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This paper proposes mathematical programming models with probabilistic constraints in order to address incident response and resource allocation problems for traffic incident management. In incident response problems, we introduce the concept of quality of service during a potential incident to give the operator the flexibility to determine the optimal dispatching policy in response to various situations. A mixed-integer programming (MIP) model with probabilistic constraints is proposed to address the dispatching problem with consideration given to the stochastic resource requirements at the sites of the potential incidents. For the resource allocation problem, assuming that the stochastic distribution of incidents over a network is given, we introduce a mathematical model to determine the number of service vehicles allocated to each depot to meet the requirements of the potential incidents by taking into account the stochastic nature of the resource requirement and incident occurrence probabilities. For completeness, relevant concepts and solution algorithms in stochastic programming are also presented. Several examples are included to demonstrate the applications of these models to real-world problems. This paper concludes with a summary of results and recommendations for future research.

**Keywords:** Traffic incident response, mathematical programming, probabilistic constraints
INTRODUCTION

Traffic incidents account for approximately 60 percent of the vehicle-hours lost due to congestion annually. It is now widely accepted that these congestion and congestion-related problems can be decreased by the proper use of an efficient incident management system. Since the initial investment, maintenance cost and operating cost of each response unit is considerable, there is a great need for reliable decision-support models to help evaluate and optimize the performance of such systems.

Increased attention in literature is focused on the incident response problem. The need for improved incident response models and the data available for developing such models are discussed in Ozbay and Kachroo (1999). Recognizing the highly stochastic nature of traffic and incident management operations, Pal and Sinha (2002), introduce a simulation model that can be used in designing a new freeway service patrol, as well as improving the operations of existing programs. In a recent paper by Ozbay and Bartin (2003), a simulation model is developed using Arena simulation package, and is used to model and examine the effects of various incident management strategies for the incident management operations on the Washington D.C. beltway network. Besides computer simulation, mathematical modeling is another frequently used approach to study incident management problems. Zografos et al. (1993) proposes an analytical framework that can minimize the freeway incident delays through the optimum deployment of traffic flow restoration units (TFRU). According to the authors of this study, this model has been proven to be an effective tool that can model and evaluate the effects of deployment of TFRU on the overall freeway incident delays. Pal and Sinha (1997) construct a MIP model to determine optimal locations for response vehicles that minimizes the annual cost of response vehicles, given the frequencies of incidents at potential sites in the network, and subject to a constraint on the maximum number of vehicles. Opportunity cost-based models proposed by Sherali et al. (1999) demonstrate that dispatching the closest available vehicle to the site of the current accident is not always optimal when considering the service to anticipated future demands. To make this model polynomial-time solvable, the number of response
vehicles required by each incident are considered to be the same, and each depot is assumed to have the same number of available vehicles. In practice, this is might not be true. Due to day-to-day uncertainties, such as response vehicle breakdown, lack of drivers, etc., it is always possible to have an insufficient number of vehicles at any given day, and, the severity of accidents also varies significantly. In general, severe incidents need more response vehicles than minor incidents.

Thus, the resource demand in an incident management problem can thus be summarized as follows:

1. It is stochastic: The available resources are stochastic and the occurrences and characteristics of incidents are stochastic.

2. It is a network problem. The incidents occur randomly over the roadway network, and the location of the depots should be chosen carefully based on the topology of the network.

3. It is a resource allocation problem. Resources should be allocated wisely among individual depots and between depots and patrol service to maximize the return on investment (ROI).

The models addressing the incident management problem in the literature either use deterministic models or computer simulation, which is time consuming especially when running multiple replications to eliminate excessive variance due to high level of stochasticities.

In this paper, we attempt to address the incident management problem, specifically, the interaction between the probabilistic occurrence rates of accidents requiring different levels of resources and the availability of adequate number of incident response units, using a stochastic programming approach. We introduce the concept of quality of service and propose mathematical programming models with probabilistic constraints to model the stochastic incident response problem. In our model, multiple potential incidents having various demands for response vehicles are allowed, and the number of available response vehicles at each depot is assumed to be non-deterministic. We also assume that the probability distributions of the future demand for resources on each route are independent of each other.

In the case of resource allocation, some areas in the transportation network may have a higher probability of experiencing serious incidents than others, due to heavy traffic conditions or complex roadway characteristics. Thus, more resources should be allocated to those depots that are located closer to these
risky areas. Given the probability distribution of the demand for resources over the network, we propose a stochastic mixed-integer programming (MIP) model to determine the optimum level of resources, and the best way to allocate the resources over the transportation network.

The remainder of this paper is organized as follows. In the next section, we introduce a traffic incident response problem formulation that captures the uncertainty of the demand for resources and the availability of the resources. A MIP model with probabilistic constraints and discrete random variables is proposed to address the dispatching problem with consideration given to the requirements at the sites of the potential incidents. For the stochastic resource allocation problem, we introduce a mixed-integer nonlinear programming (MINLP) model with probabilistic constraints. The remainder of this section is devoted to the background of mathematical programming under probabilistic constraints, including the key concept of \( p \)-level efficient points for a discrete probability distribution, and the main ideas for solving this category of problems. In the following section, we provide two examples. The simple incident response example displays the difference between our incident response model and the models that do not consider the likelihood of occurrence of potential incidents, and the impact of the quality of service concept on the service vehicle dispatching policy is illustrated. The application of the resource allocation model is demonstrated using a case study that deals with the tow truck allocation on the South New Jersey road network. This paper concludes with a brief summary of results.

