.  
**q**: Patrick is a boy.
- (14) **p**: Susan is taller than Mary.  
**q**: Susan is tall.
- (15) **p**: Jon Snow knows that Ygritte is dead.  
**q**: Ygritte is dead.
- (16) **p**: Jon Snow does not regret killing Ygritte.  
**q**: Ygritte is dead.
- (17) **p**: It is false that everyone tried to kill Templeton.  
**q**: Someone tried to kill Templeton.
- (18) **p**: Oscar and Jenny are rich.  
**q**: Oscar is rich.
- (19) **p**: Oscar and Jenny are young.  
**q**: Oscar is young.

## Tautology

Some sentences are always True. These sentences are called Tautologies. Tautologies do not entail anything.

(20) John is at home or John is not at home.

## Contradiction

Some sentences are always False. These are contradictions. Contradictions entail everything.

(21) John is at home and John is not at home.

## Presupposition

Another type of semantic relation between two sentences is **presupposition**.

In presupposition, given two sentences p and q, sentence p takes the Truth of sentence q for granted.

(22) **p:** The present king of France is bald.  
**q:** Presently, there is a king of France.

In other words, in order for p to be True or False, q must be True. Otherwise, p is **undefined**, i.e. it doesn't have a Truth Value. We call this a **Presupposition Failure**. Sentence (22p) causes a presupposition failure because France is no longer a monarchy.

## P-family Tests

Presuppositions are known to project. In other words, presuppositions keep staying, under certain environments. These are called the P-family tests. In order to make sure that some given sentence has a presupposition, we need to apply the P-family tests.

## Negation

Just add it is not the case that before a sentence to see if presupposition projects (continues to live). Alternatively, you can negate the sentence but the first option is more safe.

(23) **p: It is not the case that** the present king of France is bald.  
**q:** Presently, there is a king of France.

## Question

Turn the sentence into a question and see if the presupposition projects.

(24) **p: Is** the present king of France is bald?  
**q:** Presently, there is a king of France.

## Conditional

Put the sentence in a conditional and see if it projects.

(25) **p: If** the present king of France is bald, then...  
**q:** Presently, there is a king of France.

## Practice

Consider the following sentence pairs. Identify if the p sentences entail or presuppose the q sentences. In some cases it can be both or neither.

- (26) **p:** Vincent cleaned his car.  
**q:** Vincent has a car.
- (27) **p:** Vincent didn't clean his car.  
**q:** Vincent has a car.
- (28) **p:** Who discovered Pluto?  
**q:** Pluto was discovered.
- (29) **p:** Today is sunny.  
**q:** Today is warm.
- (30) **p:** If John discovers that Mary is in New York, he will get angry.  
**q:** Mary is in New York.
- (31) **p:** Did you stop beating your wife?  
**q:** You used to smoke.
- (32) **p:** Mary regrets smoking.  
**q:** Mary smokes.
- (33) **p:** Mary doesn't regret smoking.  
**q:** Mary smokes.
- (34) **p:** Alex is a man.  
**q:** Alex is male.
- (35) **p:** Bill doesn't like his wife.  
**q:** Bill has a wife.
- (36) **p:** Lee got a perfect score on the quiz  
**q:** Someone got a perfect score on the quiz.
- (37) **p:** It was Lee who got a perfect score on the quiz  
**q:** Someone got a perfect score on the quiz.

### Presupposition Triggers

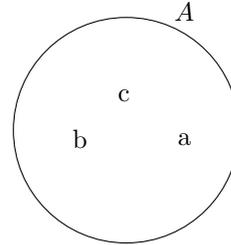
The following elements usually trigger presuppositions.

- Definite article (The)
- Factive verbs (know, regret, etc.)
- Change of state verbs (begin, stop, quit, etc.)
- Expressions of repetition (again, another, etc.)
- Clefts (It was John who ...)
- Questions

## Set Theory

A set is an unordered collection of individuals. Sets can be represented with braces (curly brackets) or Venn diagrams.

$$A = \{a, b, c\}$$



For any given set  $A$ , an individual can be either in the set  $A$  or not.

- If an individual  $x$  is in the set  $A$ , we say:  $x$  is a member of  $A$  and represent it by  $x \in A$ .
- If an individual  $x$  is not in the set  $A$ , we say:  $x$  is not a member of  $A$  and represent it by  $x \notin A$ .

