ABSTRACT

3-D shell components are used extensively in the automotive industry. Many structural topology optimization techniques were developed to reduce the total weight of shell structures while retaining its structural performance. One common approach is to utilize the concept of the design domain, such as the homogenization method and the density function approach. In this paper, a new microstructure-based design domain method is introduced to solve 3-D shell topology optimization problems. Based on physical micro-structure models, simple closed-form expressions for effective Young's modulus and effective shear modulus are rigorously derived. Using these simple relations, topology optimization problems can be formulated and solved with sequential convex approximation algorithms. Two design examples obtained from the new method are presented.

INTRODUCTION

Three-dimensional shell structures have been used intensively in the automotive industry. One of the most important design considerations in 3-D shell structures is their dynamic response. For example, lower mode frequencies are normally to be maximized to prevent noise and vibrations. One common design practice of increasing the lower mode frequencies of 3-D shell structures is to strengthen the structures with stiffeners. Therefore, the optimal design of shell stiffeners has been an active research area for laminate decades. The configuration of structural members is defined by two types of information: geometrical data and topological data. Geometrical data consists of the basic shape-defining parameters, for example, the thickness of plate or the coordinates of vertex points in a boundary. Topology data includes the connectivity relationships among geometric components, such as, the number and connectivity of nodes of stiffeners in 3-D shell structures. Earlier works utilized the sizing design variables such as the plate thickness to improve the design performance. Then, the focus had been shifted to the optimal boundary shape of structures. However, for geometry optimization, the topology of the structures is pre-defined. Thus, the result of geometry optimization is highly dependent on the initial topology.

Structural topology optimization problems can be solved by utilizing the concept of the design domain. A design domain is a union of every conceivable design space that can possess materials without violating any geometric constraints. In this approach, the design domain is divided into many design cells first and a "relaxed" problem is formulated allowing the use of "composite materials" for design cells. By solving this relaxed problem, the optimal topology can be identified. Since this type of approach is based on the design domain, it is hereby classified as the Design Domain Method (DDM). In contrast with the sizing and shape optimization of the past, the DDM rigorously optimizes the topology configurations as well as the size and shape of structures simultaneously. One of the most successful examples using this approach is the homogenization method introduced by Bendse and Kikuchi (1988). Suzuki and Kikuchi (1992) used a perforated laminate plate model with the homogenization method to solve the static layout problem. Dian and Kikuchi (1992), and Ma et al. (1993) extended this idea to dynamic problems. In the homogenization method, the material property of each design cell is computed by the homogenization theory and the optimal topology is obtained by solving a material distribution problem. An engineering variation of the DDM is called the density function approach (Mietke et al., 1993 and Yang, 1994). They use the density of each design cell as the design variables to formulate the topology optimization problem. This approach is very attractive to the engineering community because of its simplicity. But the shortcoming is the lack of theoretical support of the relationship between the density and the material property.

In this paper, a new microstructure-based DDM is introduced to solve the optimal topology problems. This new method gives simple closed-form expressions for effective Young’s modulus and effective shear modulus in terms of phase properties and volume fractions. This new
A NEW MICRO-STRUCTURE BASED DESIGN DOMAIN METHOD

The homogenization method gives a rigorous formulation for composite materials with periodic micro-structures and the density function approach leads to a very simple way in solving topology optimization problems. The goal of this paper is to present a new micro-structure based DDM. In Section 3, a sequential convex approximation method called Generalized Convex Approximation (GCA) and frequency sensitivity analysis are discussed. Finally, some numerical examples using the new method are presented.