**PROBLEM FORMULATION**

In this paper, we consider two types of incident management (IM) problems shown in Figure 1: (1) Short-term (IM) problem (operation). This is the vehicle-dispatching problem, where we attempt to determine the optimal vehicle dispatching policy not only for the known incident(s) but also for the possible incidents that might occur simultaneously; (2) Long-term (IM) problem (planning). This is the resource
allocation problem that will let transportation agencies to determine the optimal location of each depot and the optimal number of vehicles assigned to the depots.

![Figure 1](image1.png)

**Figure 1** Determining optimal incident management strategy

The example depicted in Figure 2 demonstrates the reason why we need to consider potential incidents in addition to current incidents in order to minimize the long-term incident response cost. For the current incident, we can dispatch response vehicle from depot A or depot B. Suppose we do not consider the potential incident, dispatching the response vehicle from depot A is clearly cheaper than dispatching it from depot B. However, if we consider potential incident that might occur during the clearance time of the current incident, sending response vehicle from depot B to clear the current incident could be a better choice in terms of reducing the overall response delay. Obviously, the optimal dispatching policy for the current incident depends on the occurrence probability of the potential incidents. In the following sections, we present the stochastic programming formulation of this optimization problem and its solution algorithm.

![Figure 2](image2.png)

**Figure 2** Illustrative example for considering potential incidents
Let us first introduce the mathematical notation used in this paper. Let \( G(N, L) \) be the road network, and \( N \) and \( L \) be its node and link sets, respectively. Let \( D \) denote the set of special type of nodes, the depots, from which service vehicles are dispatched, let \( F \) denote the node set where there are on going incidents, and let \( H \) denote the node set where the potential incident might occur. In addition, let \( r_i \) be the number of service vehicles available at node \( i \in D \), \( n_f \) be the number of service vehicles required by the incident occurred at node \( f \in F \), and \( N_h \) be the number of service vehicles that will be required by the potential incident that might occur at node \( h \in H \). Note that the service vehicle is an abstract concept for any type vehicles that might be needed to clear the incidents. In practice, it could be tow trucks, ambulances, police cars or fire trucks. In this paper, we consider \( N_h \) as a scalar random variable governed by a given probability distribution. We let \( t_{if} \) be the shortest travel time for a service vehicle dispatched from depot \( i \in D \) to the site of an incident at node \( f \in F \). Finally, we use \( x_{if} \) to denote the number of service vehicles dispatched from depot \( i \) to an incident at node \( f \in F \), and \( y_{ih} \) to denote the number of service vehicles in depot \( i \) reserved for a potential incident that might occur at node \( h \in H \). We employ the concept of quality of service to quantify the system reliability in the presence of traffic incidents and corresponding response resources employed to clear them. To help formalize our discussion of the quality of incident response, we offer the following definition:

**Definition-1:** Quality of Service parameter \( p \in [0, 1] \) represents the lower bound of the probability that all the resource requested by potential incidents will be satisfied.

It is worth mentioning that in the incident response problem, \( p = 0 \) signifies that the potential incident is not considered at all, while \( p = 1 \) means the maximum possible resource demand by the potential incident is taken into account. In the context of the resource allocation problem, the quality of service makes more sense if we consider it as a measure of system reliability. Higher quality of service guarantees that more incidents can be cleared in a timely manner, thus achieving higher reliability of the transportation system. The effects of quality of service on the determination of optimal incident response policy and resource allocation strategy are discussed in the subsequent sections using real-world examples.
Dispatching Response Vehicle Under Demand Uncertainty (P1)

This subsection focuses on how to dispatch response vehicles to incidents if future demand for resources is considered to be non-deterministic. Given the quality of service level $p$, the proposed model is formulated as follows:

\[
\text{(P1)} \quad \begin{align*}
\text{Minimize} & \quad \sum_{i \in D} \sum_{f \in F} t_{if} x_{if} + \sum_{i \in D} \sum_{h \in H} t_{ih} y_{ih} \\
\text{Subject to} & \quad \sum_{f \in F} x_{if} + \sum_{h \in H} y_{ih} \leq r_i, \quad \forall i \in D \tag{1b} \\
& \quad \sum_{i \in D} x_{if} = n_f, \quad \forall f \in F \tag{1c} \\
& \quad \prod_{h \in H} P \left( \sum_{i \in D} y_{ih} \geq N_h \right) \geq p \tag{1d} \\
& \quad x \geq 0, \ y \geq 0, \ \text{x and y are integer numbers.} \tag{1e}
\end{align*}
\]

The objective function minimizes the overall response cost, which includes current response and future response costs. The future response cost is the additional vehicle delay (vehicle-hours) due to potential simultaneous incidents. By including this part in the objective function, the overall response cost is minimized while the given service quality for potential simultaneous incidents is guaranteed. If we do not consider potential incidents, i.e., $p = 0$, then any $y_{ih}$ values will satisfy constraint (1d), thus minimizing the objective function forces $y_{ih} = 0$. Consequently, there is no future cost in this case. Constraint (1b) states that the total number of vehicles dispatched from a depot should be less than or equal to the number of vehicles available at that depot. Constraint (1c) states that the requirement by the current incident must be fully satisfied. Constraint (1d) requires the joint probability of meeting the response vehicle requirements at each potential incident site $f \in F$, should be greater or equal to the chosen quality of service, where $N_h$ is a discrete random variable whose distribution is assumed to be known.
**Dispatching Response Vehicle Under Resource Uncertainty**

In addition to the fact that resources demanded by the potential incidents are not deterministic, the resources available at the depots for the potential incident might be uncertain due to break-down or other unexpected factors. In this case, we can formulate the problem nearly the same as $P\text{I}$, except that 1(b) should be replaced by the following probabilistic constraint.

\[
\prod_{i \in D} P \left( \sum_{f \in F} x_{if} + \sum_{h \in H} y_{ih} \leq R_i \right) \geq q,
\]

where $R_i$ is the number of response vehicles available at depot $i$, and $q$ is the given quality of service. $R_i$ is a discrete random variable.