## Set Relations

Sets can have relations among them. We'll focus on *intersection*, *union*, *subset*, *superset* relations.

### Intersection

Consider two sets  $A$  and  $B$ . If an individual  $x$  is in both sets, then the intersection of the two sets  $A$  and  $B$  yields a set that includes  $x$ .

We represent the intersection of two sets as  $A \cap B$ .

$$A = \{a, b, c, d\}$$

$$B = \{c, d, e, f\}$$

$$A \cap B = \{c, d\}$$

### Union

Union of two sets yields a set with all the members of two sets. We represent it with  $A \cup B$ .

$$A = \{a, b, c, d\}$$

$$B = \{c, d, e, f\}$$

$$A \cup B = \{a, b, c, d, e, f\}$$

### Subset - Superset

If all the members of set  $A$  are also members of set  $B$  then  $A$  is a subset of  $B$ .

$$A \subseteq B \text{ read as } A \text{ is a subset of } B$$

If  $A$  is a subset of  $B$ , then  $B$  is a superset of  $A$ .

$$B \supseteq A \text{ read as } B \text{ is a superset of } A$$

**For Example**

$$A = \{a, b, c, d\}$$
$$B = \{c, d\}$$

$$B \subseteq A$$
$$A \supseteq B$$

**Empty Set**

A set without any members is called the empty set. The empty set is a subset of every set. The empty set can be represented as  $\{\}$  or  $\emptyset$ .

**Cardinality**

Cardinality of a set gives the number of individuals in a set. It is represented as  $|n|$ .

For example, the cardinality of the set  $A = \{a, b, c\}$  is  $|3|$ .

**Practice**

$$A = \{2, 3, 4, 5, 6, 9, 11, 33\}$$

$$B = \{2, 3, 0\}$$

$$C = \{5, 3, 11, 2\}$$

**True or False**

$$A \supseteq B$$

$$C \supseteq A$$

**Give the sets**

$$A \cap B =$$

$$A \cup B =$$

## Compositionality and Denotation

Earlier, we talked about meanings of sentences. A sentence like (38) can be either True or False.

(38) The silly boy in the park cried.

Nevertheless, we cannot know whether it is True or False, unless given a situation. On the other hand, as the speakers of the language, we know its truth conditions. We know the conditions under which sentence (38) would be True or False.

Due to our knowledge of **syntax**, we also know that a sentence is composed of its parts (namely phrases). We know that the way a set of words combine yield (un)grammaticality.

(39)  $[_{TP} [_{DP} \text{The } [_{NP} \text{silly boy in the park}] ] [_{VP} \text{fell down} ] ]$

Now that we know how to form a sentence, we want to understand what it means. We know that the meaning of a sentence is its Truth Conditions. Now, we need to derive the truth conditions of a sentence compositionally. To understand the meaning of a sentence, we need to understand the meaning of its parts.

### Denotation

Denotation is the technical form for meaning of a given phrase. Once we figure out the denotation of words and phrases, we can figure out the denotations of sentences. Let's focus on the denotation of the NP in Sentence (38).

(40)  $[_{NP} \text{silly boy in the park} ]$

The NP in (40) consists of a noun, an adjective and a prepositional phrase. Common nouns, adjectives and PPs are predicates. A predicate is simply a set of individuals. Linguists use double brackets to show denotation of a word/phrase.  $[[ ]]$

Denotation of names yield individuals. On the other hand, denotation of common nouns, adjectives and PPs yield sets of individuals.

### Situation A

#### Individuals

$[[John]] = John$

$[[Bill]] = Bill$

$[[Tom]] = Tom$

$[[Mary]] = Mary$

$[[Susan]] = Susan$

#### Predicates

$[[boy]] = \{Bill, Tom, John\}$

$[[silly]] = \{Bill, Susan, John\}$

$[[in the park]] = \{Mary, Tom, John\}$

Now that we know the denotation of each word, we can get the denotation of NPs. Denotation of NPs is merely **set intersection**.

$[[boy]] = \{Bill, Tom, John\}$

$[[silly boy]] = \{Bill, John\}$

$[[silly boy in the park]] = \{John\}$

**Practice**

Consider the following situation.