![Spherical micro-structure diagram](image)

**FIGURE 1:** The spherical micro-inclusions model of 1° DDM.

principle, we obtain, the average stress of the inclusion of the composite, \( \sigma^{(1)} \), as

\[
\begin{align*}
\sigma^{(1)} &= \sigma + \epsilon + \sigma^m \\
&= L_0 (\epsilon + \epsilon + \epsilon^m) \\
&= L_0 (\epsilon + \epsilon + \epsilon^m - \epsilon^s) \\
\end{align*}
\]

where the superscript, \( \epsilon^m \), denotes the perturbed value, and \( \epsilon^s \) is the equivalent transformation strain. Under the condition of an imposed macroscopically homogeneous stress field on the representative volume element, the average stress of the composite is defined by

\[
\sigma = c_0 \sigma^{(0)} + c_1 \sigma^{(1)}
\]

From Eq. (1) to Eq. (4), we have

\[
\epsilon = -c_1 (\epsilon^m - \epsilon^s)
\]

The perturbed strain can be represented in terms of the Eshelby's transformation tensor, \( S \), as

\[
\epsilon^m = S \epsilon^s
\]

The total strain of the composite is given by the weight mean of those of its constituents: this leads to

\[
\epsilon = \epsilon^0 + c_1 \epsilon^s
\]

where \( c_0 \) and \( c_1 \) are the volume fractions of the matrix and the inclusion, respectively. Consider a composite is subjected to a boundary traction which produces a uniform stress \( \sigma \). We introduce a comparison material with identical shape and boundary condition. The average strain produced in the comparison materials is

\[
\epsilon^0 = L_0^{-1} \sigma
\]

where \( L_0 \) is the elastic moduli tensor of the matrix, and \( L_0^{-1} \) is elastic compliance tensor. Then, the average stress of the matrix of the composite material, \( \sigma^{(0)} \), can be represented as

\[
\sigma^{(0)} = \sigma + \epsilon = L_0 (\epsilon + \epsilon)
\]
With the help of Eq. (3), the equivalent transformation strain can be found as
\[ \varepsilon = \frac{1}{2} \left( (L_1 - L_0)(c_1 + c_2 S) + L_0^{-1} (L_1 - L_0) \right)^2 \] (8)
The effective moduli of the composite, \( L \), is obtained from \( \varepsilon = L \varepsilon \), as
\[ L = \frac{L_0 (1 - c_1 L_0)(c_1 + c_2 S) + L_0^{-1} (L_1 - L_0)}{L_1 - L_0}^{-1} \] (9)
where \( L \) is the fourth-rank identity tensor.
When the inclusions are spherical and the matrix is isotropic, the material properties of the composite of spherical micro-inclusions can be obtained as,
\[ \frac{\nu}{\nu_0} = 1 + \frac{c_1 (k_1 - k_0)}{(1 - c_1) k_0 (k_1 - k_0) + k_0} \] (10)
\[ \frac{\mu}{\mu_0} = 1 + \frac{c_1 (\mu_1 - \mu_0)}{(1 - c_1) \mu_0 (\mu_1 - \mu_0) + \mu_0} \] (11)
with
\[ c_0 = 1 + \frac{\nu}{\nu_0} \] (12)
\[ 2 (1 - \nu) > 3 - 4 \nu \] (13)
where \( c_0 \) and \( \nu \) denote the bulk modulus and the shear modulus, respectively; \( \nu_0 \) is the Poisson ratio of the matrix. Using these simple closed-form relations, a mathematical programming problem of topology optimization can be formulated.

**ERALIZED CONVEX APPROXIMATION**

Structural optimization problems consist of determining the configuration of structures which obeys assigned constraints and produces an extremum for a chosen objective function. In order to solve them, we normally transform this problem into a mathematical form that can be solved by general optimization tools. Thus, problem formulation plays a crucial role in the solution process.

In the present study, a newly developed algorithm, called the Generalized Convex Approximation (GCA), is tailored specifically for the DDM. A brief review of the GCA is provided here. For further details on the GCA, please refer to the article by Chickermane and Gea (1994). The basic idea is to approximate the original function, \( f \), by a series of approximate functions as
\[ f = f_0 + \sum_i f_i \] (14)
where \( a, b, \) and \( n_i \) are constants to be determined, index \( i \) is corresponding to the design variables, \( x \). Using the GCA, a structural optimization problem is recast as

Minimize \( f = f_0 + \sum_i f_i \) (15)

Subject to
\[ g = g_0 + \sum_i g_i \leq 0 \] (16)
\[ j = 1, m \] (17)

where \( m \) represents the number of constraints.