**Resource Allocation Problem (P2)**

Unlike the previous cases described in this paper, the application of probabilistic programming to resource allocation problem for Incident Management problem is not straightforward. The goal of a resource allocation problem in the context of traffic incident management is to determine the optimal locations of each depot and the optimal fleet size of response vehicles at each depot to minimize the overall response costs, while meet the budget limit. The constraint of this problem is that the resources allocated over the network should be sufficient to adequately address the incidents. We can obtain the probability distribution of resource demands over the network by analyzing and modeling the historical incident data. Assume we know that the probability distribution of resource demand on node $h$ is governed by a random variable, $N_h$. Given the reliability requirements of this incident response system, this resource allocation problem can be formulated as a mixed-integer nonlinear programming (MINLP) model with probabilistic constraints.
We use the same notation as in the previous sections. In addition, we use $M$ to denote a big number, $B$ to denote the budget limit, and let $c_1$ and $c_2$ be the unit cost for service vehicles and the construction cost for a single depot respectively. $d_i$ is binary variables defined as follows.

$$
d_i = \begin{cases} 
1 & \text{if node } i \text{ is a depot (} r_i > 0\text{).} \\
0 & \text{otherwise.} 
\end{cases}
$$

Constraint (3c) states the reliability of this incident response system, namely, the joint probability of satisfying the resources requested by each incident should be greater than or equal to a given value. Constraint (3d) states the logical relationship between $d_i$ and $r_i$: If $r_i > 0$, then $d_i = 1$; otherwise, $d_i = 0$. Constraint (3e) states that the budget constraint must be satisfied. The objective function is the sum of all of the response costs.

A simplified version of the same problem is to determine the number of service vehicles assigned to each depot, while both the locations of depots, $D$, and the probability distribution of resource demands are given. This problem can be formulated as (P2.1).

(P2.1) Minimize $\sum_{i \in N} \sum_{h \in N} t_{ih}y_{ih}$

Subject to

$$\sum_{h \in N} y_{ih} \leq r_i, \quad \forall i \in N$$

$$P(\sum_{i \in L} y_{ih} \geq N_h) \geq p, \quad \forall h \in N$$

$$r_i \leq Md_i$$

$$c_1 \sum_{i \in N} r_i + c_2 \sum_{i \in N} d_i \leq B$$

$$x \geq 0, \quad r \geq 0, \quad x \text{ and } r \text{ are integers, } d \text{ binary.}$$
where \( r_i \) is the decision variable.

The assumption that the depot locations are known is a realistic one since it is costly to build new depots and not a very practical approach for the short and mid-term policy decision. However, it is very important to know optimal number of service vehicles needed in the given network. That is why P2.1 focuses on this version of the resource allocation problem. In the following part, we review the methods to solve these mathematical programming models with probabilistic constraints.

**Solution Algorithm for the Proposed Stochastic Optimization Problem (Prekopa, 1998)**

Stochastic programming problems, similar to the ones proposed in previous sections, are formulated based on the underlying deterministic problem in which some of the parameters are random variables. Assume that the underlying deterministic problem is as follows:

\[
\min \ c^T x \\
\text{subject to} \quad Tx \geq \xi \\
\quad A x \geq b \\
\quad x \geq 0
\]

where \( \xi = (\xi_1, \ldots, \xi_r)^T \) is a random vector. We require that \( Tx \geq \xi \) shall hold at least with some given probability \( p \in (0, 1) \), rather than for all possible realizations of the right hand side. This leads to the following probabilistic problem formulation:

\[
\min \ c^T x \\
\text{subject to} \quad P(Tx \geq \xi) \geq p, \\
\quad A x \geq b, \\
\quad x \geq 0
\]

where the symbol \( P \) denotes probability.

Because the variables in our model are integers, we dedicate more attention to the stochastic programming with discrete random constraints. Before we continue with the description of the solution algorithm, we shall first introduce the concept of a \( p \)-efficient point, defined in Prekopa (1990) as follows.
Definition-2: A point $z$ is called a p-level efficient point (pLEP) of a probability distribution function $F$, if $F(z) \geq p$ and there is no $y \leq z, y \neq z$ such that $F(y) \geq p$, where $p \in (0, 1)$.

Let $\bar{Z}$ be a discrete random vector. Note that the $i$th element of $\bar{Z}$ is a discrete random variable, and we assume the number of possible values of this discrete random variable is $k_i$. We use $z_{i,m}$ to denote the $m$th possible value of the $i$th element of $\bar{Z}$ in the increasing order, i.e., $z_{n,k_{i+1}}$ is the largest possible value of the $n$th discrete random variable of $\bar{Z}$. To enumerate the $p$-level efficient points for a multidimensional discrete probability distribution, Prekopa et al (1998) propose a recursive algorithm, which we show below for the completeness of this paper.

