

## FORMULA SHEET FOR EXAM 2, MATH 421

### Fourier series

For  $f(x)$  defined on  $[-L, L]$ , the Fourier series of  $f(x)$  is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

### Fourier coefficients

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx,$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx.$$

### Orthogonality

$$\int_{-L}^L \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = \begin{cases} 0 & m \neq n, \\ L & m = n, \end{cases}$$

$$\int_{-L}^L \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = \begin{cases} 0 & m \neq n, \\ L & m = n \neq 0, \\ 2L & m = n = 0, \end{cases}$$

$$\int_{-L}^L \cos\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = 0.$$