Here, a convex and separable subproblem is formulated. The next step is to solve this problem iteratively for the optimum values of the design variables \( x \). In the DDM, because the number of constraints are normally less than the number of design variables, a dual formulation is used. The Lagrangian corresponding to the approximate problem is formulated as
\[ L = (f_0 + \sum_i f_i) + \sum_i \lambda_i (g_i + \sum_j g_j) \] (18)
where \( \lambda \) denotes the lagrange multipliers or the dual variables. Then, the solution of the dual problem is converted to that of the primal problem for checking the convergence.

The overall iterative process employed can be summarized as follows:

1. Choose a starting point \( x_0 \) and let the iteration index \( k = 0 \).
2. Given an iteration point \( x_k \), calculate the first order derivatives of the objective and constraint functions with respect to the design variables.
3. Generate the approximated subproblems using the Generalized Convex Approximation.
4. Solve the subproblems to find an optimal solution.
5. Solve the primal problem using Newton-Raphson. If the solution does not converge, the solution of the primal problem is used as the next iteration point, the iteration index \( k \) is increased by one and the iterative process continues.

**FREQUENCY SENSITIVITY ANALYSIS**

In solving optimization problems with GCA, we must determine the effect resulting from a small perturbation in the current design on the cost and constraint functions. This is well known as sensitivity analysis. A general dynamic frequency problem can be written as \( K x = \lambda M u \), where \( \lambda \) and \( u \) represent the eigenvalue and its corresponding eigenvector, respectively. \( K \) is the stiffness matrix and \( M \) is the mass matrix. The sensitivity of dynamic frequency can be evaluated by taking the derivative of dynamic frequency equation with respect to design variable \( x \).

\[ \frac{d}{dx} (K x - \lambda M u) = 0 \]
\[ \Rightarrow (K - \lambda M) \frac{d u}{dx} + \lambda M = 0 \] (20)
By pre-multiplying \( u \) to Eqn. (20), we have the sensitivity for eigenvalue as

\[
\frac{\Delta \lambda}{\lambda} = \frac{u_i^T \frac{dK}{dz} - u_i^T \frac{dM}{dz} j_u}{u_i^T M u}
\]  

(21)

Once the dynamic frequency sensitivity is obtained, an approximated subproblem will be generated and solved by GCA. It is known that the optimizing of the single objective eigenvalue will generally result in a set of narrowing, banded eigenvalues. At some stage, the eigenvalues will be equal (repeated eigenvalues) or even switch over. Once the structure has repeated eigenvalues, the eigenvectors associated with those repeated eigenvalues are indefinite. In this case, the above formulation may generate some inaccurate sensitivity information. Although these limits many articles concerning the sensitivity analysis of repeated eigenvalue problem, it is nevertheless noted that, given the large number of design variables and the use of mathematical programming, the problem of repeated eigenvalues seldom occurs in our approach. However, the eigenvalue switch over may still happen which also bring wrong sensitivity information. In the current implementation, a multiple eigenvalue objective formulation is used in order to overcome this problem.

**DESIGN EXAMPLES**

In this section, results of two examples obtained by the new micro-structure based DDM are presented and discussed. All the examples discussed here are 3-D shell structures with the size 1.5 M by 2 M. The thickness of the shell is 1 mm. The first example is a flat plate, the second example is a ruled-surface shell. The density of shell in both examples is 7800 Kg/m^3. Young’s modulus of matrix is 10^10 and that of inclusions is 10^10.

**EXAMPLE 1: FLAT PLATE** - A flat plate with dimensions 2 M by 1.5 M is fixed at six points as shown in Fig. (2). First, we consider a design problem to maximize the lowest frequency with a given weight. This design problem is formulated as

\[
\max_{j=1, \ldots, n} \beta_j \quad \text{subject to} \quad \text{weight} \leq W
\]  

(22)

where \( W \) denotes the allowable weight of the structure. Two different allowable weights are used: 11.7 Kg and 8.8 Kg. An unstructured mass of 1/12 of the allowable weight is placed at the center of the plate. The optimal layout design are shown in Fig. (3) and Fig. (4). The dark area represents shell with stiffeners; the blank area represents shell only. The iteration histories are shown in Fig. (5) and Fig. (6), respectively. They both show that the plate requires strong stiffeners around the center to increase the fundamental frequency.