Step 0. Initialize $k \leftarrow 0$. Go to Step 1.

Step 1. Let
\[
\begin{align*}
z_{1,j_1} &= \arg \min \{y \mid F(y, z_{2,k_1+1}, \ldots, z_{n,k_{i+1}}) \geq p \} \\
z_{2,j_2} &= \arg \min \{y \mid F(z_{1,j_1}, y, z_{3,k_1+1}, \ldots, z_{n,k_{i+1}}) \geq p \} \\
\vdots \\
z_{n,j_n} &= \arg \min \{y \mid F(z_{1,j_1}, \ldots, z_{n-1,j_{n-1}}, y) \geq p \}
\end{align*}
\]

Step 2. Let $Z_p \leftarrow \{ z_{i,j_i}, \ldots, z_{n,j_n} \}$ . Go to Step 3.

Step 3. Let $k \leftarrow k+1$. If $j_1 + k > k_{i+1}$ then go to Step 5. If $j_1 + k \leq k_{i+1}$, then go to Step 4.

Step 4. Enumerate all $p$LEP’s of the function $F(z_{1,j_1}, \bar{w})$ of the variable $\bar{w}$ and eliminate in $Z_p$ and eliminate those which dominate at least one element in $Z_p$. If $Z_r$ is the set of the remaining $p$LEP’s, which may be empty, then let $Z_p \leftarrow Z_p \cup Z_r$.

Go to Step 3.

Step 5. Stop, $Z_p$ is the set of all $p$LEP’s of the function $F(\bar{z})$.

We present the pseudo code of this algorithm implemented in C++ as below:

\[
k := 0 \\
function enumerate_pLEP(n, p, k) //n is the number of dimensions, p is the given probability value \\
for I = 1 to n do \\
    z(I, k) = argmin(F, p, k) //F is the probability distribution function
\]
let \( Z(i) = z(I,*) \)  // save the resulted numbers in a vector
if \( Z(i) \) is not dominated by any members in \( Z_p \)
    Add \( Z(i) \) into \( Z_p \)
    \( k = k + 1 \)
end
enumerate_{pLEP}(n, p, k) // call the function iteratively
end
return \( Z_p \);

The time-efficiency of this enumeration algorithm is not a major concern for a realistic incident response problem, because number of routes considered for incident management in a typical transportation network is limited mainly to the major routes. This is due to the fact that major delays occur on major routes, and it is clear that maximum benefit from an incident management system will be obtained from clearing the incidents that cause the largest delays. The focus on the major routes rather than minor ones limits the size of the transportation network considerably, thus the p-level efficient points enumeration approach in this algorithm is a feasible technique. Based on this algorithm, we developed a computer program in C++, which runs very fast for the case studies presented in this paper and others with several hundred nodes similar to them.

With the set of all pLEP’s, \( Z_p = \{ z^{(1)}, z^{(2)}, \ldots, z^{(N)} \} \), an equivalent form of the problem (7) is the following:

\[
\begin{align*}
\min \quad & c^T x \\
\text{subject to} \quad & T x \geq z^{(i)}, \\
& A x \geq b \\
& x \geq 0, z^{(i)} \in Z_p.
\end{align*}
\]

(8)

A straightforward way to solve (7) is to find all p-efficient points and to process all corresponding problems, e.g., problem (7) can be solved exactly by the solution of the \( N \) linear programming problems, where the \( i \)th linear programming problem has the constraint \( T x \geq z^{(i)} \). If \( x^{(i)} \) is the optimal solution of the \( i \)th problem, and \( c^T x^{(i)} = \min_{1 \leq j \leq N} c^T x^{(j)} \), then \( x^{(i)} \) is the optimal solution of the problem shown in equation (7). For those problems with small number of pLEP’s and small matrix \( T \), this method is efficient. If the
random variables in the probabilistic constraints are \( r \)-concave (refer to Dentcheva et al, 2000 for detailed definitions and applications), we can use an equivalent formulation with more convenient structures. Many well-known one-dimensional discrete probability distributions are \( r \)-concave distributions, to name a few, Poisson distribution, geometrical distribution and binomial distribution (Prekopa, 1995). Binary random vectors and scalar integer random variables are also \( r \)-concave.

If the probability distribution function of the random variable in problem shown in equation (7) is \( r \)-concave, then an equivalent representation of (8) can be obtained as

\[
\begin{align*}
\min & \quad c^T x \\
\text{subject to} & \quad Tx \geq t \\
& \quad t \geq \sum_{i \in J} \lambda_i z^{(i)} \\
& \quad Ax \geq b \\
& \quad \sum_{i \in J} \lambda_i = 1 \\
& \quad x \geq 0, z^{(i)} \in Z_p \\
& \quad \lambda_i \geq 0, i \in J, t \text{ is integer}
\end{align*}
\]

(9)

Moreover, if the decision variable \( x \) is integer, then we can simplify (9) further into

\[
\begin{align*}
\min & \quad c^T x \\
\text{subject to} & \quad Tx \geq \sum_{i \in J} \lambda_i z^{(i)} \\
& \quad Ax \geq b \\
& \quad \sum_{i \in J} \lambda_i = 1 \\
& \quad x \geq 0, z^{(i)} \in Z_p \\
& \quad \lambda_i \geq 0, i \in J, x \text{ is integer}
\end{align*}
\]

(10)

It is worth noting that if there is no integer constraint for the decision variable \( x \), then formulation (10) cannot be used for problem (7). Actually, the objective value of (10) is the lower bound of the original problem by relaxing the constraints of (9). Since in the incident response problem discussed in this paper, the number of service vehicles dispatched should always be integer, solving the simplified formulation...
(10) leads to the same solutions for the original problem. After we obtain p-efficient points using the enumeration algorithm, problems (P1) and (P2.1) can be simplified into the formulation shown in (10), which is solvable by many software packages. In this study, we use Lindo™ (Lindo Systems Inc.) for solving our problem.