**Individuals**

$\llbracket John \rrbracket = John$

$\llbracket Bill \rrbracket = Bill$

$\llbracket Tom \rrbracket = Tom$

$\llbracket Mary \rrbracket = Mary$

$\llbracket Susan \rrbracket = Susan$

**Predicates**

$\llbracket person \rrbracket = \{Bill, Tom, John, Susan, Mary\}$

$\llbracket boy \rrbracket = \{Bill, Tom, John\}$

$\llbracket silly \rrbracket = \{Susan, John\}$

$\llbracket in\ the\ room \rrbracket = \{Mary, Tom\}$

$\llbracket tall \rrbracket = \{Mary, John\}$

$\llbracket smart \rrbracket = \{Mary, Tom\}$

**Give the denotations of the following NPs**

(41) smart boy in the room

(42) tall person in the room

(43) smart silly boy

In terms of semantic relations (entailment, tautology, contradiction, presupposition, etc.), what does *smart silly boy* remind us?

(44) smart boy

(45) boy

In terms of set theory, what is the relation between smart boy and boy?

Sometimes sets can include to a very large number of individuals. It would be hard (even impossible in some cases) to write every member of the set. Therefore, we give the description of the set.

(46)  $\text{human} = \{x \mid x \text{ is human}\}$  read as  $x$  such that  $x$  is human.

This is the set of human beings. It would be impossible to list all the names of human beings out there but we can refer to them by simply saying the name of the set.

### Practice

Consider the following sets in conjunction with the ones above. Give the denotations of rich and happy by listing their members.

(47)  $\llbracket \text{rich} \rrbracket = \{x \mid x \text{ is either tall or silly}\}$

(48)  $\llbracket \text{happy} \rrbracket = \{x \mid x \text{ is tall and smart and rich}\}$

Consider set theory relations. What do **and** and **or** look like?

### Give the denotations of the following NPs

(49) Rich person in the room

(50) Rich boy in the room

(51) Happy person

(52) Happy boy

What is the denotation of **Mary**? How is it different from the denotation of **happy person**?

## Denotation of VPs

So far, we worked on the denotation of NPs. NPs can consist of proper nouns (names), common nouns, adjectives, prepositional phrases, relative clauses etc. All these are basically words or phrases. They are just a bunch of symbols put together in a systematic way. *Semantics* is the component that associates these words/phrases with meaning. Semantics interprets these words/phrases.

We made a distinction between two types of words/phrases. These are words/phrases that denote **individuals** and words/phrases that denote **predicates** (i.e. sets of individuals). This is a crucial distinction. Names denote individuals. On the other hand, common nouns, adjectives, PPs, relative clauses are predicates.

Our overall goal is to establish the denotation of sentences. For now, we know the denotation of NPs. We want to figure out the meaning of a sentence like:

(53) The silly boy in the park cried.

The denotation of this sentence must be a **Truth Condition** because knowing the meaning of a sentence means knowing its truth conditions. We want to derive this. However, we still don't know the denotation of VPs.

VPs are **predicates**, too. They just denote sets of individuals.

(54)  $[[sleeps]] = \{ \text{John, Mary, Bill} \}$

(55)  $[[runs]] = \{ \text{Bill} \}$

(56)  $[[fell]] = \{ \text{John} \}$

## Denotations of Sentences

Now that we know the denotations of NPs and VPs, we are in a position to derive the denotation of a given sentence. The sentence in (53) is a bit complex because it has a Determiner and we don't know the denotation of a determiner, yet. So, let's start with a simpler sentence.

(57) John sleeps.

Very crudely, this sentence consists of a DP and a VP.

(58)  $[_{DP} \text{John}] [_{VP} \text{sleeps}]$

### Truth Condition of a Sentence with a Name Subject

A sentence is **True iff** (if and only is) the denotation of the DP is **in** the denotation of the VP.

In other words,

(59)  $[[Sentence]] = \text{True iff } [[DP]] \text{ is in } [[VP]].$

Yet another convention,

(60)  $[[Sentence]] = \text{True iff } [[DP]] \in [[VP]].$

Therefore,

(61)  $[[John \text{ sleeps}]] = \text{True iff } [[John]] \in [[sleeps]].$

The statement in (61) is the Truth Conditions of the sentence [John sleeps.]. Thus, it is the denotation (meaning) of the sentence [John sleeps].

