High-Resolution Networked MIMO Radar based on Sub-Nyquist Observations

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***Abstract*—A matrix completion based networked MIMO radar is proposed, which enjoys the high resolution of MIMO radar but requires a significantly smaller number of data samples. Numerical results indicate that good target recovery can be achieved with as few as 50% of the samples collected at the Nyquist rate.**

I. Introduction

Multiple-input and multiple-output (MIMO) radar systems have received considerable attention in recent years due to their superior target estimation performance. Collocated MIMO radar systems exploit waveform diversity to formulate a long virtual array with number of elements equal to the product of the number of transmit and receive antennas. As a result, they achieve higher resolution than traditional phased array radars. Compressed sensing (CS) enables MIMO radar systems to maintain their advantages while significantly reducing the sampling rates at receive antennas. This is important in scenarios in which the receive antennas are connected to the fusion center via a wireless link. In CS-based MIMO radar, the target parameters are estimated by exploiting sparsity of the targets in the angle, Doppler and range space, referred to as target space. For target estimation in CS-based radars, however, the target space needs to be discretized into a fine grid. This can introduce estimation errors when the targets fall between grid points [2].

In this paper, we propose a novel approach that maintains the advantages of CS-based MIMO radar but avoids discretization of the target space. We consider a collocated networked MIMO radar scenario in which the receive antennas forward their measurements to a fusion center. Based on the received data, the fusion center formulates a matrix, which is then used for estimating the target parameters. It can be established in this setting that when there are more receive antennas than targets and the target returns at the receive antennas are sampled at the Nyquist rate then the data matrix at the fusion center is low rank. Now assume that the receive antennas, instead of forwarding all measurements to the fusion center, send only a small fraction of their samples along with the sample indices. The fusion center in this case will have a low-rank data matrix with a large number of missing entries. The insight offered in this work is that, under certain conditions, matrix completion techniques can be applied to recover the missing entries, which can then be used in conjunction with array processing methods to obtain target information.

II. The Proposed Approach

Let us consider a collocated MIMO radar system with the following parameters: (i) (distributed) transmit and receive arrays of M_t and M_r antennas, respectively; (ii) a total of K targets, with reflection coefficients $\{\beta_k\}_{k=1}^K$, individual speeds $\{\vartheta_k\}_{k=1}^K$, and angles of arrival (and departure due to the collocated configuration) with respect to the receive array normal $\{\theta_k\}_{k=1}^K$; and (iii) $\mathbf{A}(\theta)$ and $\mathbf{B}(\theta)$ the corresponding transmit and receiving steering matrices, respectively. In this setup, we propose that each receive antenna samples the received signal at rate L/T_s and forwards only a small fraction of these samples to the fusion center. Here, T_s is the sampling time; L is the number of samples per receive antenna and it is assumed that $L \gg K$.

If the fusion center were to receive all L samples from each of the receive antennas and row-wise stack the received data into an $M_r \times L$ matrix then it can be shown that the rank of that received data matrix is only $K \ll M_r$, assuming a networked MIMO radar with large number of receive antennas. Specifically, if the fusion center were to put samples of the q-th pulse from all antennas in an $M_r \times L$ matrix $\mathbf{X}_q$, then we can express $\mathbf{X}_q,\ q = 1, ..., Q$ as:

$$\mathbf{X}_q = \mathbf{B}(\theta)\,\mathbf{\Sigma}\mathbf{D}_q\mathbf{A}^T(\theta)\,\mathbf{S} + \mathbf{W}_q = \mathbf{Z}_q + \mathbf{W}_q \qquad (1)$$

where $\mathbf{\Sigma} = \mathrm{diag}([\beta_1, ..., \beta_K])$, $\mathbf{D}_q = \mathrm{diag}(\mathbf{d}_q)$ with $\mathbf{d}_q = \left[e^{j\frac{2\pi}{\lambda}2\vartheta_1(q-1)T_{PRI}}, \ldots, e^{j\frac{2\pi}{\lambda}2\vartheta_K(q-1)T_{PRI}}\right]^T$, T_{PRI} denotes the pulse repetition interval, the matrix $\mathbf{S} = [\mathbf{s}(0T_s), ..., \mathbf{s}((L-1)T_s)]$ denotes samples of the transmitted waveforms, $\mathbf{W}_q$ is a Gaussian noise matrix, and $Q \in \mathbb{N}$ denotes the total number of transmitted pulses. In (1), the matrix $\mathbf{Z}_q$ is guaranteed to be a low-rank matrix ($K \ll M_r$) in the presence of a large number of receive antennas. When the receive antennas forward sub-Nyquist samples to the fusion center, the fusion center sees only some entries of the matrix $\mathbf{X}_q$ in (1), corresponding to the samples forwarded. Nevertheless, since $\mathbf{Z}_q$ is guaranteed to be low rank, we can leverage and extend some of the *matrix completion* techniques to enable the fusion center in obtaining a good approximation to $\mathbf{Z}_q$ from a small number of entries of $\mathbf{X}_q$. In particular, this can be accomplished by solving a relaxed nuclear norm optimization problem [1]. Once the full $\mathbf{Z}_q$ matrix is estimated, target estimation proceeds based on standard array processing techniques.

Successful matrix completion requires singular vectors of $\mathbf{Z}_q$ to satisfy certain conditions [1]. Extensive simulations indicate that maximum (absolute) entries of the left and right singular vectors of $\mathbf{Z}_q$ scale as $1/\sqrt{M_r}$ and $1/\sqrt{L}$, respectively. Therefore, depending on the number of receive antennas M_r and samples L in one pulse, we conjecture that the conditions required for successful matrix completion are satisfied for the data matrix $\mathbf{Z}_q$.

We conclude by noting that the receive nodes need to transmit both samples and indices of the samples to the fusion center so that the matrix $\mathbf{X}_q$ can be successfully completed. This can be accomplished in practice by making the fusion center dictate these indices to the receive nodes prior to forwarding of the samples to the fusion center. Such a strategy will have the added advantage of eliminating the need for the receive antennas to forward the samples' indices.

References

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