As mentioned above, in some cases, for example, for multidimensional random vectors $\xi$, the number of $p$-level efficient points can be very large and their enumeration may be difficult. It would be desirable, therefore, to avoid the complete enumeration and search for promising $p$-level efficient points only. In this case, the solution method does not find all $p$-level efficient points but builds them up subsequently by the use of the cutting plane method. The detailed discussion is given (Prekopa, 1998), but given the rather modest size of our problem, the solution approach discussed above is proved to be efficient.

**CASE STUDIES**

In this section, we present several examples to demonstrate the applications of the stochastic programming models proposed for incident management decision support. The solution of the incident response problem, namely $P_1$, is illustrated using a simple numerical example from Sherali et al (1999). The second example used to illustrate the solution of a realistic resource allocation problem, formulated as $P_2$ in this paper, is applied to a simplified version of South Jersey highway network where there is already an active incident management program. In this example, given the requirement of service quality, we determine the minimum resources allocated to the system in order to maximize the investment on return value.

**Incident Response Problem ($P_1$)**

In this section, we study the impact of a potential incident via simple example network taken from Sherali et al (1999). As illustrated in Figure 3, this small network consists of four nodes and four arcs. Nodes 1 and 2 represent depots, while nodes $f$ and $v$ represent the site of incidents. The shortest travel times
between depots and sites of incidents are $\lambda_{ij} = 25$ mins, $\lambda_{2j} = 15$ mins, $\lambda_{1v} = 20$ mins and $\lambda_{2v} = 5$ mins. The number of service vehicles available at each depot is deterministic and fixed, which is assumed to be 5 service vehicles and 3 service vehicles at depot 1 and depot 2 respectively.

Now, assume that an incident occurred at node $f$, requiring 2 service vehicles, and also assume that there is a probability of a request for additional service vehicles by another incident that might occur at node $v$. This probability is represented by the scalar probability distribution shown in Table 1. We now need to determine the optimal service vehicle dispatching policy in terms of minimizing the overall response cost.

**Table 1** Probability distribution of the number of service vehicles required by an additional incident.

<table>
<thead>
<tr>
<th>Number of service vehicles required by the additional accident</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.6</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

If we neglect the potential demand of service vehicles at node $v$, then dispatching two service vehicles from depot 2 to the site of incident, $f$, would be the optimal response strategy. On the other hand, if we consider the potential demand to a certain extent, using the knowledge introduced in previous sections, the underlying mathematical model for this small example can be formulated as follows:
\[ \text{Min } 25x_{1f} + 15x_{2f} + 20y_{1v} + 5y_{2v} \]
\[ \text{s.t. } \]
\[ x_{1f} + y_{1v} \leq 5 \]
\[ x_{2f} + y_{2v} \leq 3 \]
\[ x_{1f} + x_{2f} = 2 \]
\[ P(y_{1v} + y_{2v} \geq n_v) \geq p \]

where \(0 \leq p \leq 1\) is the given probability value. Note that \(p = 0\) means the potential demand is not considered, and \(p = 1\) means the worst case conditions of the potential incident has to be covered by the resulted dispatching strategy. Table 2 gives the optimal number of vehicles dispatched for different \(p\) levels, which is also shown in Figure 4.

### Table 2 Optimal dispatching policies at different \(p\) levels.

<table>
<thead>
<tr>
<th>(p)</th>
<th>0–0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{1f})</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(x_{2f})</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(y_{1v})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(y_{2v})</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Objective function value</td>
<td>30</td>
<td>35</td>
<td>50</td>
<td>65</td>
<td>85</td>
</tr>
</tbody>
</table>

- \(x_{1f}\) (vehicles sent from node 1 to \(f\))
- \(x_{2f}\) (vehicles sent from 2 to \(f\))
- \(y_{1v}\) (vehicles reserved for \(v\) at node 1)
- \(y_{2v}\) (vehicles reserved for \(v\) at node 2)

### Figure 4 Number of service vehicles increases with \(p\) values.
Table 2 and Figure 4 show that the total number of service vehicles involved in the incidents increases if we increase the $p$ values. Note that when the possibility of the potential incident is excluded or when we only consider the possibility of the low demand by the potential incident, straightforward optimal solution would be dispatching the closest available service vehicles to the site of the incident. In this example, both vehicles requested by the incident would be dispatched from depot 2 to the incident that occurred at node $f$. As the $p$ level increases, dispatching the closest available response vehicle might not be a wise choice. For instance, at $p = 0.8$, according to the optimal results shown in Table 2, among the two service vehicles requested by the already occurred incident at node $f$, only one of them is dispatched from the closest depot (depot 2), although there are enough service vehicles available at depot 2 in the current setting. The service vehicles that are not dispatched to node $f$ are saved for the potential demand by a potential incident at node $v$, since dispatching service vehicles from depot 2 to node $v$ is much cheaper than dispatching from depot 1. If we ignore the risk of the potential incident, we should stick to the original optimal policy. After sending the two vehicles from the closest depot, namely, depot 2, to meet the requirement of the occurred incident, if an incident occurred at node $v$ (the probability is 0.1) requested two more service vehicles, we have no choice but to send the last available vehicle from depot 2 and another vehicle has to be sent from the distant node 1 to satisfy the demand at node $v$. This results in a total response cost as high as 55 $\text{veh} \cdot \text{mins}$, compared to the obtained optimal value, 50 $\text{veh} \cdot \text{mins}$. As the $p$ value increases, more and more service vehicles at depot 2 are saved for the potential incident.