If we want to get the **Truth Value** of [John sleeps], all we have to do is to check the world (or the given situation) to see if the **individual named John** is in the **set sleeps**. For example, in the scenario given in (54) through (56), [John sleeps] would be True.

### Practice

What are the **Truth Conditions** and the **Truth Value** of the following sentence according to the situation above?

(62) Bill fell.

### Exercises

Give the truth conditions of the sentences below. Additionally, give a situation which accounts for all the desired Truth Values specified for each sentence.

### Situation

### Sentences

(63) John visited Tom.  
Truth Value = **True**  
Truth Condition =

(64) Mary is brilliant.  
Truth Value = **True**  
Truth Condition =

(65) Tom is silly.  
Truth Value = **False**  
Truth Condition =

### Challenge question

(66) The brilliant girl visited Tom.  
Truth Value = **False**  
Truth Condition =

## Generalized Quantifiers

Yesterday, we worked on the truth conditions of sentences whose subjects were proper nouns (names).

(67) John sleeps.

Deriving the truth conditions of sentences like (1) is straightforward. John is an individual, sleeps is a predicate. So sentence 1 is true iff *John* is in the set denoted by *sleeps*.

(68)  $\llbracket \text{John sleeps} \rrbracket = \text{True}$  iff  $\llbracket \text{John} \rrbracket \in \llbracket \text{sleeps} \rrbracket$

DPs are not always that simple, though. Consider cases like **every student**.

*Every* is a quantifier. Quantifiers establish relations between predicates. In other words, they take NPs and relate them to VPs.

A sentence like “**Every man laughs.**” consists of three parts. **A quantifier**, a **NP** and a **VP**.

Let’s give it a template.

Every	man	laughs
<b>Quantifier</b>	<b>NP</b>	<b>VP</b>

If a sentence is in the form “**Quantifier NP VP**”, then we use the following templates for identifying the Truth Conditions of the sentence.

### Quantifiers and their denotations

$\llbracket \text{Every } NP VP \rrbracket$	= True iff $\llbracket NP \rrbracket \subseteq \llbracket VP \rrbracket$
$\llbracket \text{No } NP VP \rrbracket$	= True iff $\llbracket NP \rrbracket \cap \llbracket VP \rrbracket = \emptyset$
$\llbracket \text{Some } NP VP \rrbracket$	= True iff $\llbracket NP \rrbracket \cap \llbracket VP \rrbracket \neq \emptyset$
$\llbracket \text{Exactly } n \text{ } NP VP \rrbracket$	= True iff $ \llbracket NP \rrbracket \cap \llbracket VP \rrbracket  = n$
$\llbracket \text{At least } n \text{ } NP VP \rrbracket$	= True iff $ \llbracket NP \rrbracket \cap \llbracket VP \rrbracket  \geq n$
$\llbracket \text{At most } n \text{ } NP VP \rrbracket$	= True iff $ \llbracket NP \rrbracket \cap \llbracket VP \rrbracket  \leq n$
$\llbracket \text{Most } NP VP \rrbracket$	= True iff $ \llbracket NP \rrbracket \cap \llbracket VP \rrbracket  >  \llbracket NP \rrbracket  / 2$

Note that  $|\cdot|$  indicates the cardinality of a set. In other words, it gives us the number of individuals in the set.

### The

**The** is a little bit different. It has a presupposition. Therefore, we can talk about its felicity condition.

$\llbracket \text{The } NP VP \rrbracket$  is felicitous iff  $|\llbracket NP \rrbracket| = 1$

This is because when we say the man, we assume that there is only one man (not 2 or 3).

### Practice

Give the truth conditions of the following sentences.

(69) Exactly three girls are rich.

(70) No boy is tall.

(71) Most books are cheap.

Consider the following sentence. Can you give its truth condition?

(72) Two girls are tall.

