Numerical Information

\[ \begin{array}{c|ccc}
\theta & 0 & \frac{\pi}{6} & \frac{\pi}{4} \\
\sin \theta & 0 & \frac{1}{2} & \frac{\sqrt{2}}{2} \\
\cos \theta & 1 & \frac{\sqrt{2}}{2} & \frac{1}{2} \\
\end{array} \]

Differentiation Formulas

\[ \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \]
\[ \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} \]

Integration Formulas

\[ \int \sec x \, dx = \ln |\sec x + \tan x| + C \]
\[ \int \csc x \, dx = \ln |\csc x - \cot x| + C \]
\[ = -\ln |\csc x + \cot x| + C \]

Trigonometric Identities

\[ 1 = \sin^2 \theta + \cos^2 \theta \]
\[ \sec^2 \theta = 1 + \tan^2 \theta \]
\[ \sin(2\theta) = 2 \sin \theta \cos \theta \]
\[ \cos \theta = \cos^2 \theta - \sin^2 \theta \]
\[ \sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta) \]
\[ \cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta) \]
\[ \sin x \cos y = \frac{1}{2} (\sin(x-y) + \sin(x+y)) \]
\[ \sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y)) \]
\[ \cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y)) \]
\[ \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \]
\[ \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \]

Midpoint Rule

\[ \int_a^b f(x) \, dx \simeq M_n \]
\[ = \Delta x (f(x_1) + f(x_2) + \cdots + f(x_n)) \]
\[ (\Delta x = \frac{b-a}{n}, x_i = a + i\Delta x, x_i = \frac{x_{i-1} + x_i}{2}) \]
Suppose \(|f''(x)| \leq K\) for \(a \leq x \leq b\). If \(E_M\) is the error in the middle point rule, then
\[ |E_M| \leq \frac{K(b-a)^3}{24n^2} \]

Trapezoidal Rule

\[ \int_a^b f(x) \, dx \simeq T_n \]
\[ = \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)) \]
\[ (\Delta x = \frac{b-a}{n}, x_i = a + i\Delta x) \]
Suppose \(|f''(x)| \leq K\) for \(a \leq x \leq b\). If \(E_T\) is the error in the middle point rule, then
\[ |E_T| \leq \frac{K(b-a)^3}{12n^2} \]

Simpson’s Rule

\[ \int_a^b f(x) \, dx \simeq S_n \]
\[ = \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)) \]
\[ (n \text{ even}, \Delta x = \frac{b-a}{n}, x_i = a + i\Delta x) \]
Suppose \(|f^{(4)}(x)| \leq K\) for \(a \leq x \leq b\). If \(E_S\) is the error in the middle point rule, then
\[ |E_S| \leq \frac{K(b-a)^5}{180n^4} \]

Length of a Plane Curve

The length of the curve \(y = f(x)\) for \(a \leq x \leq b\) is
\[ \int_a^b \sqrt{1 + (f'(x))^2} \, dx. \]
The length of the curve \(x = g(y)\) for \(c \leq y \leq d\) is
\[ \int_c^d \sqrt{1 + (g'(y))^2} \, dy. \]
The length of the parametric curve \(x = x(t), y = y(t)\) for \(a \leq t \leq b\) is
\[ \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt. \]

Area of a Surface of Revolution

Then the area of the surface generated by rotating the curve \(y = f(x)\) for \(a \leq x \leq b\) about the \(x\)-axis is
\[ \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx. \]
The area of the surface generated by rotating the curve \(x = g(y)\) for \(c \leq y \leq d\) about the \(y\)-axis is
\[ \int_c^d 2\pi g(y) \sqrt{1 + (g'(y))^2} \, dy. \]