If we consider the uncertainty of resources available at the depots, the case study problem can be presented as follows:

\[
\begin{align*}
\text{Min} & \quad 25x_{1f} + 15x_{2f} + 20y_{1v} + 5y_{2v} \\
\text{s.t.} & \quad P(x_{1f} + y_{1v} \leq R_1) \geq q \\
& \quad x_{1f} + x_{2f} = 2 \\
& \quad P(y_{1v} + y_{2v} \geq n_v) \geq p
\end{align*}
\]

$x_{1f}$, $x_{2f}$, $y_{1v}$, $y_{2v}$ are nonnegative integers
where $R_i$ is a random variable describe the uncertain number of service vehicles available at depot $i$. For illustration purposes, we assume the probability distribution of the unavailable service vehicles is according to table 3.

**Table 3** Probability distribution of the unavailable service vehicles at depot.

<table>
<thead>
<tr>
<th>Number of unavailable service vehicles</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.8</td>
<td>0.1</td>
<td>0.08</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Accordingly, the p-efficient points for various $q$ values are shown in Table 4.

**Table 4** p-efficient points.

<table>
<thead>
<tr>
<th>$q$</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of unavailable service vehicles at depot 1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Number of unavailable service vehicles at depot 2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

To illustrate the effect of this new constraint, we assume $q = 0.9$, and compute the optimal dispatching policy again for different $p$ levels. Note that at $q = 0.9$, the number of available service vehicles at depot 1 and depot 2 are $r_1 = 4$, $r_2 = 0$, or $r_1 = 2$, $r_2 = 2$. We use the second scenario, and the new results are given in Table 5.

**Table 5** Optimal dispatching policies at different $p$ levels with $q = 0.9$

<table>
<thead>
<tr>
<th>$p$</th>
<th>0~0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1f}$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>infeasible</td>
<td>infeasible</td>
</tr>
<tr>
<td>$x_{2f}$</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_{1v}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_{2v}$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Objective function value</td>
<td>30</td>
<td>45</td>
<td>60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To meet the requirement of high quality of service in the incident response and the conservative expectation of the resource availability, we might need more resources. Otherwise, the problem could be infeasible, as shown in Table 5.

**Resource Allocation Problem (P2)**

In this example we consider a portion of South New Jersey road network which consists of seven major roads depicted in Figure 5.

![Figure 5 Portion of the South Jersey roadway network used in this study.](image)

The distribution of the demand for the tow trucks is based on the analysis of the traffic incident data in South New Jersey.

**Table 6** Number of incidents on the major routes in South Jersey study network (2000 – 2001).
(Values in the parenthesis are the average numbers of trucks requested by the incident in that category)

<table>
<thead>
<tr>
<th>Category</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAZMAT (4)</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>13</td>
<td>3</td>
</tr>
</tbody>
</table>
By categorizing the traffic incidents that occurred in years 2000 and 2001 in Table 6, and assuming the average number of tow trucks demanded by each category based on the severity of incidents, as listed in the first column of Table 6, we obtain the scalar probability values which are presented in Table 7. The average travel times along each route with and without incidents are obtained by a traffic simulation software package developed at Rutgers Intelligent Transportation Systems (RITS) Laboratory, and which is based on Daganzo’s cell transmission model (Daganzo, 1994). The results are also shown in Table 7.

<table>
<thead>
<tr>
<th>Route</th>
<th>Average travel time (s)</th>
<th>Distribution of requested tow trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>without incidents</td>
<td>with incidents</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>US 30</td>
<td>754</td>
</tr>
<tr>
<td>2</td>
<td>NJ 38</td>
<td>286</td>
</tr>
<tr>
<td>3</td>
<td>NJ 42</td>
<td>554</td>
</tr>
<tr>
<td>4</td>
<td>I-76</td>
<td>242</td>
</tr>
<tr>
<td>5</td>
<td>US 130</td>
<td>1246</td>
</tr>
<tr>
<td>6</td>
<td>I-295</td>
<td>773</td>
</tr>
<tr>
<td>7</td>
<td>I-676</td>
<td>305</td>
</tr>
</tbody>
</table>

The road network is then modeled as a graph shown in Figure 6, where each node represents one route. A link between two nodes implies there is an intersection that connects these two routes.
We assume the time for the tow truck traveling from its depot to the site of the incident on the same route is the average travel time of this route under the impact of incidents. If tow trucks that belong to another route are sent from the depot, then the total traveling time of the tow truck would be the summation of the average travel time of each node along the shortest path from the depot to the site of the incident and the average traveling time with incidents of the ending nodes. Based on these assumptions and the shortest path algorithm, the travel time between every two nodes are obtained as in Table 8.

