Multinomial Logistic Regression (MLR):  
(aka polytomous-response)

- Choice when dependent variable is nominal and > 2 categories (when = 2, use binary logistic regression)
- If not sure whether categories are ordered, use MLR
- Requires fewer or weaker assumptions than ordinal logit

Model

\[
\begin{align*}
\text{Prob}(y = j) &= \frac{e^{\sum_{k=1}^{K} \beta_{jk} x_k}}{1 + \sum_{j=1}^{J-1} e^{\sum_{k=1}^{K} \beta_{jk} x_k}} \\
\end{align*}
\]

- Where j=1, 2 … J-1
- β Parameters have two subscripts: k for distinguishing x vars and j for distinguishing response categories
- There are J-1 sets of β estimates [total # of parameters = (J-1)K]
Binary dependent variable, simplifies to:

\[
\text{Prob}(y = 1) = \frac{\sum_{k=1}^{K} \beta_k x_k}{1 + e^{\sum_{k=1}^{K} \beta_k x_k}}
\]

Identical to:

\[
P(y = 1) = \frac{e^{XB}}{1 + e^{XB}}
\]

Probability for reference category

\[
P(y = J) = \frac{1}{1 + \sum_{j=1}^{J-1} e^{\sum_{k=1}^{K} \beta_k x_k}}
\]

or,

\[
P(y = J) = 1 - [\text{Prob}(y = 1) + \ldots + \text{Prob}(y = J - 1)]
\]
Multinomial model in logit form

\[
\log \left[ \frac{\text{Prob} \ (y = j)}{\text{Prob} \ (y = J)} \right] = \sum_{k=1}^{K} \beta_{jk} x_k
\]

When \( J = 2 \), simplifies to:

\[
\log \left[ \frac{\text{Prob} \ (y = 1)}{1 - \text{Prob} \ (y = 1)} \right] = \sum_{k=1}^{K} \beta_{k} x_k
\]

IIa Assumption

- Important methodological point: \textit{independence from irrelevant alternatives}
- IIa property holds that the ratio of choice probabilities of any two alternatives is not influenced systematically by any other alternative
- Example: red bus/blue bus paradox

Source: Liao, 1994, p. 50