## Exercises

- (73) If  $\llbracket \textit{cat} \rrbracket = \{\textit{Sammy, kitty, princess}\}$ ,  $\llbracket \textit{white} \rrbracket = \{\textit{Sammy}\}$  and  $\llbracket \textit{cute} \rrbracket = \{\textit{kitty}\}$ , then  $\llbracket \textit{cute kitty} \rrbracket = \{\textit{Sammy}\}$  or  $\{\textit{kitty}\}$ . (True/False)
- (74) If  $\llbracket \textit{cat} \rrbracket = \{\textit{Sammy, kitty, princess}\}$ ,  $\llbracket \textit{white} \rrbracket = \{\textit{Sammy}\}$  and  $\llbracket \textit{cute} \rrbracket = \{\textit{kitty}\}$ , then  $\llbracket \textit{cute white cat} \rrbracket = \{\textit{Sammy}\}$  or  $\{\textit{kitty}\}$ . (True/False)
- (75) “Bill went to NY to see a play” entails “Bill went to NY and saw a play”. (T/F)
- (76) Suppose there are three individuals, Mary, Suzy and Jane. Only Mary and Suzy are tall. Jane and Suzy are blond. Mary is lazy. Given this situation, what would be the semantic values of the following expressions:
- $\llbracket \textit{lazy and blond} \rrbracket =$
  - $\llbracket \textit{Suzy} \rrbracket =$
  - Which of the following is a superset of *girls*?
    - girls
    - tall and lazy
    - blond girl
    - none of the above
- (77) Which of the following contains contradictory sentences?
- ‘John is not tall’ - ‘John is short’.
  - ‘Most girls wished for a white pony’ - ‘Some girl wished for a black pony’
  - ‘All of John’s friends are silly.’ - ‘Some of John’s friends are not silly.’
  - ‘Every brown dog barks loudly’ - ‘Some brown dog barks loudly’
  - ‘The milk is hot or at room temperature’ - ‘The milk is cold’
- (78) Provide the semantic values for the five items below, such that the sentences in (a) to (d) have the given truth values:
- Every dog and some cat is white. (T)
  - No dog is black or furry. (F)
  - Two dogs and two cats are white. (T)
  - No cat is furry. (T)

$\llbracket \textit{cat} \rrbracket =$	$\llbracket \textit{white} \rrbracket =$
$\llbracket \textit{dog} \rrbracket =$	$\llbracket \textit{furry} \rrbracket =$
$\llbracket \textit{black} \rrbracket =$	

- (79) The two following questions are based off the following sentences:
- Most Ling 201 students studied hard for Midterm.
  - John, who is a Ling 201 student, studied hard for Midterm.

Sentence (a) entails sentence (b). (T/F)

If your answer above is ‘True’, describe a **situation** in which Sentence (a) and Sentence (b) have the same truth values. If your answer above is ‘False’, describe a situation in which Sentence (a) and Sentence (b) have different truth values.

- (80) Consider the following situation:  
 $\llbracket Tom \rrbracket = \text{Tom}$ ,  $\llbracket Mary \rrbracket = \text{Mary}$ ,  $\llbracket Paul \rrbracket = \text{Paul}$ ,  $\llbracket Susan \rrbracket = \text{Susan}$ ,  $\llbracket John \rrbracket = \text{John}$ ,  
 $\llbracket girl \rrbracket = \{\text{Mary, Susan}\}$   $\llbracket boy \rrbracket = \{\text{Tom, Paul, John}\}$   
 $\llbracket studied\ hard\ for\ Ling\ 201\ test \rrbracket = \{\text{Tom, Mary}\}$   
 $\llbracket has\ missed\ breakfast \rrbracket = \{\text{Tom, Paul, Mary, Susan}\}$   
 $\llbracket has\ work\ after\ class \rrbracket = \{\text{Mary, Susan}\}$
- a. Those who have work after class are a subset of those who have missed breakfast. (T/F)  
 b.  $\llbracket studied\ hard\ for\ Ling\ 201\ test \rrbracket$  is a superset of the intersection of  $\llbracket has\ missed\ breakfast \rrbracket$  and  $\llbracket boy \rrbracket$ . (T/F)  
 c.  $\llbracket Tom \rrbracket$  is a member of the set intersection of  $\llbracket studied\ hard\ for\ Ling\ 201\ test \rrbracket$  and  $\llbracket has\ work\ after\ class \rrbracket$ . (T/F)

- (81) Suppose **erc** is a quantifier in English with the following denotation:  
 $\llbracket Erc\ NP\ VP \rrbracket = \text{True iff at least 3 individuals in } \llbracket NP \rrbracket \notin \llbracket VP \rrbracket$

### Situation X

$\llbracket Tom \rrbracket = \text{Tom}$ ,  $\llbracket Ed \rrbracket = \text{Ed}$ ,  $\llbracket Paul \rrbracket = \text{Paul}$ ,  $\llbracket John \rrbracket = \text{John}$ ,

$\llbracket boy \rrbracket = \{\text{Tom, Paul, John, Ed}\}$

$\llbracket smart \rrbracket = \{\text{Tom}\}$

- a. Is 'Erc boys are smart' True or False in Situation B?