<table>
<thead>
<tr>
<th></th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
<tr>
<td>US 30</td>
<td>6041</td>
<td>1067</td>
<td>1917</td>
<td>5360</td>
<td>2420</td>
<td>5892</td>
<td>5180</td>
</tr>
<tr>
<td>NJ 38</td>
<td>6327</td>
<td>313</td>
<td>2203</td>
<td>5646</td>
<td>1952</td>
<td>6178</td>
<td>5466</td>
</tr>
<tr>
<td>NJ 42</td>
<td>7142</td>
<td>2168</td>
<td>616</td>
<td>4855</td>
<td>1765</td>
<td>5692</td>
<td>5222</td>
</tr>
<tr>
<td>I-76</td>
<td>6588</td>
<td>1614</td>
<td>858</td>
<td>4301</td>
<td>1908</td>
<td>5380</td>
<td>4673</td>
</tr>
<tr>
<td>US 130</td>
<td>7287</td>
<td>1559</td>
<td>1407</td>
<td>5547</td>
<td>1666</td>
<td>6384</td>
<td>5914</td>
</tr>
<tr>
<td>I-295</td>
<td>6814</td>
<td>1840</td>
<td>1389</td>
<td>5074</td>
<td>2439</td>
<td>5138</td>
<td>5441</td>
</tr>
<tr>
<td>I-676</td>
<td>6364</td>
<td>1372</td>
<td>1163</td>
<td>4606</td>
<td>2213</td>
<td>5685</td>
<td>4426</td>
</tr>
</tbody>
</table>

The number of tow trucks sent to the site of the incident is determined by the attributes of the incident. If the number of tow trucks requested by the incident exceeds the number of the available tow trucks in the
nearest depot, then the rest of the trucks would be sent from the next nearest depot with idle trucks. With
the probability distribution of incidents and the average travel time at hand, we will proceed with resource
allocation analysis on South Jersey roadway network in two steps: First, we assume that tow truck depots
exist along these routes, and we just need to determine how many tow trucks need to assigned to each
depot (problem \( P2.1 \)). Second, we determine the locations of the depots and number of tow trucks
assigned to them.

Now, assuming there is a depot constructed on each route, to maximize the return on investment, we need
to determine the minimum number of tow trucks allocated to each route, while satisfying the given
requirement of the quality of service. Let \( y_{ij} \) be the number of tow trucks sent from the depot on route \( i \) to
the site of incident occurred on route \( j \), and \( t_{ij} \) be the according travel time. The objective function of this
problem can be presented as

\[
MIN \sum_{i=1}^{2} \sum_{j=1}^{7} t_{ij} Y_{ij}.
\] (12)

If the service guarantee level \( p \) is 0.9, referring to the algorithm presented in previous part, the efficient
points are obtained in Table 9.

<table>
<thead>
<tr>
<th>Point number</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
Let $n_{kj}$ be the number of tow trucks on route $j$ according to $k$th efficient points, and let $\lambda_k$ be the coefficients of $k$th efficient point which satisfies $\sum_{k=1}^{15} \lambda_k = 1$. If we do not consider the budget limit, then the main constraints of this optimization problem are

$$\sum_{i=1}^{7} y_{ij} \geq \sum_{k=1}^{15} \lambda_k n_{kj} \quad j = 1, \ldots, 7$$

$$\sum_{k=1}^{15} \lambda_k = 1$$

$$\sum_{j=1}^{7} y_{ij} \leq r_i \quad i = 1, \ldots, 7$$

$y, r$ are integers.

Using Lindo 6.1 (Lindo Systems Inc.) to solve this problem, we achieve the optimal resource allocation strategy. Similarly, we can compute the optimal number of tow trucks assigned to each routes for different $p$ values which is summarized in Table 10.

<table>
<thead>
<tr>
<th>$p$</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>0.7</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>0.9</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>25</td>
</tr>
</tbody>
</table>

Note that the numbers of tow trucks we obtained in table 10 are very conservative numbers. In practice, the probability of having more than one incident simultaneously is quite low. Without lowering the quality of service, to avoid expensive costs due to the purchase of tow trucks and expensive maintenance cost for a large fleet, what the traffic management center (TMC) usually does is to sign service contract with private companies. For example, to achieve 0.9 quality of service level, the TMC can probably have 4 tow trucks of its own, which is the maximum number of tow trucks a single accident might request, while all of the rest of 21 tow trucks could be from private contractors.
Figure 7 Optimal number of tow trucks assigned to each routes for different $p$ levels.

Figure 7 visually depicts the results presented in Table 10. Without surprise, to improve the quality of service more tow trucks are required. However, it is worth to note that the number of tow trucks assigned to each route does not increase simultaneously when we target higher quality of service. For instance, if we raise the $p$ value from 0.5 to 0.9, US30 keeps the same number of tow trucks while NJ38 increase its need for more tow trucks rapidly. This happens as a result of the overall travel time from US30 being much longer than NJ38. To cover the potential incidents, assigning more tow trucks to NJ38, instead of US30, is a wiser choice. The number of tow trucks even reduced when $p$ increase, for example, when $p$ increases from 0.7 to 0.9, the number of tow trucks assigned to US130 decreased from 4 to 3. However, the total number of tow trucks increased from 21 to 25. If the budget and operational limitations can only afford to sign contracts with 21 tow trucks, then the quality of service for the whole system cannot exceed 0.7.