Then, we consider a multiple frequencies design problem with the same flat plate. Three frequencies are to be maximized. The objective function of this problem is formulated as a weighted sum of the first three frequencies with the same weighting factors. Two different allowable weights are used here again. The results from DDM of their iteration histories are shown in Fig. (7) to Fig. (10). Comparing with single frequency design problem, we found that the multiple frequencies design prefers an interconnected structure. Also, the lowest frequency of the single frequency formulation is better (higher) than those of the multiple frequency formulation; This is price one has to pay when more than one frequency are to be maximized.

During the iteration, the single frequency design tends toward a set of narrow banded eigenfrequencies, and the eigenfrequency switch over between the objective eigenfrequency and the second eigenfrequency is highly likely. On the other hand, the multiple frequency design generate a set of wide distances eigenfrequencies that the switch over is unlikely.

**EXAMPLE 2: RULED-SURFACE SHELL** - A ruled-surface shell has a base 2 M by 1.5M. The longer sides of the shell are straight and the shorter sides are lifted up 2 cm. The shell is fixed at six points, three points each along the longer sides, as shown in Fig. (11). In this example, we consider only single frequency design problem with allowable weight 11.7 Kg. First, an unstructured mass, 1.95 Kg, is placed at the center of the shell. The result from DDM is shown in Fig. (12). In contrast with the flat plate design, the result shows interlinked stiffeners are a feasible design to support the ruled-surface shell.

Second, unstructured mass is removed from the shell and only the weight of shell is considered. The optimal topology is shown in Fig. (13). We found that the interlinked stiffeners disappear and two straight stiffeners are clearly identified by the DDM. This design is very reasonable because the self-weight of the structure can be easily carried by two ribs; more stiffeners are required only at the vicinity of the bases.

**CONCLUSIONS**

In the paper, a new micro-structure based DDM to solving 3-D shell topology optimization problems has been reported. This new approach has been derived from a spherical micro-inclusion model and resulted in very simple closed-form expressions for the effective Young’s modulus and the effective shear modulus. The "relaxed" topology optimization problem can then be solved by general mathematical programming tools. Two design examples solved by this approach have been presented. It is expected that this approach will provide a new and simple alternative for solving 3-D topology optimization problems.

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REFERENCES


FIGURE 2: Flat plate has the size 2 by 1.5 M and is fixed at six points along the longer sides.

FIGURE 3: The single frequency optimal topology of flat plate with an allowable mass 11.7 Kg.

FIGURE 4: The single frequency optimal topology of flat plate with an allowable mass 8.9 Kg.
FIGURE 5: The iteration history of the single frequency design with an allowable mass 11.7 Kg.

FIGURE 6: The iteration history of the single frequency design with an allowable mass 6.8 Kg.

FIGURE 7: The multiple frequencies optimal topology of a flat plate with an allowable mass 11.7 Kg.

FIGURE 8: The multiple frequencies optimal topology of a flat plate with an allowable mass 8.3 Kg.
FIGURE 9: The iteration history of the multiple frequencies design with an allowable mass 11.7 Kg.

FIGURE 10: The iteration history of the multiple frequencies design with an allowable mass 8.8 Kg.

FIGURE 11: A ruled-surface shell has a base 2 M by 1.5M. The longer sides of the shell are straight and the shorter sides are lifted up 2 cm. The shell is fixed at six points, three points each along the longer sides.

FIGURE 12: The optimal layout of curved ruled-surface with 11.7 Kg allowable mass and unstructural mass 1.95 Kg.

FIGURE 13: The optimal layout of curved ruled-surface without unstructural mass.