Modifying just the set denoted by *smart*, how would you make 'Erc boys are smart' to have the opposite truth value in Situation X to the one you indicated above?

## Downward Entailment

Consider the following sentences:

(82) John ran.

(83) John ran fast.

What is the entailment relation between two sentences. What are the subset superset relations?

Now, consider the following sentences:

(84) Nobody ran.

(85) Nobody ran fast.

What is the entailment relation between the two sentences? What are the subset superset relations?

Contexts where entailment relations are reversed like above because of elements like *nobody*, *not*, *few*, *at most* are called **downward entailing contexts**.

## Denotation of Sentences and Quantifiers

We have already discussed that denotations of sentences are Truth Conditions. We can understand a sentence without knowing its Truth Value (True or False) because we can understand its Truth Conditions. Our goal has been to derive the Truth Conditions of sentences *compositionally*. We made a distinction between two types of sentences: Sentences whose subjects are proper nouns (i.e. names) and sentences whose subjects have a common noun and a Quantifier.

### Sentences with proper noun subjects

Our general formula for sentences with proper noun subjects is given in (86).

$$(86) \quad \llbracket NP VP \rrbracket = \text{True iff } \llbracket NP \rrbracket \in \llbracket VP \rrbracket$$

### Sentences whose subjects have Quantifiers

$$\begin{aligned} \llbracket \text{Every } NP VP \rrbracket &= \text{True iff } \llbracket NP \rrbracket \subseteq \llbracket VP \rrbracket \\ \llbracket \text{No } NP VP \rrbracket &= \text{True iff } \llbracket NP \rrbracket \cap \llbracket VP \rrbracket = \emptyset \\ \llbracket \text{Some } NP VP \rrbracket &= \text{True iff } \llbracket NP \rrbracket \cap \llbracket VP \rrbracket \neq \emptyset \\ \llbracket \text{Exactly } n NP VP \rrbracket &= \text{True iff } | \llbracket NP \rrbracket \cap \llbracket VP \rrbracket | = n \\ \llbracket \text{At least } n NP VP \rrbracket &= \text{True iff } | \llbracket NP \rrbracket \cap \llbracket VP \rrbracket | \geq n \\ \llbracket \text{At most } n NP VP \rrbracket &= \text{True iff } | \llbracket NP \rrbracket \cap \llbracket VP \rrbracket | \leq n \\ \llbracket \text{Most } NP VP \rrbracket &= \text{True iff } | \llbracket NP \rrbracket \cap \llbracket VP \rrbracket | > | \llbracket NP \rrbracket | / 2 \end{aligned}$$

### Practice

Suppose **erc** is a quantifier in English with the following denotation:

$$(87) \quad \llbracket \text{Erc } NP VP \rrbracket = \text{True iff at least 3 individuals in } \llbracket NP \rrbracket \notin \llbracket VP \rrbracket$$

### Situation X

$$\llbracket \text{Tom} \rrbracket = \text{Tom}, \quad \llbracket \text{Ed} \rrbracket = \text{Ed}, \quad \llbracket \text{Paul} \rrbracket = \text{Paul}, \quad \llbracket \text{John} \rrbracket = \text{John},$$

$$\llbracket \text{boy} \rrbracket = \{\text{Tom}, \text{Paul}, \text{John}, \text{Ed}\}$$

$$\llbracket \text{smart} \rrbracket = \{\text{Tom}\}$$

a. Is 'Erc boys are smart' True or False in Situation B?

Modifying just the set denoted by *smart*, how would you make 'Erc boys are smart' to have the opposite truth value in Situation X to the one you indicated above?

## Entailment Relations Among Sets

In previous lectures, we discussed entailment relations between sentences. For example, (88) entails (89) but not vice versa.

$$(88) \quad \text{Patrick is a smart boy.}$$

$$(89) \quad \text{Patrick is a boy.}$$

We have been working on denotations of sentences and we have been using **set theoretic relations** to identify denotations of sentences. In fact, we can also identify entailment relations with sets. Note that denotations of NPs like *smart boy* is established by intersecting the denotations of *smart* and *boy*.

$$(90) \quad \llbracket \text{boy} \rrbracket = \{\text{Tom}, \text{Paul}, \text{John}, \text{Ed}\}$$

$$(91) \quad \llbracket \text{smart} \rrbracket = \{\text{Tom}, \text{Mary}, \text{Susan}, \text{Ed}\}$$

$$(92) \quad \llbracket \text{smart boy} \rrbracket = \{\text{Tom}, \text{Ed}\}$$

*Smart boy* is always a subset of *boy*, or *smart*. Thus, if Partick is a smart boy, then he is definitely a boy because *smart boy* is always a subset of *boy*. Such entailment relations where a subset entails a superset are called **upward entailment**.

Now, consider the following sentences:

- (93) Nobody ran.  
 (94) Nobody ran fast.

What is the entailment relation between the two sentences? What are the subset superset relations?

Contexts where entailment relations are reversed like above because of elements like *nobody*, *not*, *few*, *at most* are called **downward entailing contexts**. In a downward entailing context, the superset entails the subset.

### Practice

Take a look at the sentence pairs below. Identify the entailment relations between the sentence pairs. Is the relation downward entailing or upward entailing?

- (95) a. Every Italian student smokes.  
       b. Every Student smokes.  
 (96) a. Some student is Italian and tall.  
       b. Some student is Italian.  
 (97) a. No student smokes.  
       b. No Italian student smokes.  
 (98) a. Few unsullied run.  
       b. Few unsullied run fast.  
 (99) a. At most three students watch GoT.  
       b. At most three students watched GoT last night.

### Why does directionality in entailment matter?

Words (or phrases) like *ever*, *any*, *give a damn*, *at all* are called **negative polarity items**. These words denote some negativity and they occur in certain contexts. They don't occur freely.

Take a look at the following sentences and see if you can come up with a generalization about the distribution of the NPIs (negative polarity items).

- (100) No student did any work.  
 (101) \*Some student did any work.  
 (102) Few people have ever seen the Niagara Falls.  
 (103) \*John saw any bird.  
 (104) John didn't see any bird.

## Scope Ambiguities

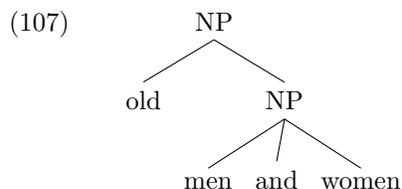
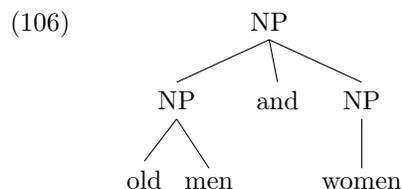
Consider the following phrase:

(105) old men and women

This phrase is ambiguous. It can mean:

- a. [old men] and some women
- b. old men and old women

The ambiguity results from the two possible syntactic structures that can be linearized in the same way.



The difference in meaning is caused by the **scope** of the adjective *old*. In (106), *old* takes scope over only *men* while in (107), *old* takes scope over both *men* and *women*. Ambiguities like this are called scope ambiguities.

We can define scope as the range of words (or phrases) a given expression has control over. In (106) *old* controls the meaning of *men* while in (107), *old* controls the meaning of *men* and *women*.

**As a general rule**, a scope bearing element takes scope over the elements that are in its sister. So, in (106), the only element in the scope of *old* is *men* while in (107), the elements that are in the scope of *old* are *men* and *women*.

## Quantifiers, Operators, and Scope

Natural language is full of ambiguities and one of the main reasons for such ambiguities is scope. Quantifiers like *every*, *some*, *no*, *two*, *etc.* and operators like *not* are scope bearing elements. These elements take scope over their sisters and interact with their meaning. For example the negation operator *not*, takes scope over its complement and negates it.

(108) I will go home.

(109) I will [**not** go home.]

When we get two or more scope bearing elements in a sentence, we get ambiguity (most of the time).

(110) Everyone loves someone.

- a. There is one person *x* such that everyone loves *x*.
- b. For every person *x*, there is a person *y* such that *x* loves *y*.

### Another example

(111) A guard stands in front of every gate.

- a. There is one guard *x* such that *x* stands in front of every gate.
- b. For every gate *y*, there is one guard *x* such that *x* stands in front of *y*.

These are scopal ambiguities. The ambiguities in these sentences arise because of the quantifiers *every* and *some* (some here refers to *a/someone*).

When *every*, which is called the *universal quantifier* ( $\forall$ ) takes scope over *some*, called the *existential quantifier* ( $\exists$ ), we get a **distributed** reading. In other words, we get the (b) readings in sentences (110) and (111).

When the existential quantifier takes scope over the universal quantifier, we get the collective reading. In other words, we get the (a) readings in (110) and (111).

### Question

How do we get the collective reading in (110) and the distributed reading in (111)? What does this tell us about the grammar in general? Think about the Y model of grammar.

## Binding Theory

In general we have three types of Noun Phrases. These are **R-expressions**, **Anaphors**, and **Pronouns**.

### R-expressions

R-expression means “a referring expression”. Consider the following sentence:

(112) Marissa typed the assignment in in L<sup>A</sup>T<sub>E</sub>X.

In this sentence *Marissa* and *the assignment* are R-expressions. They refer to entities in the world.

### Anaphors

An anaphor is an NP that obligatorily gets its meaning from another NP in the sentence.

(113) Sam slapped **himself** when he saw Gilly.

Typical anaphors are *himself*, *herself*, *themselves*, *myself*, *yourself*, *ourselves*, *yourselves*, and *each other*.

### Pronouns

Pronouns are NPs that can optionally get their meaning from another NP in the sentence but can also get it from the context or from the previous sentence.

(114) Selina Kyle said that **she** saw Bruce Wayne.

Typical pronouns include: *he*, *she*, *it*, *I*, *you*, *me*, *we*, *they*, *us*, *him*, *her*, *them*, *his*, *her*, *your*, *my*, *our*, *their*, and *one*.

### Antecedent

An antecedent is an NP that gives its meaning to another NP.

(115) **Susan** saw *herself* in the mirror.

In this sentence, *Susan* is the antecedent and *herself* is the anaphor.

### Coindexation

Traditionally, linguists have used **indices** to indicate co-reference or disjoint reference. We write indices as letters or numbers in the subscript form.

(116) Susan<sub>*i*</sub> saw herself<sub>*i*</sub>.

In (116), the same indices show that the two NPs refer to the same entity.

(117) Susan<sub>*i*</sub> saw her<sub>*k*</sub>.

In (117), there are two indices that are different. Hence, this indicates that *Susan* and *her* refer to two distinct entities.

Sometimes linguists use more than one index in a given NP. For example:

(118) Susan<sub>*i*</sub> saw herself<sub>*i*</sub>/<sub>*\*k*</sub>.

In (118), the subscripts show that *herself* refers to Susan (*i* index) and it cannot refer to anything else than Susan (*\*k* index).

(119) Susan<sub>*i*</sub> saw her<sub>*\*i*</sub>/<sub>*k*</sub>.

In (119), the indices show that *her* cannot refer to Susan. It must refer to someone else in the context.

## Binding

When the meaning of an NP is controlled by another NP, we call this binding. We say *x* binds *y* (*x* and *y* are just placeholders).

The distribution of NPs (R-expressions, anaphors, and pronouns) is not free. They occur in complementary environments. This seems to be universally true (in all languages). By looking at the following sentences, can you predict their distribution? Can you come up with a theory that tells us where each element can occur.

(120) Every princess<sub>*i*</sub> appreciates herself<sub>*i*</sub>.

(121) \*Every princess<sub>*i*</sub> appreciates her<sub>*i*</sub>.

(122) \*Every princess<sub>*i*</sub> knows that we appreciate herself<sub>*i*</sub>.

(123) Every princess<sub>*i*</sub> knows that we appreciate her<sub>*i*</sub>.

(124) \*Heidi<sub>*i*</sub> saw Susan<sub>*i*</sub>.

(125) \*She<sub>*i*</sub> kissed Heidi<sub>*i*</sub>.

(126) \*She<sub>*i*</sub> said that Heidi<sub>*i*</sub> was a disco queen.

**Binding Principle A:** \_\_\_\_\_ must be bound by an antecedent in the same clause (smallest TP).

**Binding Principle B:** \_\_\_\_\_ cannot be bound by an antecedent in the same clause (smallest TP).

**Binding Principle C:** \_\_\_\_\_ must be free. They cannot be bound at all.