Now, considering a brand-new incident management system, we need to determine the location of the depots and number of tow trucks assigned to each depot. Our objective function is still to minimize the incident response cost while subject to certain budget limits. By assuming the annual operation cost of a single tow trucks is $10000, the annual cost of a depot is $100,000, and the total annual budget for this tow truck-depot system is $500,000, then the constraints for this problems are
\[
\sum_{i=1}^{7} y_{ij} \geq \sum_{k=1}^{15} \lambda_k n_{kj} \quad j = 1, \ldots, 7
\]
\[
\sum_{k=1}^{15} \lambda_k = 1
\]
\[
\sum_{j=1}^{7} y_{ij} \leq r_i \quad i = 1, \ldots, 7
\]
\[
r_i \leq 40 \cdot d_i \quad i = 1, \ldots, 7
\]
\[
\sum_{i=1}^{7} r_i + 10 \sum_{i=1}^{7} d_i \leq 50
\]

\(y, r_i\) are integers, \(d_i\) binary.

Similarly, we use the Lindo system to solve the problem above. Since the version of Lindo we are using can only handle maximum 50 integer variables, we relax \(y\)’s as continuous variables and keep \(r_i\) as integer and \(d_i\) as binary variables. Here, we take \(M = 40\), since with the capital we have we can only handle 40 response vehicles in one depot. The \(y\) values we obtain in our optimal solution turned out to be integers automatically. Table 11 shows the results we got for different \(p\) levels.

### Table 11: Optimal location of depots and number of tow trucks assigned to them for different \(p\) levels.

<table>
<thead>
<tr>
<th>(p)</th>
<th>US 30</th>
<th>NJ 38</th>
<th>NJ 42</th>
<th>I-76</th>
<th>US 130</th>
<th>I-295</th>
<th>I-676</th>
<th>Total trucks</th>
<th>Total Depots</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>0.7</td>
<td>6</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21</td>
<td>2</td>
</tr>
<tr>
<td>0.9</td>
<td>6</td>
<td></td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td>2</td>
</tr>
</tbody>
</table>

We can see the total number of tow trucks we obtained in table 11 are same as table 10, but much less depots are needed to reach the same level of quality of service. This result shows that if we could design an incident management system from scratch and locate depots optimally, we can reduce our costs due to the less number of depots needed.
SUMMARY AND DISCUSSIONS

In this paper, we proposed mathematical programming models with probabilistic constraints to stress the importance of future service demands for the incident response problem. The probabilistic constraints are derived from the uncertainty in the number of service vehicles requested by the potential incidents and the number of service vehicles available at the depot. Quality of service is introduced to measure to what extent the potential incidents should be considered in determining the response policy. High quality of service means the potential incidents are taken seriously in the choice of response policy for the current set of incidents, and vice versa. To put it in another way, high quality of service for the potential incidents is a conservative practice, while low quality of service means high risk. If the potential incidents really occur, this will cause higher response costs compared to a high quality of service value approval. We assume the distributions of the resources requested by the potential incidents are known, and independent from each other, from which efficient points could be obtained for a given requirement of the quality of service. The impact of the quality of service is also studied in this paper. The total response costs rise as the quality of service increases, since more service vehicles are required to guarantee higher quality of service. In this case, although the number of service vehicles at each depot involved in the incident response is non-decreasing, the destination of the service vehicles dispatched could change. To lower the overall response costs, dispatchers sometimes need to make sacrifices for the potential incidents by not using the closest available resources for the current incidents.

Considering the probability distribution of the incidents over a transportation network, we also constructed probabilistic programming models to address the resource allocation problem for the incident management system. We consider a distributed-depot configuration, in which we need to determine the minimum number of service vehicles allocated to each depot, in order to meet the reliability requirements of this system. This offers the planners the capability of considering inherent stochasticities of the system when they attempt to determine the solution that maximizes the return on the investment, while remaining within certain budget constraints. In the case study of the South New Jersey road network example, we derived the probability distribution of the demand for tow trucks along the eight major roads from the
analysis of two years of incident data in this area. Based on these distributions, probabilistic mathematical models were constructed, and the minimum number of tow trucks allocated to each major road for various reliability requirements was obtained. For the planning of a new incident management system, our case study shows how to determine the locations of depots and the number of service vehicles assigned to the depots within the budget limits.

The mathematical programming models with probabilistic constraints discussed in this paper are solved using the concepts and algorithms presented in section 2. We also developed a computer program to enumerate all of the efficient points for any multidimensional scalar random variables. Comparing to the models in the literature, our model has following features:

1. Multiple-incident, multiple-response (MIMR) problem can be solved conveniently using our formulation, while it can be much more complicated if we use opportunity cost model (Sherali, 1999) or simulation based models.
2. The concept of quality of service offers the flexibility in measuring the impact of the potential incidents. This adds to the realism of the model by being able to take into account various accident occurrence probabilities.
3. Our model can also capture the uncertainty of the resource availability and can help the decision makers to better understand the reliability of the incident management policies they adopt.
4. More importantly, this model can accurately capture the varying probabilities of simultaneous incident occurrences. The analysis of real-world incident data shows that the probability of incident occurrence is quiet different and our model can take this variance into account.

In brief, the models presented in this paper can be used by state Departments of Transportation to evaluate and improve existing Incident Management programs and to plan the future ones.
REFERENCES:


